Bloch-Nordsieck Model and Critical Fermions

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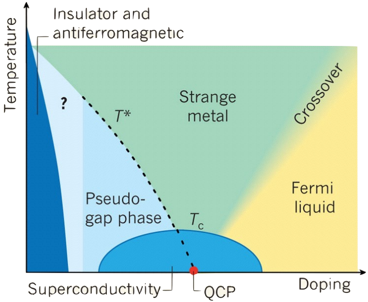
Outline

- Critical Fermions
- Bloch-Nordsieck Model
- Results
- Next steps

Critical Fermions

Interesting materials such as high-T_c superconductors can not be described by Fermi-liquid theory.
They are believed to be described by a quantum critical point.



We wish to deform the Fermi liquid to get something else.

Patch Theory

We will study a Fermi surface coupled to a gapless boson.

$$\mathcal{L} = \bar{\psi}(\partial_t - \epsilon(i\nabla) + \mu)\psi + \phi(\partial_t^2 - \nabla^2)\phi + \lambda\phi\bar{\psi}\psi \tag{1}$$

Expand around the Fermi-surface

$$\mathcal{L} = \bar{\psi}(\partial_t - v_f\partial_\perp - \kappa\partial_\parallel^2)\psi + \phi(\partial_t^2 - \nabla^2)\phi + \lambda\phi\bar{\psi}\psi \tag{2}$$

This model has been studied before[1][2]

[1] Quantum Phase Transitions, Subir Sachdev

[2] 1410.6814, A. L. Fitzpatrick, S Kachru, J. Kaplan, S. Raghu, G Torroba, H. Wang

IR Divergencies

The IR is strongly coupled in $d = 2$.

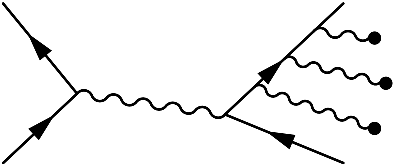
The massless boson gives rise to infrared singularities in perturbative treatments of this model.

These have been circumvented by using different bubble resummations, cut-offs, or self-consistent damping of the fermion propagator.

We wish to resolve the IR divergence in a more controlled manner.

Soft Photons in QED

Final states in QED scattering processes have an infinite number of soft photons.



Their total energy is finite, $\sum$  $< \infty$.
This was shown by F. Bloch and A. Nordsieck in 1937.

Bloch-Nordsieck Model

To show this they used an approximate model of QED now called the Bloch-Nordsieck (BN) model:

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\Psi}(i\gamma_\mu D^\mu - m)\Psi \tag{3}$$

$$\Big\downarrow$$

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\Psi}(iu_\mu D^\mu - m)\Psi \tag{4}$$

This approximation was valid for the type of processes they were studying.

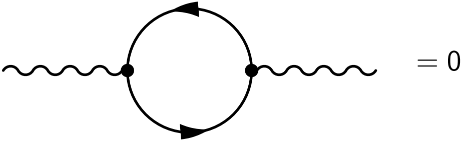
It is also seen to closely resemble the patch theory of a Fermi surface coupled to a critical boson.

Bloch-Nordsieck Model

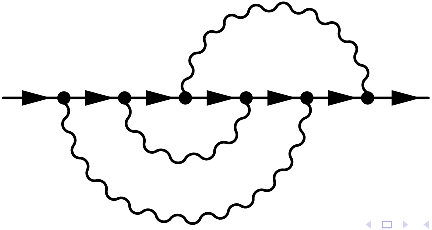
This type of Lagrangian gives a free fermion propagator with only one pole.

$$G = \frac{1}{m - up - i\epsilon} \tag{5}$$

which is purely retarded. This means that all closed fermion loops are 0



The two point function consists of diagrams of the form



Exact Solution

One can exactly calculate the fermion 2-point function in this model without calculating any Feynman diagrams.

The solution makes use of the absence of fermion loops and that the fermion dispersion is linear.

Preliminary Results

We obtain an anomalous dimension ($d = 3$):

$$G_{R,f,3}(k) \propto \frac{1}{(\omega + i\epsilon + v_f k_x)^{1+\frac{\lambda^2}{8\pi(1-v_f^2)}}} \tag{6}$$

We do not obtain any Fermi velocity running. This is in contrast to results by others, the Fermi velocity has been seen to flow to 0 in the same model[3].

[3] G. Torroba, H. Wang, 1406.3029

Next steps

- Add Fermi-surface curvature
- Corrections to absence of fermion Loops: Landau-damped bosons, ...
- Fermi-velocity renormalization

Solving Bloch-Nordsieck Model

The Bloch-Nordsieck model is solvable meaning we can calculate the Fermion two-point function non-perturbatively. The solution proceeds along these lines

$$\begin{aligned} G_{E,f}(x,x') &= N\int [\mathrm{d}\phi][\mathrm{d}\bar{\psi}][\mathrm{d}\psi]\bar{\psi}(x)\psi(x')\mathrm{e}^{-S_E} \\ &= -N\int [\mathrm{d}\phi]\mathrm{e}^{-S_b}\,\mathrm{det}[G_{E,f}^{-1}[\phi](x,x')]\,G_{E,f}[\phi](x,x') \end{aligned} \tag{7}$$

$$(\partial_\tau - iv_f\partial_x - \frac{\kappa}{2}\nabla_y^2 + \lambda\phi(x))G_{E,f}[\phi](x,x') = \delta(x-x'). \tag{8}$$

$$G_{R,f}[\phi](k) = -i\int_0^\infty \mathrm{d}\nu \mathrm{e}^{i\nu(\omega+i\epsilon+v_f k_x)+\lambda\int_0^\nu \mathrm{d}\nu'\phi(-\nu' u)} \tag{9}$$