

(1) **Dynamical Compact Binary Assembly** Dr. Insane doesn't understand all this focus on binary compact object mergers. He thinks that direct collisions of single neutron stars in clusters with each other will make wonderful burst sources. He has requested that you work out the numbers for him. Suppose that you consider a dense globular cluster, such that in the center the number density of neutron stars is  $10^6 \text{ pc}^{-3}$  and there are 1000 total neutron stars per cluster. Suppose that each neutron star has a radius of 10 km and mass of  $1.5 M_{\odot} = 3 \times 10^{33} \text{ g}$ , and that the typical random speed in the cluster is 10 km/s. To within an order of magnitude, calculate how often two neutron stars in a given cluster will hit each other. If there are  $10^2$  such clusters in a Galaxy, how often will this happen in the universe? Note: be careful when you calculate the cross section for collisions, because gravitational focusing is important. Thus your first step will be to determine the impact parameter  $b$  that results in a collision: if an object's velocity vector would have a closest approach  $b$  if it continued in a straight line, then  $b$  would be the impact parameter. Then the cross section is  $\pi b^2$ . If gravitational focusing were unimportant then  $b \approx R$ , where  $R$  is the radius of the neutron star, but you should find that  $b \gg R$  in this case.

(2) **Devouring Black Holes** One model for short hard gamma-ray bursts is the tidal disruption of a neutron star by a black hole. If this tidal disruption happens inside the ISCO, the whole neutron star falls in at once and it is doubtful whether any emission can last for more than a few milliseconds. However, for tidal disruption outside the ISCO there is at least a possibility that a longer-lived disk could form, as would be required to explain the observed durations of tenths of a second. In this problem, you should write a computer program to calculate the maximum mass of a black hole that would disrupt a neutron star outside the ISCO, as a function of the hole's dimensionless spin parameter  $j = a/M = cJ/GM^2$ . Assume that the neutron star has a mass of  $1.4M_{\odot}$  and a radius of 10 km. Also assume that the spacetime is Kerr with the hole's spin parameter, so that you can compute the ISCO radius using the formulae given in the file entitled: *Kerr.pdf*.

In determining the tidal radius, you may use Newtonian formulae to calculate when the difference in gravitational acceleration across the neutron star from the black hole is greater or less than the acceleration from self-gravity of the star.