2d-4d Correspondences

Sam van Leuven*

*Universiteit van Amsterdam, Institute for Theoretical Physics,

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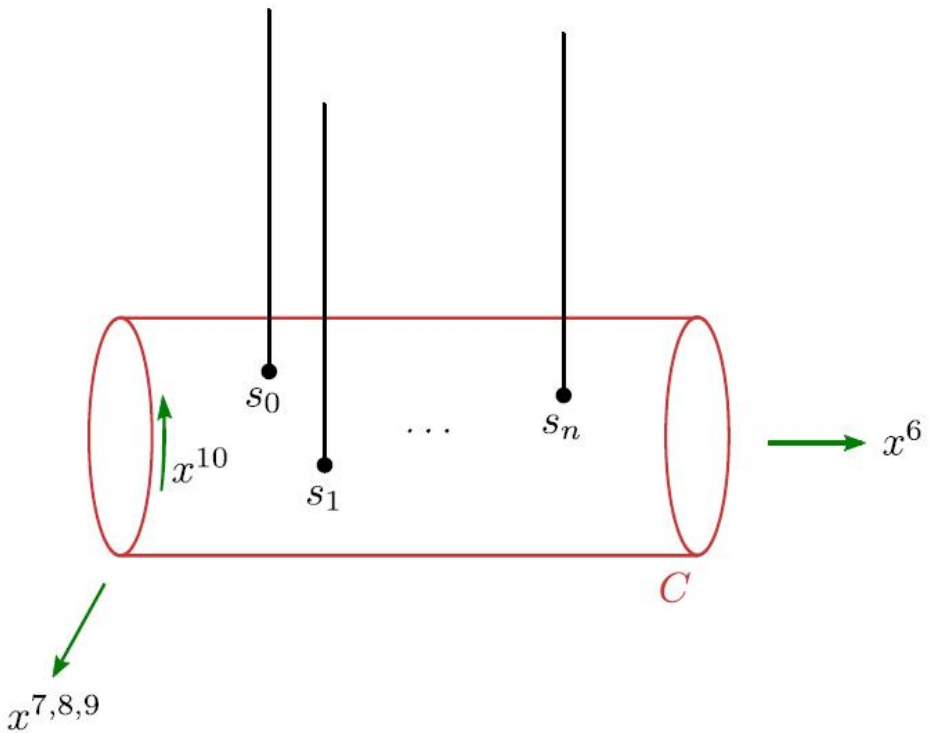
Under supervision of E.P. Verlinde

Class $\mathcal{S}$ Theories

Class $\mathcal{S}$ theories are **4d super Yang-Mills theories** constructed in M-theory:

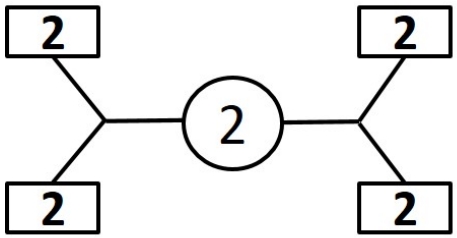
- N M5 branes wrap a Riemann surface C_g of **genus** g and spacetime
- n transverse M5s intersect the N M5s creating n **punctures**

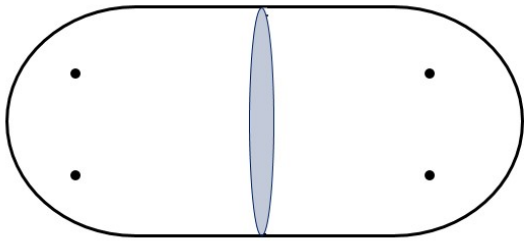


Example: $N = 2$

At **low energies** and **small area** two M5 branes describe an $\mathcal{N} = 2$ SCFT characterized by $C_{n,g}$: Gaiotto[09]

- Number of $SU(2)$ **gauge groups** determined as $n - 3 + 3g$
- Number of $SU(2)$ **flavour symmetry groups** determined as n
- Seiberg-Witten curve is a double cover of $C_{n,g}$





AGT Correspondence

Original correspondence finds **equivalence**[1] between:

Alday, Gaiotto, Tachikawa[09]

- Conformal blocks $\mathcal{F}_{1\dots n}$ of the n-point correlator of **Liouville theory** on C_g
- Instanton partition function Z_{inst} of **4d gauge theory** associated to $C_{n,g}$ on $\mathbb{R}^4_{\epsilon_1,\epsilon_2}$ Nekrasov[02]

[1] Up to some irrelevant constants

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and

- Liouville three-point functions C_{123}
- Perturbative partition function Z_{pert}

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Specific Example

- Liouville **four-point function** on $C_0 = S^2$:

$$\langle V_1(\infty) V_2(1) V_3(q) V_4(0) \rangle =$$
$$\int_0^\infty dp |q|^{2\Delta_p} C_{12p} C_{p34} \mathcal{F}_1{}^2_p{}^3_4(q) \bar{\mathcal{F}}_1{}^2_p{}^3_4(\bar{q})$$

- **Partition function** on S^4: Pestun[07]

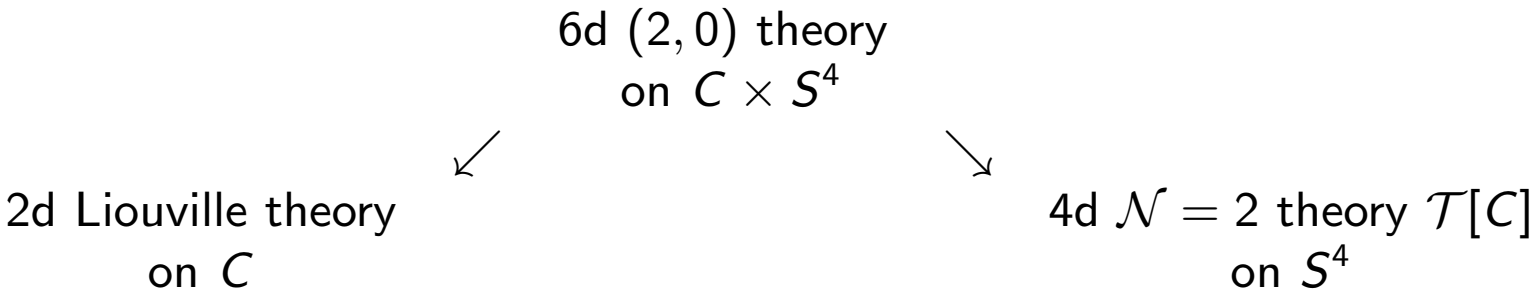
$$Z_{S^4}(m_i, \tau) =$$
$$\int da |q|^{2a^2} Z_{1-loop}(m_1, m_2, a) Z_{1-loop}(a, m_3, m_4) Z_{inst}(q) \bar{Z}_{inst}(\bar{q})$$

- Crossing symmetry CFT 'proves' **S-duality** of the gauge theory!

6d Perspective

How to understand the AGT correspondence?

- 6d A_{N-1} $(2,0)$ interacting CFT
- Worldvolume theory of N coincident M5 branes
- No Lagrangian description



Observations and Questions

- Both the S^4 theory and recently found $S^2 \times S^2$ theory correspond to CFTs with central charge $c = 26$, related to 2d quantum gravity! Bawane,Bonelli,Ronzani,Tanzini[14]
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- Is there any sense in integrating over the gauge coupling? Cheng,Dijkgraaf,Vafa[10]
- Why does Liouville appear?
  In particular: could it have a gravitational origin?
- More precisely: the Liouville/Toda central charge was reproduced from the anomaly-polynomial of the $\mathcal{N} = (2, 0)$ theory. Is there a relation between anomaly inflow and Liouville? Alday,Benini,Tachikawa[09]