Neutrino

Phenomenology

Boris Kayser INSS August, 2015 Part 1

NASA Hubble Photo

What Are Neutrinos Good For?

Energy generation in the sun starts with the reaction —

$$p + p \to d + e^{+} + \nu$$

Spin: $\frac{1}{2} \quad \frac{1}{2} \quad 1 \quad \frac{1}{2} \quad \frac{1}{2}$

Without the neutrino, angular momentum would not <u>be conserved</u>.

Uh, oh



The Neutrinos

Neutrinos and photons are by far the most abundant elementary particles in the universe. There are 340 neutrinos/cc.

The neutrinos are spin -1/2, electrically neutral, leptons.

The only known forces they experience are the weak force and gravity.

This means that they do not interact with other matter very much at all. Thus, neutrinos are difficult to detect and study.

Their weak interactions are successfully described by the Standard Model.

The Neutrino Revolution (1998 – ...)

Neutrinos have nonzero masses!

Leptons mix!

The Origin of Neutrino Mass

The fundamental constituents of matter are the *quarks*, the *charged leptons*, and the *neutrinos*.

The discovery and study of the *Higgs boson* at CERN's Large Hadron Collider has provided strong evidence that the *quarks* and *charged leptons* derive their masses from an interaction with the *Higgs field*.

Most theorists strongly suspect that the origin of the *neutrino* masses is different from the origin of the *quark* and charged lepton masses.

The Standard-Model *Higgs field* is probably still involved, but there is probably something more something way outside the Standard Model.

More later

The discovery of neutrino mass and leptonic mixing comes from the observation of *neutrino flavor change (neutrino oscillation)*.

The Physics of Neutrino Oscillation

— Preliminaries

The Neutrino Flavors

There are three flavors of charged leptons: e , μ , τ

There are three known flavors of neutrinos: v_e, v_μ, v_τ

We *define* the neutrinos of specific flavor, v_e , v_{μ} , v_{τ} , by W boson decays:



As far as we know, when a neutrino of given flavor interacts and turns into a charged lepton, that charged lepton will always be of the same flavor as the neutrino.



The weak interaction couples the neutrino of a given flavor only to the charged lepton of the same flavor.

Neutrino Flavor Change ("Oscillation") If neutrinos have masses, and leptons mix, we can have —



Give a v time to change character, and you can have

for example: $v_{\mu} \longrightarrow v_{e}$

The last 17 years have brought us compelling evidence that such flavor changes actually occur.

Flavor Change Requires Neutrino Masses

There must be some spectrum of neutrino mass eigenstates v_i :



Mass $(v_i) \equiv m_i$

Flavor Change Requires *Leptonic Mixing*

The neutrinos $v_{e,\mu,\tau}$ of definite flavor $(W \rightarrow e v_e \text{ or } \mu v_{\mu} \text{ or } \tau v_{\tau})$ must be superpositions of the mass eigenstates:

$$V_{\alpha} > = \sum_{i} U^{*}_{\alpha i} |_{V_{i}} > .$$
Neutrino of flavor
$$\alpha = e, \mu, \text{ or } \tau$$

$$U^{*}_{\alpha i} |_{V_{i}} > .$$
Neutrino of definite mass m_{i}

$$Matrix$$

Notation: ℓ denotes a charged lepton. $\ell_e \equiv e, \ell_\mu \equiv \mu, \ell_\tau \equiv \tau$.

Since the only charged lepton v_{α} couples to is ℓ_{α} , the 3 v_{α} must be orthogonal.

To make up 3 orthogonal v_{α} , we must have at least 3 v_i . Unless some v_i masses are degenerate, all v_i will be orthogonal.

Then — $\delta_{\alpha\beta} = \left\langle v_{\alpha} \middle| v_{\beta} \right\rangle = \left\langle \sum_{i} U_{\alpha i}^{*} v_{i} \middle| \sum_{i} U_{\beta j}^{*} v_{j} \right\rangle \begin{bmatrix} \text{Inis says that} \\ U \text{ is unitary,} \\ \text{but note the} \end{bmatrix}$ $=\sum_{\alpha i} U_{\alpha i} U_{\beta j}^* \left\langle v_i \middle| v_j \right\rangle = \sum_{\alpha i} U_{\alpha i} U_{\beta i}^*$

This says that but note the unitary U may not be 3×3 .

Leptonic mixing is easily incorporated into the Standard Model (SM) description of the ℓvW interaction.

For this interaction, we then have —

Semi-weak
coupling
$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} v_{L\alpha} W_{\lambda}^{-} + \overline{v}_{L\alpha} \gamma^{\lambda} \ell_{L\alpha} W_{\lambda}^{+} \right)$$
$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau\\i=1,2,3}} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} v_{L i} W_{\lambda}^{-} + \overline{v}_{L i} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$
Taking mixing into account

The SM interaction conserves the Lepton Number L, defined by $L(v) = L(\ell^{-}) = -L(\overline{v}) = -L(\ell^{+}) = 1$.



Slides on The Physics of Neutrino Oscillation go here.

Neutrino Flavor Change In Matter



Coherent forward scattering via this W-exchange interaction leads to an extra interaction potential energy —

$$V_{W} = \begin{cases} +\sqrt{2}G_{F}N_{e}, & v_{e} \\ -\sqrt{2}G_{F}N_{e}, & \overline{v_{e}} \end{cases}$$

Fermi constant ______ Electron density

This raises the effective mass of v_e , and lowers that of $\overline{v_e}_{18}$.

The fractional importance of matter effects on an oscillation involving a vacuum splitting Δm^2 is —



The matter effect —

- Grows with neutrino energy E

- Is sensitive to $Sign(\Delta m^2)$

— Reverses when ν is replaced by $\overline{\nu}$

This last is a "fake CP violation" that has to be taken into account in searches for genuine CP violation.

Evídence For Flavor Change

Neutrinos

Evidence of Flavor Change

Solar Reactor (Long-Baseline) Compelling Compelling

Atmospheric Accelerator (Long-Baseline) Compelling Compelling

Accelerator, Reactor, and Radioactive Sources (Short-Baseline)

"Interesting"

KamLAND Evidence for O^Scilatory Behavior

The KamLAND detector studies $\overline{v_e}$ produced by Japanese nuclear power reactors ~ 180 km away.

For KamLAND, $x_{Matter} < 10^{-2}$. Matter effects are negligible.

The \overline{v}_e survival probability, $P(\overline{v}_e \rightarrow \overline{v}_e)$, should oscillate as a function of L/E following the vacuum oscillation formula.

In the two-neutrino approximation, we expect —

$$P(\overline{v}_e \rightarrow \overline{v}_e) = 1 - \sin^2 2\theta \sin^2 \left[1.27 \Delta m^2 \left(eV^2 \right) \frac{L(km)}{E(GeV)} \right]$$



 $L_0 = 180$ km is a flux-weighted average travel distance.

 $P(\overline{v}_e \rightarrow \overline{v}_e)$ actually oscillates!



The (Mass)² Spectrum





What Tritium β Decay Measures

Tritium decay: ${}^{3}H \rightarrow {}^{3}He + e^{-} + \overline{v}_{i}$; i = 1, 2, or 3

There are 3 distinct final states.

The amplitudes for the production of these 3 distinct final states contribute *incoherently*.

$$BR({}^{3}H \rightarrow {}^{3}He + e^{-} + \overline{v}_{i}) \propto |U_{ei}|^{2}$$

In ${}^{3}H \rightarrow {}^{3}He + e^{-} + \overline{v}_{i}$, the bigger m_{i} is, the smaller the maximum electron energy is.

There are 3 separate thresholds in the β energy spectrum.

The β energy spectrum is modified according to -

$$(E_0 - E)^2 \Theta[E_0 - E] \Rightarrow \sum_i |U_{ei}|^2 (E_0 - E) \sqrt{(E_0 - E)^2 - m_i^2} \Theta[(E_0 - m_i) - E]$$

$$\int_i^{i} Maximum \beta \text{ energy when}$$
there is no neutrino mass
$$\beta \text{ energy}$$

Present experimental energy resolution is insufficient to separate the thresholds.

Measurements of the spectrum bound the average neutrino mass —

$$\left\langle m_{\beta} \right\rangle = \sqrt{\sum_{i} \left| U_{ei} \right|^2 m_i^2}$$

Presently:
$$\langle m_{\beta} \rangle < 2 \text{ eV}$$

Leptonic Mixing

Mixing means that —

$$|v_{\alpha}\rangle = \sum_{i} U^{*}_{\alpha i} |v_{i}\rangle.$$
Neutrino of flavor
$$\alpha = e, \mu, \text{ or } \tau$$
Neutrino of definite mass m_{i}

Inversely,
$$|v_i\rangle = \sum_{\alpha} U_{\alpha i} |v_{\alpha}\rangle$$
.

Flavor- α fraction of $v_i = |U_{\alpha i}|^2$.

When a v_i interacts and produces a charged lepton, the probability that this charged lepton will be of flavor α is $|U_{\alpha i}|^2$. Experimentally, the flavor fractions are —



Observations We Can Use To Understand This Picture

The Disappearance of Atmospheric ν_{μ}



Isotropy of the ≥ 2 GeV cosmic rays + Gauss' Law + No ν_{μ} disappearance

$$\Rightarrow \frac{\phi_{\nu_{\mu}}(\mathrm{Up})}{\phi_{\nu_{\mu}}(\mathrm{Down})} = 1 \ .$$

But Super-Kamiokande finds for $E_v > 1.3 \text{ GeV}$ —





the solar splitting is largely invisible. Then—

$$\frac{P(\nu_{\mu} \rightarrow \nu_{\mu})}{\frac{1}{2}} \approx 1 - 4 |U_{\mu3}|^{2} (1 - |U_{\mu3}|^{2}) \sin^{2} \left[1.27 \Delta m_{atm}^{2} \frac{L(km)}{E(GeV)} \right] \\
\frac{1}{2} \qquad 1 \longrightarrow |U_{\mu3}|^{2} = \frac{1}{2} \frac{1}{2} \qquad 34$$

Reactor – Neutrino Experiments and $|U_{e3}|^2 = \sin^2\theta_{13}$

Reactor \overline{v}_e have $E \sim 3$ MeV, so if $L \sim 1.5$ km,

$$\sin^{2}\left[1.27\Delta m^{2}(eV^{2})\frac{L(km)}{E(GeV)}\right]$$
 will be sensitive to --

$$\Delta m^2 = \Delta m_{\rm atm}^2 = 2.4 \times 10^{-3} {\rm eV}^2 \approx \frac{1}{400} {\rm eV}^2$$

but not to —

$$\Delta m^2 = \Delta m_{\rm sol}^2 = 7.5 \times 10^{-5} {\rm eV}^2 \approx \frac{1}{13,000} {\rm eV}^2$$

Then —

$$P(\overline{\nu}_e \rightarrow \overline{\nu}_e) \approx 1 - 4|U_{e3}|^2 (1 - |U_{e3}|^2) \sin^2 \left[1.27\Delta m_{\text{atm}}^2 \frac{L(\text{km})}{E(\text{GeV})}\right]$$

Measurements by the Daya Bay, RENO, and Double CHOOZ reactor neutrino experiments, (and by the T2K accelerator neutrino experiment)



The Change of Flavor of Solar ν_e

Nuclear reactions in the core of the sun produce v_e . Only v_e .

The Sudbury Neutrino Observatory (SNO) measured, for the high-energy part of the solar neutrino flux:

$$v_{sol} d \rightarrow e p p \Rightarrow \phi_{v_e}$$

 $v_{sol} d \rightarrow v n p \Rightarrow \phi_{v_e} + \phi_{v_{\mu}} + \phi_{v_{\tau}}$ (v remains a v)

From the two reactions,

$$\frac{\phi_{\nu_e}}{\phi_{\nu_e} + \phi_{\nu_{\mu}} + \phi_{\nu_{\tau}}} = 0.301 \pm 0.033$$

For solar neutrinos, $P(v_e \rightarrow v_e) = 0.3 = |U_{e2}|^2$.

The Behavior of Reactor \overline{v}_e In KamLAND



Constructing the Approximate Mixing Matrix (A Blackboard Exercise)

The result —

 $U \approx \begin{bmatrix} V_{1} & V_{2} & V_{3} \\ \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{bmatrix} V_{\mu}$ V_{μ}



Parametrizing the 3 X 3 Unitary Leptonic Mixing Matrix

Caution: We are *assuming* the mixing matrix U to be $3 \ge 3$ and unitary.

$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau\\i=1,2,3}} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \overline{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

$$(CP)\left(\overline{\ell}_{L\alpha}\gamma^{\lambda}U_{\alpha i}\nu_{Li}W_{\lambda}^{-}\right)(CP)^{-1} = \overline{\nu}_{Li}\gamma^{\lambda}U_{\alpha i}\ell_{L\alpha}W_{\lambda}^{+}$$

Phases in *U* will lead to CP violation, unless they are removable by redefining the leptons.



When $|\ell_{\alpha}^{-}\rangle \rightarrow |\mathbf{e}^{i\varphi}\ell_{\alpha}^{-}\rangle, \ \mathbf{U}_{\alpha i} \rightarrow \mathbf{e}^{-i\varphi}\mathbf{U}_{\alpha i}, \text{ all } \mathbf{i}$

Thus, one may multiply any column, or any row, of U by a complex phase factor without changing the physics.

Some phases may be removed from U in this way.

When the Neutrino Mass Eigenstates Are Their Own Antiparticles

When this is the case, processes that do not conserve the lepton number $L \equiv #(Leptons) - #(Antileptons)$ can occur.

Example:



The amplitude for any such *L*-violating process contains an extra factor.

When we phase-redefine v_i to remove a phase from U, that phase just moves to the extra factor.

It does not disappear from the physics.

Hence, when $\overline{v}_i = v_i$, *U* can contain extra physically-significant phases.

These are called Majorana phases.

How Many Mixing Angles and *CP* Phases Does U Contain?

Real parameters before constraints:	
Unitarity constraints — $\sum_{i} U_{\alpha i}^{*} U_{\beta i} = \delta_{\alpha \beta}$	
Each row is a vector of length unity:	- 3
Each two rows are orthogonal vectors:	-6
Rephase the three ℓ_{α} :	- 3
Rephase two v_i , if $\overline{v_i} \neq v_i$:	- 2
Total physically-significant parameters:	
Additional (Majorana) \mathcal{CP} phases if $\overline{v}_i = v_i$:	2

How Many Of The Parameters Are Mixing Angles?

The *mixing angles* are the parameters in U when it is *real*.

U is then a three-dimensional rotation matrix.

Everyone knows such a matrix is described in terms of 3 angles.

Thus, U contains 3 mixing angles.

<u>Summary</u>

	CP phases	CP phases
Mixing angles	if $\overline{\nu}_i \neq \nu_i$	if $\overline{\nu}_i = \nu_i$
3	1	3

The Lepton Mixing Matrix U

The leptonic mixing matrix
$$U$$
 is -
 v_1
 v_2
 v_3
 $e_1 \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$
×diag $(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$
Majorana phases

The Majorana CP Phases

The phase α_i is associated with neutrino mass eigenstate v_i :

 $U_{\alpha i} = U_{\alpha i}^{0} \exp(i\alpha_i/2)$ for all flavors α .

 $\begin{array}{l} \operatorname{Amp}(v_{\alpha} \rightarrow v_{\beta}) = \sum_{i} U_{\alpha i}^{*} \exp(-im_{i}^{2}L/2E) \ U_{\beta i} \\ \text{is insensitive to the Majorana phases } \alpha_{i} \ . \end{array}$ $\begin{array}{l} \operatorname{Only the phase } \delta \operatorname{can cause CP violation in} \\ \operatorname{neutrino oscillation.} \end{array}$

There Is Nothing Special About θ_{13}

All mixing angles must be nonzero for *CP* in oscillation.

For example —

$$P(\bar{v}_{\mu} \rightarrow \bar{v}_{e}) - P(v_{\mu} \rightarrow v_{e}) = 2\cos\theta_{13}\sin2\theta_{13}\sin2\theta_{12}\sin2\theta_{23}\sin\delta$$

$$\times \sin\left(\Delta m^{2}_{31}\frac{L}{4E}\right)\sin\left(\Delta m^{2}_{32}\frac{L}{4E}\right)\sin\left(\Delta m^{2}_{21}\frac{L}{4E}\right)$$

In the factored form of U, one can put
$$\delta$$
 next to θ_{12} instead of θ_{13} .