Universiteit Utrecht

Matrix Quantum Mechanics

and the S^1/Z_2 orbifold

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What is Matrix Quantum Mechanics

- Matrix Quantum Mechanics (gauged) is a $0+1$ dimensional quantum field theory of $N \times N$ Hermitian matrices denoted by $M(t)$ and a non dynamical gauge field A_t.
- The Partition function is:

$$e^{-W} = \int \mathcal{D}M(t) \exp\left[-N \int_{-\infty}^{\infty} dt \mathrm{Tr}\left(\frac{1}{2}\left(D_t M\right)^2 + \frac{1}{2}M^2 - \frac{\kappa}{3!}M^3\right)\right] \tag{1}$$

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- It is convenient to split $M = U\Lambda U^\dagger$ where $\Lambda(t)$ is diagonal and U unitary.
- One then picks up a Jacobian

$$\mathcal{D}M = \mathcal{D}U \prod_{i=1}^{N} d\lambda_i \Delta^2(\Lambda), \quad \Delta(\Lambda) = \prod_{i<j}(\lambda_i - \lambda_j) \tag{2}$$

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- The gauge field will also lead to the decoupling of the angular dof's U from the path integral (it projects to the singlet sector).

Fermionic Well

It is convenient to pass to the Hamiltonian description. The Hamiltonian using the covariant laplacian becomes:

$$H = -\frac{1}{2\beta^2 \Delta(\lambda)} \frac{d^2}{d\lambda_i^2} \Delta(\lambda) + V(\lambda_i) \ , \quad \beta = \frac{N}{\kappa^2} \sim \frac{1}{\hbar} \tag{3}$$

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We define an "antisymmetric" wavefunction $\Psi(\lambda)$ as $\Psi(\lambda) \equiv \Delta(\lambda)\chi_{sym}(\lambda)$. The Schrödinger equation now reads:

$$\left(-\frac{1}{2\beta^2} \frac{d^2}{d\lambda_i^2} + V(\lambda_i)\right) \Psi(\lambda) = E\Psi(\lambda). \tag{4}$$

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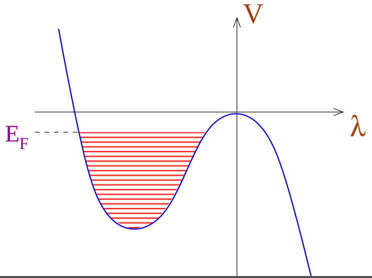
- We fill up the potential with N fermions. The energy spacing is $\sim \beta^{-1}$. By tuning the coupling constant κ, there is a critical value κ_c after which the eigenvalues will start to spill.



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In the double scaling limit, the matrix model perturbative expansion changes from the double parameter expansion of $(1/N, \kappa)$ to the single parameter expansion of $\mu_r \equiv N(\kappa - \kappa_c)^{(2-\Gamma)/2}$:

$$Z(\mu_r) = \sum_g \mu_r^{2-2g} f_g. \tag{7}$$

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In the free fermion language we send $\beta \to \infty,\ \mu \to 0,\ \mu_r = \beta\mu$ with μ the distance of the fermi level from the top of the potential.

Some comments, Open-Closed duality

- Matrices naturally arise in the study of dynamics of D-Branes.
- It can be shown that String theories with c=1 contain D_0 branes whose excitations are simply a Tachyon and a 1-d gauge field.

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- The tachyon potential in bosonic string theories is unbounded from below -the D branes can decay and produce closed strings (SFT).
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- This connection strengthens in the double scaling limit $g_s \to 0, N \to \infty, \mu = fixed$, where one focuses at the top of the potential barrier.
- In the same limit, the Matrix model describes also the physics of continuous Riemmann surfaces (closed string Liouville description).
- This is the analogue of the Maldacena decoupling limit in this simple case.
- MQM describes the dynamics of N D_0 branes anchored at the strong coupling region of the dual closed string Liouville description.

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- For a circle the partition function is:

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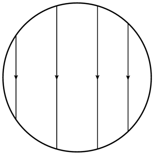
where f_n is a function of $R + R^{-1}$, and $\mu_r = \beta\mu_0$ is the genus expansion parameter.

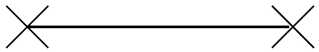
- This expression is T-dual $R \to \frac{1}{R}, \quad \mu_r \to R\mu_r$.
- We can also match the second term with the Liouville computation of the torus partition function:

$$Z_1 = -\frac{1}{24}\left(R + \frac{1}{R}\right)\log\mu_0, \quad (9)$$

S^1/Z_2 Orbifold

The idea is to make a similar computation for the orbifold. In string theory we get extra twisted states from the orbifold fixed points.





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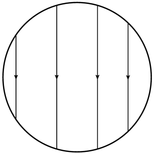

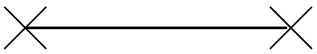

To implement orbifolding on the matrix model we gauge the Z_2 reflection symmetry by combining it with a Z_2 subgroup of the gauge group:

$$\Omega = \begin{pmatrix} -1_{n\times n} & 0 \\ 0 & 1_{(N-n)\times(N-n)} \end{pmatrix} * \tag{10}$$

with $*f(t) = f(-t)*, \quad *\partial_t = -\partial_t *, \quad 0 \leq n \leq \frac{N}{2}$, and then requiring:

$$\Omega A(t) \Omega^{-1} = -A(t), \quad \Omega M(t) \Omega^{-1} = M(t) \tag{11}$$

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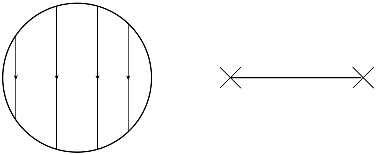

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This naturally splits the matrices into (even/odd) blocks that need to satisfy different boundary conditions. Gauge invariance is now under $U(n)\times U(N-n)$, thus we get two separate sets of n and $N-n$ fermions.

Directions

- Compute Partition function (Generalised statistics?) - T-duality?
- Compare with the Liouville torus Partition function on the orbifold.
- Supersymmetric case (OB)
- Analytic continuation -Pick specific value of n for the initial/final state of a Big bang Big crunch universe?
- Second quantization (SFT)?

References

I. Klebanov: hep-th/9108019v2 , (general basic review)

Boulatov - Kazakov: hep-th/0012228v1 , (non-singlets, different representations)

Yu Nakayama: hep-th/0402009v7 , (general review, Liouville oriented)

Ramgoolam - Waldram: hep-th/9805191 , (D0 branes on compact orbifolds)

Thank you!