Basics of QCD
Lecture 1: the core ingredients

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ICTP–SAIFR school on QCD and LHC physics
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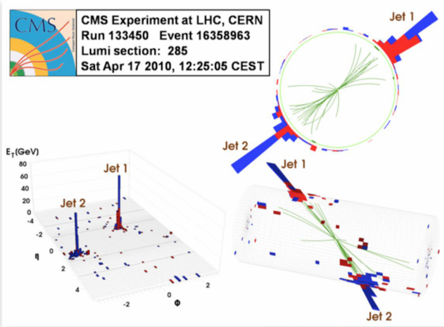
QUANTUM CHROMODYNAMICS

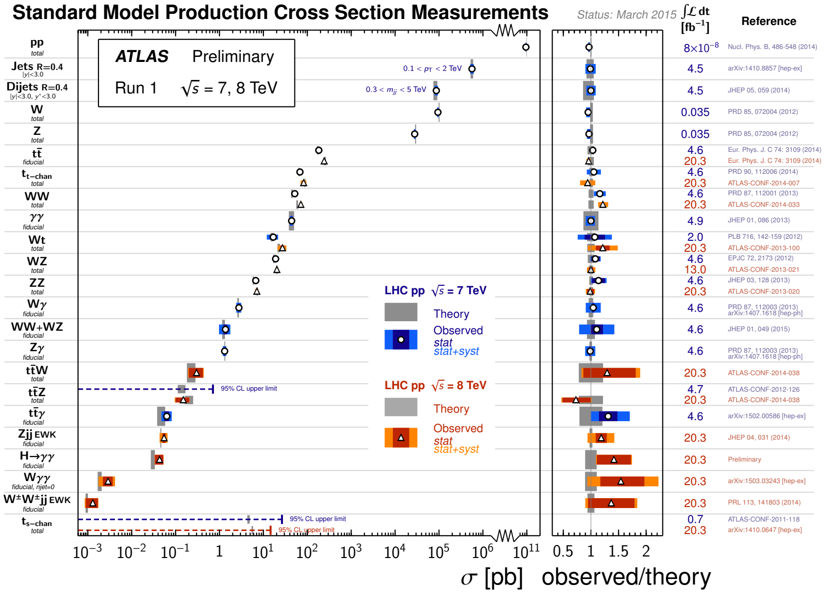
The theory of quarks, gluons and their interactions

It's central to all modern colliders.

(And QCD is what we're made of)



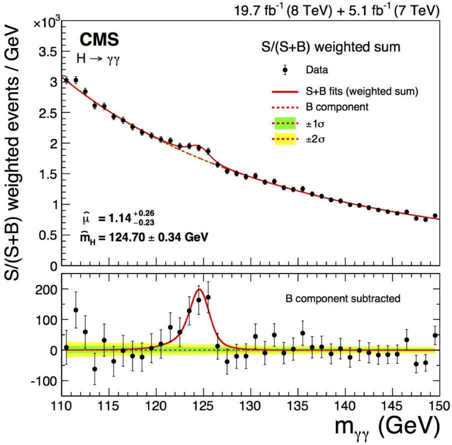
QCD predictions v. data for many processes



QCD for Higgs physics

QCD is especially relevant in order to deduce information about the **Higgs boson**.

[cf. lectures by Andre Sznajder & Claude Duhr]



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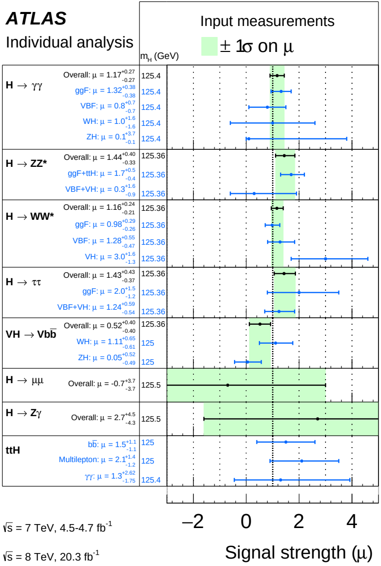
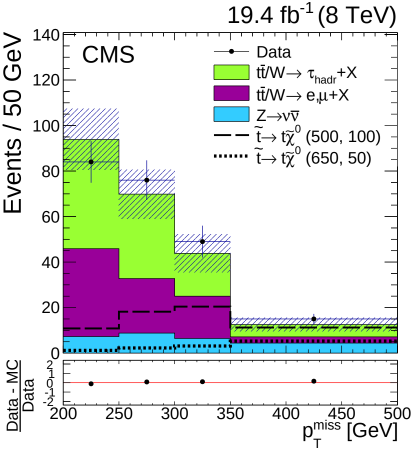


QCD and searches for new physics

Identification and/or exclusion of potential new physics relies in diverse ways on a QCD-based understanding of signals and backgrounds.

Search for stop squarks



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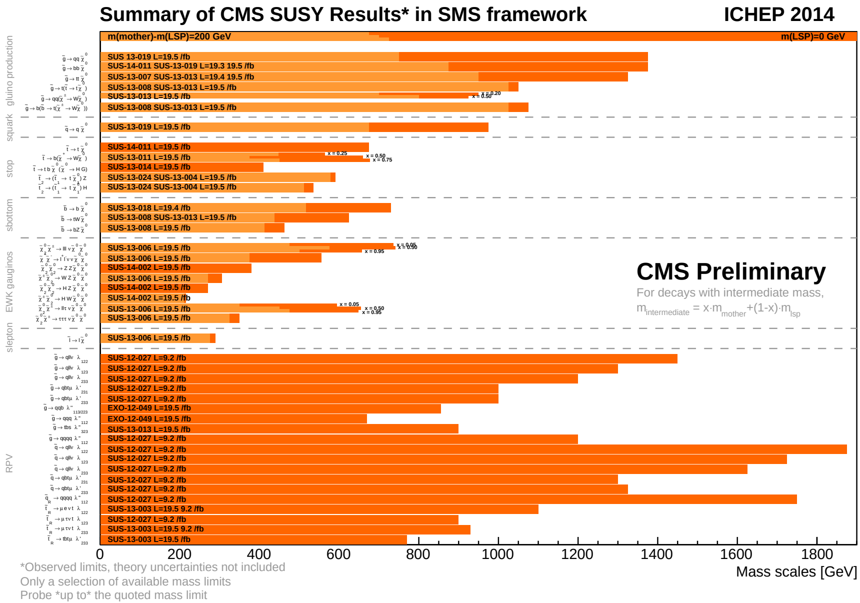

Aims of the course

The school will cover many aspects of QCD, from the advanced calculation techniques to the experimental consequences.

This course:

A reminder of what QCD is

A few core equations & concepts
(running coupling, collider cross sections infrared divergences and infrared safety)

More depth on topics not covered in other lectures
(parton distribution functions, jets)

The ingredients of QCD

- Quarks (and anti-quarks): they come in 3 colours
- Gluons: a bit like photons in QED

  But there are 8 of them, and they're colour charged
- And a coupling, α_s, that's not so small and runs fast

  At LHC, in the range 0.08(@ 5 TeV) to $\mathcal{O}(1)$(@ 0.5 GeV)

Lagrangian + colour

Quarks — 3 colours: $\psi_a = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$

Quark part of Lagrangian:

Let's write down QCD in full detail

(There's a lot to absorb here — but it should become more palatable as we return to individual elements later)

A representation is: $t^A = \frac{1}{2}\lambda^A$,

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

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$$\mathcal{L}_q = \bar{\psi}_a (i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t^C_{ab} \mathcal{A}^C_\mu - m) \psi_b$$

$SU(3)$ local gauge symmetry $\leftrightarrow$ 8 $(= 3^2 - 1)$ generators $t^1_{ab} \dots t^8_{ab}$ corresponding to 8 gluons $\mathcal{A}^1_\mu \dots \mathcal{A}^8_\mu$.

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Lagrangian: gluonic part

Field tensor: $F_{\mu\nu}^A = \partial_\mu \mathcal{A}_\nu^A - \partial_\nu \mathcal{A}_\nu^A - g_s\, f_{ABC} \mathcal{A}_\mu^B \mathcal{A}_\nu^C$ $\qquad [t^A, t^B] = i f_{ABC} t^C$

f_{ABC} are structure constants of $SU(3)$ (antisymmetric in all indices — $SU(2)$ equivalent was ϵ^{ABC}). Needed for gauge invariance of gluon part of Lagrangian:

$$\mathcal{L}_G = -\frac{1}{4} F_A^{\mu\nu} F^{A\,\mu\nu}$$

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Exercise

Consider gauge transformations

$$\psi \to U(x)\psi\,, \quad U(x) = e^{iu^A(x)t^A}$$

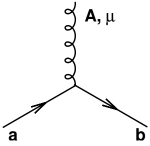
How should the gluon field A^C_μ transform in order for $\mathcal{L}_q$ to be gague invariant. Show that with the same transformations, $\mathcal{L}_G$ is gague invariant.

Perturbation theory

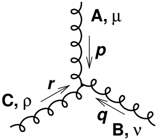
Relies on idea of order-by-order expansion small coupling, $\alpha_s \ll 1$

$$\alpha_s + \underbrace{\alpha_s^2}_{\text{small}} + \underbrace{\alpha_s^3}_{\text{smaller}} + \underbrace{\ldots}_{\text{negligible?}}$$

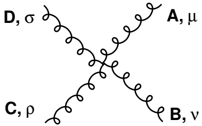
Interaction vertices of Feynman rules:



$-ig_s t^A_{ba}\gamma^\mu$



$$-g_s f^{ABC}[(p-q)^\rho g^{\mu\nu} + (q-r)^\mu g^{\nu\rho} + (r-p)^\nu g^{\rho\mu}]$$

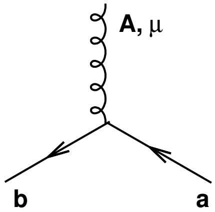


$$-ig_s^2 f^{XAC} f^{XBD}[g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\gamma}] + (C,\gamma) \leftrightarrow (D,\rho) + (B,\nu) \leftrightarrow (C,\gamma)$$

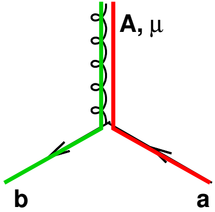
These expressions are fairly complex, so you really don't want to have to deal with too many orders of them! i.e. α_s had better be small...

[Zvi Bern will show you how to do things more easily!]

What do Feynman rules mean physically?



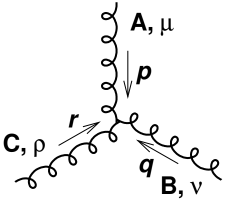
$\bar\psi_b(-ig_s t^A_{ba}\gamma^\mu)\psi_a$



$$\underbrace{\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}}_{\bar\psi_b}\underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{t^1_{ab}}\underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{\psi_a}$$

A gluon emission **repaints** the quark colour.
A gluon itself carries colour and anti-colour.

What does "ggg" Feynman rule mean?



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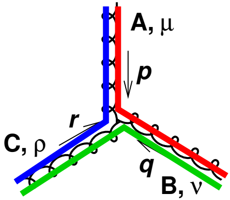
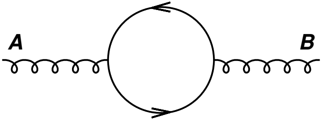


A gluon emission also repaints the gluon colours.

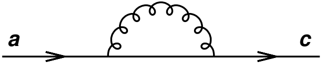
Because a gluon carries colour + anti-colour, it emits $\sim$ twice as strongly as a quark (just has colour)

Quick guide to colour algebra

$$\mathrm{Tr}(t^A t^B) = T_R \delta^{AB}\,, \quad T_R = \frac{1}{2}$$



$$\sum_A t^A_{ab} t^A_{bc} = C_F \delta_{ac}\,, \quad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$$



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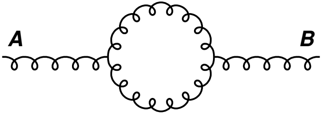

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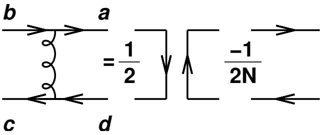

$N_c \equiv$ number of colours $= 3$ for QCD

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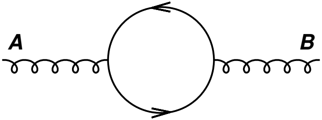

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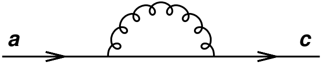

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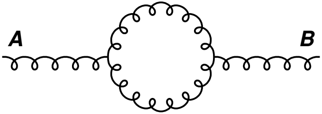

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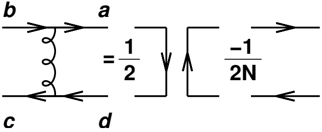

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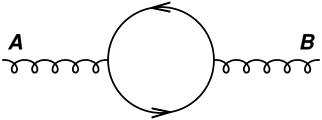

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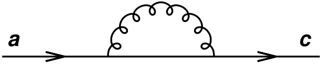

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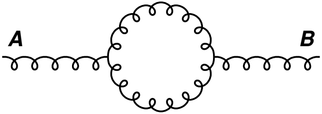

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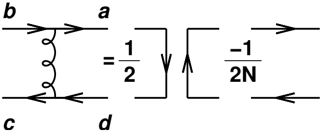

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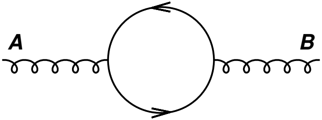

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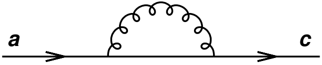

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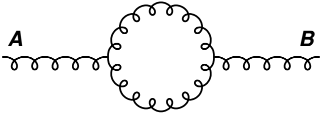

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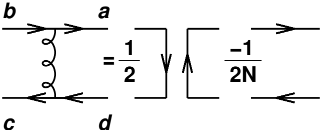

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Exercise

Use the Fierz identity (fourth line of previous slide) to derive the second line.

The running coupling

All couplings run (QED, QCD, EW), i.e. they depend on the momentum scale (Q^2) of your process.

The evolution equation for the QCD coupling, $\alpha_s(Q^2)$, is:

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = \beta(\alpha_s)\,, \qquad \beta(\alpha_s) = -\alpha_s^2(b_0 + b_1\alpha_s + b_2\alpha_s^2 + \ldots)\,,$$

$$b_0 = \frac{11 C_A - 2 n_f}{12\pi}\,, \qquad b_1 = \frac{17 C_A^2 - 5 C_A n_f - 3 C_F n_f}{24\pi^2} = \frac{153 - 19 n_f}{24\pi^2}$$

Note sign: Asymptotic Freedom, due to gluon to self-interaction

- At high scales Q, coupling becomes small
  ➥quarks and gluons are almost free, interactions are weak
- At low scales, coupling becomes strong
  ➥quarks and gluons interact strongly — confined into hadrons
  Perturbation theory fails.

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Running coupling (cont.)

Solve $Q^2 \frac{\partial \alpha_s}{\partial Q^2} = -b_0 \alpha_s^2 \quad \Rightarrow \quad \alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + b_0 \alpha_s(Q_0^2) \ln \frac{Q^2}{Q_0^2}} = \frac{1}{b_0 \ln \frac{Q^2}{\Lambda^2}}$

$\Lambda \simeq 0.2$ GeV (aka Λ_{QCD}) is the fundamental scale of QCD, at which coupling blows up.

- Λ sets the scale for hadron masses
  (NB: Λ not unambiguously defined wrt higher orders)
- Perturbative calculations valid for scales $Q \gg \Lambda$.

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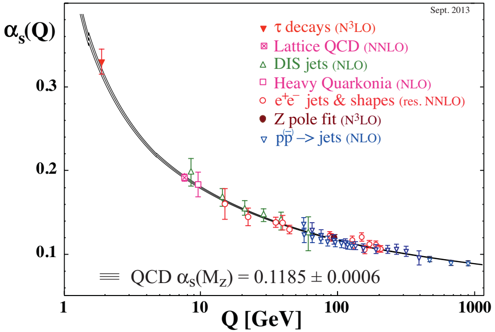

Nobel prize citation (annotated by Skands)

"What this year's Laureates discovered was something that, at first sight, seemed completely contradictory. The interpretation of their mathematical result was that the closer the quarks are to each other, the *weaker* is the 'colour charge'. When the quarks are really close to
*1 each other, the ~~force~~ charge is so weak that they behave almost as free particles. This phenomenon is called 'asymptotic freedom'. The converse is true when the quarks move apart:
*2 the ~~force~~ potential becomes stronger when the distance increases."

Nobelprize.org
The Official Web Site of the Nobel Prize

The Nobel Prize in Physics 2004
David J. Gross, H. David Politzer, Frank Wilczek

David J. Gross | H. David Politzer | Frank Wilczek

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek *"for the discovery of asymptotic freedom in the theory of the strong interaction"*.

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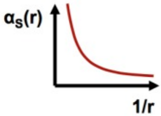


*1 The force still goes to ∞ as r → 0 (Coulomb potential), just less slowly

*2 The potential grows linearly as r→∞, so the force actually becomes constant (even this is only true in "quenched" QCD. In real QCD, the force eventually vanishes for r>>1fm)

P. Skands

Exercise

There is a freedom in defining the "scheme" for α_s. E.g.

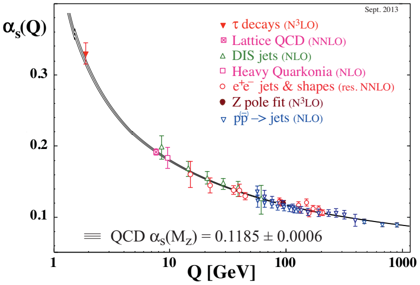
$$\alpha_s^{\text{scheme-B}} = \alpha_s^{\text{scheme-A}} + c_B(\alpha_s^{\text{scheme-A}})^2$$

where c_B is some scheme-conversion coefficient.

The β-function coefficients, usually given in the so-called $\overline{\text{MS}}$ renormalisation scheme, are known up to b_3.

Which of the b_0, b_1, b_2, etc. coefficients is modified if the scheme is changed?

A dilemna



- We want to investigate collisions at high energies ($\sim$ 100 GeV to few TeV), where the coupling is small $\rightarrow$ perturbative methods are the natural choice
- But the LHC collides protons, $m \simeq 0.94$ GeV, which definitely involve strong coupling physics

There is no escape from non-perturbative physics

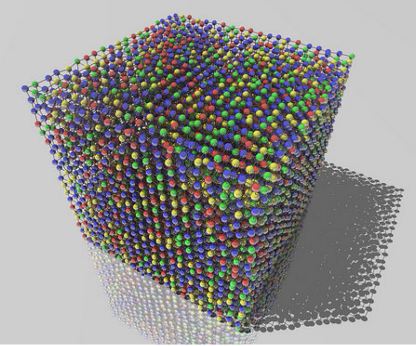
Lattice QCD

- Put all the quark and gluon fields of QCD on a 4D-lattice

  NB: with imaginary time

- Figure out which field configurations are most likely (by Monte Carlo sampling).

- You've solved QCD



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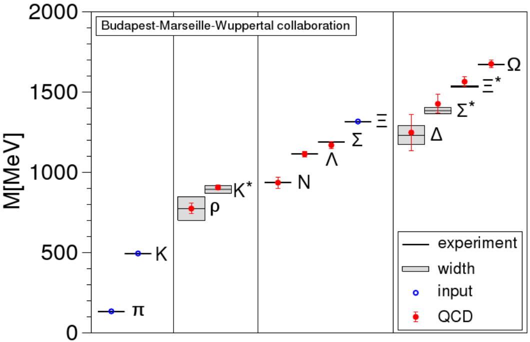
Lattice hadron masses

Lattice QCD is great at calculation static properties of a single hadron.

E.g. the hadron mass spectrum



Durr et al '08

Lattice for LHC?

How big a lattice do you need for an LHC collision @ 14 TeV?

Lattice spacing: $\dfrac{1}{14\ \text{TeV}} \sim 10^{-5}\ \text{fm}$

Lattice extent:

- non-perturbative dynamics for quark/hadron near rest takes place on timescale $t \sim \dfrac{1}{0.5\ \text{GeV}} \sim 0.4\ \text{fm}/c$
- But quarks at LHC have effective boost factor $\sim 10^4$
- So lattice extent should be ~ 4000 fm

Total: need $\sim 4 \times 10^8$ lattice units in each direction, or 3×10^{34} nodes total.

Plus clever tricks to deal with high particle multiplicity, imaginary v. real time, etc.

Neither lattice QCD nor perturbative QCD can offer a full solution to using QCD at colliders

What the community has settled on is

1) **factorisation** of initial state non-perturbative problem

from

2) the "**hard process**," calculated perturbatively

supplemented with

3) non-perturbative modelling of final-state hadronic-scale processes ("**hadronisation**").