Structure Functions at small-x from String Theory

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Main idea and results

Using AdS/CFT to study DIS from mesons

DIS at strong coupling $+ \mathcal{N} = 2$ SYM / D3D7 brane model: holographic mesons[1]

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Dual description by coupling brane excitations to a current from the boundary

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Hadronic tensor from superstring theory

$\Downarrow$

Calculations at small-x (Bjorken parameter)

Our results

- Eight structure functions $F_i(q^2, x)$ for polarized vector mesons at small-x.
- Callan-Gross type relations: $F_2 \sim 2xF_1$ and $b_2 \sim 2xb_1$.
- Similar results for different Dp-brane models $\rightarrow$ general results? and for QCD?

[1] M. Kruczenski et all (2003).

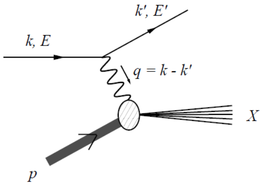

Figure 1: The typical interaction in the DIS inclusive process between a lepton and a Hadron via a virtual photon. The state X is not measured.

Some Definitions

- $q = k - k'$ is the momentum transfer and $y \equiv \frac{P \cdot q}{P \cdot k}$ is the fractional energy loss of the lepton.
- $x \equiv -\frac{q^2}{2P \cdot q}$ is the Bjorken parameter $(0 \leq x \leq 1)$.
- Small x regime : $e^{\sqrt{\lambda}} << x << 1/\sqrt{\lambda}$.

DIS is the study of the lepton-hadron scattering when $q^2 \to \infty$, with x fixed.

The differential cross section is given

$$\frac{d\sigma}{dx\, dy\, d\phi} = \frac{e^2}{16\pi^2 q^4} y\, l^{\mu\nu}(k, k', \epsilon) W_{\mu\nu}(P, q, \zeta) \;, \boxed{W^{\mu\nu} \sim < P|J^\mu J^\nu|P' >}$$

where the leptonic tensor $l^{\mu\nu} \approx 2\left(k^\mu k^{\nu'} + k^\nu k^{\mu'} - g^{\mu\nu} k \cdot k' - i\epsilon^{\mu\nu\alpha\beta} q_\alpha s_\beta^{lep}\right)$ is easy to obtain from pQED. Current conservation implies $q_\mu l^{\mu\nu} = q_\nu l^{\mu\nu} = 0$.

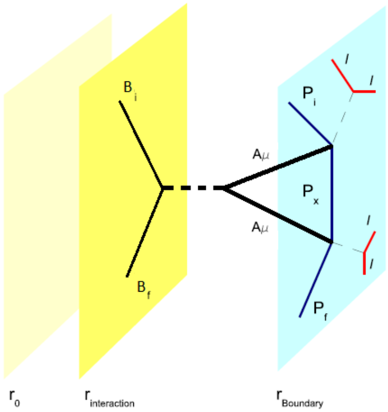

Figure 2: Picturing the relation between the DIS process and the holographic scattering one.

$\boxed{D3D7 \text{ brane model}/\mathcal{N} = 2 \text{ SYM}}$

The perturbation induced by the electro-magnetic currents J^μ from the boundary is a metric fluctuation of the form

$$h^{MN} = \frac{1}{2}\left(A^M v^N + A^N v^M\right)$$

where A^m is a $U(1)$ gauge field and v^i is a killing vector on the sphere [2].

These fluctuations will interact with **scalar or vector brane fields** (transversal or longitudinal brane fluctuations)

[2] J. Polchinski and M. Strassler (2002)

Finally, we obtain the following results for the structure functions (at leading order)

$$g_1 = -2g_2 = \frac{1}{4x^2}(I_1 + I_0) \quad , \quad F_1 = \frac{1}{12x^2} I_1 \ , \ F_2 = \frac{1}{6x}(I_1 + I_0)$$
$$b_1 = \frac{1}{4x^2} I_1 \quad , \quad b_2 = -\frac{3}{2} b_3 = 3b_4 = \frac{1}{2x}(I_1 + I_0)$$

where

$$I_1 \propto \left(\frac{q}{\Lambda}\right)^{-2\Delta+2} \frac{1}{\sqrt{gN}} I_{1,2\Delta+3} \, , \, I_0 =\propto \left(\frac{q}{\Lambda}\right)^{-2\Delta+2} \frac{1}{\sqrt{gN}} I_{0,2\Delta+3} \, ,$$

and since $\frac{I_{0,2\Delta+3}}{I_{1,2\Delta+3}} = \frac{2\Delta+3}{\Delta+2}$ one recovers relations of the Callan-Gross type:

$$\boxed{F_2 = 2xF_1 \left(1 + \frac{I_0}{I_1}\right)} \, , \, \boxed{b_2 = 2xb_1 \left(1 + \frac{I_0}{I_1}\right)} .$$

- DIS scattering of leptons from spin-0 and spin-1 mesons at small x at strong coupling and in the large N limit has been investigated in terms of superstring theory (in the large N limit).

- For polarized vector mesons the 8 **structure functions** were obtained, along with the **Callan-Gross type relations**[3]

$$F_2(x) \sim 2xF_1(x) \text{ and } b_2(x) \sim 2xb_1(x).$$

- This results have similarities for all the different Dp-brane models. **This could be a signal of a universal behaviour for confining gauge theories with a dual description in terms of probe Dp-branes**.

- **Future work**: calculating the differential cross section for DIS and apply this techniques to other process, using the OPE of vertex operators and other techniques in order to describe the string theory scattering.

Thank you for listening! Any questions?

[3] M. Schvellinger ,E. Koile and S. Macaluso (2011,2013) found $F_2 = 2F_1$ and $b_2 = 2b_1$ at $x \sim 1$.

Thank you for listening! Any questions?

Some important steps

- Using the optical theorem to relate DIS and forward Compton scattering, in order to relate our calculations and the hadronic tensor.
- Calculate the leading amplitude and finding an effective action S_{eff} (first in flat space-time).
- Obtaining the field solutions on the curved background ($AdS_5 \times S^3$) with boundary conditions.
- Inserting these solutions in the S_{eff} and folding the amplitude in AdS (since the interaction can be considered local) [4].
- Using the AdS/CFT dictionary $\rightarrow n_\mu n_\nu \tilde{W}^{\mu\nu} \sim \frac{\delta^2}{\delta(b.c.)^2} S_{eff}^{AABB}$.
- Comparing the resulting $W^{\mu\nu}$ with the most general one and extract the structure functions F, b and g.

[4] Polchinski et all (2006)

$\mathcal{N}=2$ SYM gauge theory:

- It is derived from $\mathcal{N}=4$ SYM with 8 broken susy.
- **It has matter in the fundamental rep. and mesons, like QCD**. However recall that QCD is logarithmically running in the UV.
- At high energies it becomes conformal, so we can use the gauge-string duality.
- It possesses a mass gap of order m/λ with $m \propto L \rightarrow$ light mesons at strong coupling.

The D3D7 brane model[5]:

- $N_f << N$ probe D7-branes are introduced in an $AdS_5 \times S^5$ background:

$$\begin{array}{ccl} D3 & : & 0\,1\,2\,3\,-\,-\,-\,-\,-\,- \\ D7 & : & 0\,1\,2\,3\;\,4\;\,5\;\,6\;\,7\;-\;- \end{array}$$

- Mass comes from separation in the 89-plane (L).
- In the interaction region the induced metric can be approximated by $AdS_5 \times S^3$.
- We will use $N_f = 1$ and some other models[6]: $D4D8\overline{D8}$ and $D4D6\overline{D6}$.

[5] Kruczenski et all. (2003), A. Karch and E. Katz (2002)

[6] Sakai and Sugimito (2004) y citar el otro

Open-closed superstring amplitudes

Formally, both for scalar and vector mesons one should compute the two open *vs* two closed strings amplitude from a vertex operator worldsheet integral on the disk of the form[7]

$$A_{string}(h_1, h_4, \epsilon_2, \epsilon_3) = \int_{\partial \mathbf{H}_+} dx \int_{\partial \mathbf{H}_+} dy \int_{z \in \mathbf{H}_+} dz d\bar{z} \int_{w \in \mathbf{H}_+} dw d\bar{w} \, \langle c(z) \widetilde{c}(\bar{z}) \times$$

$$: V_c^{(-1,-1)}(z, \bar{z}; h_{1\mu\nu}, k_1) : : V_c^{(0,0)}(w, \bar{w}; h_{4\mu\nu}, p_4) : (c(x) - c(y)) : V_o^{(0)}(x, \epsilon_{2\mu}, k_2) :$$

$$: V_o^{(0)}(y, \epsilon_{3\mu}, k_3) : \rangle,$$

This gives a sum of terms with an α'-independent kinematic term and a pre-factor that carries the α' dependence of the form

$$\mathcal{A}_4^{2o2c} = \mathcal{P}_1^{2o2c} \, \mathcal{K}_1^{2o2c} + \mathcal{P}_2^{2o2c} \, \mathcal{K}_2^{2o2c} + \cdots .$$

For us, $\boxed{|t| << 1 << s}$. Only the one that has a $\frac{1}{\tilde{t}}$ pole in the pre-factor and a kinetic term that can be obtained from supergravity is important in this regime.

[7] S. Stieberger (2009).

Supergravity Feynman Diagrams

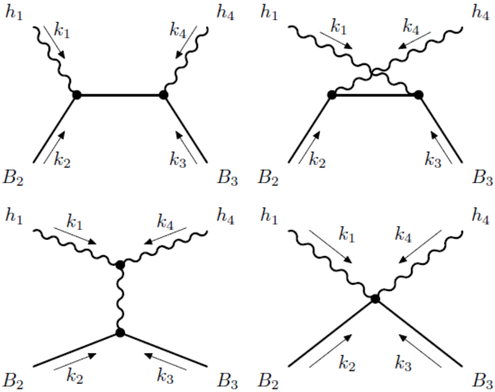

Figure 3: The *s*-, *u*- and *t*-channel diagrams together with the contact interaction. They all appear in the supergravity calculation.

Expantion of the Dirac-Born-Infeld action

We begin with the action

$$S_{DBI} = -T_7 \int d^8\xi \sqrt{-det\left(\hat{g}_{ab} + (2\pi\alpha')G_{ab}\right)}\,,$$

where $G_{ab} = \partial_a B_b - \partial_b B_a$ and choose the static gauge

$$H_{ab} \to \hat{g}_{ab} \equiv g_{ab} + 2g_{I(a}\partial_{b)}X^I + g_{IJ}\,\partial_a X^I \partial_b X^J,$$
$$g_{ab} = \eta_{ab} + 2\kappa h_{ab}\ ,\ g_{IJ} = \delta_{IJ} + 2\kappa h_{IJ}\ ,\ g_{aI} = 2\kappa h_{aI}.$$

By expanding (1) we get the usual propagator for B_a and the interaction lagrangians

$$\begin{aligned} L_{BB} &= -\frac{T_7}{4} G^{ab} G_{ab}\ ,\ \ L_{hBB} = T_7\kappa\left[\frac{1}{4} hG^{ab}G_{ab} + h^{ab}G_{bc}G_a^c\right]\ , \\ L_{hhBB} &= T_7\kappa^2\left[\frac{1}{8}h^2 G^{ab}G_{ab} + 2hh^{ab}G_{bc}G_a^c - \frac{1}{4}h^{ab}h_{ab}G^{cd}G_{cd} \right. \\ &\quad \left. - h^{ab}h^{cd}G_{bc}G_{da} - 2h^{ab}h_{bc}G^{cd}G_{da}\right]. \end{aligned}$$

Folding the action into curved space-time

When x is not exponentially small the string size is the interaction can be considered **local** and so we can *fold* our flat spacetime results into the AdS interaction [8]. Thus, from the effective action our starting point we consider our starting point

$$n_\mu n_\nu Im_{exc} T^{\mu\nu} = \frac{\pi\alpha'}{8} \sum_{m=1}^{\infty} \int d\Omega_3 \, dr \, \sqrt{-g} v_i v^i G^{*mq}(P) G_q^n(P) F_{mp}^*(q) F_n^p(q) \delta\left(m - \frac{\alpha' \tilde{s}}{4}\right) ,$$

where all indices are contracted with the full $p+1$-dimensional metric. The integration in $x^0, \ldots, x^3$ has already been used to set the momenta by conservation, and the solutions are

$$\begin{aligned} A_\mu(q) &= n_\mu f(r) e^{iq\cdot x} \ , \ f(r) = \frac{qR^2}{r} K_1\left(\frac{qR^2}{r}\right) \\ A_r(q) &= \frac{-iq\cdot n}{q^2} f'(r) e^{iq\cdot x} \ , \ B_r(P) = 0 \\ B_\mu^l(P) &= \frac{\zeta_\mu}{\Lambda} \frac{c_i^l}{\Lambda R^3} \left(\frac{r}{\Lambda R^2}\right)^{-\Delta} Y^l(\Omega_3) e^{iP\cdot x} = \frac{\zeta_\mu}{\Lambda} X^l(P) \end{aligned}$$

The last lines come from the asymptotic form of the solutions in the $D3D7$ model. [9]

[8] J. Polchinski et all (2006)

[9] M. Kruczenski et all (2003).

Substituing the solutions one gets

$$
\begin{aligned}
& Im_{exc}\, T^{\mu\nu} = \frac{\pi\alpha'}{8}\sum_{m=1}^{\infty}\int d\Omega_3 dr\delta\left(m-\frac{\alpha'\tilde{s}}{4}\right)\times \\
& P^{\mu}P^{\nu}\times\left[\frac{R^4|X|^2}{r^4\Lambda^2}\left(\frac{R^4}{r^4}q^2f^2+(f')^2\right)(\zeta\cdot\zeta^*)\right]+\left[\frac{1}{2}(\zeta^{*\mu}\zeta^{\nu}+\zeta^{*\nu}\zeta^{\mu})\right. \\
& \left.+\frac{1}{2}(\zeta^{*\mu}\zeta^{\nu}-\zeta^{*\nu}\zeta^{\mu})\right]\times\frac{R^4|X|^2}{r^4\Lambda^2}\left[\frac{R^4}{r^4}P^2q^2f^2+P^2(f')^2+\frac{\Sigma^2}{r^2}q^2f^2+\frac{\Sigma^2 r^2}{R^4}(f')^2\right] \\
& +\eta^{\mu\nu}\times\frac{R^4|X|^2}{r^4\Lambda^2}\left[\frac{R^4}{r^4}f^2(\zeta\cdot\zeta^*)(P\cdot q)^2+\left(\frac{R^4}{r^4}P^2+\frac{\Sigma^2}{r^2}\right)f^2(q\cdot\zeta)(q\cdot\zeta^*)\right. \\
& \left.+(\zeta\cdot\zeta^*)\left(\frac{r^2\Delta^2}{R^4}(f')^2+2\frac{\Delta}{r}ff'(P\cdot q)\right)\right],
\end{aligned}
$$

Comments on the exponentially small x region

This parameter region characterized by $x \ll \exp -\sqrt{gN}$ is more complicated because it goes deeper into string theory issues:

- The locality approximation breaks down. This is because the $(\alpha' \tilde{s})^{\alpha' \tilde{t}}$ that we had set to 1 cannot be neglected because we must take into account the transverse momentum transfer. Thus one has to include

$$m^{\alpha' \tilde{t}/2} \sim (\alpha' \tilde{s})^{\alpha' \tilde{t}/2} \sim x^{-\alpha' \tilde{t}/2} \sim x^{-\alpha' \nabla^2/2}.$$

- The differential operator acts on the solutions as a diffusion operator in the r direction, coming from the growth of the strings.

- The odd q^2- dependence both in the structure functions obtained in the previous regimes ($x \gg 1/\sqrt{gN}$ and $1/\sqrt{gN} \gg x \gg \exp -\sqrt{gN}$) is fixed, and so is the divergent moment issue.

In the last ten years a lot of progress has been made in this direction, specially in the spin zero (glueball) case. [10]

[10] J. Polchinski et all. (2006), R. Brower et all. (2007 and 2012), and references therein.