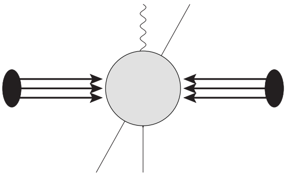
Part 4: Multiple interactions and hadronisation.

a) Multiparton interactions
b) Hadronisation

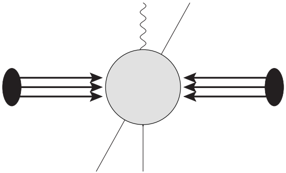
Back to the big picture: Some questions...



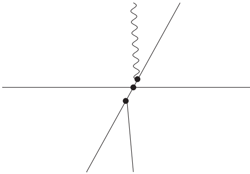
Say an event contains one boson and *three or four* jets. Where do these particles come from?

Back to the big picture: Some questions...

Detector event



Perturbative scattering



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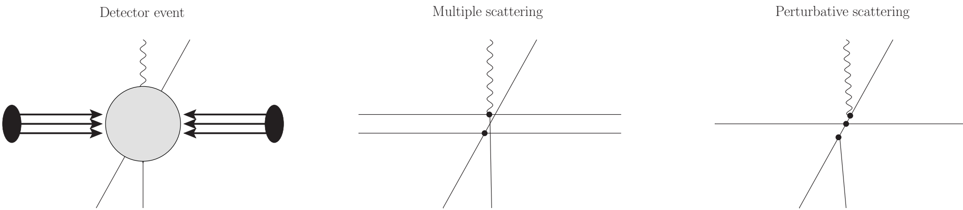
By now, we know quite well how to get these jets by dressing a complicated hard scattering. But when does this apply?

What if two jets merge? What if the boson and a jet are collinear? What if the jets have a low transverse momentum? What if pairs are back-to-back?

Back to the big picture: Some questions...



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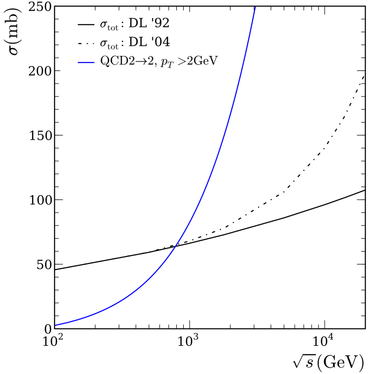
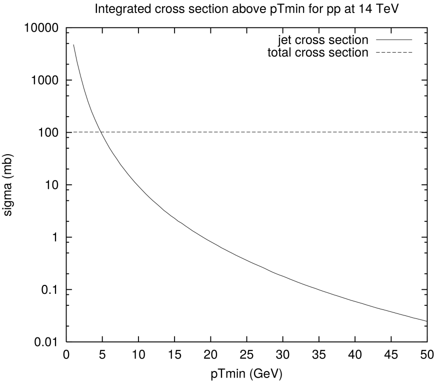
What if two jets merge? What if the boson and a jet are collinear? What if the jets have a low transverse momentum? What if pairs are back-to-back?

When colliding composite objects, many constituent scatterings "compete" for the collision energy – **and multiple scattering can look like single complicated scatterings!**

The dijet process



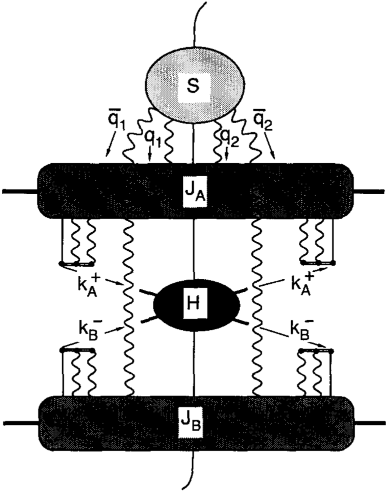


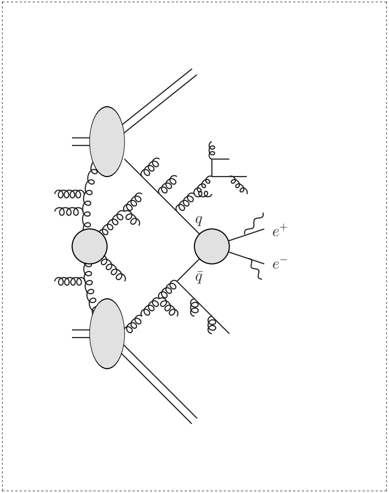
Perturbative cross section

$$\sigma(pp \to jj + X) = \int_{p_{\perp min}}^{\frac{E_{cm}}{2}} dx_1 dx_2 f_1(x_1) f_2(x_2) \frac{d\hat{\sigma}}{dp_\perp} dp_\perp > \sigma(pp \to anything) \text{ for } \frac{p_{\perp min}}{E_{cm}} \to 0$$

as $f(x)$ not small (enough) for low $x \approx \frac{p_{\perp min}}{E_{cm}}$ to suppress $\frac{p_{\perp min}}{E_{cm}} \to 0$ divergence!

Still consistent with perturbative QCD: PDFs are the *inclusive* probability to find parton at x, with all other interactions above $x \approx \frac{p_{\perp min}}{E_{cm}}$ integrated out!



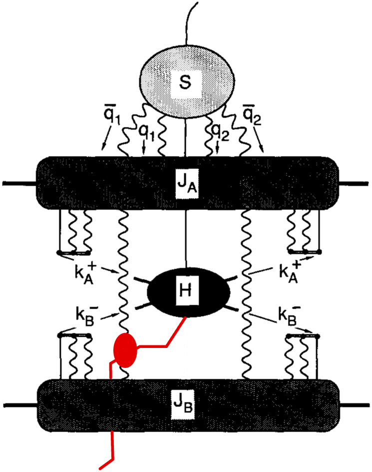


Exclusive observables (i.e. not integrating everything out) "see" these additional interactions!
$\Longrightarrow$ An average $\langle n(p_{\perp min})\rangle$ scatterings accompany one scattering above $p_{\perp min}$, so that

$$\sigma^{inc}(p_{\perp min}, E_{cm}) = \langle n(p_{\perp min})\rangle \cdot \sigma^{inel}(p_{\perp min}, E_{cm})$$

where

$$\sigma^{inel} < \sigma(pp \to anything)$$



Multiple interactions between the composite protons are supported by 30 years of evidence:

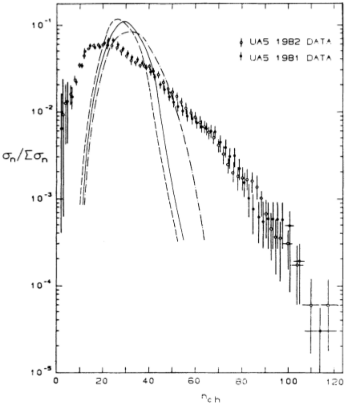


FIG. 3. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low p_T only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.

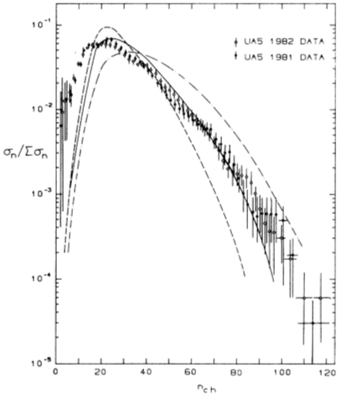


FIG. 5. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs impact-parameter-independent multiple-interaction model: dashed line, $p_{T\min}=2.0$ GeV; solid line, $p_{T\min}=1.6$ GeV; dashed-dotted line, $p_{T\min}=1.2$ GeV.

Question: Can't we just overlay many scatterings to approximate the result? Just like we do for Pile-Up?
Answer: **No!**

For large $p_\perp$, model must preserve the perturbative hard scattering cross section, otherwise factorisation of inclusive cross section violated!

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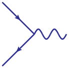
Answer: **No!**

For large $p_\perp$, model must preserve the perturbative hard scattering cross section, otherwise factorisation of inclusive cross section violated!

Solution: Subtract what you add!
For every additional scattering, we need "virtual corrections"

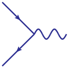
This should sound familiar from PS unitarity / multi-jet merging.

"Virtual corrections" for second scattering



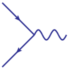
$\mathcal{O}(S_H)$

"Virtual corrections" for second scattering



$$\left[\mathcal{O}\left(S_H\right) \quad + \int$$ 

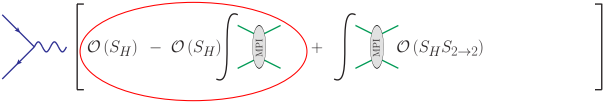
$$\mathcal{O}\left(S_H S_{2\to 2}\right) \right]$$



$\Big[\ \mathcal{O}(S_H) - \mathcal{O}(S_H)\int$

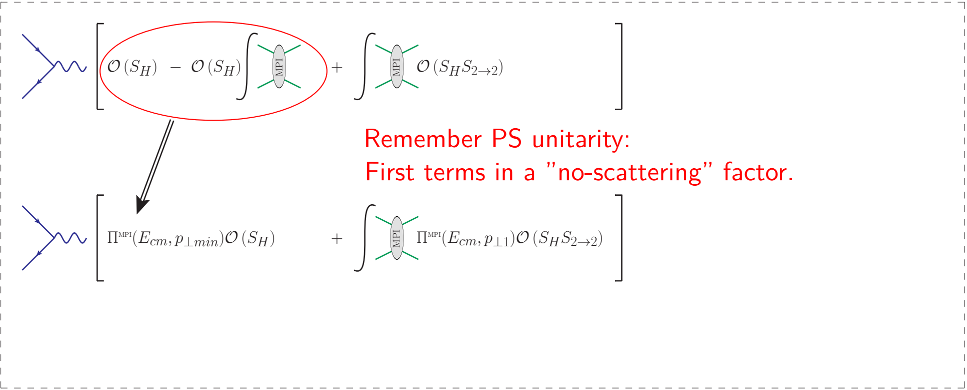
$+ \int$

$\mathcal{O}(S_H S_{2\to 2})\ \Big]$

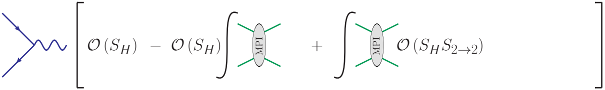


Remember PS unitarity:
First terms in a "no-scattering" factor.

"Virtual corrections" for second scattering



"Virtual corrections" for multiple scatterings



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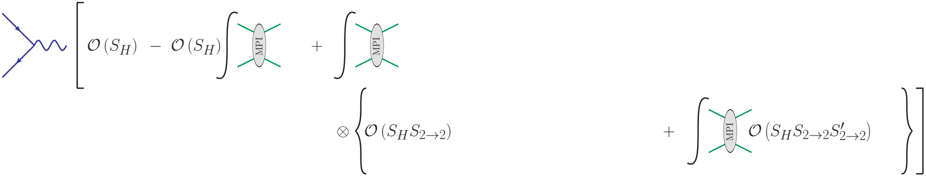

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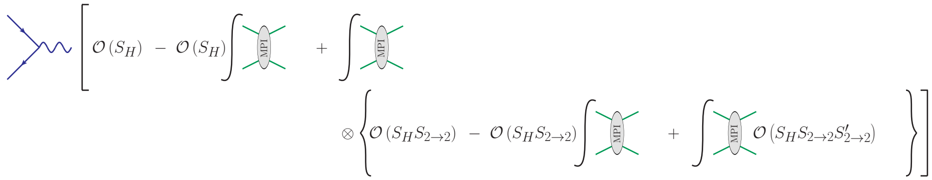

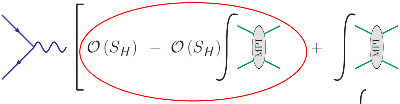

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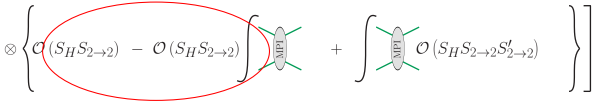

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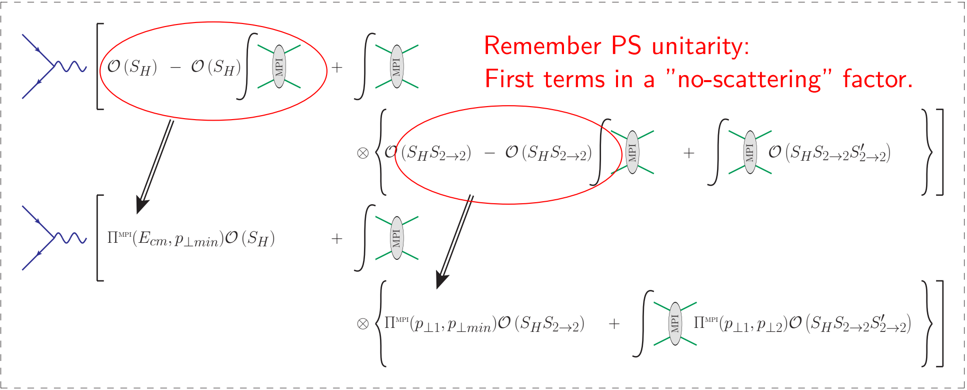


...everything looks like a parton shower. Assume

$$\delta p_\perp \langle n(p_\perp)\rangle \equiv \text{Probability for scattering with } p_\perp \in [p_{\perp min}, p_{\perp min} + \delta p_\perp].$$

Then the probability of no scattering is

$$1 - \delta p_\perp \langle n(p_\perp)\rangle$$

or, if $\delta p_\perp$ is divided into m parts, and the scattering probabilities are independent

$$[1 - \delta p_\perp / m \ \langle n(p_\perp)\rangle]^m \underset{m\to\infty}{\rightarrow} \exp\left(-\int_{p_{\perp min}}^{p_{\perp min}+\delta p_\perp} dp_\perp \langle n(p_\perp)\rangle\right) \equiv \Pi^{\mathsf{MPI}}(p_{\perp min}+\delta p_\perp, p_{\perp min})$$

We can define a no-additional scattering probability which contains "all-order virtual corrections" – just like a parton shower Sudakov factor.

$\Longrightarrow$ Can recycle the PS algorithm to produce additional scatterings.

The probability of a hardest scattering at $p_{\perp 1}$ is

$$\Pi^{\mathsf{MPI}}(p_{\perp 0}, p_{\perp 1})\ \langle n(p_{\perp 1})\rangle$$

The probability of having a second hardest scattering at $p_{\perp 2} < p_{\perp 1}$ is

$$\int_{p_{\perp 2}}^{p_{\perp 0}} dp_{\perp 1} \Pi^{\mathsf{MPI}}(p_{\perp 0}, p_{\perp 1})\ \langle n(p_{\perp 1})\rangle\ \Pi^{\mathsf{MPI}}(p_{\perp 1}, p_{\perp 2})\ \langle n(p_{\perp 2})\rangle$$

Show that the probability of having any partonic scattering (provided the protons scatter) is given by σ^{inc}, i.e. by the perturbative result!

Hints: Look at n scatterings with $p_{\perp n} < p_{\perp n-1} < \cdots < p_{\perp 0}$, use the properties of the exponential functions, you can find a short form for nested integrals by finding a differential equation for the sum of all nested integrals, and thinking about solutions to linear differential equations.

Parameters of Multiparton Interaction models

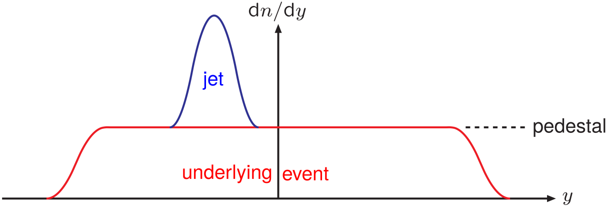
$\Rightarrow$ Perturbative MPI *model* keeping the inc. cross section. Unknowns:

- MPI probability $\langle n(p_\perp)\rangle = \frac{\sigma^{inc}}{\sigma^{inel}}$ with σ^{inel} taken from data (tuned)
- Most MPI very soft, but σ^{inc} is still divergent for $p_{\perp min} \to 0$, i.e. needs regulator $\Longrightarrow$ Extra parameter $p_{\perp 0}$
- Regulator should be larger if E_{cm} becomes larger (to not violate the total cross section) $\Longrightarrow$ Parameters for energy scaling of $p_{\perp 0}$
- ...and some technical parameters specific to implementation.

Current MPI models are much more complicated and differ significantly:
HERWIG: Pick interactions prior to running according to Poissonian,
SHERPA: MPI after hard process evolution in a $p_\perp$-ordered sequence.
PYTHIA: MPI + ISR + FSR combined in one single $p_\perp$ sequence.

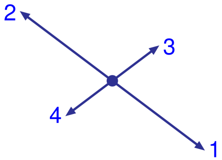
Note: For a complete picture of the *total* cross section, MPI supplemented by non-perturbative dynamics (diffractive physics)

What MPI does for (to?) you



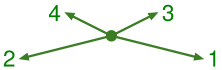
Multiple interactions model the "underlying event" that is present in any hard scattering event. All hadron collider measurement can be sensitive to MPI. However, MPI can be assessed because of its typical kinematics:

Double Parton Scattering



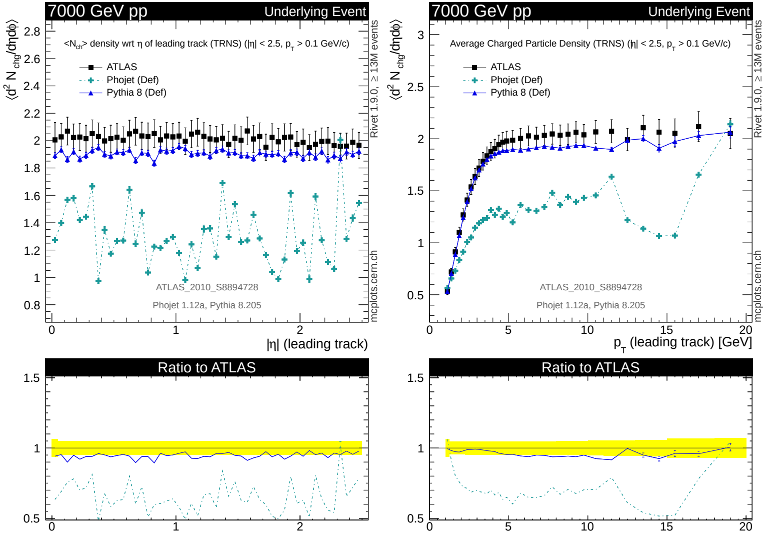
$$|\mathbf{p}_{\perp 1} + \mathbf{p}_{\perp 2}| \approx 0$$
$$|\mathbf{p}_{\perp 3} + \mathbf{p}_{\perp 4}| \approx 0$$

Double BremsStrahlung



$$|\mathbf{p}_{\perp 1} + \mathbf{p}_{\perp 2}| \gg 0$$
$$|\mathbf{p}_{\perp 3} + \mathbf{p}_{\perp 4}| \gg 0$$

MPI "perpendicular" to the hard scattering



Activity uniform in rapidity. More particles for harder scatterings ⇒ Trigger bias, and harder collisions more "central" ⇒ MPI have impact parameter dependence. PHOJET not MPI, but based on Pomeron picture.

However, our result still contains coloured partons.
$\Rightarrow$ Need to convert to hadrons! Two prescriptions in use:

Cluster

Form hadrons by decaying "preconfined" colourless clusters of partons.

Gluons split non-perturbatively to $q\bar{q}$

Many-parameter energy-momentum structure.

Few-parameter flavour chemistry.

Used in HERWIG, SHERPA

String

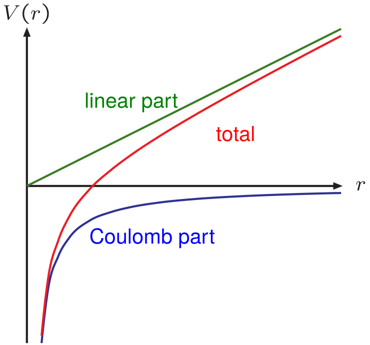
Colour flux tubes (strings, junctions) between partons break to form hadrons.

Gluons are kink on string.

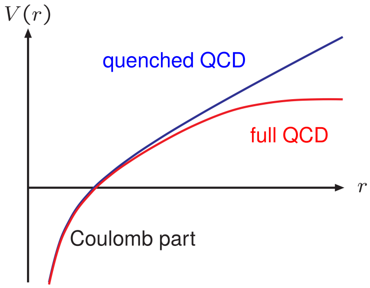
Few-parameter energy-momentum structure.

Many-parameter flavour chemistry.

Used in PYTHIA, EPOS (?)



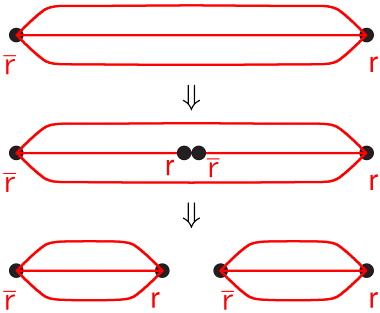
Potential between two quarks assumed linear
⇒ Constant force per unit length (just like a string / flux tube)
⇒ Confining force.



Mesons have yo-yo modes while strings break before yo-yo point.

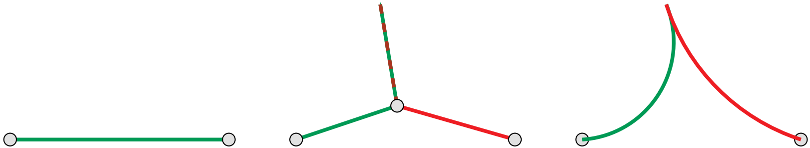
⇒ Linear potential flattens off.

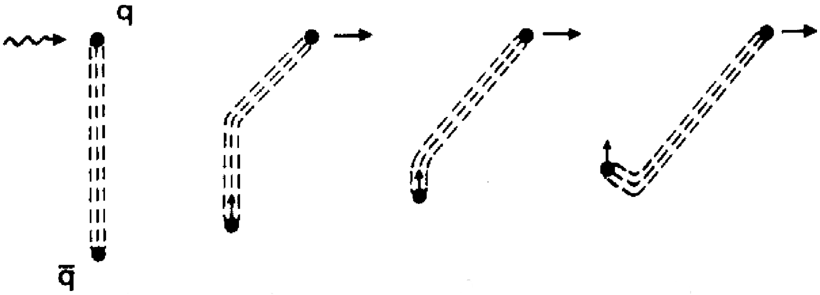
⇒ Breaking gives back-to-back particle production in string CM frame.



But we don't only have quarks! Three possibilities when adding gluons:

- Singlett: Gluon does not change colour field. Very unlikely.
- Junction: Gluon is new type string, attached to old string in a junction. Needs new parameters.
- Kink: Exists on massless relativistic string. No extra parameters.

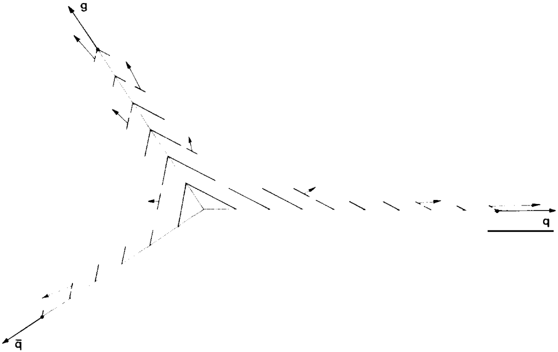




What is a kink?

- Large, instantaneous momentum transfer at initial time
  $\rightarrow$ Stretches string in one direction.
- Kink is connected to two string segments
  $\rightarrow$ Looses energy twice as fast as "endpoint quarks", like gluon
  $(C_A/C_F = 9/4\ (N=3),\ 2\ (N \rightarrow \infty))$
- By causality, string segments fragment as before. String + Kink system fragments as any other string would.

String effect



The addition of gluons leads to the string effect:

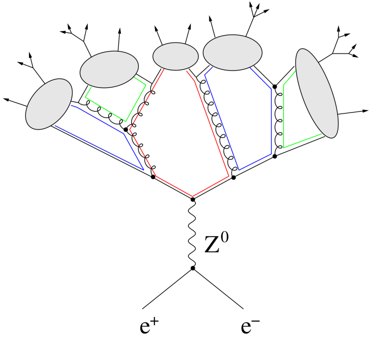
Gluon kink drags string along, while decoupling the quark pair
⇒ Hadron production along qg and $g\bar{q}$ strings.
⇒ 4 hadron production regions, two of them from the "kink" end.
⇒ 3 jets in event frame, almost no hadrons opposite of kink.

⇒ Dynamical coherence effect!

Note: Color coherence prevents gluon production at comparable angles!
⇒ Approximates string effect at perturbative level.
⇒ Can get away with simpler non-perturbative model?

Cluster model:

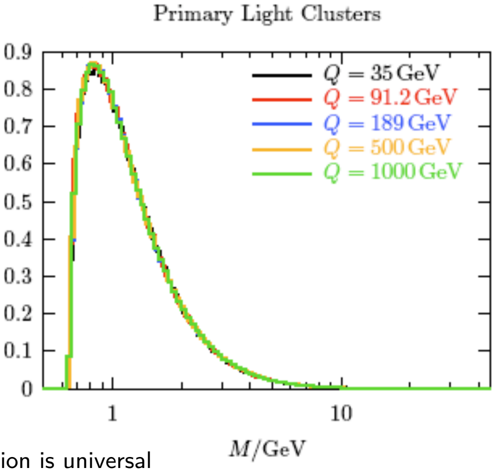
◇ Use perturbative calculation that preserves coherence.
◇ Convert gluons to quarks non-perturbatively.
◇ Collect quarks into colour singlett *preconfined* "clusters".
◇ Clusters decay isotropically into two hadrons.
◇ Heavy clusters need to be split in string-like fashion.



Cluster mass distribution

The main motivation of the cluster model is colour preconfinement.
Following the flow of colour in the $N_C \to \infty$ limit, we find:

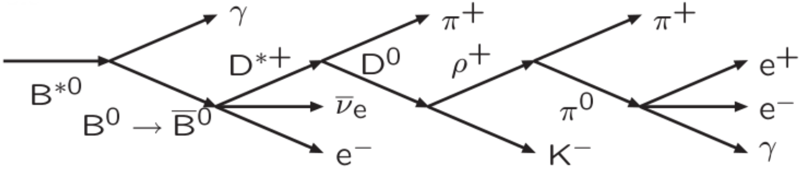
1. Colour-singlet parton pairs are close in phase space.
2. Mass of singlet clusters almost independent of hard scattering scale.



$\Longrightarrow$ Hadronisation is universal

$\Longrightarrow$ Fix parameters at LEP, then "predict" at another collider.

But we're still not there yet! Fragmentation can produce excited hadrons, which will then decay. For example



Most particles are produced in this part.

⇒ Process has to be modelled for the correct jet structure by
...Hadronic matrix elements for some (important) decays.
...PDG decay tables for others. If tables are incomplete, be creative.

Summary of Part 4: Soft physics and hadronisation

- Prediction incomplete before assessing/including non-pert. effects.
- Soft physics:
  - *2 partons* $\rightarrow$ *2 partons* cross section naively exceeds total cross section.
  - Factorisation hints this may be due to additional scatterings.
  - Multiparton interaction models attempt to describe hadron-collider data by "resolving" these additional scatterings.
  - Models fulfill some consistency conditions, e.g. should yield the inclusive cross section when additinal scattering are integrated out.
  - Models come with a handful of parameters.
- Hadronisation:
  - A complete event generation needs to convert partons to hadrons.
  - To generic models exist: The string model and the cluster model.
  - In Lund string model, the constant color field between quarks is split into smaller pieces by string breaking. Incudes coherence effects non-perturbatively.
  - In cluster model, quarks form preconfined color singlet clusters which decay isotropically. The perturbative inputs must include coherence.
  - Hadronisation produces excited hadrons which have to be decayed.

Summary of Event Generator Lectures

- To make the most of (collider) data, we want an accurate representation of our physics model
  $\Rightarrow$ Event generation.

- Event generation is split up into handy bits:
  Hard cross section (perturbative)
  Parton shower resummation (perturbative)
  Multiparton interactions (non-perturbative, not factorisable)
  Hadronisation (non-perturbative, factorisable)

- Parton shower resummation relies on probabilities to produce all-order results, but only for soft / collinear emissions.

- Many improvements in place for accurate jet modelling – by combining with many precise fixed order calculations.

- No first-principle results for non-perturbative components, but still important feature. Modelling requires physics insight!

References

Factorisation and soft gluons:
Collins, Soper, Sterman (Nucl. Phys. B 308 (1988) 833)
The book: Collins, Perturbative Quantum Chromodynamics

Multiparton interactions:
The original article: Sjostrand, van Zijl (PRD D36 (1987) 2019)
A good introduction to the HERWIG model: M. Bähr, Underlying Event Simulation in the Herwig++ Event Generator, Dissertation, ITP Karlsruhe (see `https://www.itp.kit.edu/prep/phd/PSFiles/Diss_Baehr.pdf`)

Hadronisation:
The Lund string model: Andersson, Gustafson, Ingelman, Sjostrand (Phys.Rept. 97 (1983) 31)
Cluster model: Webber (Nucl. Phys. B 238 (1984) 492)