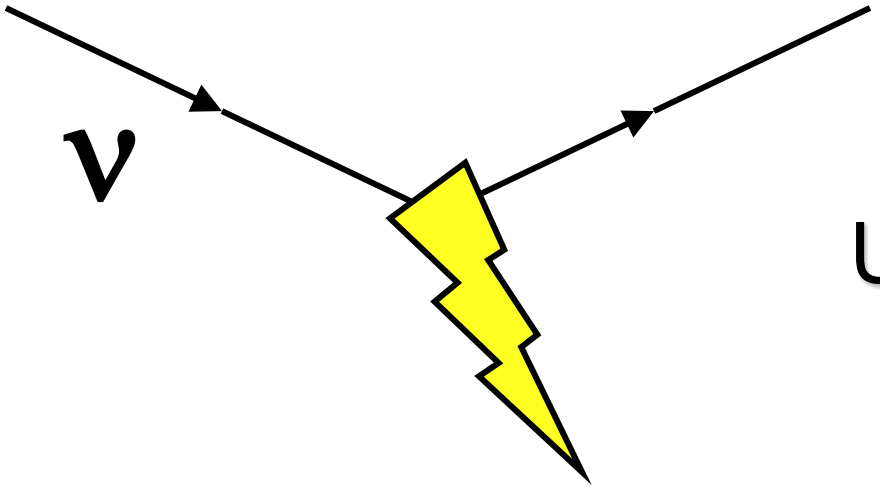
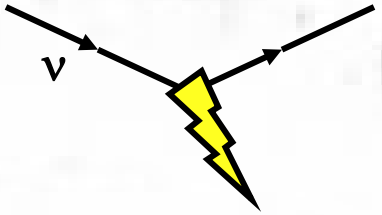
Interactions of Neutrinos



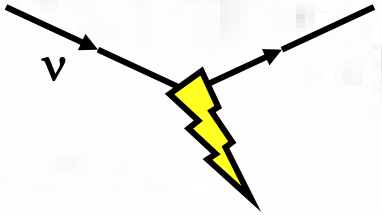
Kevin McFarland
University of Rochester
INWS 2015
São Paulo, Brasil
21-24 August 2015

Outline



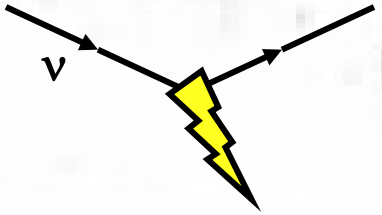
- Brief Motivation for and History of Measuring Interactions
- Weak interactions and neutrinos
  - Elastic and quasi-elastic processes, e.g., νe scattering
  - Complication of Targets with Structure
  - Deep inelastic scattering (νq) and UHE neutrinos
  - Quasielastic and nearly elastic scattering
- Special problems at accelerator energies
  - Nuclear Effects
  - Generators, theory and experimental data
- Conclusions

Focus of These Lectures



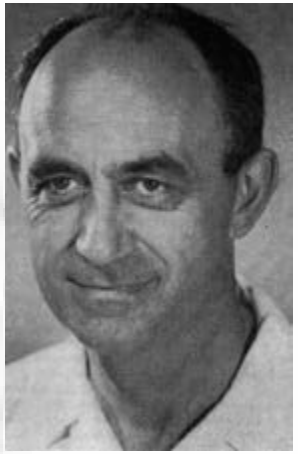
- This is not a comprehensive review of all the interesting physics associated with neutrino interactions
- Choice of topics will focus on:
  - Cross-sections useful for studying neutrino properties
  - Estimating cross-sections
  - Understanding the most important effects qualitatively or semi-quantitatively
  - Understanding how we use our knowledge of cross-sections in experiments

Weak Interactions



- Current-current interaction Fermi, Z. Physik, 88, 161 (1934)

$$\mathcal{H}_w = \frac{G_F}{\sqrt{2}} \mathcal{J}^\mu \mathcal{J}_\mu$$

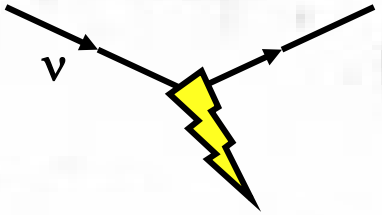


  - Paper famously rejected by *Nature:*
    "it contains speculations too remote from reality to be of interest to the reader"
- Prediction for neutrino interactions
  - If $n \rightarrow p e^- \bar{\nu}$, then $\bar{\nu} p \rightarrow e^+ n$
  - Better yet, it is robustly predicted by Fermi theory
    - Bethe and Peirels, Nature 133, 532 (1934)
  - For neutrinos of a few MeV from a reactor, a typical cross-section was found to be

$$\sigma_{\bar{\nu} p} \sim 5 \times 10^{-44} \text{cm}^2$$

    This is wrong by a factor of two (parity violation)

How Weak is This?

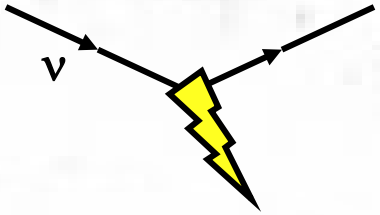


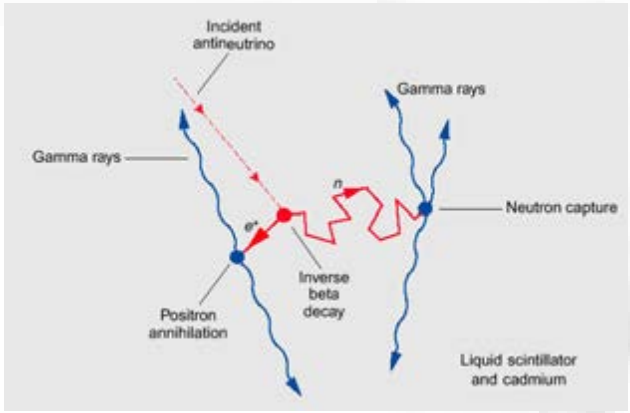
- $\sigma \sim 5 \times 10^{-44} cm^2$ compared with
  - $\sigma_{\gamma p} \sim 10^{-25}$ cm^2 at similar energies, for example
- The cross-section of these few MeV neutrinos is such that the mean free path in steel would be 10 light-years

"I have done something very bad today by proposing a particle that cannot be detected; it is something no theorist should ever do."

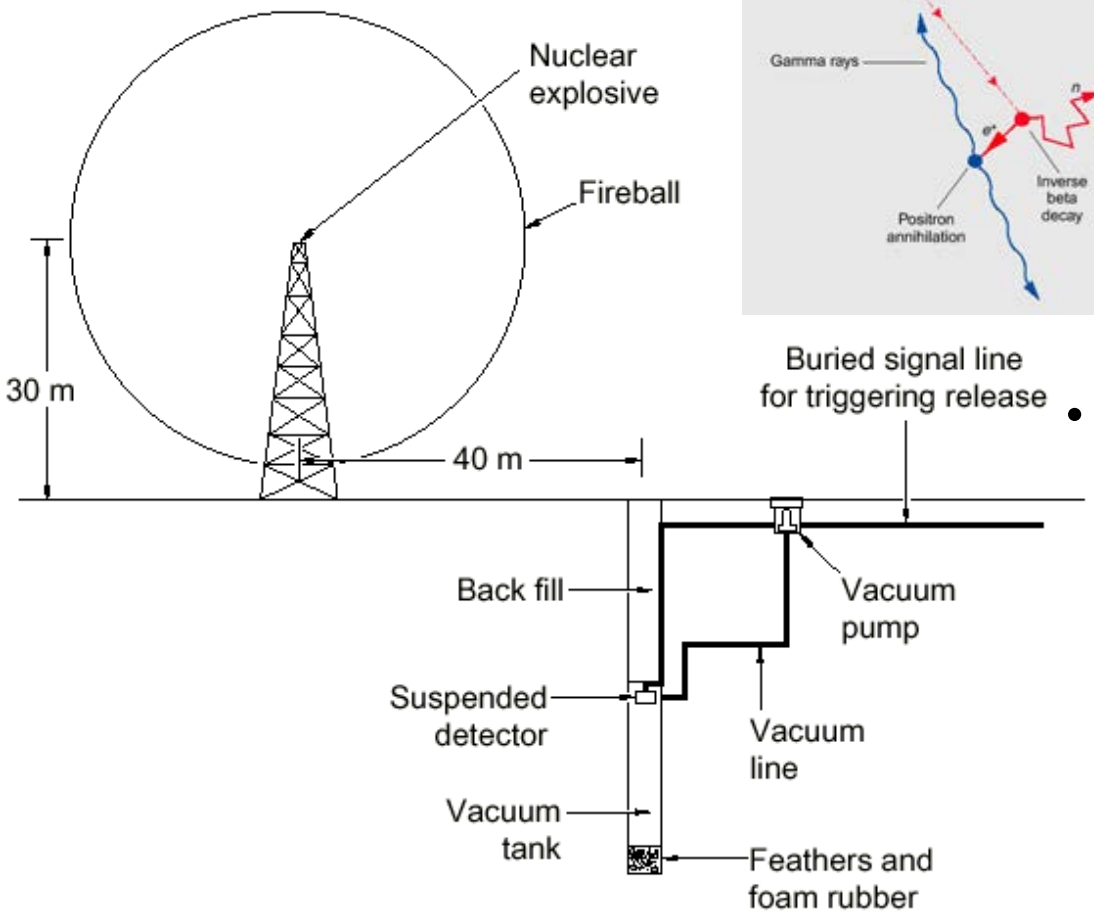
Wolfgang Pauli

Extreme Measures to Overcome Weakness (Reines and Cowan, 1946)



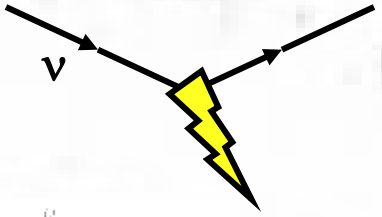


$$\bar{\nu}\, p \rightarrow e^{+} n$$

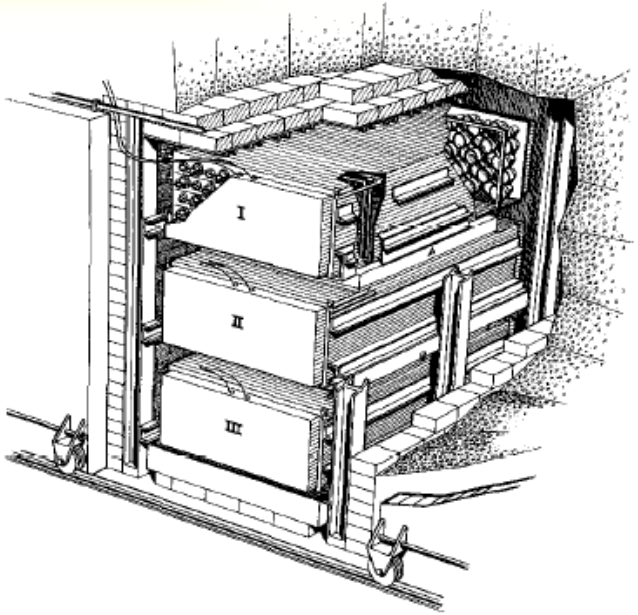


- Why inverse neutron beta decay?
  - clean prediction of Fermi weak theory
  - clean signature of prompt gammas from e^{+} plus delayed neutron signal.
    - Latter not as useful with bomb source.

Discovery of the Neutrino



- Reines and Cowan (1955)
  - Chose a constant source, nuclear reactor (Savannah River)
  - 1956 message to Pauli: ”We are happy to inform you [Pauli] that we have definitely detected neutrinos…”
  - 1995 Nobel Prize for Reines



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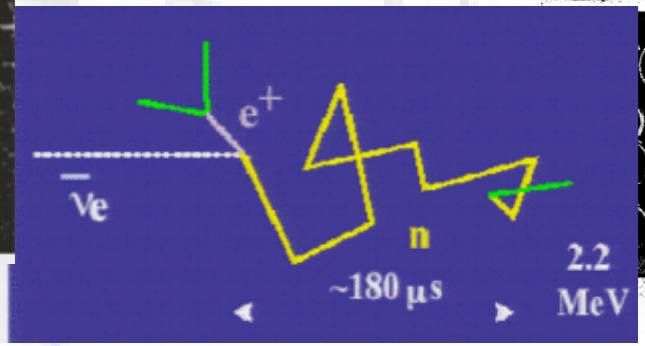
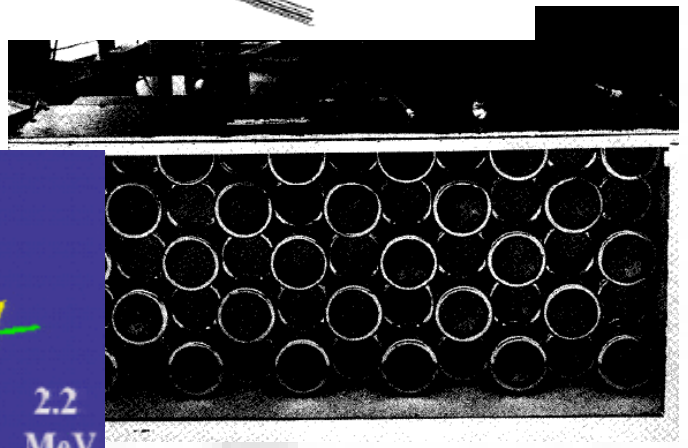
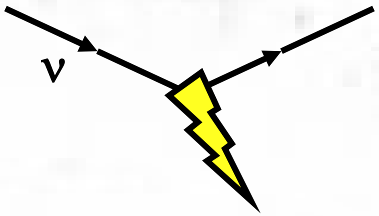




Better than the Nobel Prize?



Frederick REINES and Clyde COWAN
Box 1663, LOS ALAMOS, New Mexico

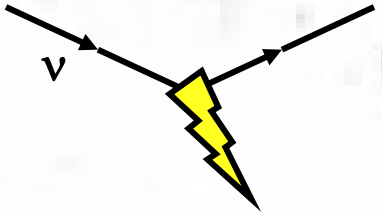
Thanks for message. Everything comes to him who knows how to wait.

Pauli

Thanks for the message. Everything comes to him who knows how to wait.

erl. 15.6.18 / 15.35h
[illegible]

Lecture Questions



- You've been listening to a lot of lectures
- Lectures are a hard format for active learning
- I like to ask my audience questions in lectures.
- Here's a warm up

PHYSICAL REVIEW VOLUME 97, NUMBER 3 FEBRUARY 1, 1955

Attempt to Detect the Antineutrinos from a Nuclear Reactor by the $Cl^{37}(\bar{\nu},e^-)A^{37}$ Reaction*

RAYMOND DAVIS, JR.
Department of Chemistry, Brookhaven National Laboratory, Upton, Long Island, New York
(Received September 21, 1954)

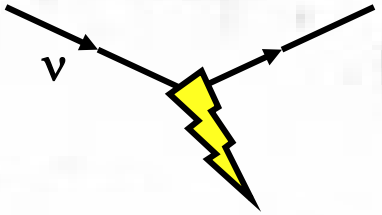
Tanks containing 200 and 3900 liters of carbon tetrachloride were irradiated outside of the shield of the Brookhaven reactor in an attempt to induce the reaction $Cl^{37}(\bar{\nu},e^-)A^{37}$ with fission product antineutrinos. The experiments serve to place an upper limit on the antineutrino capture cross section for the reaction of 2×10^{-46} cm² per atom. Cosmic-ray-induced A^{37} was observed and the production rate measured at 14 100 feet altitude and sea level. Measurements with the 3900-liter container shielded from cosmic rays with 19 feet of earth permit placing an upper limit on the neutrino flux from the sun.

I. INTRODUCTION

THERE have been a number of experiments performed in the past to detect the neutrino by scattering processes and nuclear interactions.[1] The most sensitive of these experiments serve to place a limit on the scattering cross section for neutrinos on electrons of less than 14×10^{-46} cm²/electron and for nuclear interaction of less than 10^{-39} cm²/atom. Recently Reines and Cowan of the Los Alamos Laboratory performed an experiment with a large hydrocarbon liquid scintillator having a high sensitivity for detecting the interaction $\bar{p}(\bar{\nu},e^+)n$ within the liquid.[2] Measurements were made with this scintillator located adjacent to the Hanford reactor within a shield designed to absorb other radiations from the reactor to which the scintillator was sensitive. Under these conditions decay a neutrino (ν) is emitted which may be formally distinguished from an antineutrino ($\bar{\nu}$) which accompanies negative beta emission. A nuclear reactor emits antineutrinos which arise from the negative beta decays of fission products. In our experiment an attempt is made to observe an inverse electron capture process which requires neutrinos, using a source emitting antineutrinos. If neutrinos and antineutrinos are identical in their interactions with nucleons one should be able to observe the process upon carrying the experiment to the required sensitivity. However, if neutrinos and antineutrinos differ in their interactions with nucleons one would not expect to induce the reaction $Cl^{37}(\bar{\nu},e^-)A^{37}$. A positive experiment of this type would show that these particles are not to be distinguished in their nuclear reactions. A negative experiment

Raymond Davis first tried out his chlorine experiment at a reactor, looking for $\bar{\nu}+{}^{37}Cl\rightarrow{}^{37}Ar+e^-$. Same as his solar neutrino experiment. He didn't find it. Why?

Lecture Question: Warm Up



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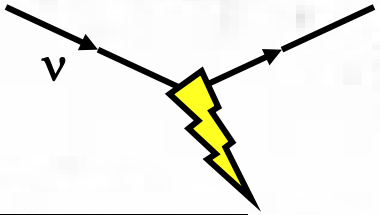
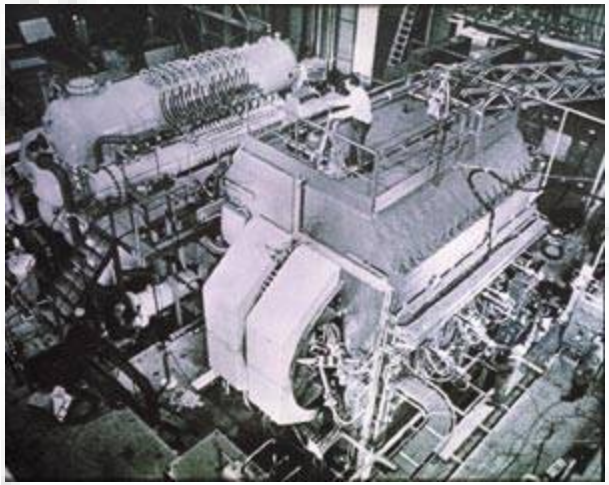
- Subsequent questions will mostly be multiple choice and require some short calculations.
- Paper may be helpful.
- Please participate!

Another Neutrino Interaction Discovery



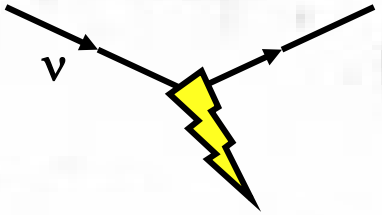
- Neutrinos only feel the weak force
  - a great way to study the weak force!
- Search for neutral current
  - arguably the most famous neutrino interaction ever observed is shown at right

$$\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$$



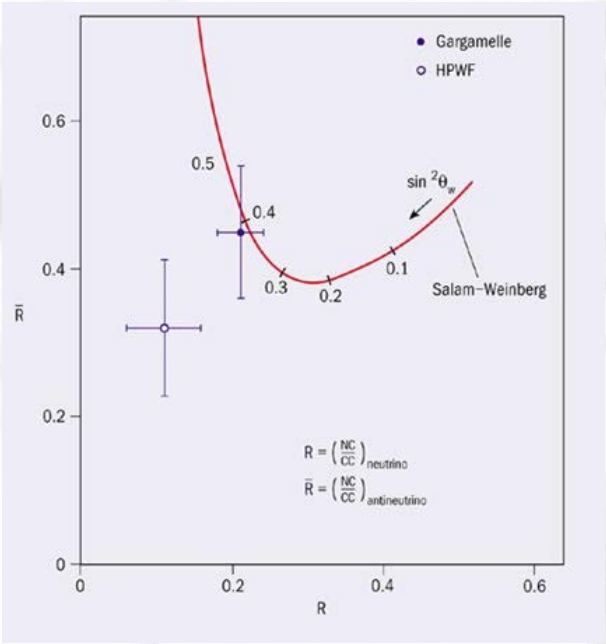
Gargamelle, event from neutral weak force

An Illuminating Aside



- The "discovery signal" for the neutral current was really neutrino scattering from nuclei
  - usually quoted as a ratio of muon-less interactions to events containing muons

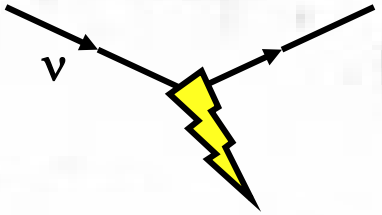
$$R^{\nu} = \frac{\sigma(\nu_\mu N \to \nu_\mu X)}{\sigma(\nu_\mu N \to \mu^- X)}$$



- But this discovery was complicated for 12-18 months by a lack of understanding of neutrino interactions
  - backgrounds from neutrons induced by neutrino interactions outside the detector
  - not understanding fragmentation to high energy hadrons which then "punched through" to fake muons

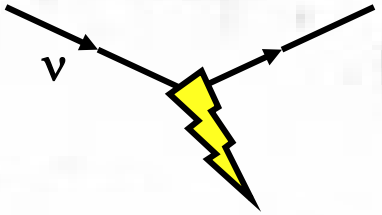
Great article: P. Gallison, Rev Mod Phys 55, 477 (1983)

The Future: Interactions and Oscillation Experiments

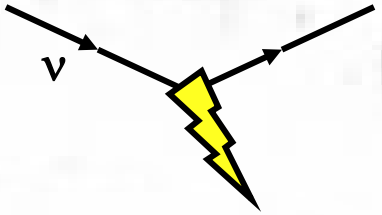


- Oscillation experiments point us to a rich physics potential at L/E~400 km/GeV *(and L/E~N·(400 km/GeV) as well)*
  - mass hierarchy, CP violation
- But there are difficulties
  - transition probabilities as a function of energy must be precisely measured for mass hierarchy and CP violation
  - the neutrinos must be at difficult energies of 1-few GeV for electron appearance experiments, few-many GeV for atmospheric neutrino and τ appearance experiments.
  - or use neutrinos from reactors… *"past is prologue" – B.S.*
- Our generations lack neutrino flavor measurements in which distinguishing 1 from 0 or 1/3 buys a trip to Stockholm
  - Difficulties are akin to neutral current experiments

What are the potential problems from interactions?

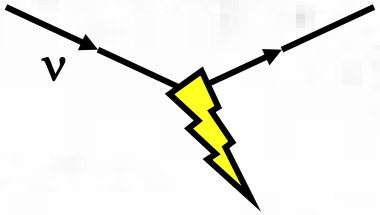


- As you have learned from Boris, for a fixed baseline oscillation experiment, the relationship between oscillation parameters and event rate depends on flavor and E_ν, both of which we measure from the final state
- *Energy reconstruction*
  - Final state particles and their production from a nuclear target determine ability to reconstruct E_ν
- *Signal rate for different flavors*
- *Backgrounds*
  - Copiously produced pions have an annoying habit of faking leptons ($\pi^0 \rightarrow e$ or $\pi^\pm \rightarrow \mu$) in realistic detectors
  - Important to understand rate and spectrum of pions



Calculating Neutrino Interactions from Electroweak Theory

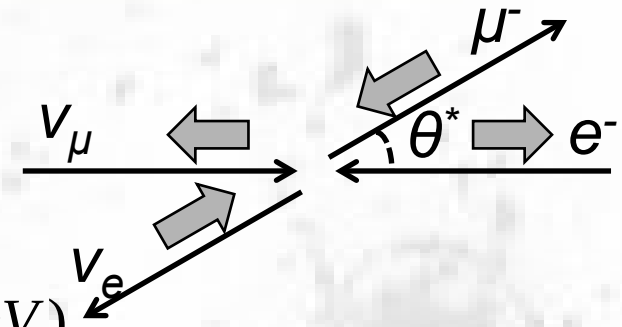
Neutrino-Electron (cont'd)



$$\sigma_{TOT} \propto Q^2_{\max} = s$$

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi}$$

$$= 17.2 \times 10^{-42} cm^2 / GeV \cdot E_\nu (GeV)$$

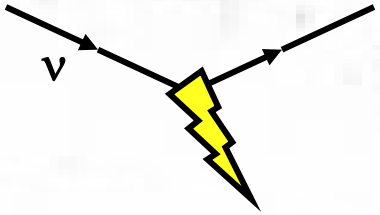


- Why is it proportional to beam energy?

$$s = (\underline{p}_{\nu_\mu} + \underline{p}_e)^2 = m_e^2 + 2 m_e E_\nu \text{ (e}^- \text{ rest frame)}$$

- Proportionality to energy is a generic feature of point-like scattering!
  - because $d\sigma/dQ^2$ is constant (at these energies)

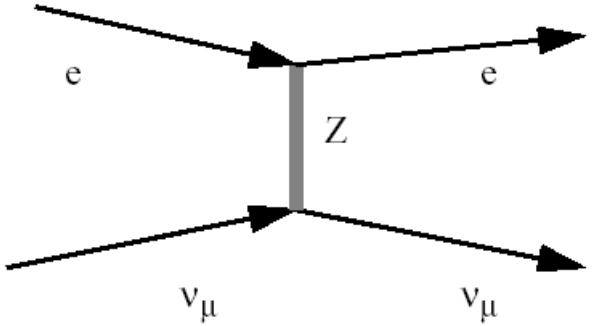
Neutrino-Electron (cont'd)



- Elastic scattering:

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

  - Recall, EW theory has coupling to left *or* right-handed electron
  - Total spin, J=0,1



Z Couplings	g_L	g_R
ν_e , ν_μ , ν_τ	1/2	0
e , μ , τ	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$
u , c , t	$1/2 - 2/3\ \sin^2\theta_W$	$-2/3\ \sin^2\theta_W$
d , s , b	$-1/2 + 1/3\ \sin^2\theta_W$	$1/3\ \sin^2\theta_W$

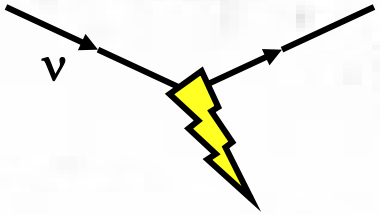
- Electron-Z^0 coupling
  - Left-handed: $-1/2 + \sin^2\theta_W$

$$\sigma \propto \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \sin^4\theta_W\right)$$

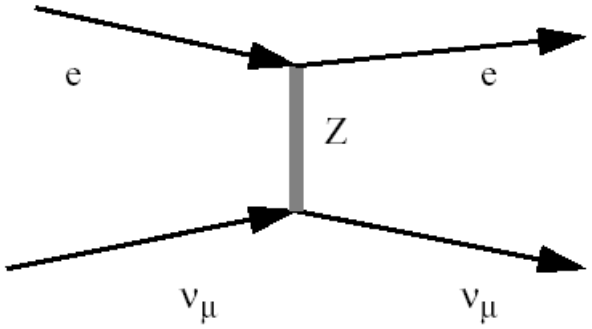
  - Right-handed: $\sin^2\theta_W$

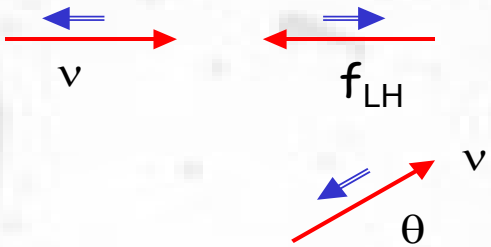
$$\sigma \propto \frac{G_F^2 s}{\pi}\left(\sin^4\theta_W\right)$$

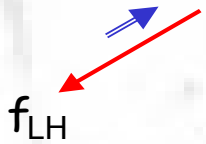
Neutrino-Electron (cont'd)



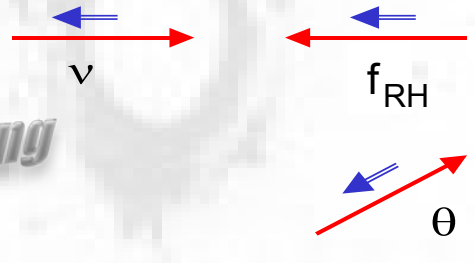
- What are relative contributions of scattering from left *and* right-handed electrons?

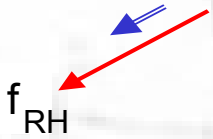






$$\frac{d\sigma}{d\cos\theta} = \text{const}$$

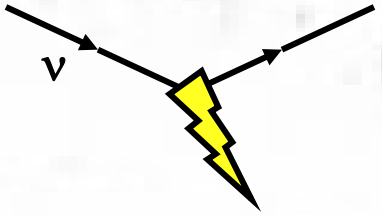




$$\frac{d\sigma}{d\cos\theta} = \text{const} \times \left(\frac{1+\cos\theta}{2}\right)^2$$

Backwards scattering is disfavored

Neutrino-Electron (cont'd)



- Electron-Z^0 coupling $\sigma \propto \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \sin^4\theta_W\right)$
  - (LH, V-A): $-1/2 + \sin^2\theta_W$
  - (RH, V+A): $\sin^2\theta_W$ $\sigma \propto \frac{G_F^2 s}{\pi}\left(\sin^4\theta_W\right)$

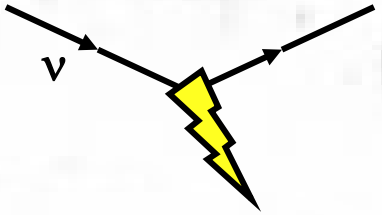
Let y denote inelasticity.
Recoil energy is related to
CM scattering angle by

$$y = \frac{E_e}{E_\nu} \approx 1 - \tfrac{1}{2}(1 - \cos\theta)$$

$$\int dy \frac{d\sigma}{dy} = \begin{cases} \text{LH:} & \int dy = 1 \\ \text{RH:} & \int (1-y)^2 dy = 1/3 \end{cases}$$

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \frac{4}{3}\sin^4\theta_W\right) = 1.4\times10^{-42}\, cm^2/GeV \cdot E_\nu(GeV)$$

Flavors and ve Scattering



The reaction

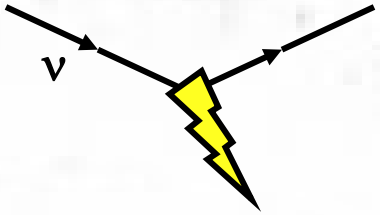
$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

has a much smaller cross-section than

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

Why?

Flavors and ve Scattering



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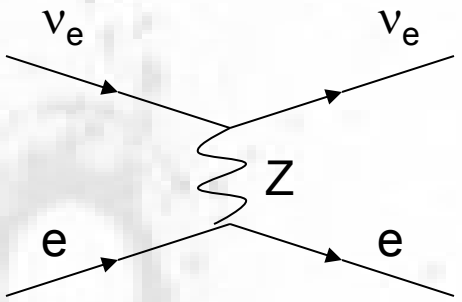
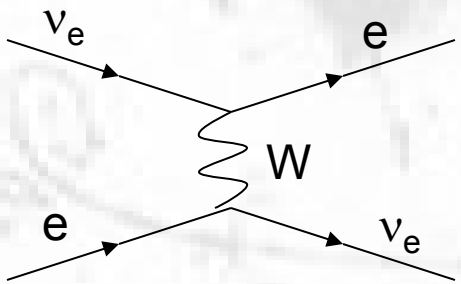
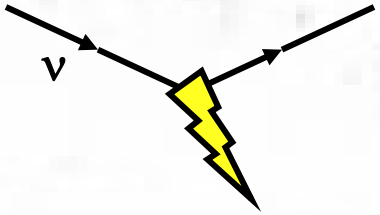


$$\nu_e + e^- \rightarrow \nu_e + e^-$$

has a second contributing reaction, charged current



Flavors and ve Scattering



Let's show that this increases the rate

(Recall from the previous pages…

$$\sigma_{TOT} = \int dy \frac{d\sigma}{dy}$$

$$= \int dy \left[\frac{d\sigma^{LH}}{dy} + \frac{d\sigma^{RH}}{dy} \right]$$

$$= \sigma_{TOT}^{LH} + \tfrac{1}{3}\sigma_{TOT}^{RH}$$

$$\sigma_{TOT}^{LH} \propto \left| \text{total coupling}_{e^-}^{LH} \right|^2$$

For electron…	LH coupling	RH coupling
Weak NC	-1/2+ $\sin^2\theta_W$	$\sin^2\theta_W$
Weak CC	-1/2	0

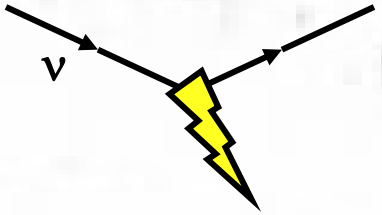
)

We have to show the interference between CC and NC is constructive.

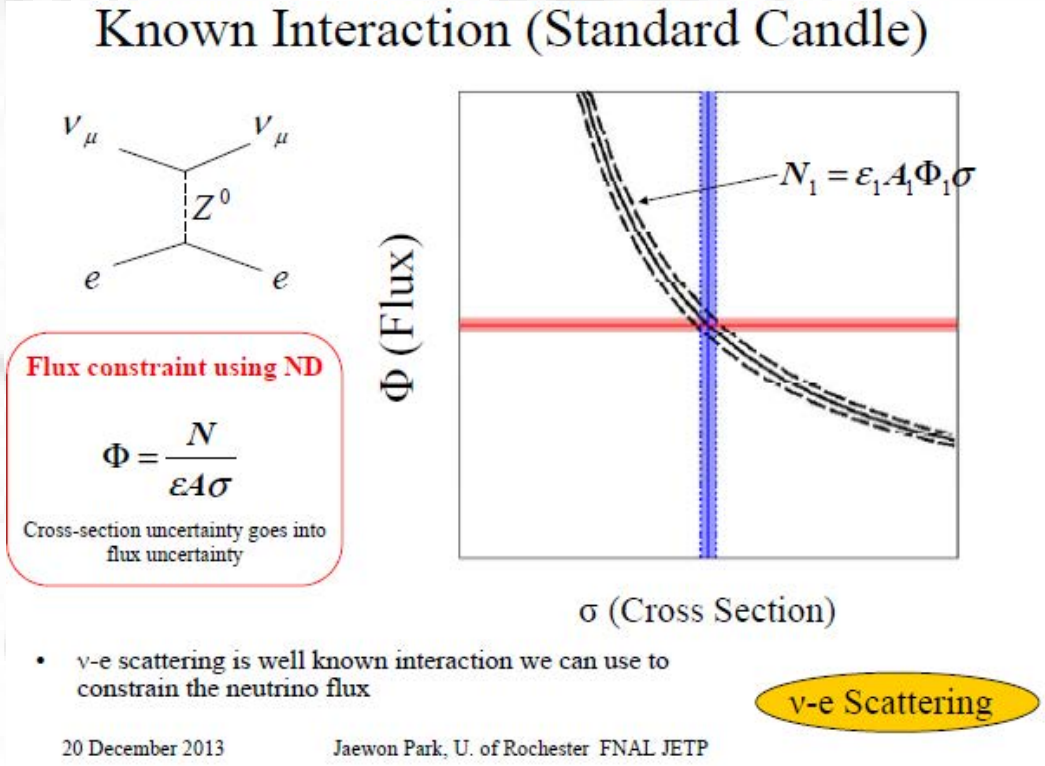
The total RH coupling is unchanged by addition of CC because there is no RH weak CC coupling

There are two LH couplings: NC coupling is -1/2+$\sin^2\theta_W$ ≈ -1/4 and the CC coupling is -1/2. We add the associated amplitudes… and get -1+$\sin^2\theta_W$ ≈ -3/4

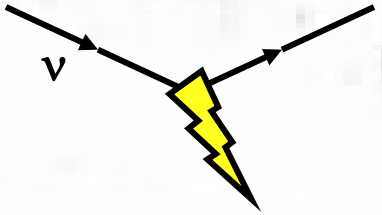
Who Cares about ν-e Elastic Scattering?



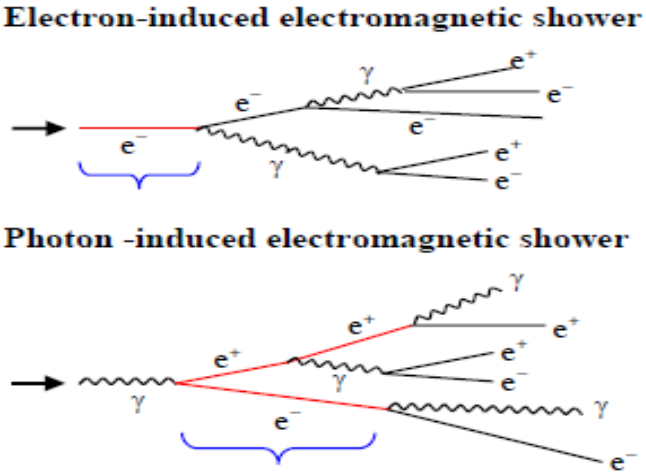
- I just spent ~10^{-6} of your life span telling you about a reaction whose rate is 500x10^{-6} of the leading reaction for accelerator neutrinos
  - Was this a good deal?
  - I'll argue yes… maybe…
- This reaction, as we will see, is nearly unique in being predicted to a fraction of a % precision

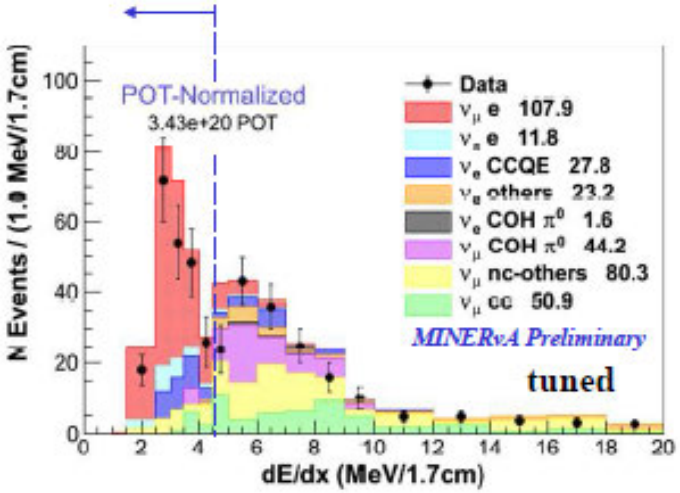


Who Cares… (cont'd)

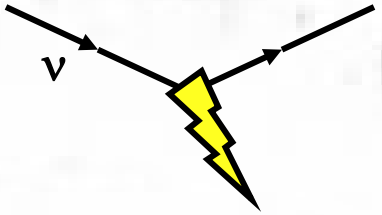


- Not easy to do. Reaction is rare and the detector is filled with photons from π^0 decays which can be easily confused with electrons
  - But electrons from $\nu + e^- \rightarrow \nu + e^-$ are very forward (because of small Q^2_{max}) and electromagnetic showers from photons & electrons are subtly different

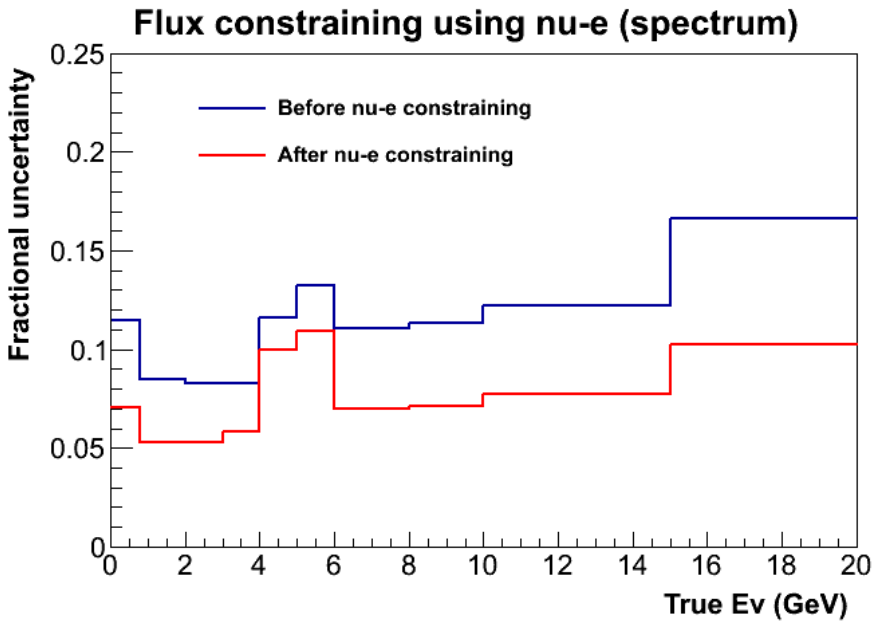




Who Cares… (cont'd)

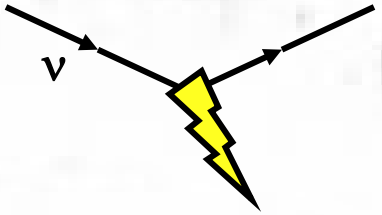


- In this example (MINERvA) the number of events is small, so impact on the uncertainty of neutrino flux is modest today
  - ~10%→6%

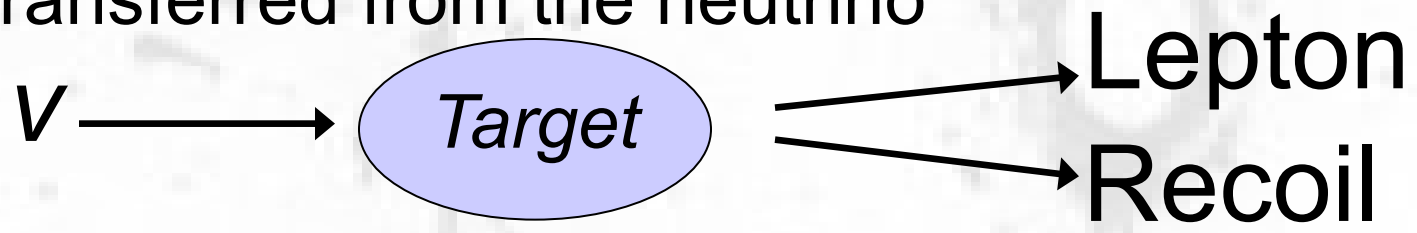


- But for NOvA-era and LBNF-era beams, another order of magnitude in events makes this the leading method for measuring neutrino flux

Final state mass effects

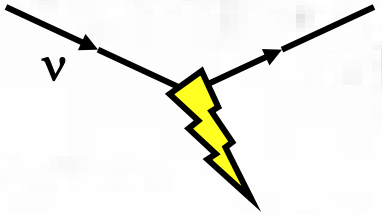


- As always, we detect neutrino interactions only in the final state.
- Creation of that final state may require energy to be transferred from the neutrino



  - In charged-current reactions, where the final state lepton is charged, this lepton has mass
  - The recoil may be a higher mass object than the initial state, or it may be in an excited state

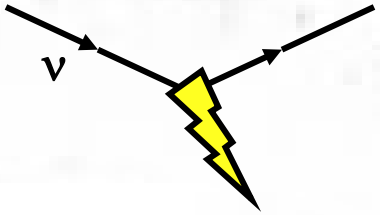
Thresholds and Processes



Process	Considerations	Threshold (typical)
$\nu N \rightarrow \nu N$ (elastic)	Target nucleus is free and recoil is very small	none
$\nu_e n \rightarrow e^- p$	In some nuclei (mostly metastable ones), this reaction is exothermic if proton not ejected	None for free neutron & some other nuclei.
$\nu e \rightarrow \nu e$ (elastic)	Most targets have atomic electrons	~ 10eV – 100 keV
anti-$\nu_e p \rightarrow e^- n$	$m_n > m_p$ & m_e. Typically more to make recoil from stable nucleus.	1.8 MeV (free p). More in nuclei.
$\nu_\ell n \rightarrow \ell^- p$ (quasielastic)	Final state nucleon is ejected from nucleus. Massive lepton	~ 10s MeV for ν_e +~100 MeV for ν_μ
$\nu_\ell N \rightarrow \ell^- X$ (inelastic)	Must create additional hadrons. Massive lepton.	~ 200 MeV for ν_e +~100 MeV for ν_μ

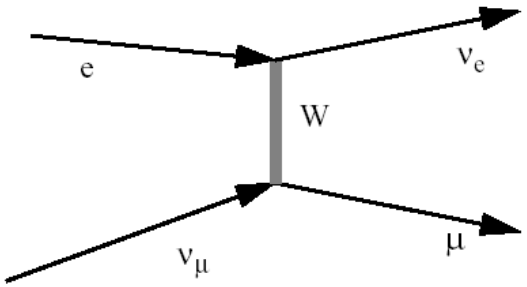
- Energy of neutrinos determines available reactions, and therefore experimental technique

Lepton Mass Effects



- Let's return to Inverse μ–decay:

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$



  - What changes in the presence of final state mass?
    - o pure CC so always left-handed
    - o BUT there must be finite Q^2 to create muon in final state!

$$Q^2_{\min} = m^2_\mu$$

$$\sigma_{TOT} \propto \int_{Q^2_{\min}}^{Q^2_{\max}} dQ^2 \frac{1}{(Q^2 + M_W{}^2)^2}$$

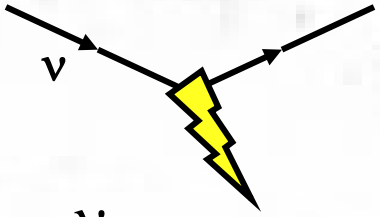
$$\approx \frac{Q^2_{\max} - Q^2_{\min}}{M_W{}^4}$$

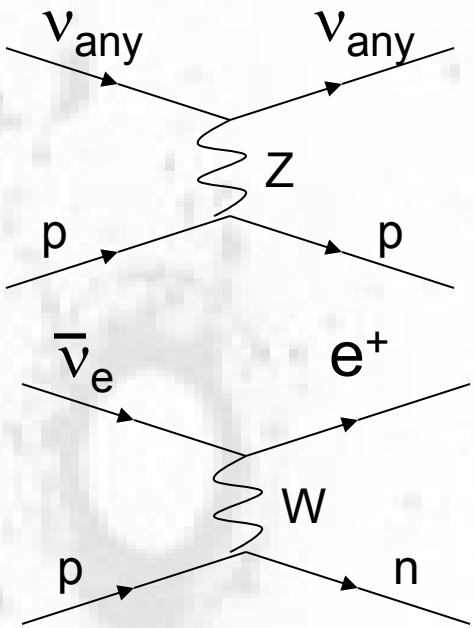
$$\sigma_{TOT} = \frac{G_F^2 (s - m_\mu^2)}{\pi}$$

$$= \left[\sigma_{TOT}^{(\text{massless})}\right]\left(1 - \frac{m_\mu^2}{s}\right)$$

  - see a suppression scaling with **(mass/CM energy)²**
    - o This can be generalized…

Enough about electrons…





- Imagine now a nucleon target
  - Neutrino-proton elastic scattering:

$$\nu_e + p \rightarrow \nu_e + p$$

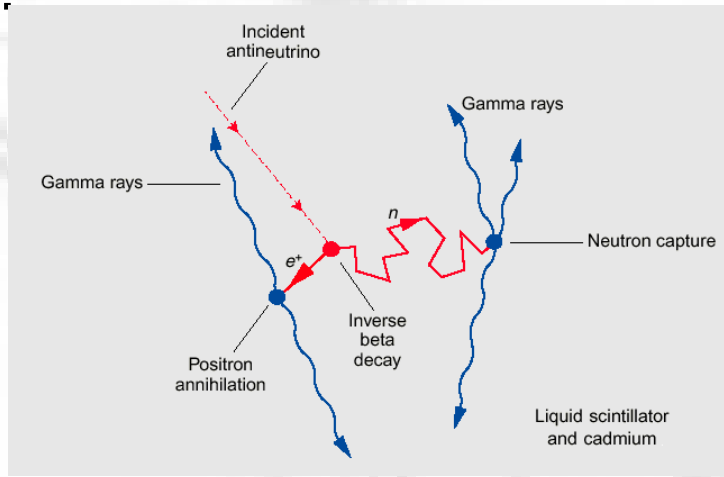
  - "Inverse beta-decay" (IBD):

$$\bar{\nu}_e + p \rightarrow e^+ + n$$

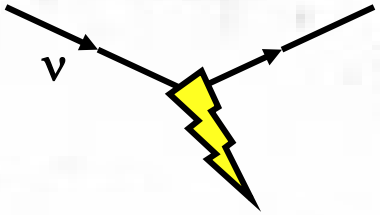
  - and "stimulated" beta decay:

$$\nu_e + n \rightarrow e^- + p$$

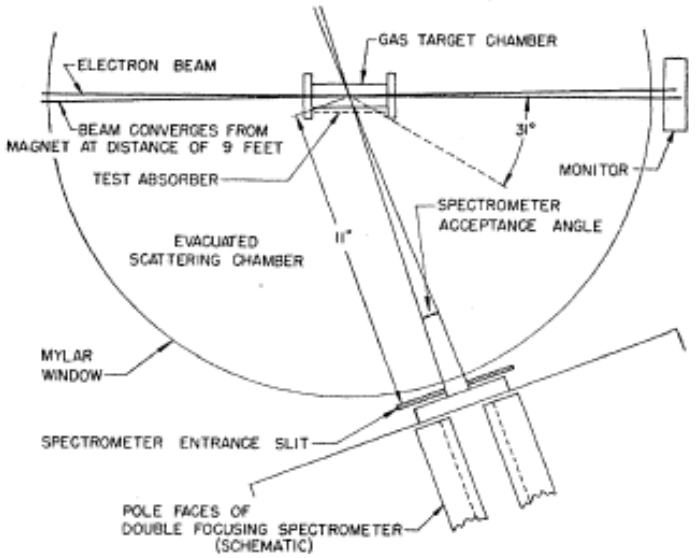
  - Recall that IBD was the Reines and Cowan discovery signal



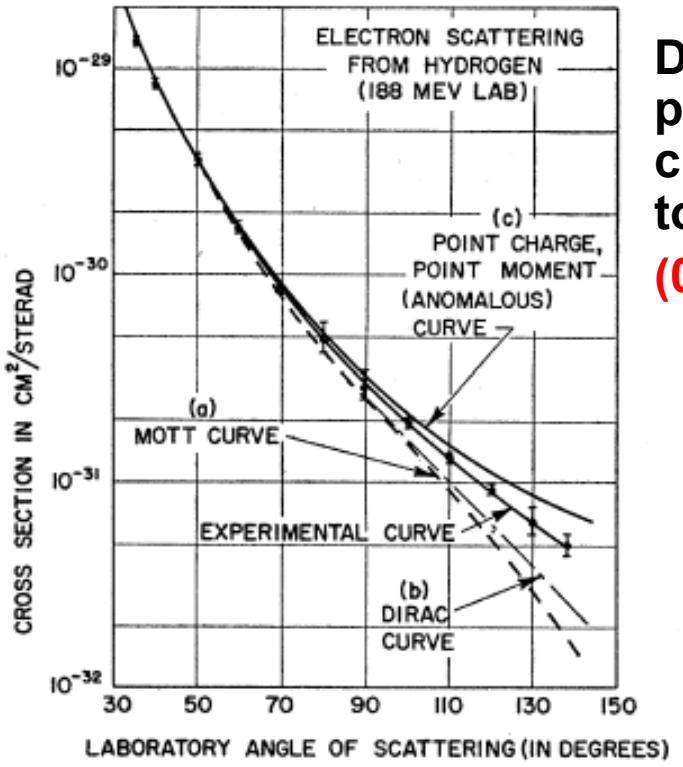
Proton Structure



- How is a proton different from an electron?
  - anomalous magnetic moment, $\kappa \equiv \frac{g-2}{2} \not\approx 0$
  - "form factors" related to finite size

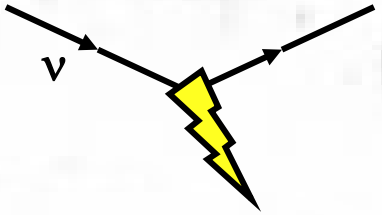


McAllister and Hofstadter 1956
188 MeV and 236 MeV electron beam from linear accelerator at Stanford

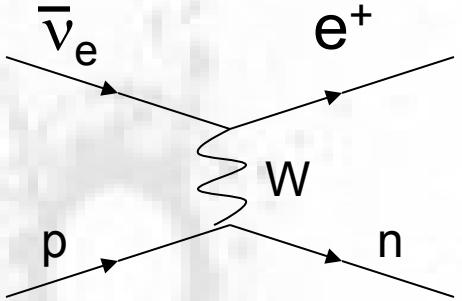


Determined proton RMS charge radius to be
(0.7±0.2) x10^{-13} cm

Final State Mass Effects



- In IBD, $\bar{\nu}_e + p \rightarrow e^+ + n$, have to pay a mass penalty *twice*
  - *M_n-M_p≈1.3 MeV, M_e≈0.5 MeV*
- What is the threshold?
  - kinematics are simple, at least to zeroth order in M_e/M_n → heavy nucleon kinetic energy is zero

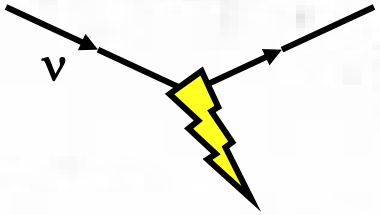


$$s_{\text{initial}} = (\underline{p}_\nu + \underline{p}_p)^2 = M_p^2 + 2M_p E_\nu \text{ (proton rest frame)}$$

$$s_{\text{final}} = (\underline{p}_e + \underline{p}_n)^2 \approx M_n^2 + m_e^2 + 2M_n\left(E_\nu - \left(M_n - M_p\right)\right)$$

- Solving… $$E_\nu^{\min} \approx \frac{\left(M_n + m_e\right)^2 - M_p^2}{2M_p} \approx 1.806 \text{ MeV}$$

Final State Mass Effects (cont'd)



- Define δE as E_ν-E_ν^{min}, then

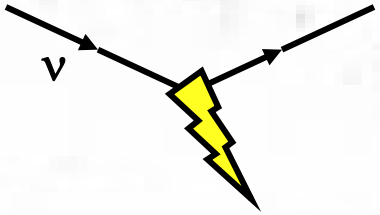
$$\begin{aligned} s_{\text{initial}} &= M_p^2 + 2M_p\left(\delta E + E_\nu^{\min}\right) \\ &= M_p^2 + 2\delta E \times M_p + \left(M_n + m_e\right)^2 - M_p^2 \\ &= 2\delta E \times M_p + \left(M_n + m_e\right)^2 \end{aligned}$$

- Remember the suppression generally goes as

$$\xi_{\text{mass}} = 1 - \frac{m_{\text{final}}^2}{s} = 1 - \frac{\left(M_n + m_e\right)^2}{\left(M_n + m_e\right)^2 + 2M_p \times \delta E}$$

$$= \frac{2M_p \times \delta E}{\left(M_n + m_e\right)^2 + 2M_p \times \delta E} \approx \begin{cases} \delta E \times \dfrac{2M_p}{\left(M_n + m_e\right)^2} & \text{low energy} \\ 1 - \dfrac{\left(M_n + m_e\right)^2}{2M_p^2}\dfrac{M_p}{\delta E} & \text{high energy} \end{cases}$$

Putting it all together…



$$\sigma_{TOT} = \frac{G_F^2 s}{\pi} \times \cos^2 \vartheta_{\text{Cabibbo}} \times \left(\xi_{\text{mass}}\right) \times \left(g_V{}^2 + 3 g_A{}^2\right)$$

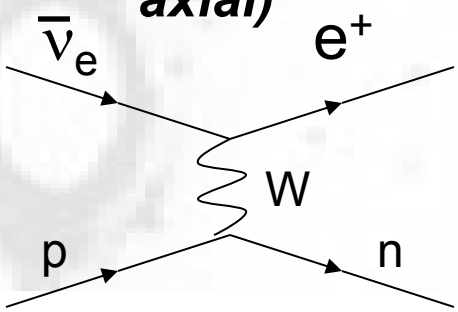
quark mixing!

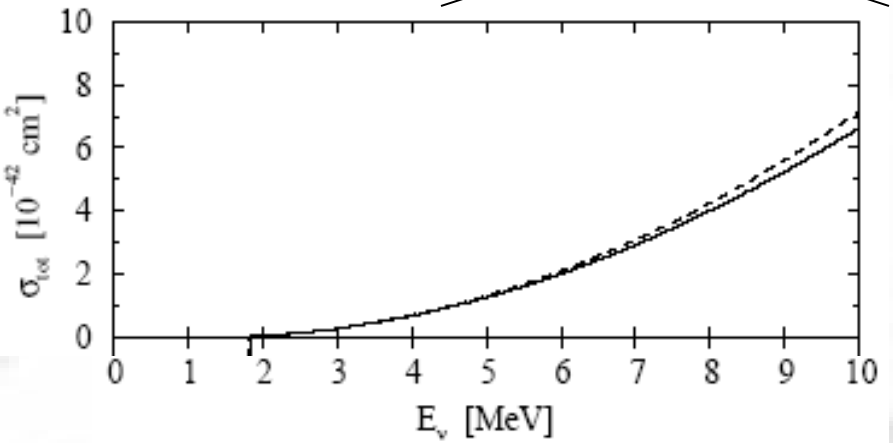
final state mass suppression

proton form factors (vector, axial)

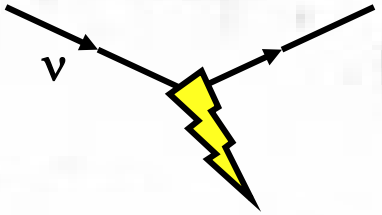
- mass suppression is proportional to δE at low E_ν, so quadratic near threshold
- vector and axial-vector form factors (for IBD usually referred to as f and g, respectively)

  g_V, $g_A \approx 1, 1.26$.
  - FFs, $\theta_{Cabibbo}$, best known from τ_n



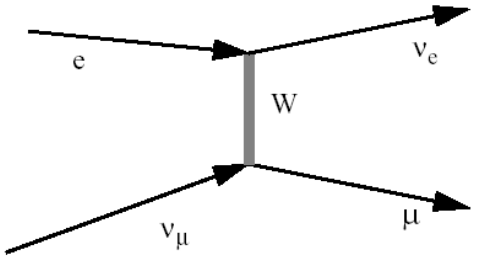


Lecture Question #2: Quantitative Lepton Mass Effect



- **Which is closest to the minimum beam energy in which the reaction**

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

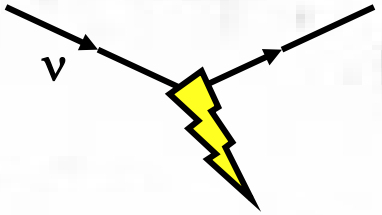


can be observed?

(a) 100 MeV (b) 1 GeV (c) 10 GeV

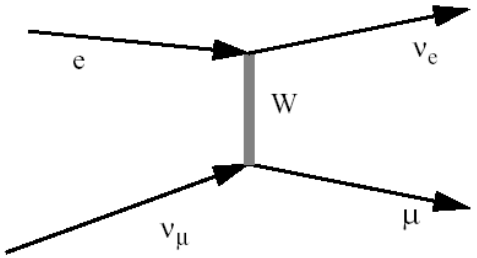
(It might help you to remember that $Q^2_{\text{min}} = m^2_\mu$
or you might just want to think about the total CM energy required to produce the particles in the final state.)

Lecture Question #2: Quantitative Lepton Mass Effect



- **Which is closest to the minimum beam energy in which the reaction**

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$



can be observed?

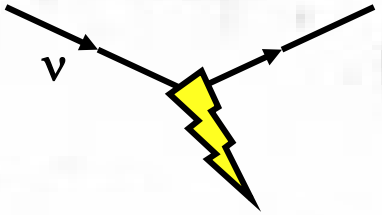
$Q^2_{\min} = m_\mu^2$ **(a) 100 MeV (b) 1 GeV (c) 10 GeV**

$$Q^2 < s = (\underline{p}_e + \underline{p}_\nu)^2$$

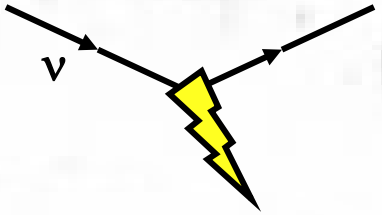
$$= (m_e + E_\nu, 0, 0, \sqrt{E_\nu{}^2 - m_\nu{}^2})^2 \approx m_e^2 + 2m_e E_\nu$$

$$\therefore E_\nu > \frac{m_\mu^2}{2m_e} \approx 10.9 \text{ GeV}$$

Summary of First Lecture… and Next Topic

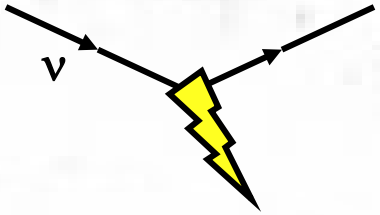


- We calculated νe^- scattering and Inverse Beta Decay (IBD) cross-sections!
- In point-like weak interactions, key features are:
  - $d\sigma/dQ^2$ is ≈ constant.
    - Integrating gives $\sigma \propto E_\nu$
  - LH coupling enters w/ $d\sigma/dy \propto 1$, RH w/ $d\sigma/dy \propto (1-y)^2$
    - Integrating these gives 1 and 1/3, respectively
  - Lepton mass effect gives minimum Q^2
    - Integrating gives correction factor in σ of $(1-Q^2_{min}/s)$
  - Structure of target can add form factors
- High energy point-like ν-quark scattering ("deep inelastic scattering") and *what's in between*…

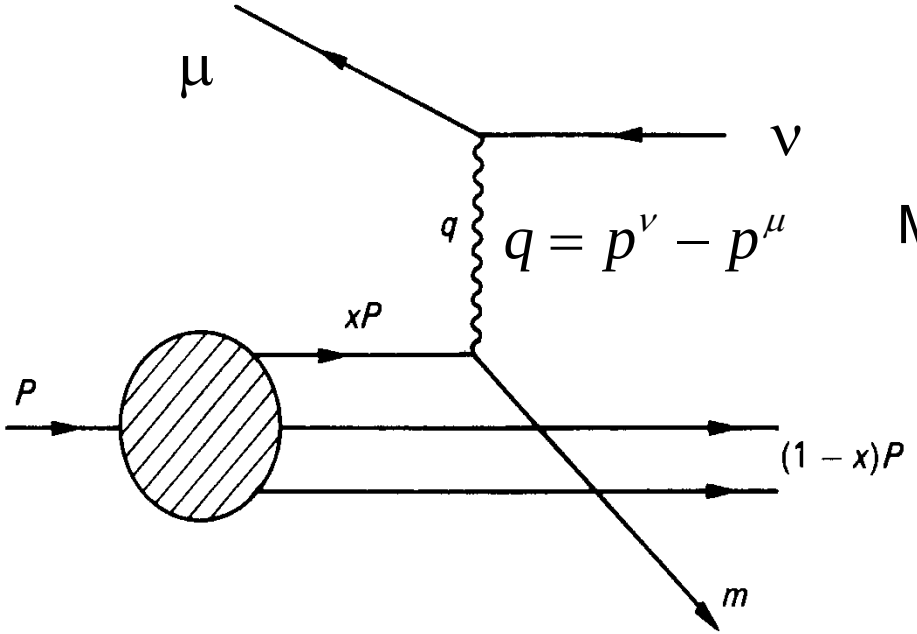


Neutrino-Nucleon Deep Inelastic Scattering

High Energy Limit and Quark-Parton model of DIS



In “infinite momentum frame”, xP is four momentum of partons inside the nucleon



Neutrino scatters off a parton (a quark) inside the nucleon

Mass of target quark

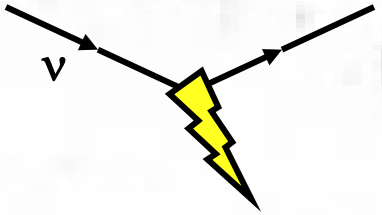
$$m_q^{\,2} = x^2 P^2 = x^2 M_T^2$$

Mass of final state quark

$$m_{q'}^{\,2} = (xP + q)^2$$

$$x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2M_T \nu}$$

So why is cross-section so large?



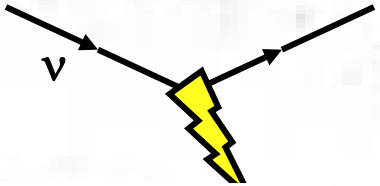
- (at least compared to νe^- scattering!)
- Recall that for neutrino beam and target at rest

$$\sigma_{TOT} \approx \frac{G_F^2}{\pi} \int_0^{Q^2_{\max} \equiv s} dQ^2 = \frac{G_F^2 s}{\pi}$$

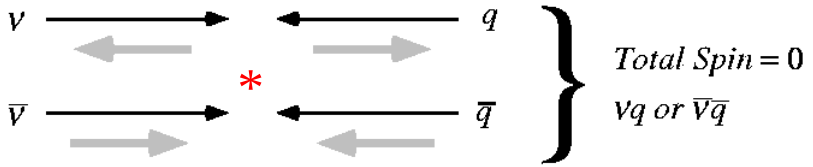
$$s = m_e^{\ 2} + 2 m_e E_\nu$$

- But we just learned for DIS that effective mass of each target quark is $m_q = x m_{\text{nucleon}}$
- So much larger target mass means larger σ_{TOT}

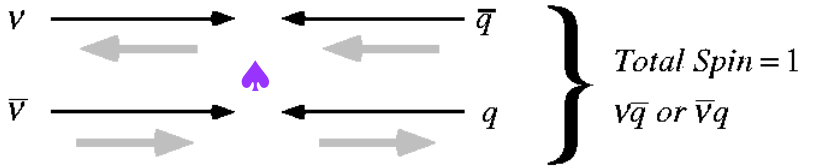
Chirality, Charge in CC ν-q Interaction



- Massless limit for simplicity
- Total spin determines inelasticity distribution
  - Familiar from neutrino-electron scattering



Flat in y



$1/4(1+\cos\theta^*)^2 = (1-y)^2$

$\int(1-y)^2dy=1/3$

point-like scattering implies linear with energy

$$\frac{d\sigma^{\nu p}}{dxdy} = \frac{G_F^2 s}{\pi}\left(x\overset{*}{d}(x) + x\overset{\spadesuit}{\bar{u}}(x)(1-y)^2\right)$$

$$\frac{d\sigma^{\bar{\nu} p}}{dxdy} = \frac{G_F^2 s}{\pi}\left(x\overset{*}{\bar{d}}(x) + x\overset{\spadesuit}{u}(x)(1-y)^2\right)$$

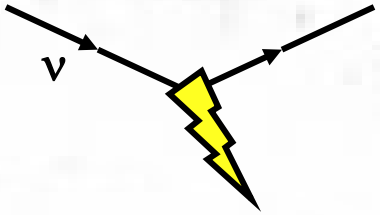
but what is this "u(x)" and "d(x)"?

- Neutrino/Anti-neutrino CC each produce particular Δq in scattering

$$\nu d \to \mu^- u$$

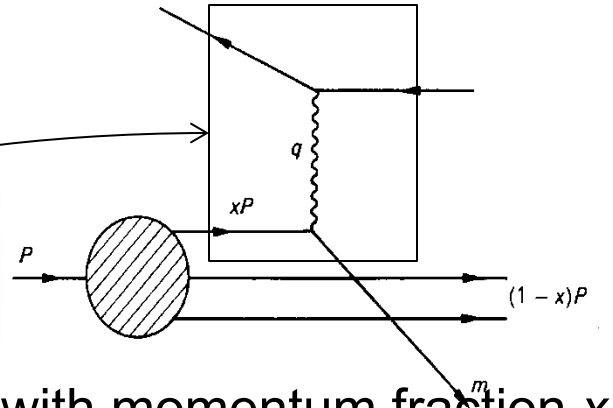
$$\bar{\nu} u \to \mu^+ d$$

Factorization and Partons



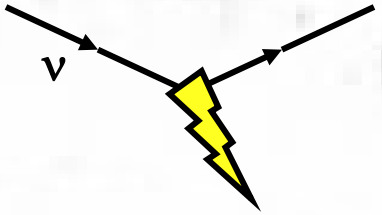
- Factorization Theorem of QCD allows cross-sections for hadronic processes to be written as:

$$\sigma(l+h \to l+X)$$
$$= \sum_q \int dx \sigma(l+q(x) \to l+X) q_h(x)$$



  - $q_h(x)$ is the probability of finding a parton, q, with momentum fraction x inside the hadron, h. It is called a parton distribution function (PDF).
  - PDFs are universal
  - PDFs are not (yet) calculable from first principles in QCD
- "Scaling": parton distributions are largely independent of Q^2 scale, and depend on fractional momentum, x.

Complication: Charged Current to Neutral Current

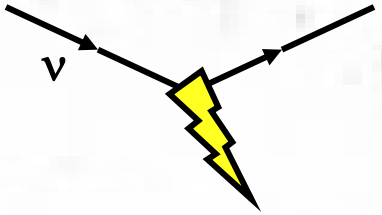


- We previously saw how to generalize from charged current to neutral current in νe^- scattering
  - Right handed current couples to target (but not neutrino)
  - Complicated couplings
  - For neutral current case, scattering from all flavors of quarks because there is no charge carried by boson

$$\frac{d\sigma^{\nu p,CC}}{dxdy} = \frac{G_F^2 s}{\pi} x\left(d(x) + \bar{u}(x)(1-y)^2\right)$$

$$\frac{d\sigma^{\nu p,NC}}{dxdy} = \frac{G_F^2 s}{\pi}\left(\begin{array}{l} x d_L^2 d(x) + d_R^2 \bar{d}(x) + u_L^2 u(x) + u_R^2 \bar{u}(x) \\ +(1-y)^2\left(d_R^2 d(x) + d_L^2 \bar{d}(x) + u_R^2 u(x) + u_L^2 \bar{u}(x)\right)\end{array}\right)$$

Simplification: Isoscalar Targets



- Heavy nuclei are roughly neutron-proton isoscalar
  - OK, more neutrons than protons, but it's closer to 1:1 than 2:1 or 0:1
- Isospin symmetry implies $u_p = d_n, d_p = u_n$

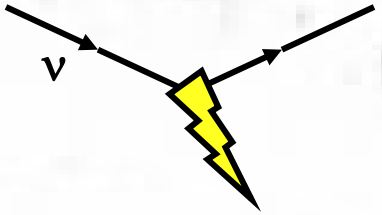
$$\frac{d\sigma^{\nu N,CC}}{dxdy} = \frac{G_F^2 s}{\pi} x\left(u(x) + d(x) + \left(\bar{u}(x) + \bar{d}(x)\right)(1-y)^2\right)$$

$$= \frac{G_F^2 s}{\pi} x\left(q(x) + \bar{q}(x)(1-y)^2\right)$$

$$\frac{d\sigma^{\bar{\nu} N,CC}}{dxdy} = \frac{G_F^2 s}{\pi} x\left(\bar{u}(x) + \bar{d}(x) + \left(u(x) + d(x)\right)(1-y)^2\right)$$

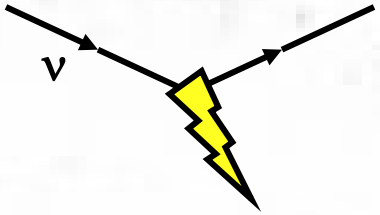
$$= \frac{G_F^2 s}{\pi} x\left(\bar{q}(x) + q(x)(1-y)^2\right)$$

Brief Summary of Neutrino-Quark Scattering so Far

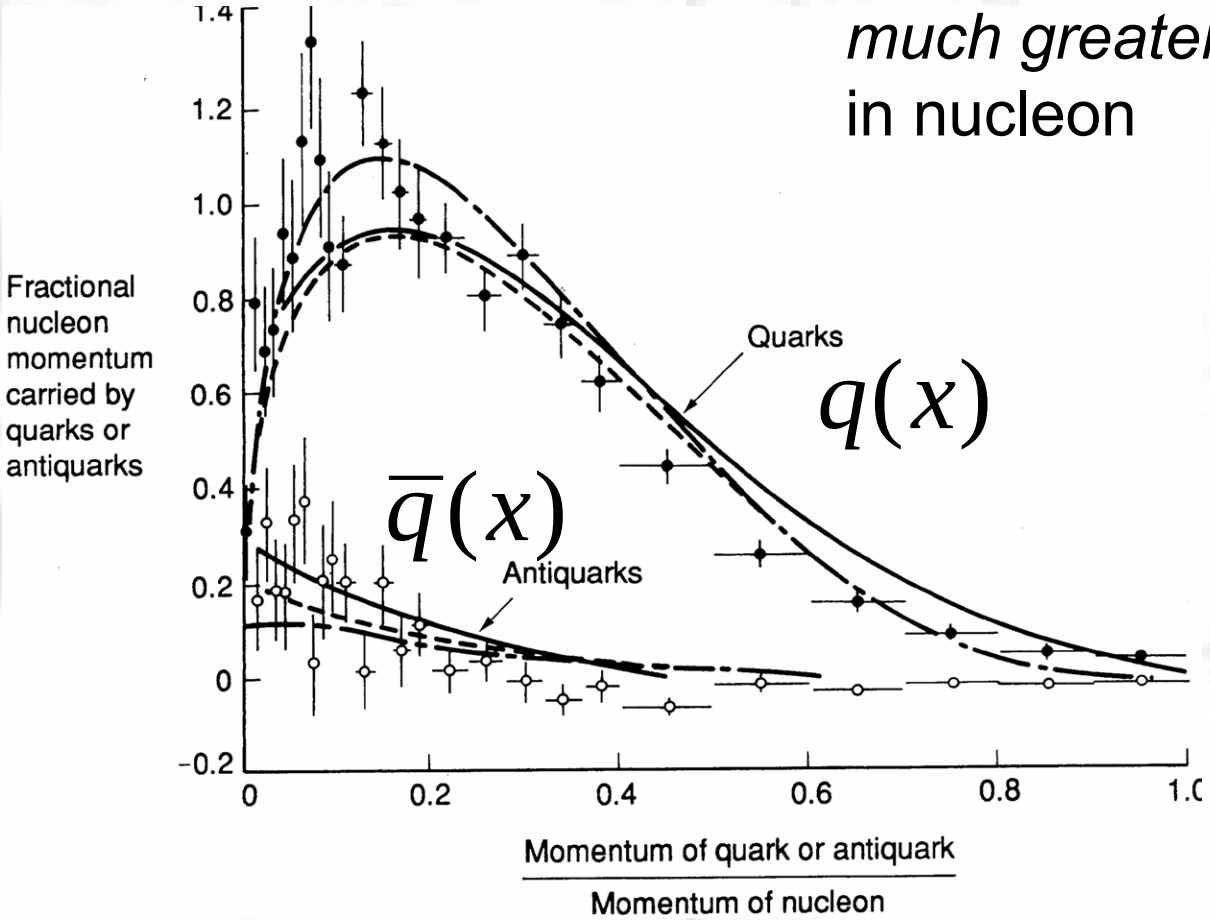


- $x \equiv Q^2/2M_T\nu$ is the fraction of the nucleon 4-momentum carried by a quark in the infinite momentum frame
  - Effective mass for struck quark, $M_q = \sqrt{(x\underline{P})^2} = xM_T$
  - Parton distribution functions, q(x), incorporate information about the "flux" of quarks inside the hadron
- Quark and anti-quark scattering spin:
  - νq and $\overline{\nu q}$ are spin 0, isotropic
  - $\nu\bar{q}$ and $\nu\bar{q}$ are spin 1, backscattering is suppressed
- Neutrinos and anti-neutrinos pick out definite quark and anti-quark flavors (charge conservation)
  - Isoscalar targets re-average over flavors

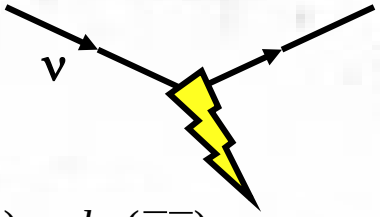
Momentum of Quarks & Antiquarks



- Momentum carried by quarks *much greater* than anti-quarks in nucleon



y distribution in Neutrino CC DIS



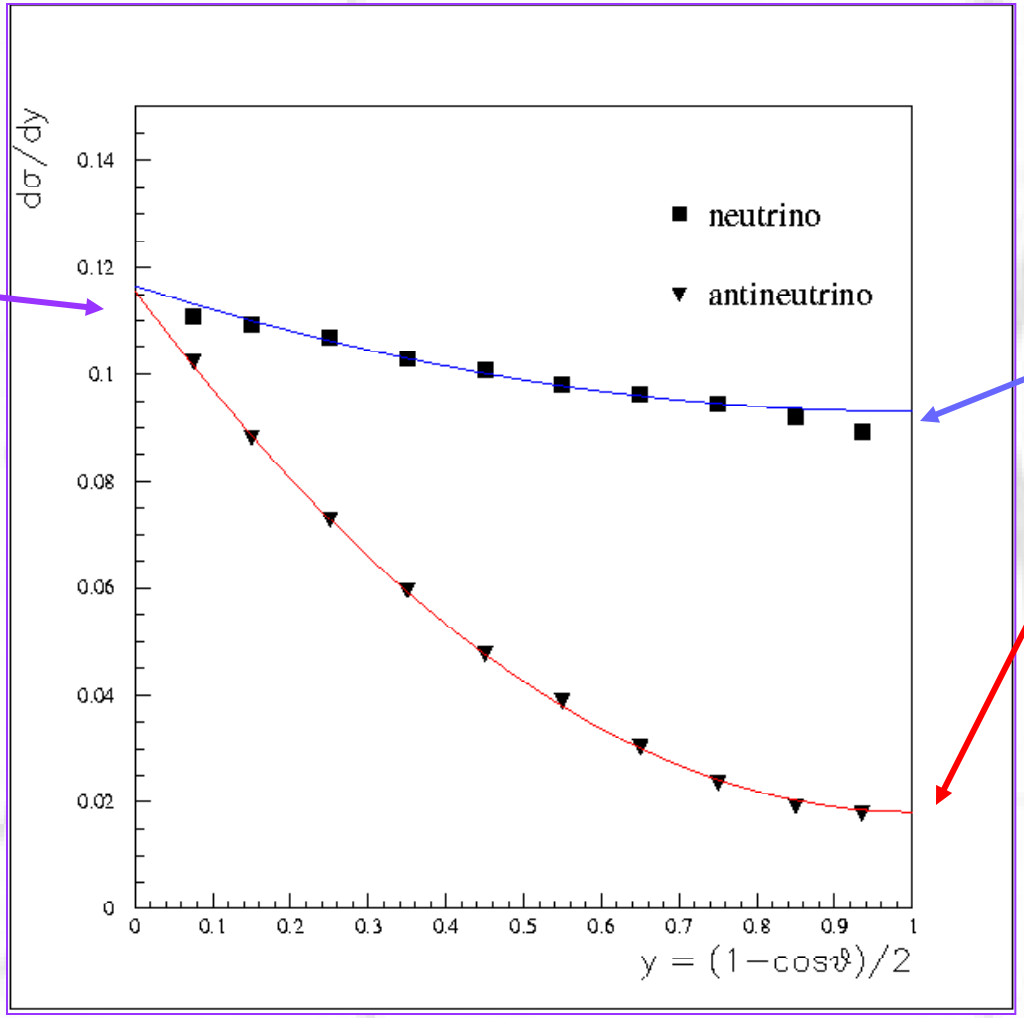
$$\frac{d\sigma(\nu q)}{dxdy} = \frac{d\sigma(\bar{\nu}\bar{q})}{dxdy} \propto 1$$

$$\frac{d\sigma(\nu \bar{q})}{dxdy} = \frac{d\sigma(\bar{\nu} q)}{dxdy} \propto (1-y)^2$$

At y=0:

Quarks & anti-quarks

Neutrino and anti-neutrino identical



At y=1:

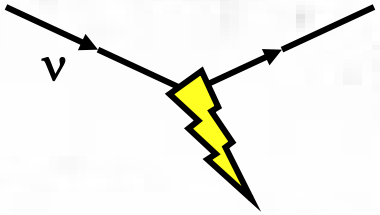
Neutrinos see only quarks.

Anti-neutrinos see only anti-quarks

Averaged over protons and neutrons,

$$\sigma^{\overline{\nu}} \approx \frac{1}{2}\sigma^{\nu}$$

Lecture Question #3: Neutrino and Anti-Neutrino $\sigma^{\nu N}$



- **Given that $\sigma_{CC}^{\bar{\nu}N} \approx \frac{1}{2}\sigma_{CC}^{\nu N}$ in the DIS regime (CC)**

and that $\frac{d\sigma(\nu q)}{dx} = \frac{d\sigma(\bar{\nu}\bar{q})}{dx} = 3\frac{d\sigma(\nu\bar{q})}{dx} = 3\frac{d\sigma(\bar{\nu}q)}{dx}$

for CC scattering from quarks or anti-quarks of a given momentum,

and that cross-section is proportional to parton momentum, what is the approximate ratio of anti-quark to quark momentum in the nucleon?

(a) $\bar{q}/q \sim 1/3$ **(b)** $\bar{q}/q \sim 1/5$ **(c)** $\bar{q}/q \sim 1/8$

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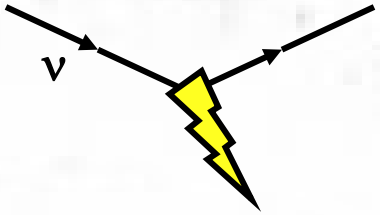

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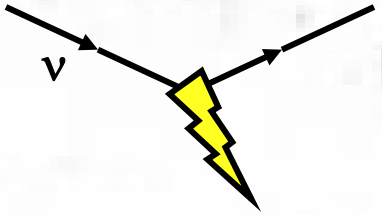


- **Given:** $\sigma_{CC}^{\bar{\nu}N} \approx \frac{1}{2}\sigma_{CC}^{\nu N}$ **in the DIS regime (CC)**

and $$\frac{d\sigma(\nu q)}{dx} = \frac{d\sigma(\bar{\nu}\bar{q})}{dx} = 3\frac{d\sigma(\nu\bar{q})}{dx} = 3\frac{d\sigma(\bar{\nu}q)}{dx}$$

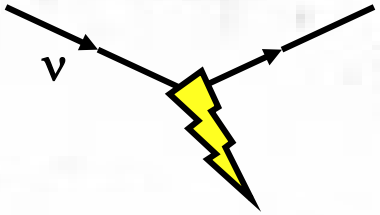
$$\sigma_\nu = \int_{q,\bar{q}} dx\left(\frac{d\sigma(\nu q)}{dx} + \frac{d\sigma(\nu\bar{q})}{dx}\right)$$

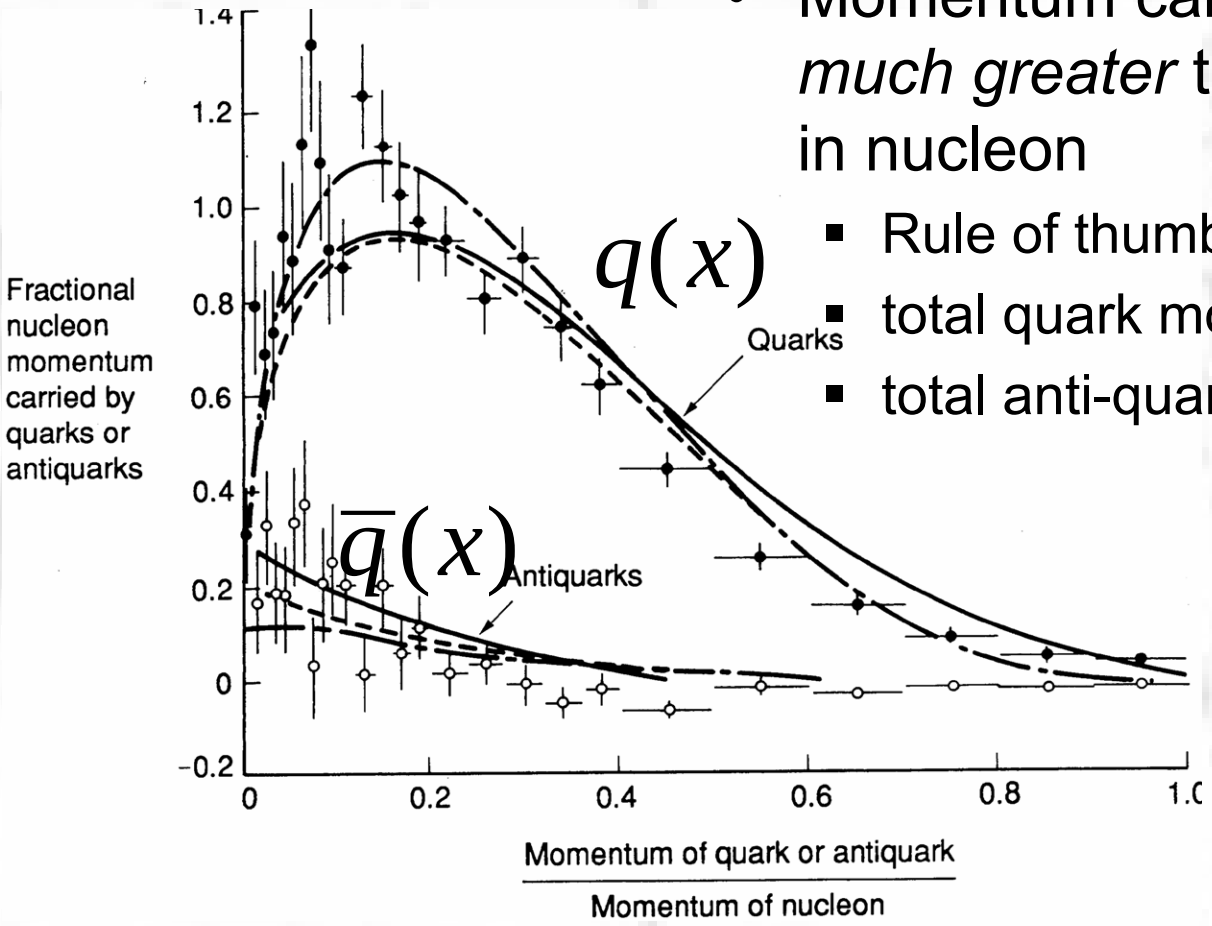
$$\sigma_{\bar{\nu}} = \int_{q,\bar{q}} dx\left(\frac{d\sigma(\bar{\nu}q)}{dx} + \frac{d\sigma(\bar{\nu}\bar{q})}{dx}\right) = \int_{q,\bar{q}} dx\left(\frac{d\sigma(\nu q)}{3dx} + \frac{3d\sigma(\nu\bar{q})}{dx}\right)$$

$$\therefore \int_{q,\bar{q}} dx\left(\frac{d\sigma(\nu q)}{dx} + \frac{d\sigma(\nu\bar{q})}{dx}\right) = 2\int_{q,\bar{q}} dx\left(\frac{d\sigma(\nu q)}{3dx} + \frac{3d\sigma(\nu\bar{q})}{dx}\right)$$

$$\frac{1}{3}\int_q dx\frac{d\sigma(\nu q)}{dx} = 5\int_{\bar{q}} dx\frac{d\sigma(\nu\bar{q})}{dx} = \frac{5}{3}\int_{\bar{q}} dx\frac{d\sigma(\bar{\nu}\bar{q})}{dx}$$

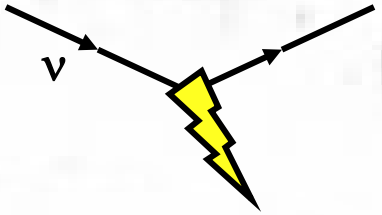
Momentum of Quarks & Antiquarks



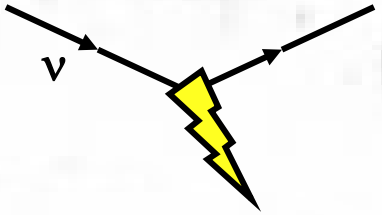


- Momentum carried by quarks *much greater* than anti-quarks in nucleon
  - Rule of thumb: at Q^2 of 10 GeV^2:
  - total quark momentum is 1/3,
  - total anti-quark is 1/15.

Deep Inelastic Scattering: Conclusions and Summary

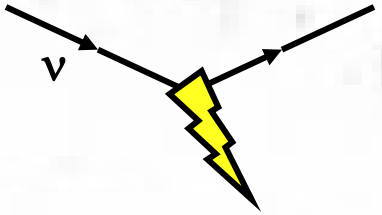


- Neutrino-quark scattering is elastic scattering!
  - complicated by fact that quarks live in nucleons
  - *and, as we will discuss later, nucleons in nuclei!*
- But with those caveats, this is another scatttering cross-section we can “calculate”
- Supplemental material (posted at end of slides):
  - structure functions
  - scaling violations of partons (more partons with lower mometum at higher Q^2)
  - mass effects for tau neutrino interactions and production of charm quarks

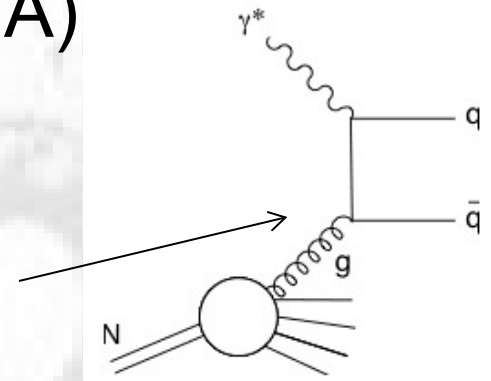


Ultra-High Energy Cross-Sections

Ultra-High Energies

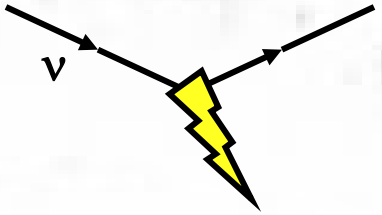


- At energies relevant for UHE Cosmic Ray studies (e.g., IceCube, Antares, ANITA)
  - ν-parton cross-section is dominated by high Q^2, since $d\sigma/dQ^2$ is constant
    - o at high Q^2, gluon radiation and splitting lead to more sea quarks at fewer high x partons (see supplemental material: scaling violations)
    - o see a rise in σ / E_ν from growth of sea at low x
    - o neutrino & anti-neutrino cross-sections nearly equal
  - *Until* $Q^2 \gg M_W^2$, then propagator term starts decreasing and cross-section stops growing linearly with energy



$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

Lecture Question #4: Where does σ Level Off?



- *Until* $Q^2 \gg M_W^2$, then propagator term starts decreasing and cross-section becomes constant

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

- To within a few orders of magnitude, at what beam energy for a target at rest will this happen?

(a) $E_\nu \sim 10\text{TeV}$ **(b)** $E_\nu \sim 10,000\text{TeV}$ **(c)** $E_\nu \sim 10,000,000\text{TeV}$

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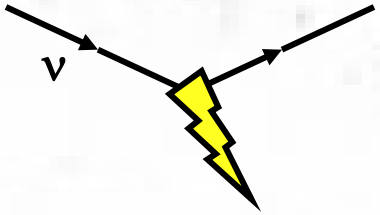

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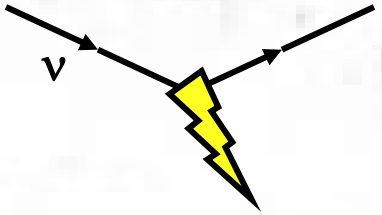

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$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

- At what beam energy for a target at rest will this happen?

$$Q^2 < s_{\text{nucleon}} = m^2_{\text{nucleon}} + 2E_\nu m_{\text{nucleon}}$$

$$Q^2 < s_{\text{nucleon}} \approx 2E_\nu m_{\text{nucleon}}$$

$$\frac{M_W^2}{2m_{\text{nucleon}}} < E_\nu$$

Q^2 limit is s. So won't start to plateau until $s > M_W^2$

$$\therefore E_\nu \gtrsim \frac{(80.4)^2\ \text{GeV}^2}{2(.938)\text{GeV}} \sim 3000\text{GeV}$$

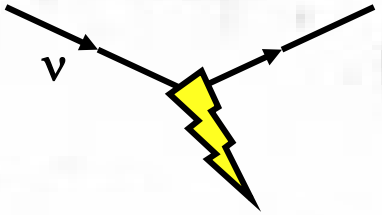
Bonus point realization…

In reality, that is only correct for a parton at x=1. Typical quark x is much less, say ~0.03

$$\frac{M_W^2}{2m_{\text{nucleon}}x} < E_\nu$$

$$\therefore E_\nu \gtrsim \frac{3000\text{GeV}}{0.03} \sim 100\text{TeV}$$

Ultra-High Energies



- ν-parton cross-section is dominated by high Q^2, since $d\sigma/dQ^2$ is constant
  - at high Q^2, scaling violations have made most of nucleon momentum carried by sea quarks
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  - neutrino & anti-neutrino cross-sections nearly equal
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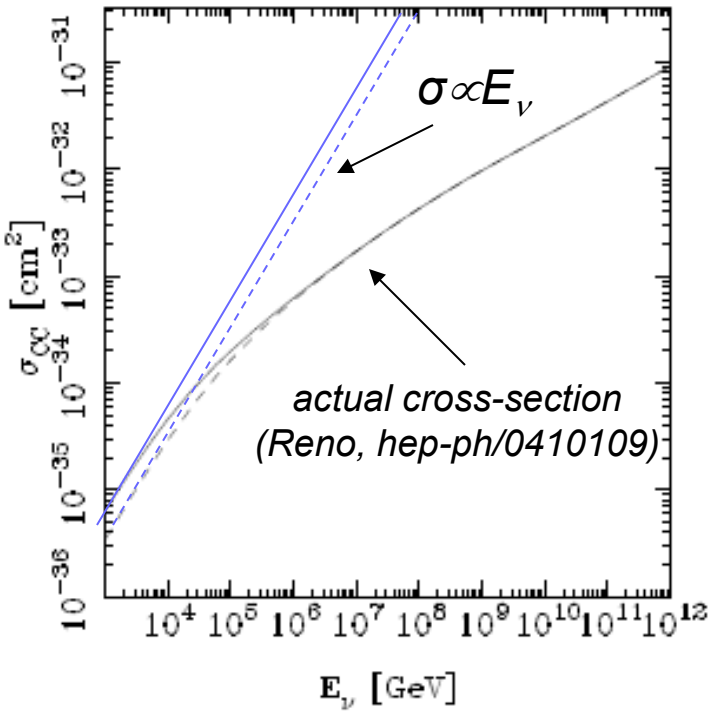
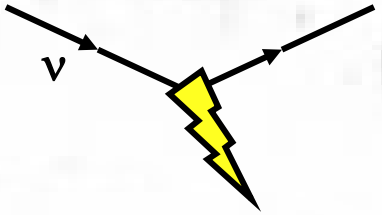
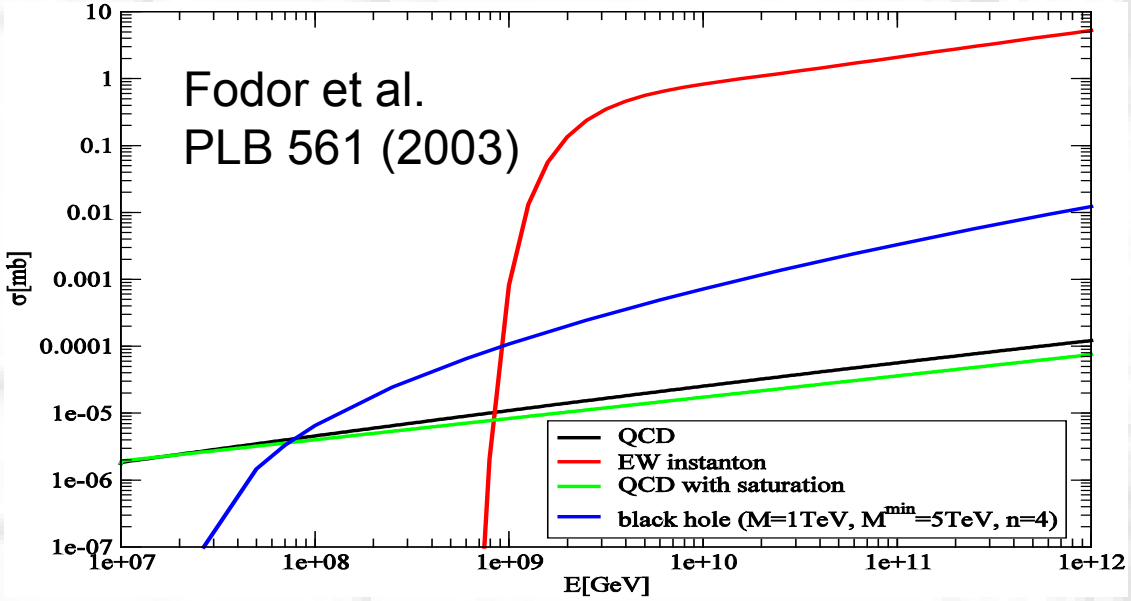


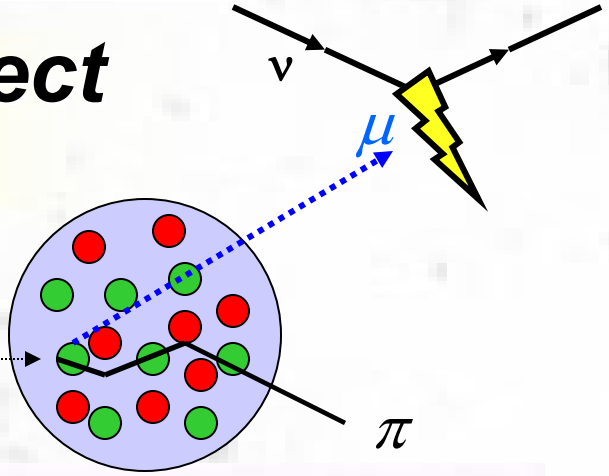
Example: Ultra-High Energies



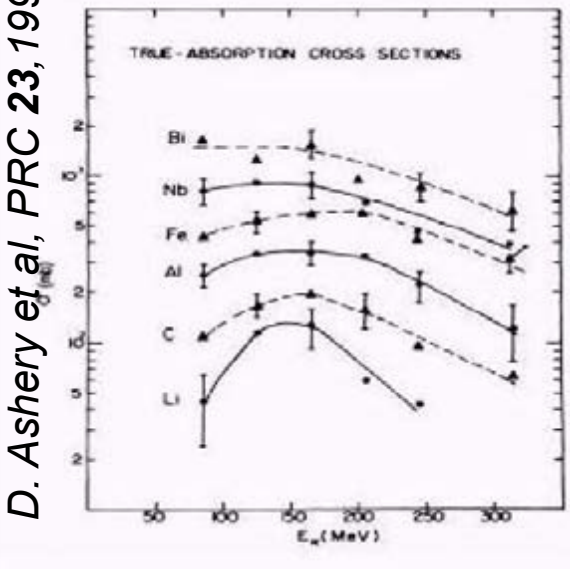
- At UHE, can we reach thresholds of non-SM processes?
  - E.g., structure of quark or leptons, black holes from extra dimensions, etc.
  - Then no one knows what to expect…

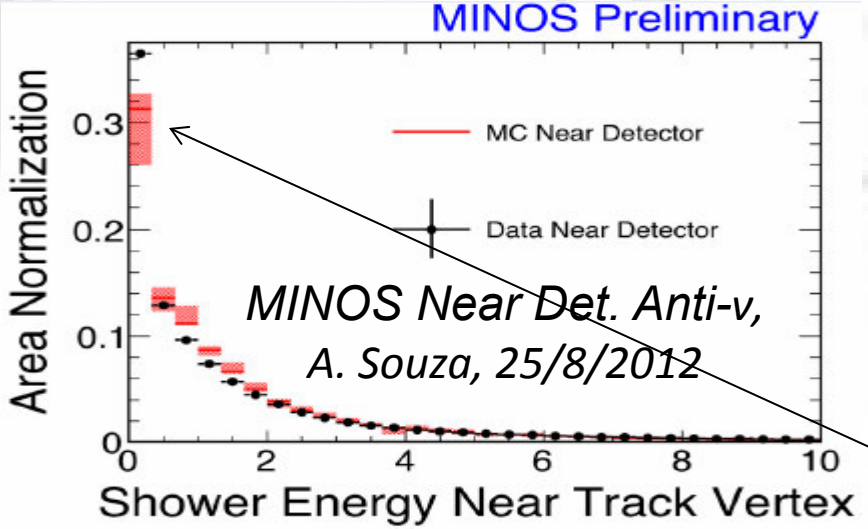


How do cross-sections effect oscillation analysis?



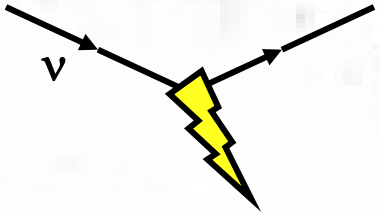
- ν_μ disappearance (high energy)
- Visible Energy in a calorimeter is NOT the ν energy transferred to the hadronic system
  - π absorption, π re-scattering, final state rest mass effect the calorimetric response
    - Can use external data to constrain



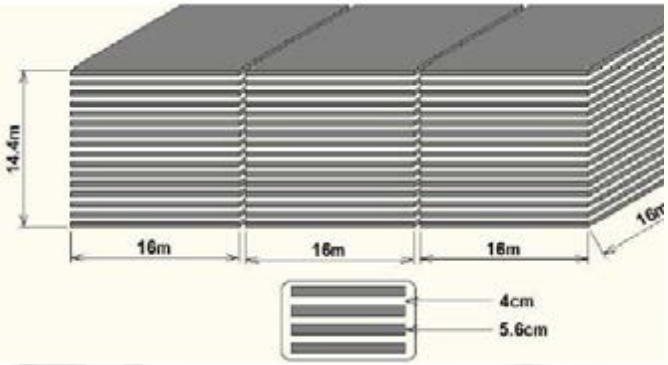


- *At very high energies, particle multiplicities are high and these effects will average out*
- *Low energy is more difficult*

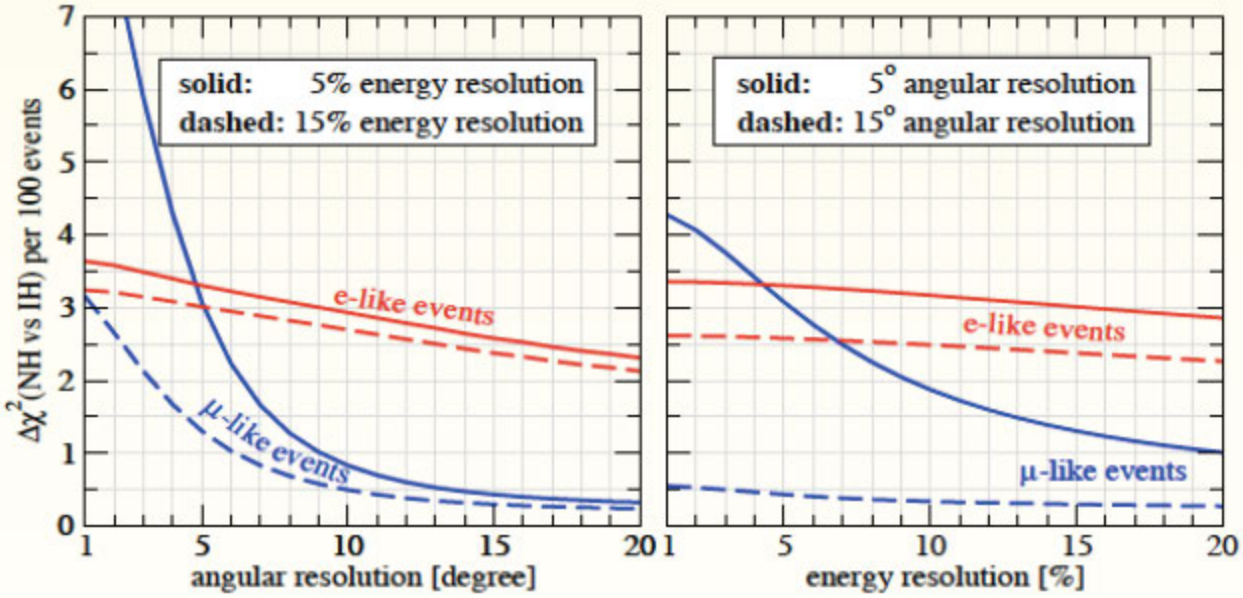
How do cross-sections effect oscillation analysis?



- In the case of INO ICAL, need good energy and angle resolution to separate normal and inverted hierarchy
  - Best sensitivity requires survival probability in both E_ν and L

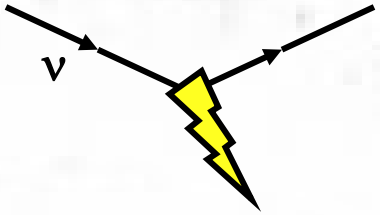


- Interaction models are understanding of detector response both needed to optimize resolution

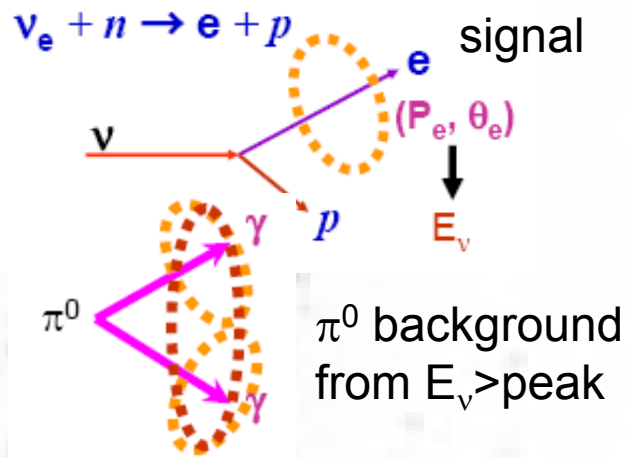


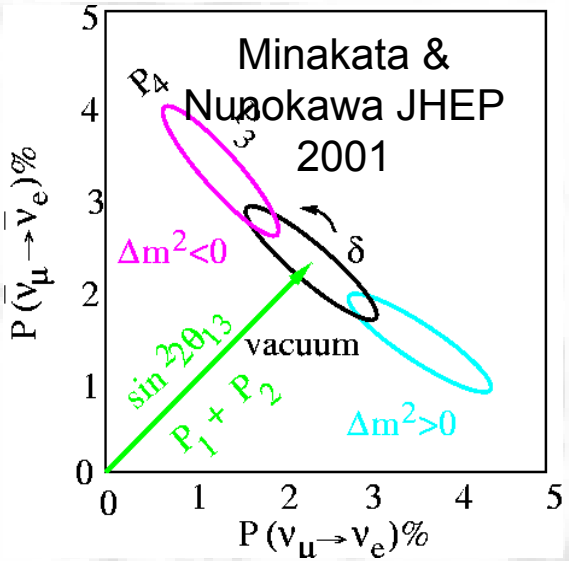
Petcov, Schwetz, hep-ph/0511277

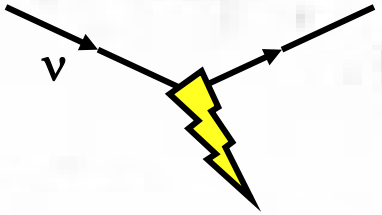
How do cross-sections effect oscillation analysis?



- ν_e appearance
  - different problem: signal rate is very low so even rare backgrounds contribute!
- Remember the end goal of electron neutrino appearance experiments
- Want to compare two signals with two different sets of backgrounds and signal reactions
  - with sub-percent precision
  - Requires precise knowledge of background and signal reactions

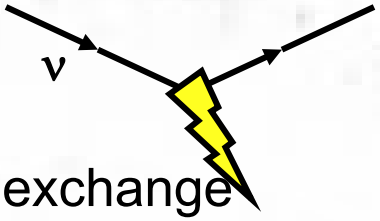




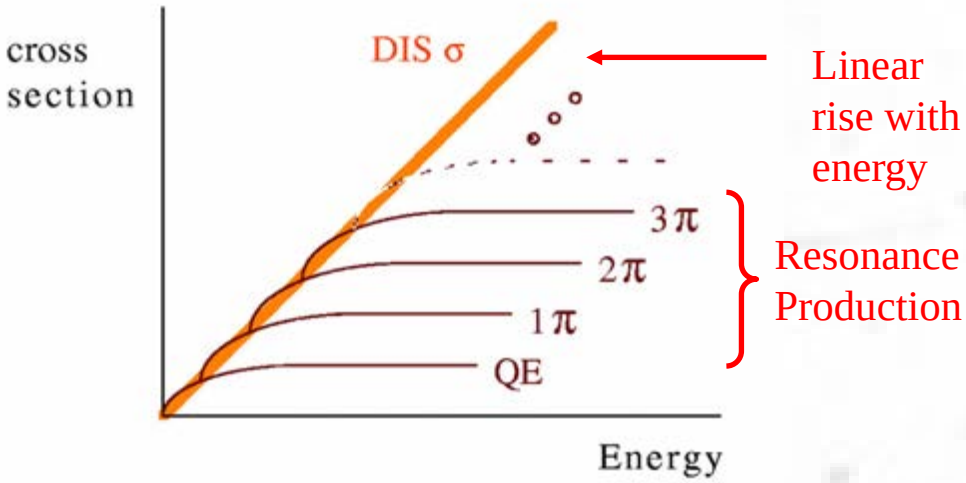


Models for GeV Cross-Sections

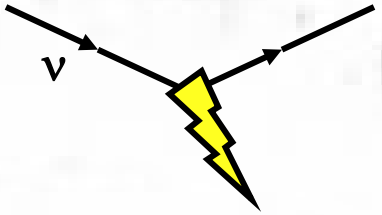
Neutrino-Nucleon Scattering



- Charged - Current: $W^{\pm}$ exchange
  - CC Elastic Scattering *(sometimes called "quasi-elastic" since neutron targets are only found in nuclei)*
    (Target changes but no break up)
    $\nu_\mu + n \rightarrow \mu^- + p$
  - Baryon Resonance Production:
    (Target goes to excited state)
    $\nu_\mu + n \rightarrow \mu^- + p + \pi^0$ (N^* or Δ)
    $n + \pi^+$
  - Deep-Inelastic Scattering:
    (Nucleon broken up)
    $\nu_\mu + \text{quark} \rightarrow \mu^- + \text{quark'}$
- Neutral - Current: Z^0 exchange
  - Elastic Scattering:
    (Target unchanged)
    $\nu_\mu + N \rightarrow \nu_\mu + N$
  - Baryon Resonance Production:
    (Target goes to excited state)
    $\nu_\mu + N \rightarrow \nu_\mu + N + \pi$ (N^* or Δ)
  - Deep-Inelastic Scattering
    (Nucleon broken up)
    $\nu_\mu + \text{quark} \rightarrow \nu_\mu + \text{quark}$

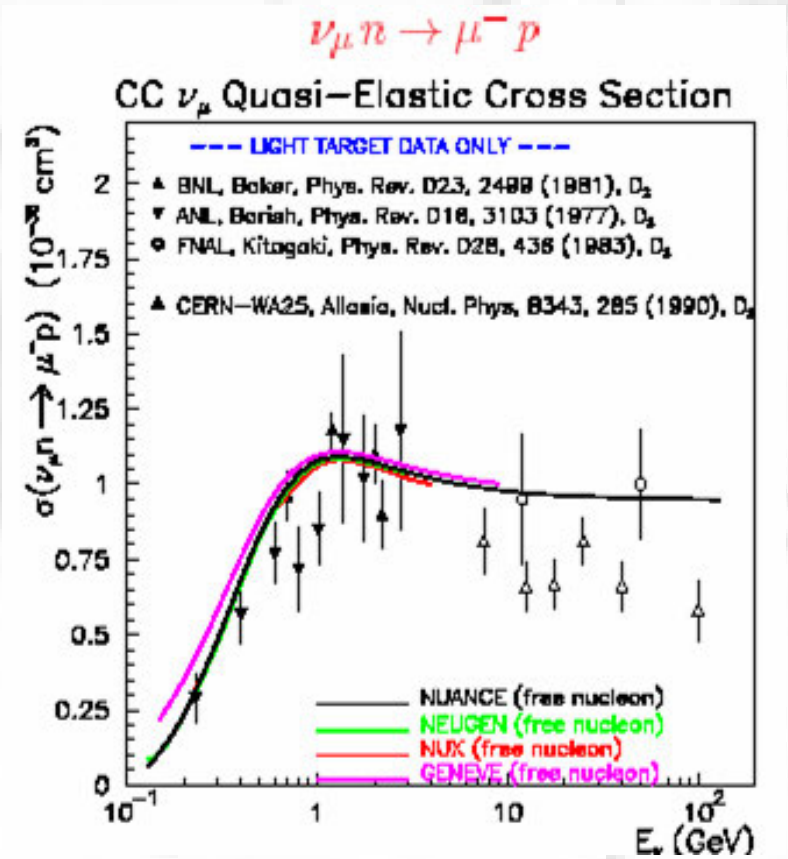


(Quasi-)Elastic Scattering



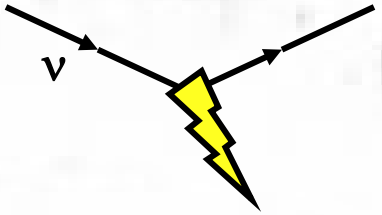
- Elastic scattering leaves a single nucleon in the final state
  - CC quasi-elastic *("quasi" since neutrons are in nuclei)* is easier to observe

$$\nu n \to l^- p$$
$$\bar{\nu} p \to l^+ n$$
$$\overset{(-)}{\nu} N \to \overset{(-)}{\nu} N$$



- State of data on "free-ish" neutrons (D_2) is marginal
  - No free neutrons implies nuclear corrections
  - Low energy statistics poor
- Cross-section is calculable
  - But depends on incalculable form-factors of the nucleon
- Theoretically and experimentally constant at high energy
  - 1 GeV^2 is ~ a limit in Q^2

What was that last cryptic remark?



- Theoretically and experimentally constant at high energy
  - 1 GeV^2 is ~ a limit in Q^2

- Inverse μ–decay:

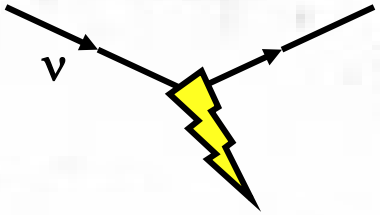
$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

a maximum Q^2 independent of beam energy $\Rightarrow$ constant σ_{TOT}

$$\sigma_{TOT} \propto \int_0^{Q^2_{max}} dQ^2 \frac{1}{(Q^2 + M_W{}^2)^2} \approx \frac{Q^2_{max}}{M_W{}^4}$$

- OK, but why does cross-section have a Q^2_{max} limit?
  - If Q^2 is too large, then the probability for the final state nucleon to stay intact (elastic scattering) becomes low
  - This information is encoded in “form factors” of the nucleons

Elastic Scattering (cont'd)



- As with IBD, nucleon structure alters cross-section
  - Can write down in terms of all possible "form factors" of the nucleon allowed by Lorentz invariance

$$\nu n \to l^- p$$
$$\bar{\nu} p \to l^+ n$$
$$\overset{(-)}{\nu} N \to \overset{(-)}{\nu} N$$

C.H. Llewellyn Smith, Phys. Rep. 3C, 261 (1972)

$$\frac{d\sigma}{dQ^2}\left(\begin{matrix}\nu n\to l^- p\\ \bar{\nu}p\to l^+ n\end{matrix}\right) = \left[A(Q^2) \mp B(Q^2)\frac{s-u}{M^2} + C(Q^2)\frac{(s-u)^2}{M^4}\right] \times \frac{M^2 G_F^2 \cos^2\theta_c}{8\pi E_\nu^2}$$

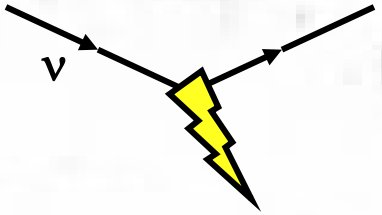
$$A(Q^2) = \frac{m^2+Q^2}{4M^2}\left[\left(4+\frac{Q^2}{M^2}\right)|F_A|^2 - \left(4-\frac{Q^2}{M^2}\right)|F_V^1|^2 + \frac{Q^2}{M^2}\xi|F_V^2|^2\left(1-\frac{Q^2}{4M^2}\right) + \frac{4Q^2 ReF_V^{1*}\xi F_V^2}{M^2} - \frac{Q^2}{M^2}\left(4+\frac{Q^2}{M^2}\right)|F_A^3|^2 - \frac{m^2}{M^2}\left(|F_V^1+\xi F_V^2|^2 + |F_A+2F_P|^2 - \left(4+\frac{Q^2}{M^2}\right)\left(|F_V^3|^2+|F_P|^2\right)\right)\right],$$

$$B(Q^2) = \frac{Q^2}{M^2}ReF_A^*\left(F_V^1+\xi F_V^2\right) - \frac{m^2}{M^2}Re\left[\left(F_V^1 - \frac{Q^2}{4M^2}\xi F_V^2\right)^* F_V^3 - \left(F_A - \frac{Q^2 F_P}{2M^2}\right)^* F_A^3\right] \text{ and}$$

$$C(Q^2) = \frac{1}{4}\left(|F_A|^2 + |F_V^1|^2 + \frac{Q^2}{M^2}\left|\frac{\xi F_V^2}{2}\right|^2 + \frac{Q^2}{M^2}|F_A^3|^2\right).$$

Occupants of the form factor zoo: F^1_V, F^2_V are vector form factors; F_A is the axial vector form factor; F_P is the pseudo-scalar form factor; F^3_V and F^3_A are form factors related to currents requiring G-parity violation, small?

Elastic Scattering (cont'd)



- Form factors representing second class currents, F^3_V and F^3_A, are usually assumed to be zero
- Pseudoscalar form factor, F_P, can be calculated from F_A with reasonable assumptions (Adler's theorem and the Goldberger-Treiman relation)
- The leading form factors, F^1_V, F^2_V and F_A, are approximately dipole in form

$$F_V(q^2) \sim \frac{1}{(1-q^2/M_V^2)^2} \qquad F_A(q^2) = \frac{F_A(0)}{(1-q^2/M_A^2)^2}$$

← "dipole approximation"

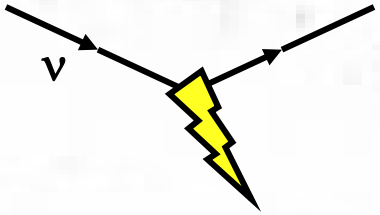
$M_V \approx 0.71$ GeV
$M_A \approx 1.01$ GeV
$F_A(0) \approx -1.267$
$F_V(0)$ is charge of proton

} parameters determined from data

n.b.: we've seen $F_V(0)$ and $F_A(0)$ before in IBD discussion (g_V and g_A)

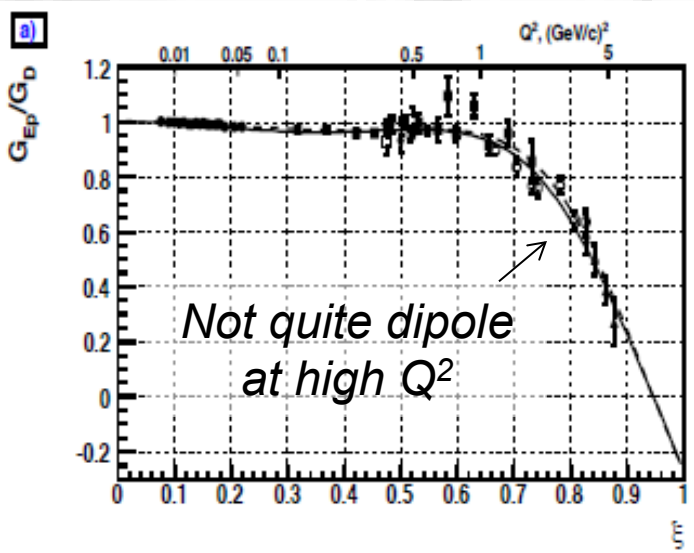
- Note that those masses which "cut off" the form factor are of order 1 GeV, so form factors are low beyond 1 GeV^2

Elastic Scattering (cont'd)



Vector form factors

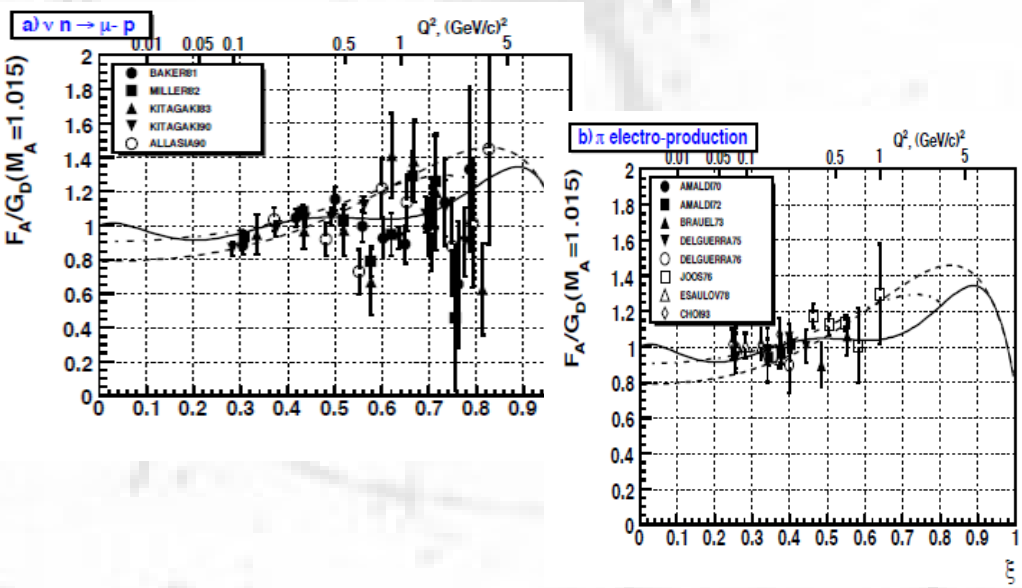
- Measured in charged lepton scattering



e.g., Bradford-Bodek-Budd-Arrington ("BBBA"), Nucl.Phys.Proc.Suppl.159:127-132,2006

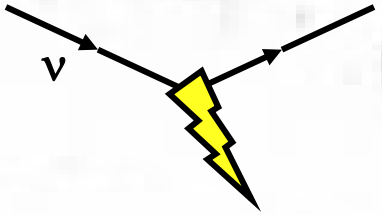
Axial vector form factors

- Measured in pion electro-production & neutrino scattering



Bodek, Avvakumov, Bradford and Budd, J. Phys. Conf. Ser. 110, 082004 (2008).

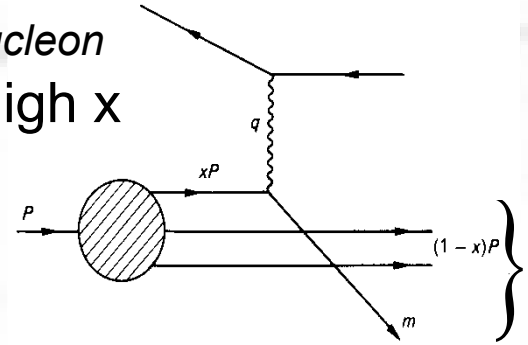
Low W, the Baryon Resonance Region

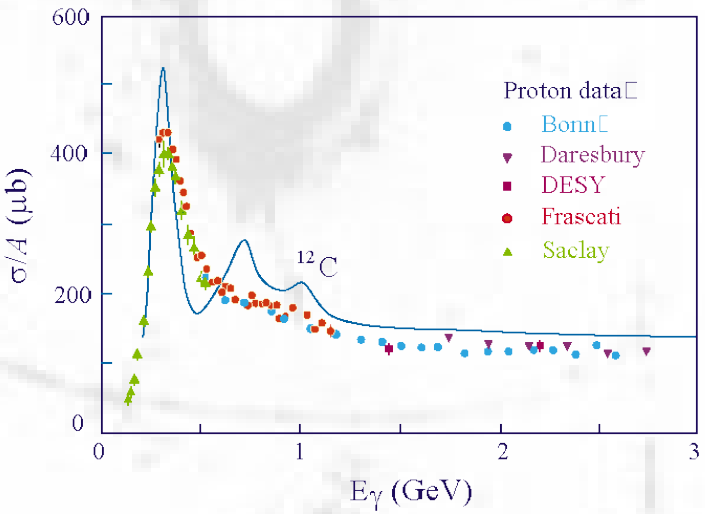


- Intermediate to elastic and DIS regions is a region of resonance production
  - Recall mass^2 of hadronic final state is given by

$$W^2 = M_T^2 + 2M_T\nu - Q^2 = M_T^2 + 2M_T\nu(1-x)$$

  - At low energy, nucleon-pion states dominated by N* and Δ resonances
- Leads to cross-section with significant structure in *W* just above $M_{nucleon}$
  - Low ν, high x

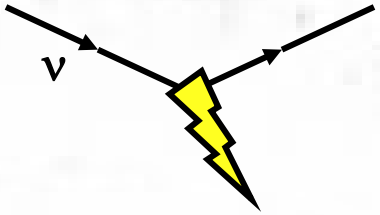




W^2

photoabsorption vs E_γ. Line shows protons. More later…

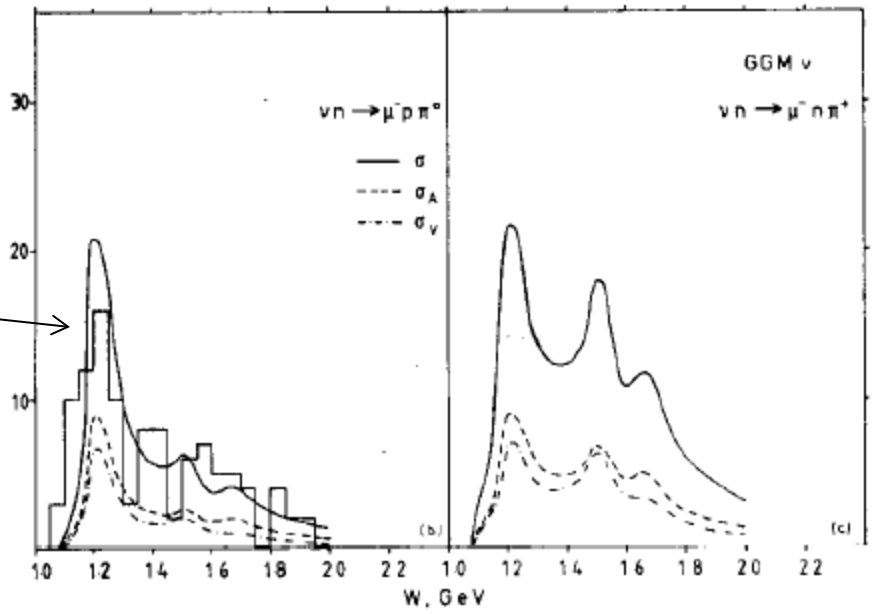
The Resonance Region



- Models of the resonance region are complicated
  - In principle, many baryon resonances can be excited in the scattering and they all can contribute
  - They de-excite mostly by radiating pions

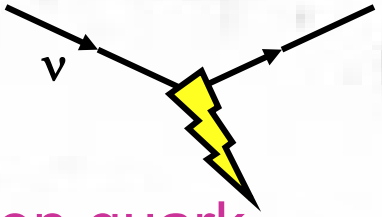
Nucleon Resonances below 2 GeV/c^2 according to Ref. [4]

Resonance Symbol[a]	Central mass value M [MeV/c^2]	Total with Γ_0[MeV]	Elasticity $x_E = \pi\mathcal{N}$ branching ratio	Quark-Model/ SU_6-assignment
$P_{33}(1234)$	1234	124	1	$^4(10)_{3/2}$ [56, 0^+]$_0$
$P_{11}(1450)$	1450	370	0.65	$^2(8)_{1/2}$ [56, 0^+]$_2$
$D_{13}(1525)$	1525	125	0.56	$^2(8)_{3/2}$ [70, 1^-]$_1$
$S_{11}(1540)$	1540	270	0.45	$^2(8)_{1/2}$ [70, 1^-]$_1$
$S_{31}(1620)$	1620	140	0.25	$^2(10)_{1/2}$ [70, 1^-]$_1$
$S_{11}(1640)$	1640	140	0.60	$^4(8)_{1/2}$ [70, 1^-]$_1$
$P_{33}(1640)$	1640	370	0.20	$^4(10)_{3/2}$ [56, 0^+]$_2$
$D_{13}(1670)$	1670	80	0.10	$^4(8)_{3/2}$ [70, 1^-]$_1$
$D_{15}(1680)$	1680	180	0.35	$^4(8)_{5/2}$ [70, 1^-]$_1$
$F_{15}(1680)$	1680	120	0.62	$^2(8)_{5/2}$ [56, 2^+]$_2$
$P_{11}(1710)$	1710	100	0.19	$^2(8)_{1/2}$ [70, 0^+]$_2$
$D_{33}(1730)$	1730	300	0.12	$^2(10)_{3/2}$ [70, 1^-]$_1$
$P_{13}(1740)$	1740	210	0.19	$^2(8)_{3/2}$ [56, 2^+]$_2$
$P_{31}(1920)$	1920	300	0.19	$^4(10)_{1/2}$ [56, 2^+]$_2$
$F_{35}(1920)$	1920	340	0.15	$^4(10)_{5/2}$ [56, 2^+]$_2$
$F_{37}(1950)$	1950	340	0.40	$^4(10)_{7/2}$ [56, 2^+]$_2$
$P_{33}(1960)$	1960	300	0.17	$^4(10)_{3/2}$ [56, 2^+]$_2$
$F_{17}(1970)$	1970	325	0.06	$^4(8)_{7/2}$ [70, 2^+]$_2$



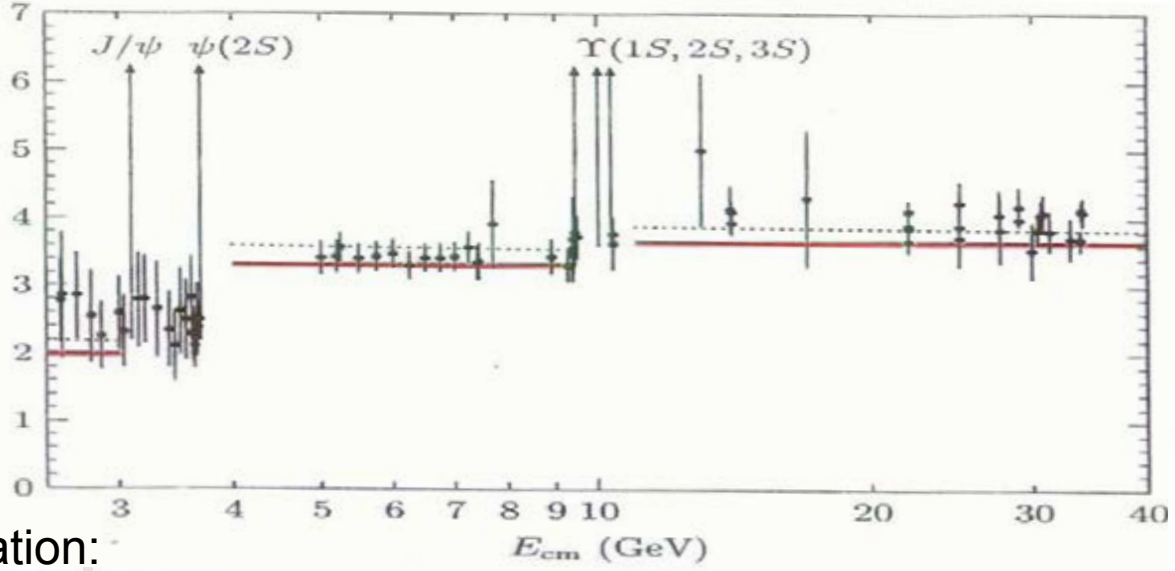
D. Rein and L. Sehgal, Ann. Phys. 133, 79 (1981)

Quark-Hadron Duality



- Bloom-Gilman Duality is the relationship between quark and hadron descriptions of reactions. It reflects:
  - link between *confinement* and *asymptotic freedom*
  - transition from *non-perturbative* to *perturbative* QCD

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

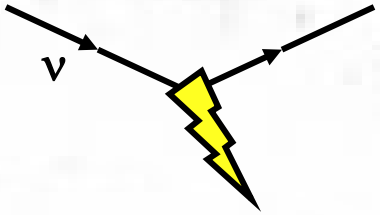


quark-parton model calculation:

$$R = N_C \sum_{q\text{'s} > m_q^2} \left(Q_q^{EM}\right)^2 + O(\alpha_{EM} + \alpha_S)$$

but of course, final state is really sums over discrete hadronic systems

Duality and ν

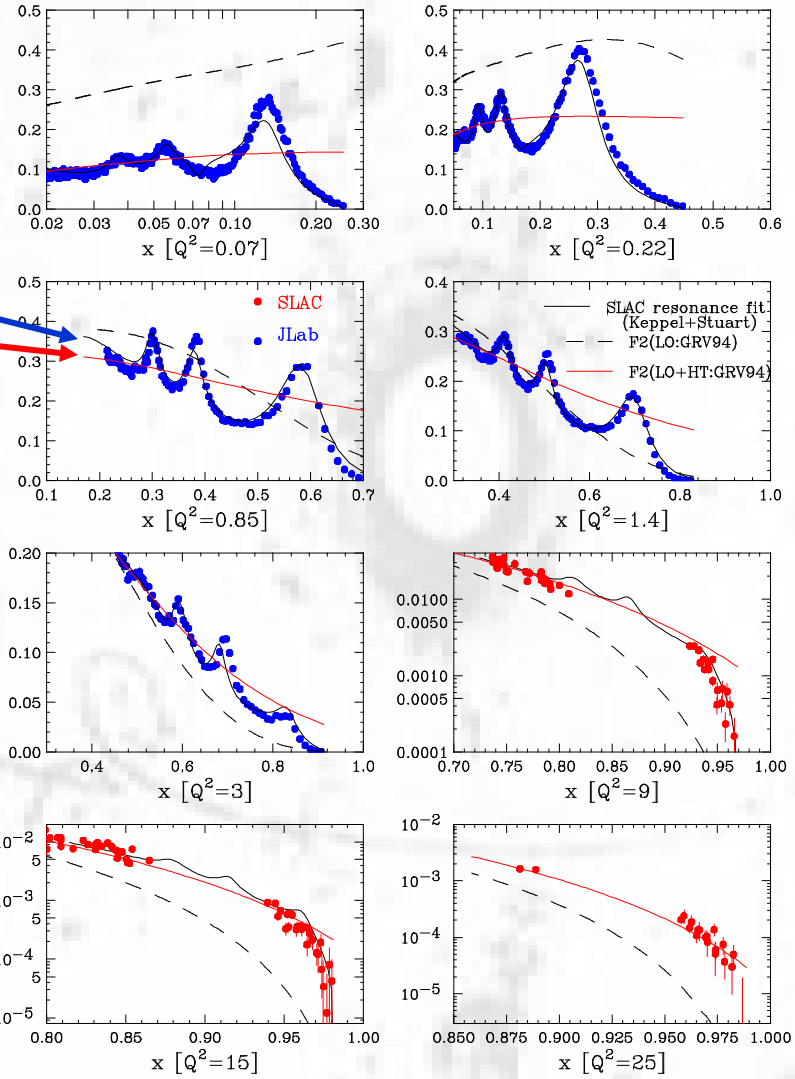


$$W^2 = M_T^2 + Q^2\left(\frac{1}{x} - 1\right)$$

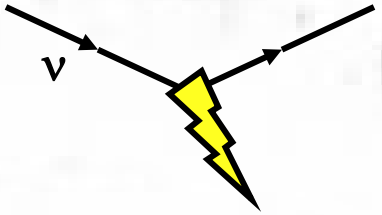
Low Q^2 data

DIS-Style PDF prediction

- Governs transition between resonance and DIS region
- Sums of discrete resonances approaches DIS cross-section
- Bodek-Yang: *Observe in electron scattering data; apply to ν cross-sections*



Duality's Promise



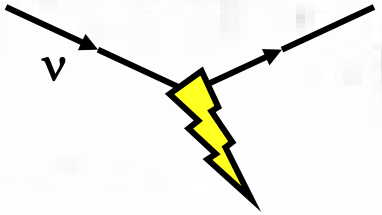
- In principle, a duality based approach can be applied over the entire kinematic region
- The problem is that duality gives "averaged" differential cross-sections, and not details of a final state

Microphysical models of exclusive processes

Duality models based on data of inclusive rates

- Microphysical models may lack important physics, but duality models may not predict all we need to know
  - How to scale the mountain between the two?

Lecture Question #5: Duality meets Reality



A difficulty in relating cross-sections of electron scattering (photon exchange) to charged-current neutrino scattering ($W^{\pm}$ exchange) is that some e-scatting reactions have imperfect ν-scattering analogues.

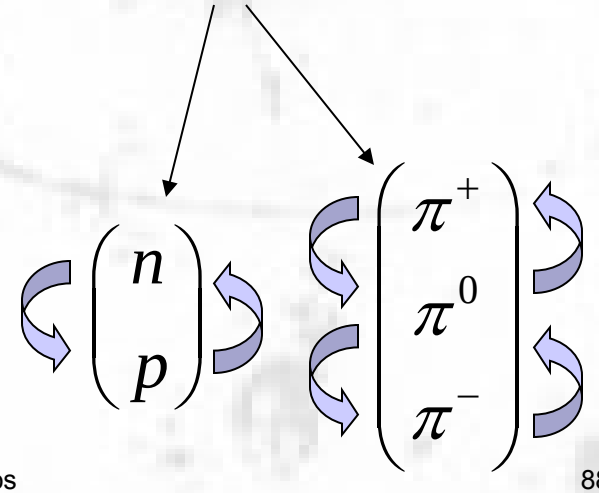
Write all possible ν_μ CC reactions involving the same target particle and isospin rotations of the final state for each of the following…

(a) $e^- n \to e^- n$

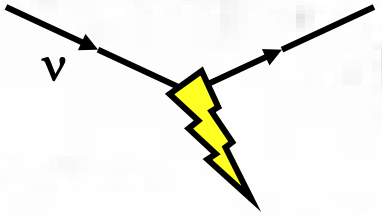
(b) $e^- p \to e^- p$

(c) $e^- p \to e^- n \pi^+$

(d) $e^- n \to e^- p \pi^-$



Lecture Question #5: Duality meets Reality



Write all possible ν reactions involving the same target particle and isospin rotations of the final state for each of the following…

(a) $e^- n \to e^- n$

$\nu_\mu n \to \mu^- p$

(b) $e^- p \to e^- p$

there are none!

(c) $e^- p \to e^- n\pi^+$

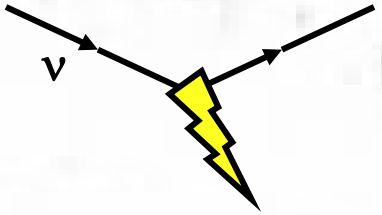
$\nu_\mu p \to \mu^- p\pi^+$

(d) $e^- n \to e^- p\pi^-$

$\nu_\mu n \to \mu^- n\pi^+$

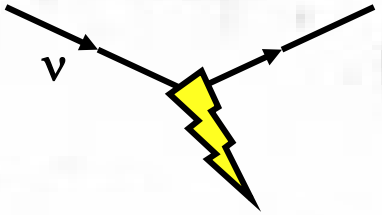
$\nu_\mu n \to \mu^- p\pi^0$

Summary of Second Lecture… and Next Topics

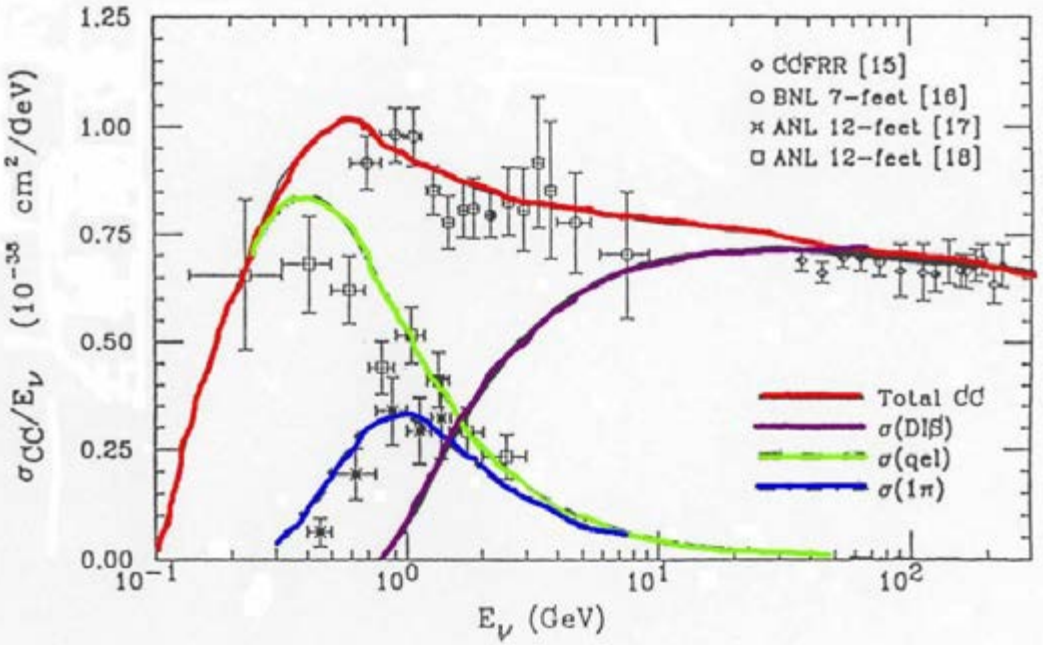


- We (slowly) extended what we learned about νe^- scattering to (anti)ν-(anti)quark scattering and calculated in the inelastic high energy limit
  - Fully predicted cross-section, up to quark distributions inside nucleon (PDFs)
  - Discussed implications for Ice Cube energy neutrinos
- We then tried to build the elastic and barely-inelastic neutrino-nucleon cross-sections *ab initio*
  - Lots of form factors and baryon resonances. Complex!
- Duality between quark and hadron pictures can help extend calculations in deep inelastic limit to Δ resonance dominated regime

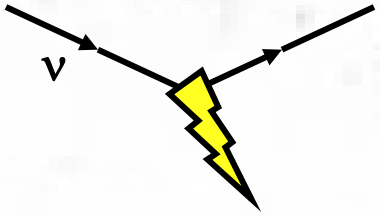
Building a Unified Model

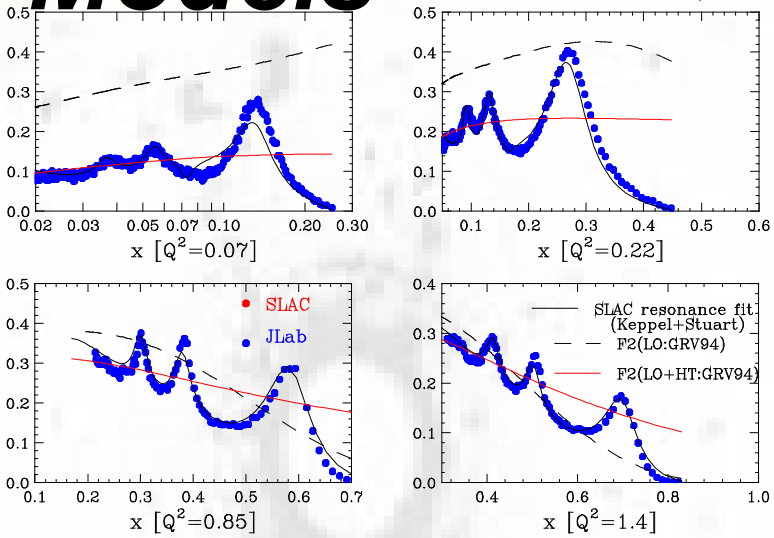


- In the relevant energy regime around 1 GeV, need a model that smoothly manages exclusive (elastic, resonance) to inclusive (DIS) transition
- Duality argues that the transition from the high W part of the resonance region (many resonances) to deep inelastic scattering should be smooth.

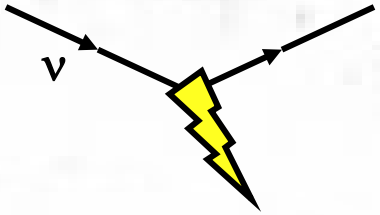


Exclusive Resonance Models and Duality Models



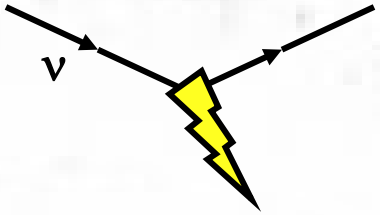


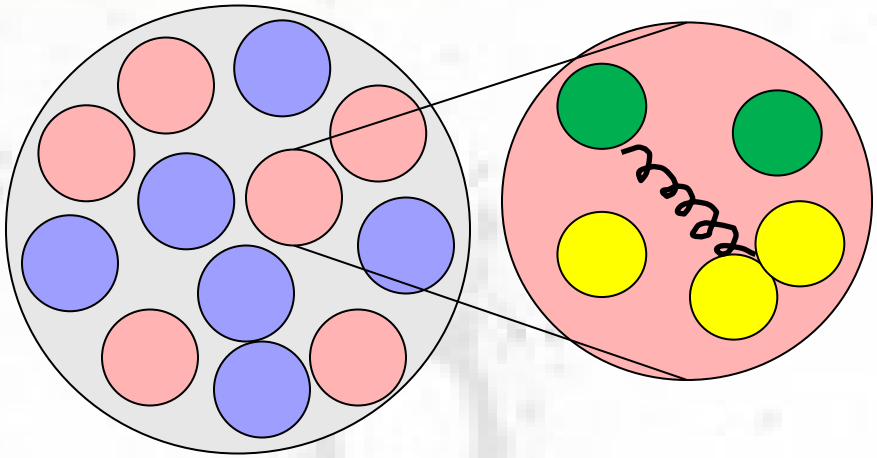
- Duality models agree with inclusive data by construction
  - However, in a generator context, have to add details of final state
- Typical approach (GENIE, NEUT and NUANCE) is to use a resonance model (Rein & Sehgal) below W<2 GeV, and duality + string fragmentation model for W>2 GeV
  - This is far from an idea solution
  - Discrete resonance model (probably) disagrees with total cross-section data below W<2 GeV and is difficult to tune
  - Average cross-section at high W does agree with data, but final state simulation is of unknown quality and difficult to tune also.



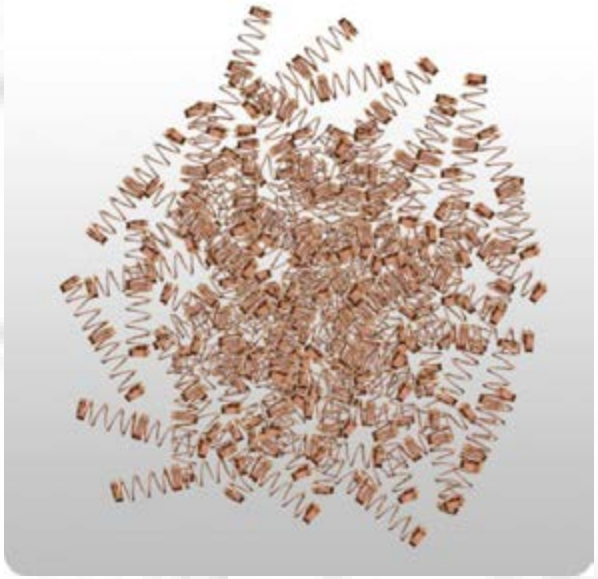
From Nucleons to Nuclei

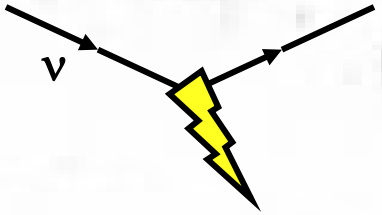
Why are Nuclei So Difficult?





- The fundamental theory allows a complete calculation of neutrino scattering from quarks
- But those quarks are in nucleons (PDFs), and those nucleons are in a strongly interacting tangle
- Imagine calculating the excitations of a pile of coupled springs. Very hard in general.

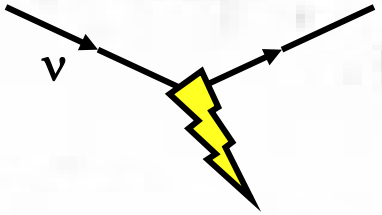




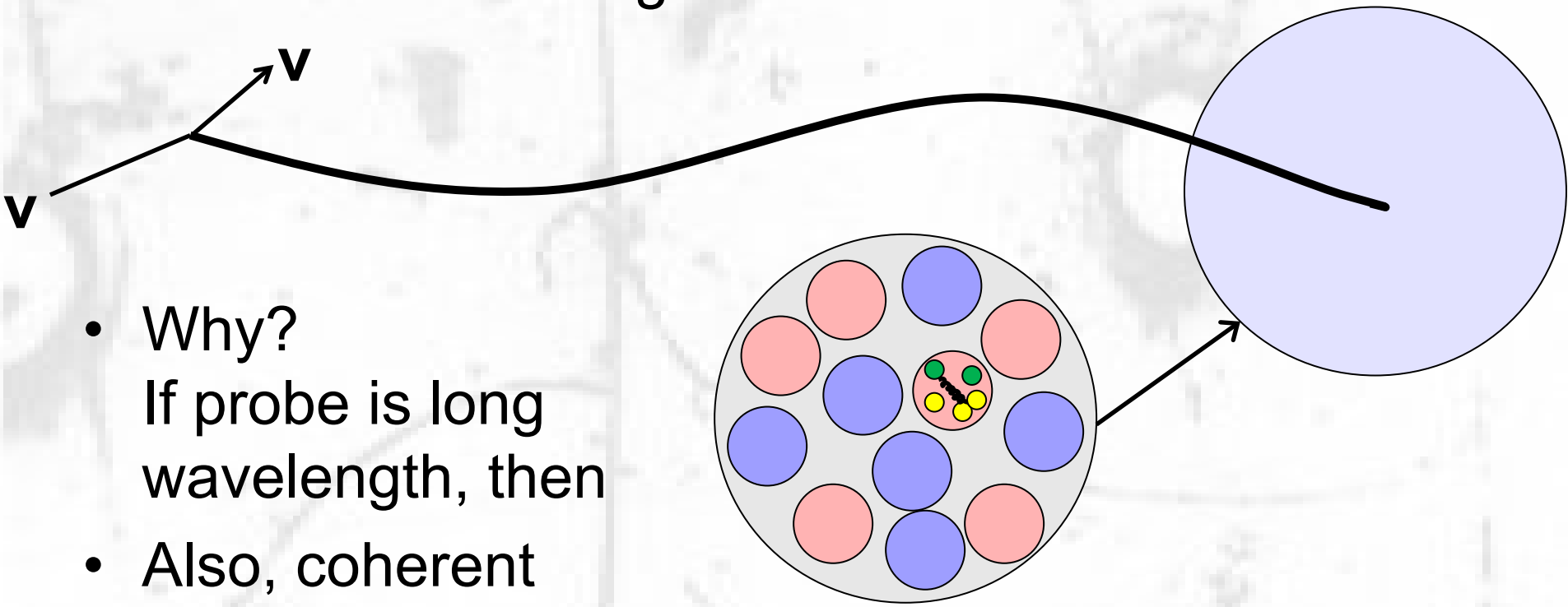
Coherent Neutrino-Nucleus Scattering

("CEvNS", says Kate Scholberg. But the "C" in "Coherent" is the first "C" in "Cachaça", not the "ç". So isn't that pronounced like "Kevin's"?)

Coherent and Elastic

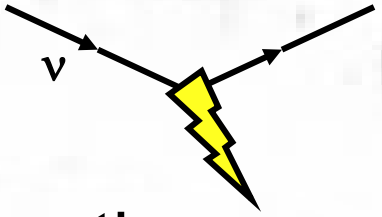


- Here is a limit in which, in principle, we can calculate scattering from the nucleus



- Why?
  If probe is long wavelength, then
- Also, coherent
  implies significant enhancement of rate

Coherence Condition



- Wavelength of probe, must be much larger than target, so momentum transfer: $Q \ll 1/R$
- If coherent, *amplitudes* from nucleons add
  - Therefore rate goes as (#nucleons)^2
- Limited momentum transfer, means limited kinetic energy of recoil: $T_{\max} \ll 1/M_A R^2$
  - Typical nuclear size in "natural" units ~ 100 MeV, so maximum recoil energy is ~100 keV or less for ^{40}Ar

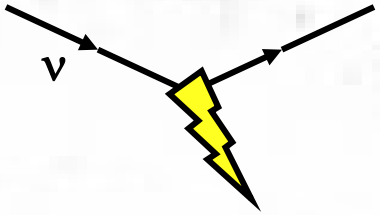
$$T \approx \frac{Q^2}{2M_A} \text{ for } Q \ll M_A$$

$$\frac{d\sigma}{dT} = \frac{G_F^2}{4\pi}\underbrace{\left[N - Z\left(1 - 4\sin^2\theta_W\right)\right]^2}\left(1 - \frac{M_A T}{2E_\nu^2}\right)\left(F(Q^2)\right)^2$$

Weak NC coupling : nearly zero for proton

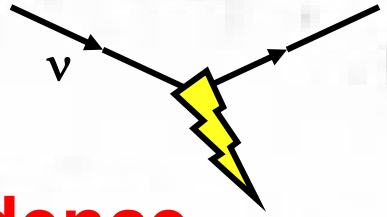
Form factor with coherence condition… goes to 0 except for very low Q^2

Comments on Coherent Nuclear Scattering



- No one has ever observed this because of the difficulties of finding such low recoils in nuclear matter
  - Most promising approaches have much in common with dark matter detectors
- Very useful practically if this can be overcome since it is a reaction perfect for “counting” neutrinos from a beam, a reactor, etc.

Lecture Question #6



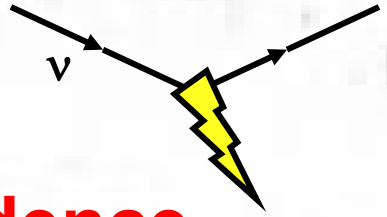
I would be willing to assert at high confidence that the discovery of neutrinos from the big bang would earn you a Nobel prize.

Coherent scattering has no threshold, so can use it to detect neutrinos with energies ~1 meV

What makes this difficult?

$$Q \ll \frac{1}{R} \Rightarrow T_{\max} \ll \frac{1}{M_A R^2} \qquad T \approx \frac{Q^2}{2M_A} \text{ for } \mathrm{Q} \ll \mathrm{M}_A$$

$$\frac{d\sigma}{dT} = \frac{G_F^2}{4\pi}\left[N - Z\left(1 - 4\sin^2\theta_W\right)\right]^2 \left(1 - \frac{M_A T}{2E_\nu^2}\right)\left(F(Q^2)\right)^2$$



Lecture Question #6

I would be willing to assert at high confidence that the discovery of neutrinos from the big bang would earn you a Nobel prize.

Coherent scattering has no threshold, so can use it to detect neutrinos with energies ~1 meV

What makes this difficult to detect?

The maximum momentum that can be transferred to a heavy stationary target is no more than twice the lab frame momentum.

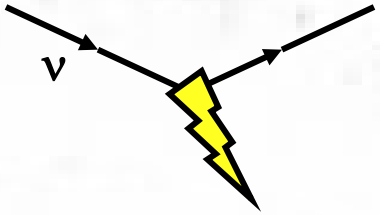
$$T \approx \frac{Q^2}{2M_A} \text{ for } Q \ll M_A$$

So

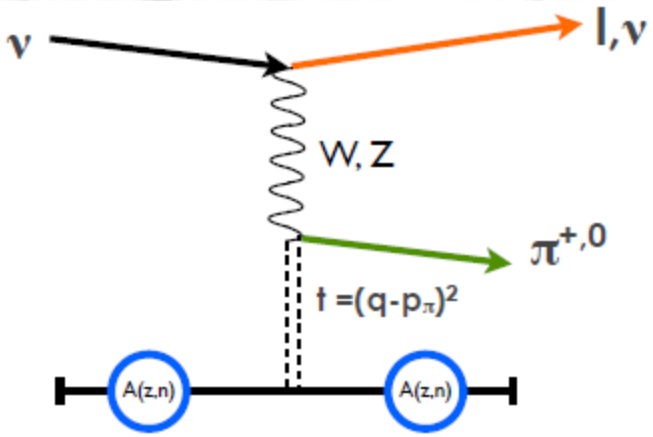
$$T \approx \frac{Q^2}{2M_A} < \frac{2p_\nu^{\,2}}{M_A} < 10^{-15}\,\text{eV}$$

Bummer! I was looking forward to that sauna.

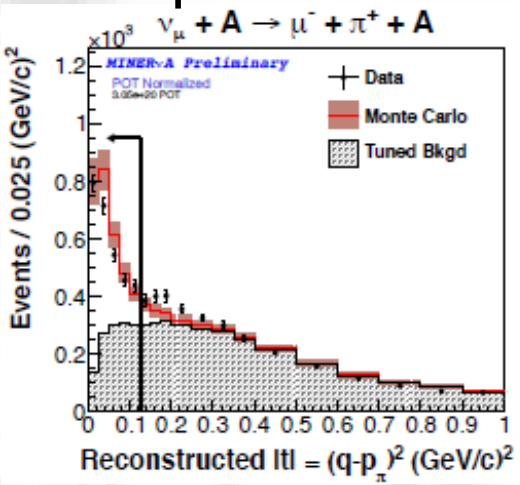
Coherent and Inelastic?



- What does that even mean?
- A long wavelength probe of the nucleus can interact with an off-shell W or Z, turning it into a pion!
  - Firing a gun at a bubble, leaving it intact, but breaking apart the bullet?



- Gives energetic leading pion which is a potential lepton background in less capable detectors
- Model independent features: low momentum transfer, |t|, to target and no recoil activity

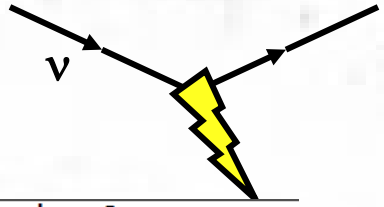


$$E_\nu = E_\mu + E_\pi$$

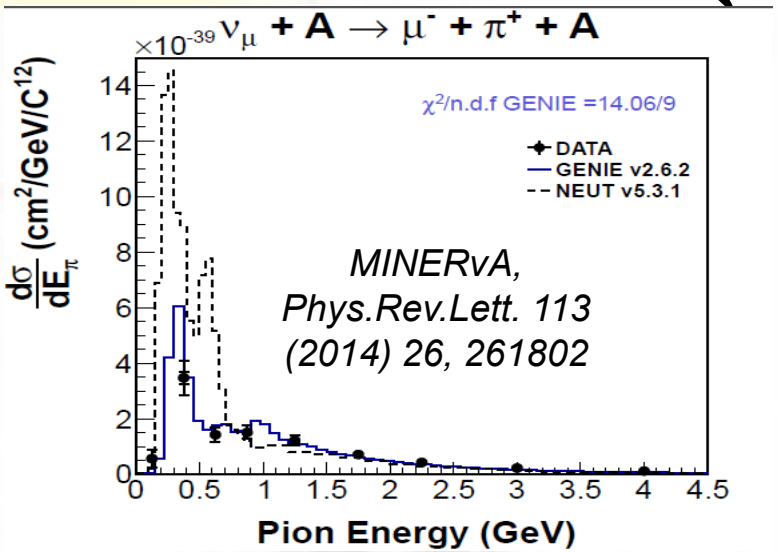
$$Q^2 = 2E_\nu(E_\mu - P_\mu cos\theta_\mu) - m_\mu^2$$

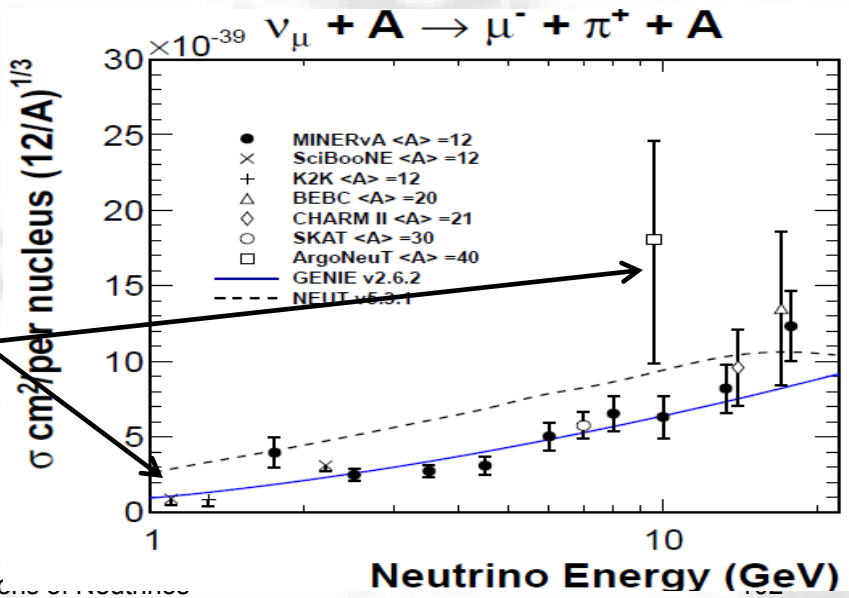
$$|t| = -Q^2 - 2(E_\pi^2 + E_\nu p_\pi cos\theta_\pi - p_\mu p_\pi cos\theta_{\mu\pi}) + m_\pi^2$$

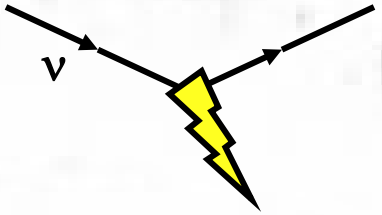
Coherent Pion Data



- Recent MINERvA measurement shows predictions overestimate low energy pions
- Biggest effect at low E_ν
- Explains non-observations at K2K and SciBooNE?
- Note also recent ArgoNeuT measurement on Ar (low statistics), *Phys Rev. Lett 113 (2014) 261801*

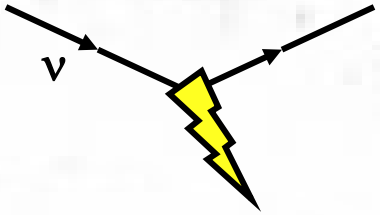






Inverse Beta Decay and Related Reactions in Nuclei

Recall: Inverse Beta Decay



$$\frac{d\sigma}{d\cos\theta} = \frac{G_F^2 s}{\pi} \times \cos^2 \vartheta_{\text{Cabibbo}} \times (\xi_{\text{mass}}) \times \left(g_V{}^2 (1+\beta_e \cos\theta) + 3 g_A{}^2 \left(1 - \frac{\beta_e}{3}\cos\theta\right)\right)$$

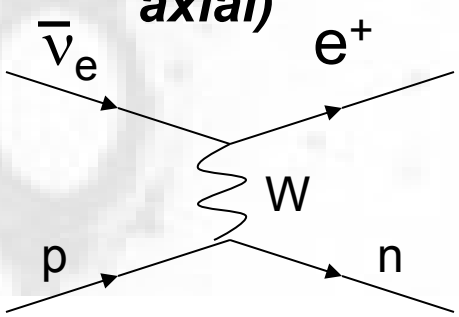
quark mixing!

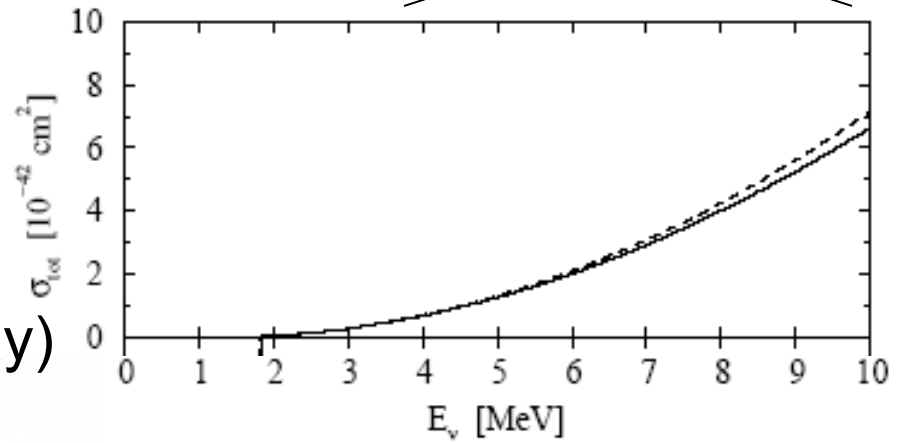
final state mass suppression

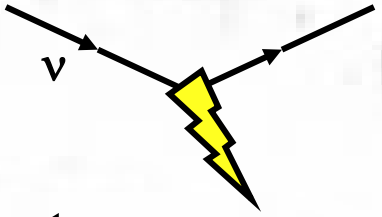
proton form factors (vector, axial)

- mass suppression is proportional to δE at low E_ν, so quadratic near threshold
- vector and axial-vector form factors (for IBD usually referred to as f and g, respectively)

  g_V, $g_A \approx 1, 1.26$.
  - FFs, $\theta_{Cabibbo}$, best known from τ_n (neutron beta decay)



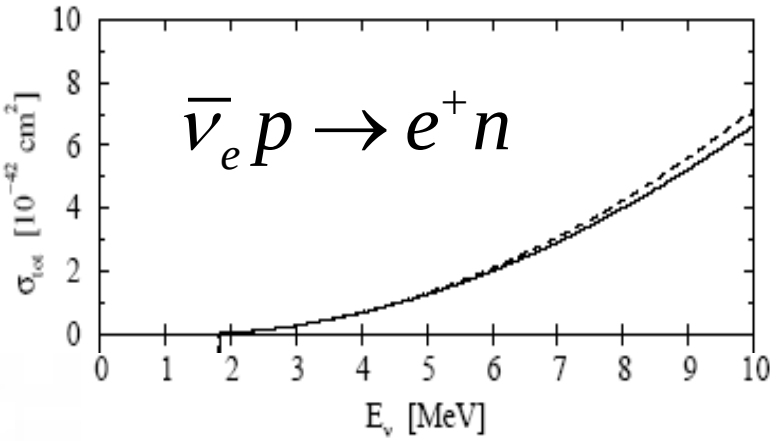


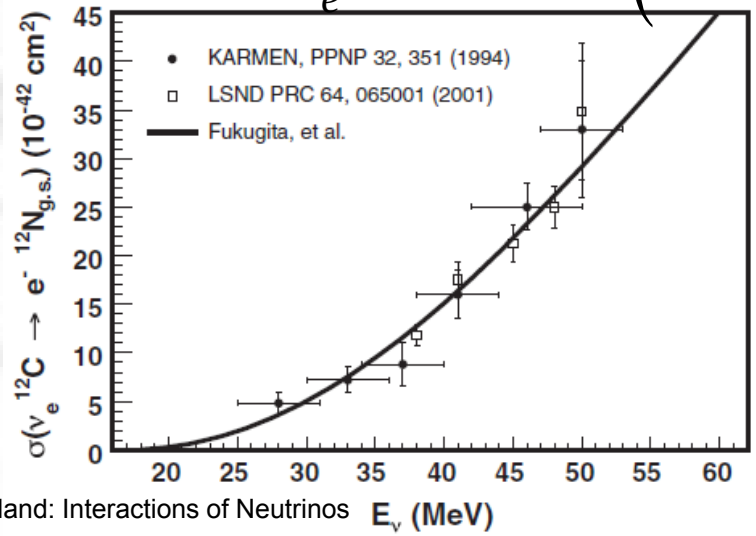


Inside a Nucleus

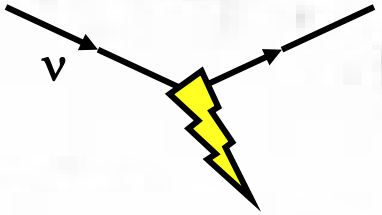
- Near threshold, have to account for discrete excitations of final state nucleus
  - If reaction is inclusive, then this is a sum over states
  - That can be difficult if many states are involved
- Exclusive reactions behave like free nucleon beta decay, but with a different threshold

$$\nu_e\ ^{12}C \rightarrow e^- \left(^{12}N\right)_{\text{ground state}}$$





Nuclei for Solar Neutrinos



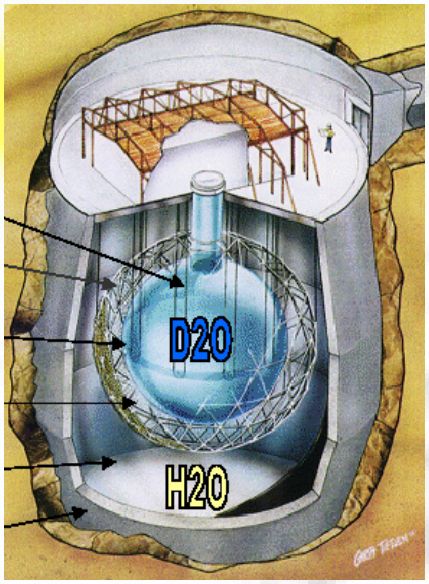
- Here are some nuclei historically important for Solar neutrino experiments. Low thresholds.

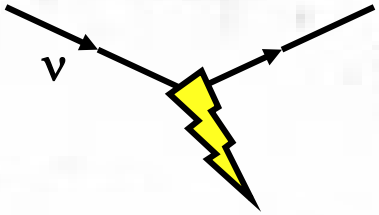
Experiment	*Nuclear Target*	*Reaction*	σ_0 $[10^{-46}\text{cm}^2]$	ΔE_{nucl} [MeV] (no det. Thres.)
GALLEX/GNO SAGE	$^{71}Ga_{33}$	$\nu_e + {}^{71}Ga \rightarrow e^- + {}^{71}Ge$	$8.611 \pm 0.4\%$ **(GT)**	0.2327
HOMESTAKE	$^{37}Cl_{17}$	$\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$	1.725 **(F)**	0.814
SNO	$^{2}H_{1}$	$\nu_e + {}^{2}H \rightarrow e^- + p + p$	**(GT)**	1.442
DUNE, ICARUS, etc.	$^{40}Ar_{18}$	$\nu_e + {}^{40}Ar \rightarrow e^- + {}^{40}K^*$	148.58 **(F)** … 44.367 **(GT_2)** … 41.567 **(GT_6)** …	1.505 +

table courtesy F. Cavana

SNO

- Three reactions for observing ν from sun (E_ν ~ few MeV





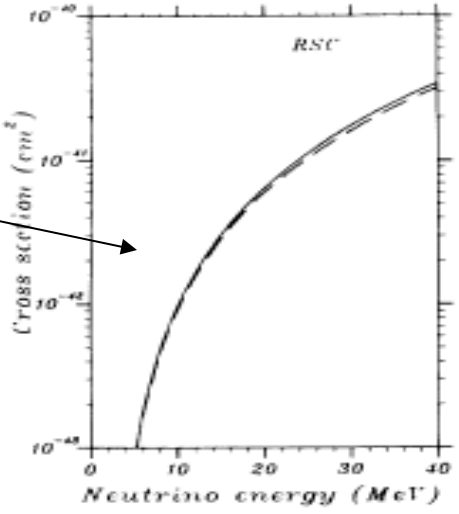
ES $\nu_x + e^- \Rightarrow \nu_x + e^-$

- ^{2}H, ^{16}O binding energies are 13.6eV, ~1 keV.
- Therefore, e$^-$ are "free". $\sigma \propto E_\nu$

NC $\nu_x + d \Rightarrow p + n + \nu_x$

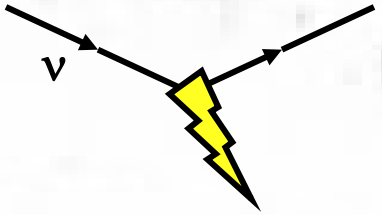
CC $\nu_e + d \Rightarrow p + p + e^-$

Deuteron binding energy is 2.2 MeV



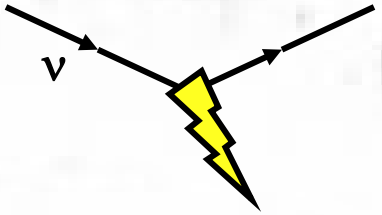
- Energy threshold of a few MeV for neutral current. Less for the charged current because $m_n > m_p + m_e$

(Bahcall, Kubodara, Nozawa, PRD38 1030)

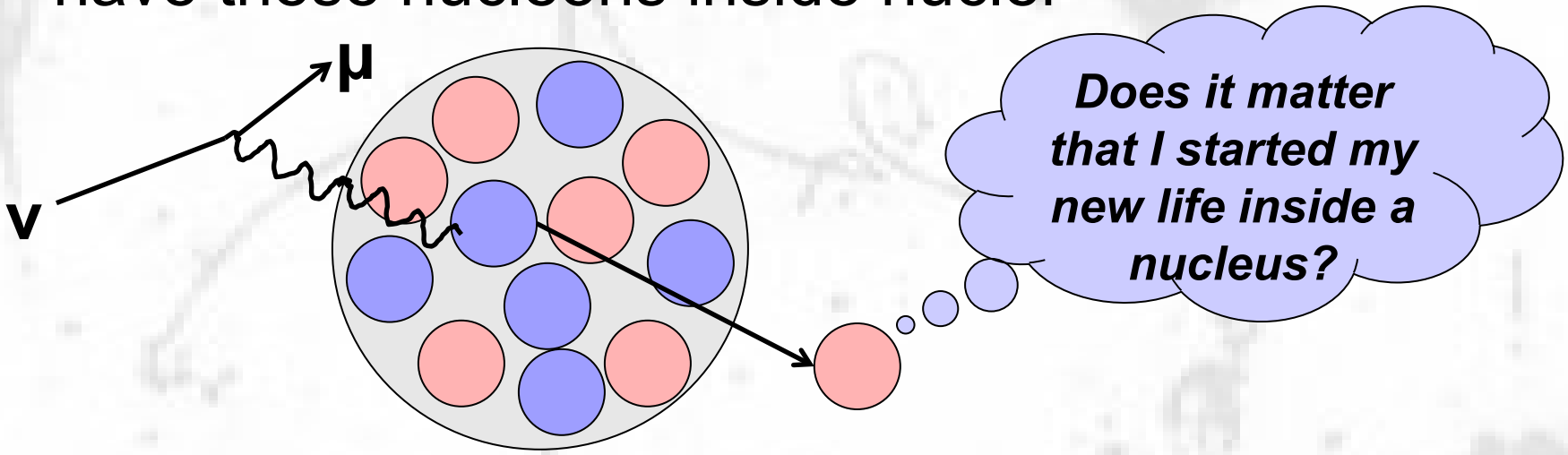


GeV Cross-Sections on Nucleons in a Nucleus

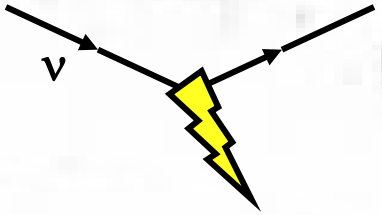
Elastic? Fantastic!



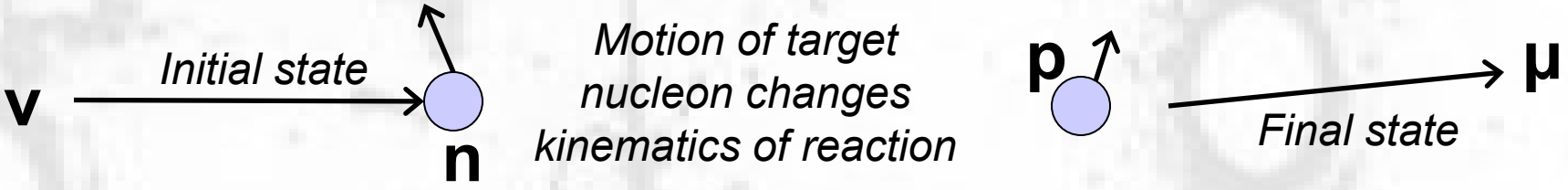
- Last time, we showed that the elastic scattering of neutrinos from nucleons is (nearly) predicted
  - Charged-current reaction allows tagging of neutrino flavor and reconstruction of energy
- Unfortunately, practical neutrino experiments have these nucleons inside nuclei



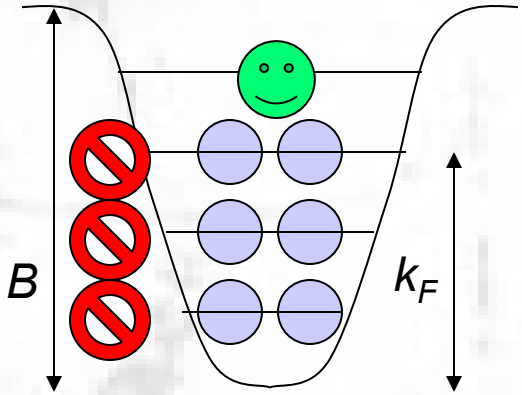
Fermi Motion, Binding and Pauli "Blocking"



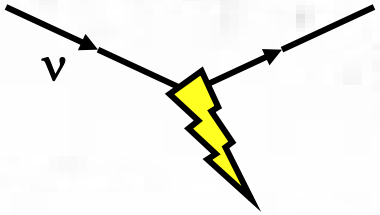
- In a nucleus, target nucleon has some initial momentum which modifies the observed scattering
  - Simple model is a "Fermi Gas" model of nucleons filling available states up to some initial state Fermi momentum, k_F



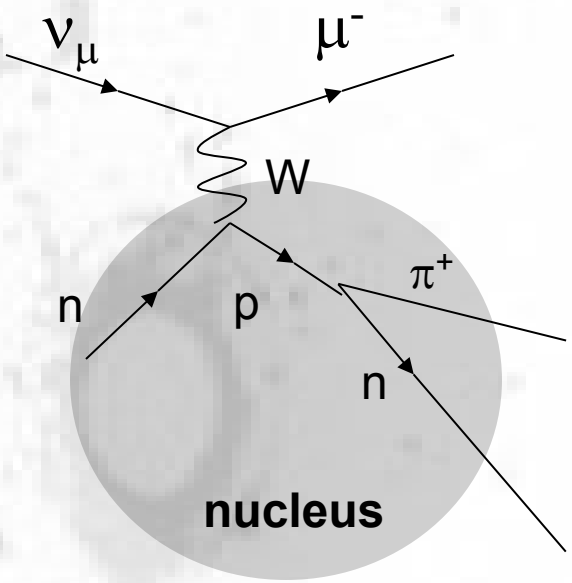
- The nucleon is bound in the nucleus, so it take energy to remove it
- Pauli blocking for nucleons not escaping nucleus… states are already filled with identical nucleon



"Final State" Interactions



- The outgoing nucleon could create another particle as it travels in nucleus
  - If it is a pion, event would appear inelastic
- Also other final states can contribute to apparent "quasi-elastic" scattering through absorption in the nucleus…
  - kinematics may or may not distinguish the reaction from elastic



- Theoretical uncertainties in these reactions are large
  - At least at the 10% level. More on this later.
  - If precise knowledge is needed for target (e.g., water, liquid argon, hydrocarbons), dedicated measurements will be needed
    - Most relevant for low energy experiments, i.e., T2K

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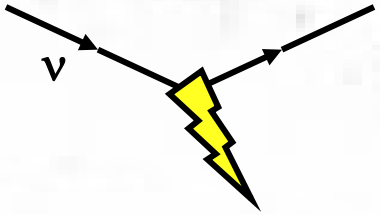

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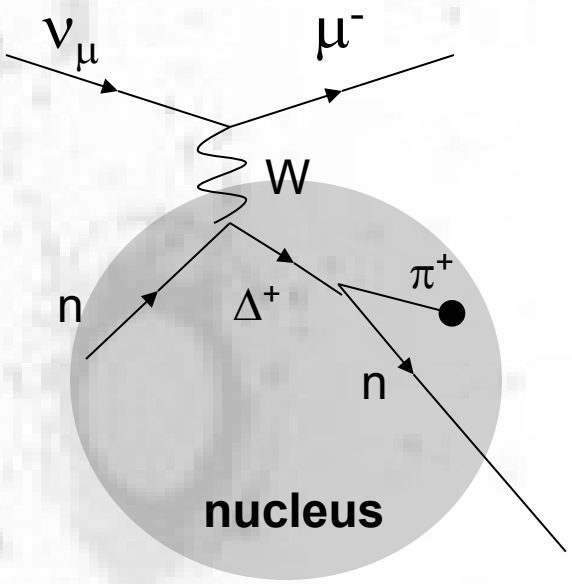

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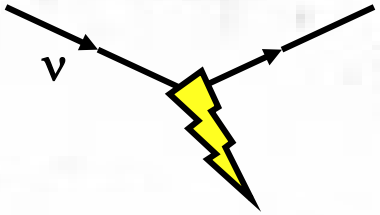
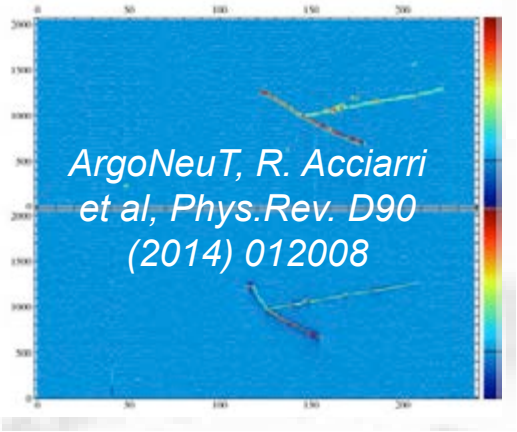
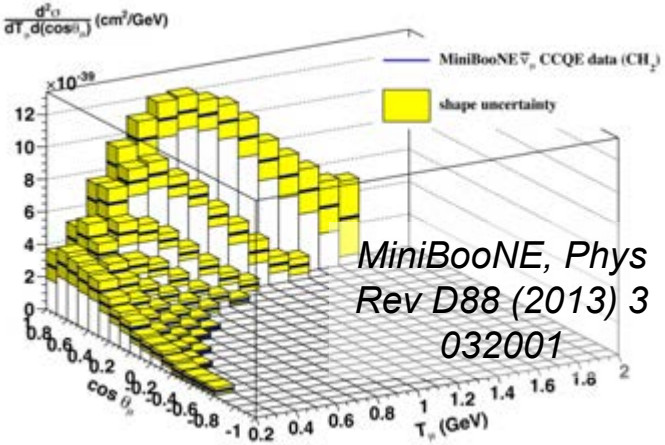
Many Measurements

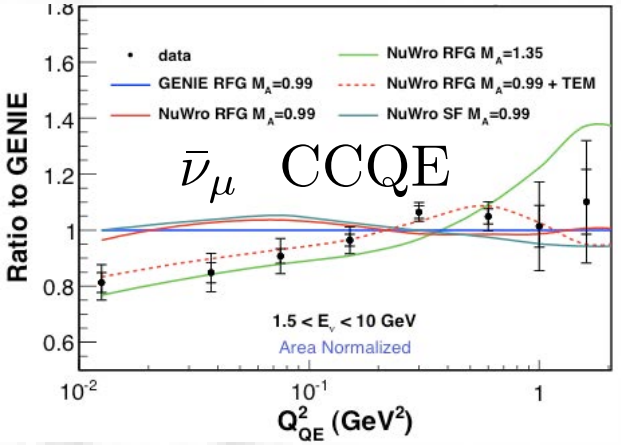


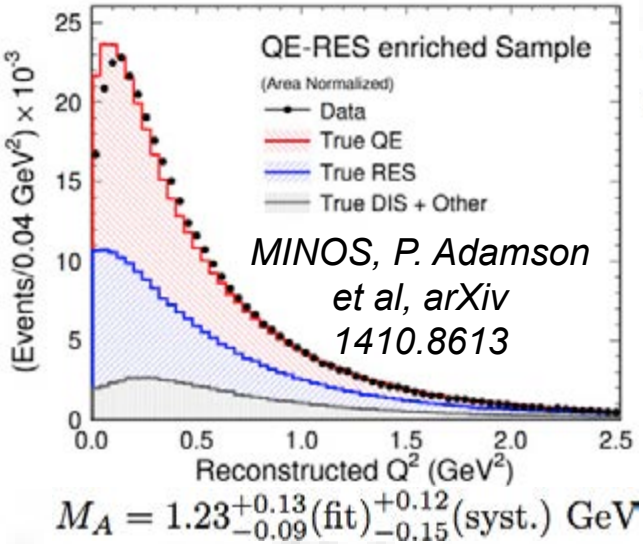


ArgoNeuT, R. Acciarri et al, Phys.Rev. D90 (2014) 012008

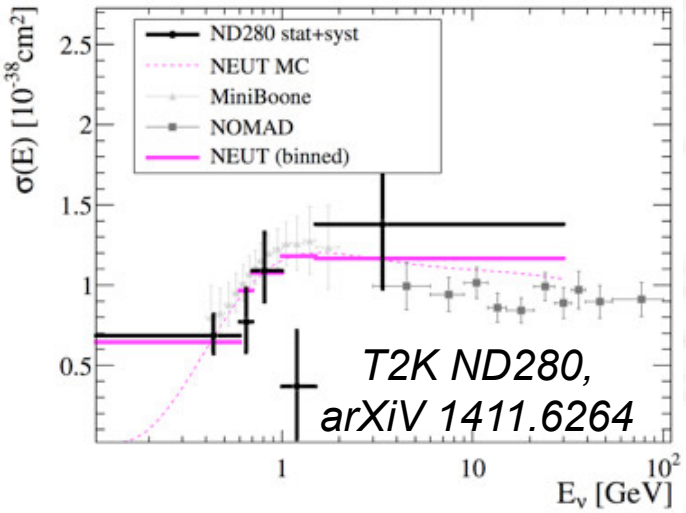


MiniBooNE, Phys Rev D88 (2013) 3 032001



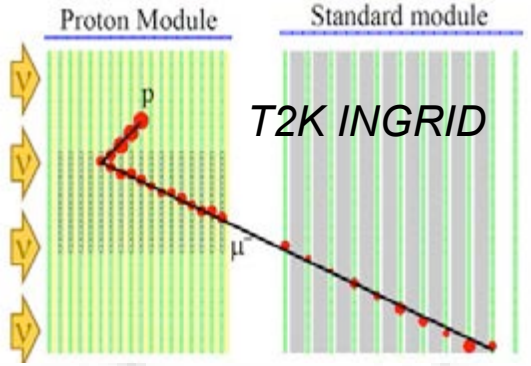


MINOS, P. Adamson et al, arXiv 1410.8613



T2K ND280, arXiV 1411.6264

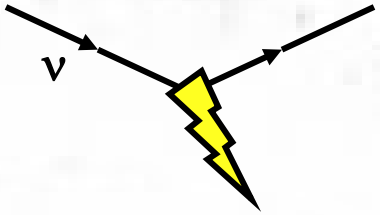
MINERvA, Phys Rev. Lett. 111, 002051 and 002052 (2013)



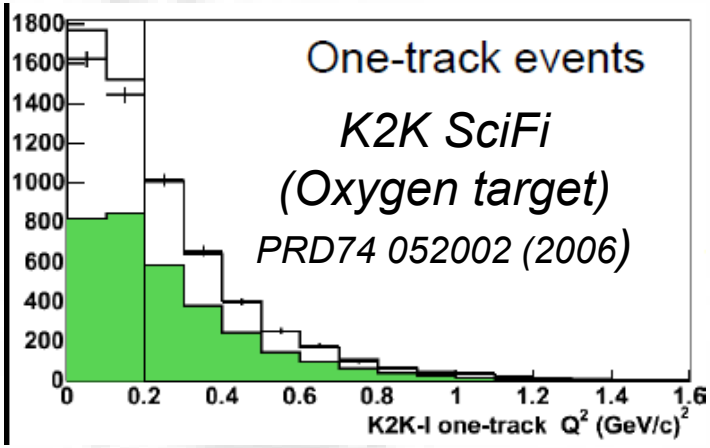
T2K INGRID

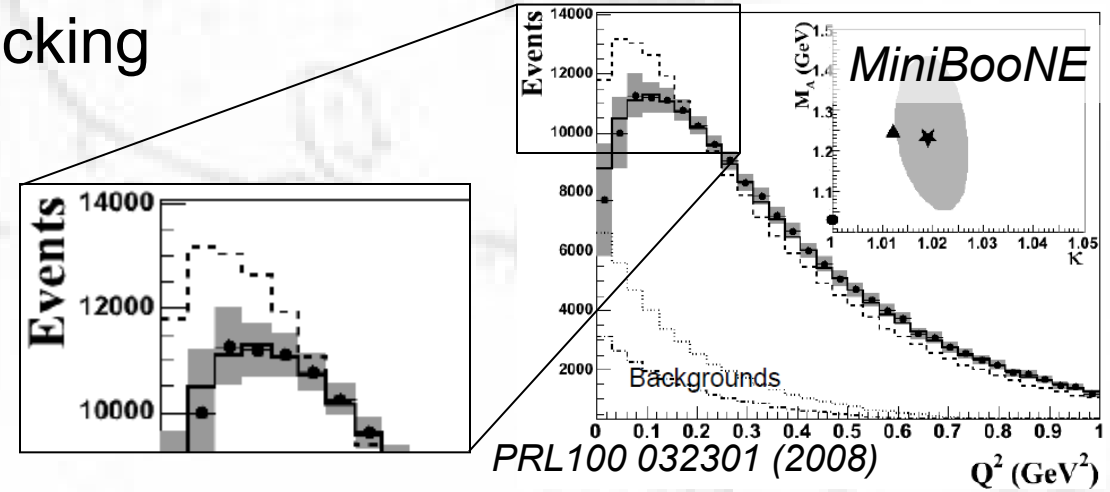
- *I need to pick and choose some highlights*

Measurements of CCQE on Nuclei: Backgrounds

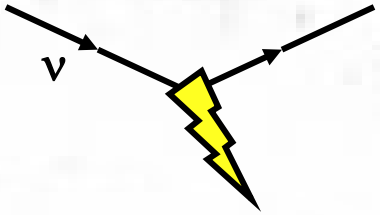


- K2K famously observed a "low Q^2 deficit" in its analysis
- MiniBooNE originally had a significant discrepancy at low Q^2 as well
  - Original approach was to enhance Pauli blocking to "fix" low Q^2
  - Was resolved by tuning single pion background to data w/ pions

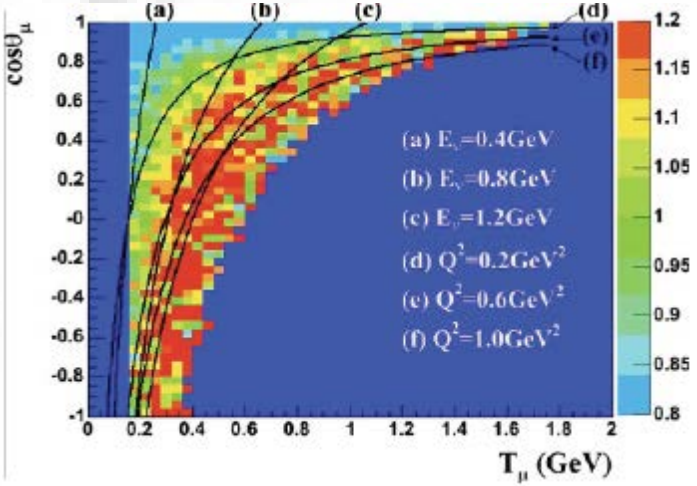


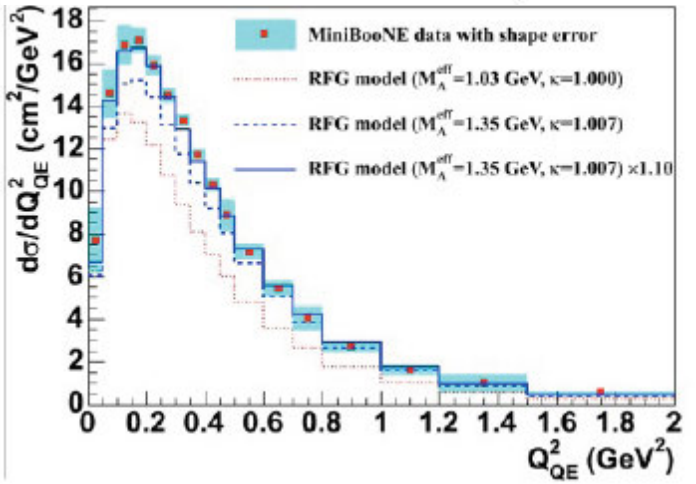


MiniBooNE *(Phys. Rev. D81 092005, 2010)*

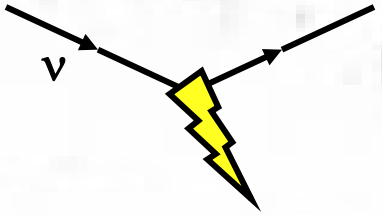


- Oil Cerenkov detector (carbon), views only muon
- Fit to observables, muon energy & angle find a discrepancy with expectation from free nucleons
- It looks like a distortion of the Q^2 distribution
- MiniBooNE fits for an "effective" axial mass, M_A, higher than expected
  - Good consistency between total cross-section and this Q^2 shape in this high M_A explanation



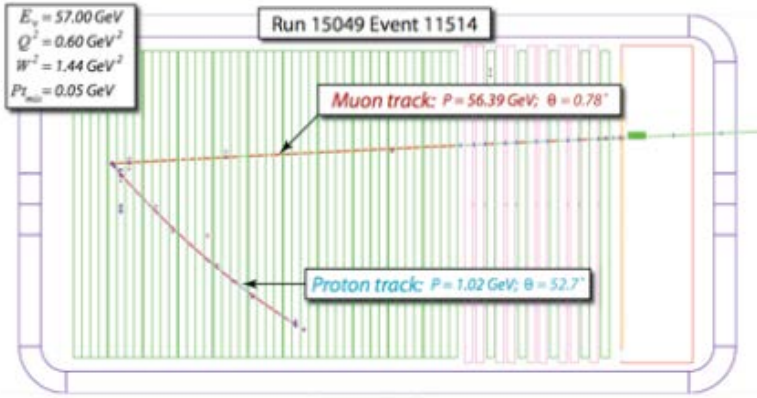


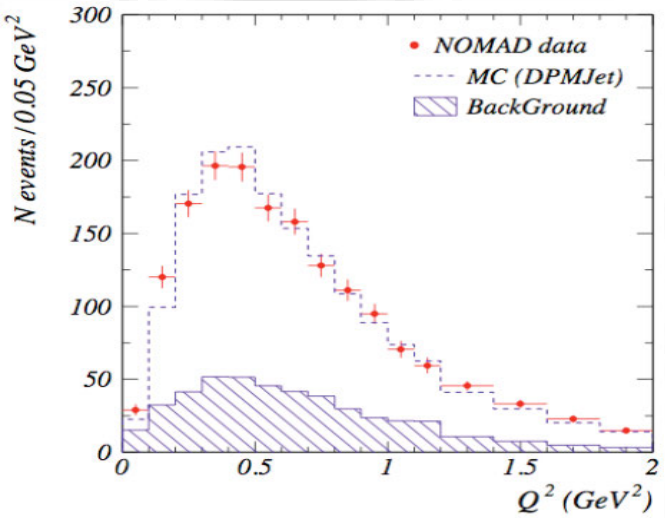
NOMAD *(Eur.Phys.J.C63:355-381,2009)*



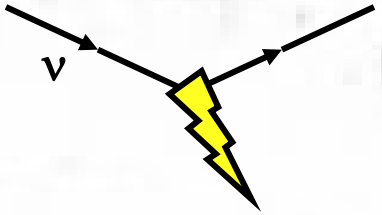
- Like MiniBooNE, target is mostly carbon (drift chamber walls)
- Reconstruct both recoiling proton and muon
- Total cross-section and Q^2 distribution are both consistent with expectation from free nucleon
- *Two experiments, same target, but different energies and reconstruction…*

  … incompatible results?

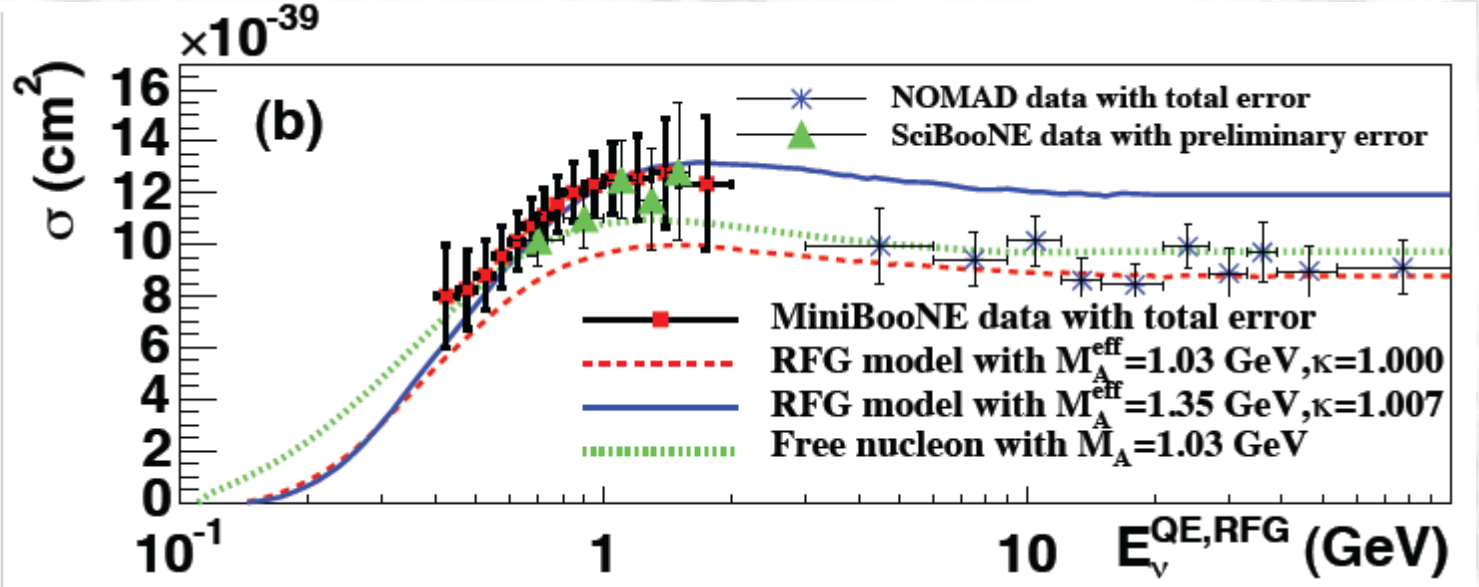




MiniBooNE and NOMAD

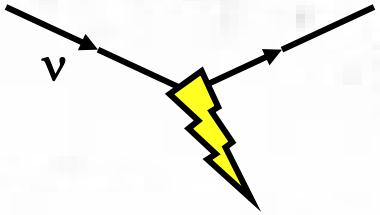


- Current data cannot be fit by a single prediction for low energy data (BooNEs) and high energy data (NOMAD)
  - In effective dipole form-factor picture, different "M_A"
  - Free nucleon M_A is ~1 GeV from both pion electroproduction and neutrino scattering on deuterium
- Detail: MiniBooNE measures μ only, NOMAD μ+p

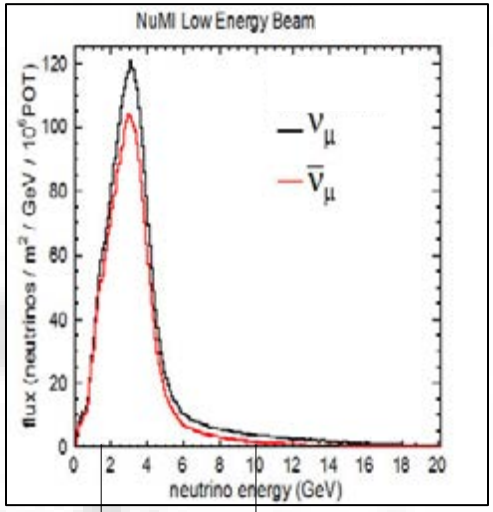


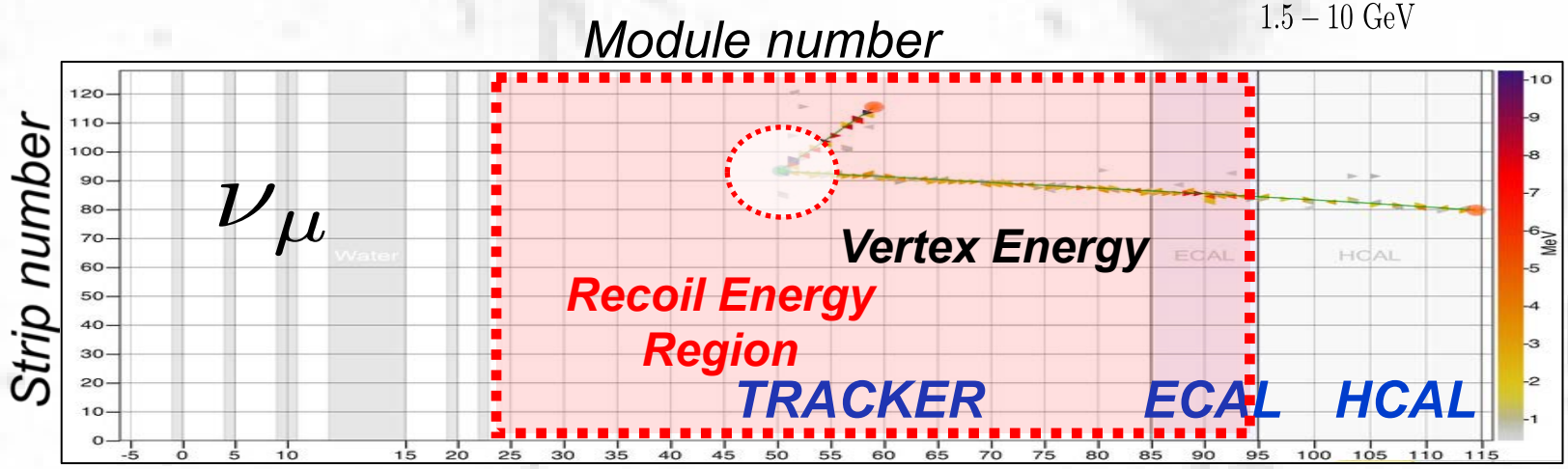
Plot courtesy of T. Katori

MINERvA CCQE on Carbon

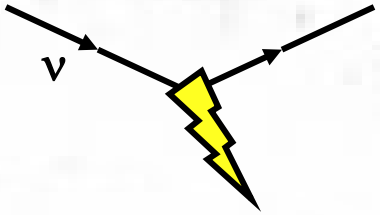


- MINERvA has measured CCQE in neutrino and anti-neutrino beams
  - Flux integrated from 1.5 to 10 GeV. It's a measurement "near" 3.5 GeV
- Sample is selected by muon and "low" calorimetric recoil away from vertex

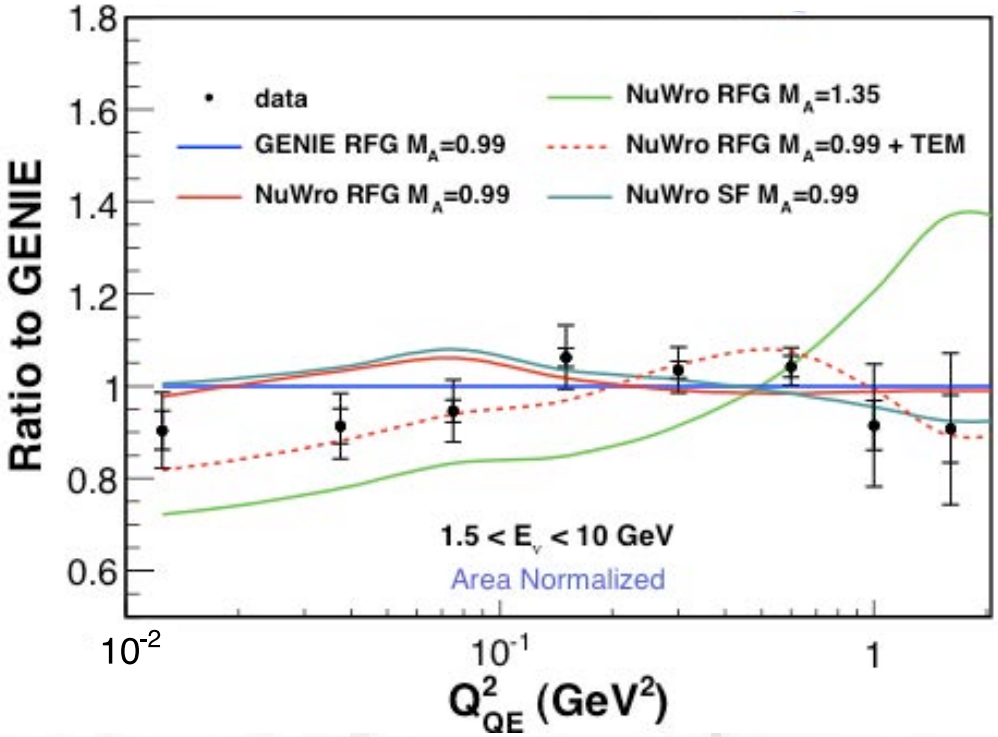




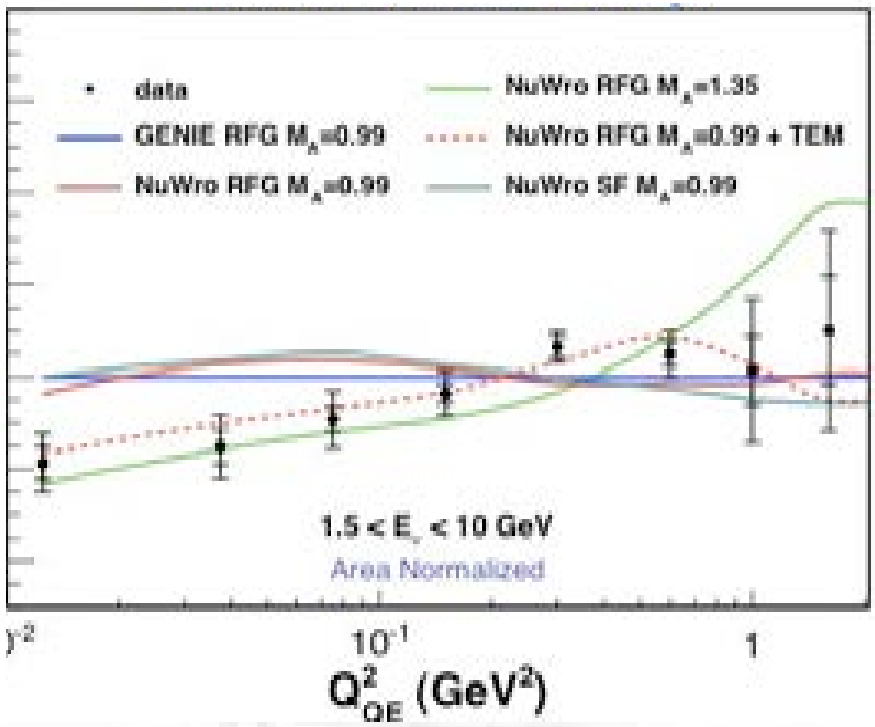
$d\sigma/dQ^2$ Shape



ν_μ CCQE

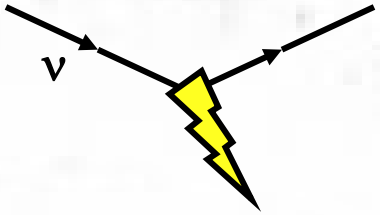


$\bar{\nu}_\mu$ CCQE

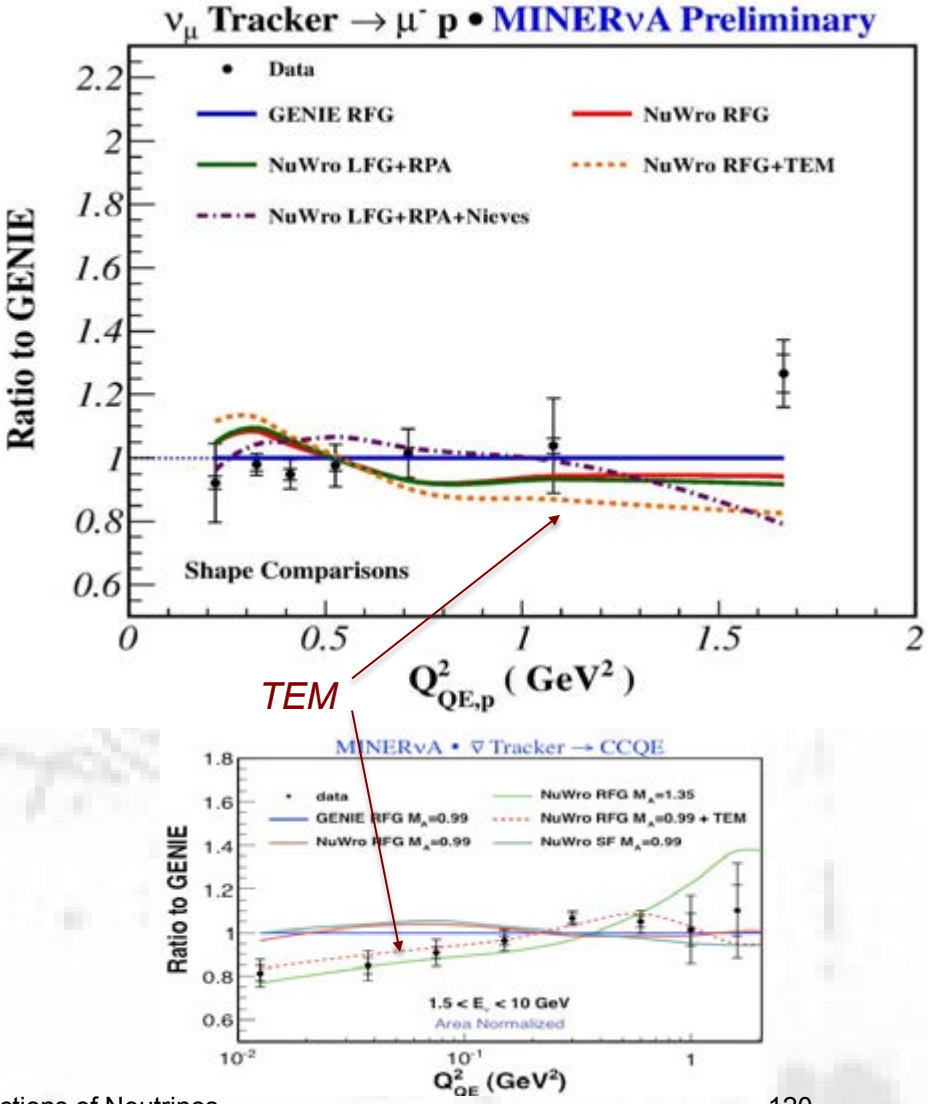


- Q^2 distribution doesn't agree well with "high effective M_A", but there is a clear disagreement with free nucleon result
- Best fit is to "transverse enhancement model"

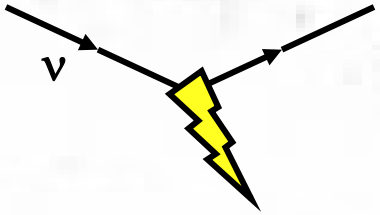
MINERvA μ+proton CCQE



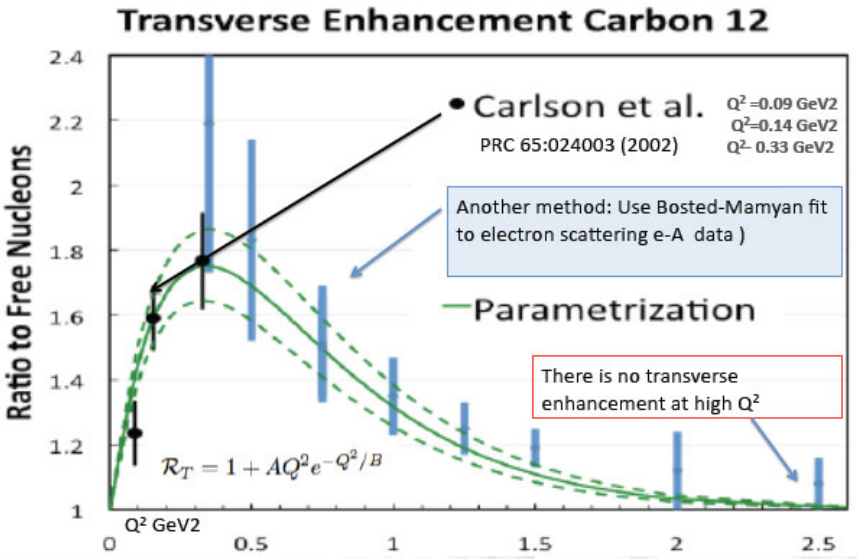
- MINERvA has also done a NOMAD-like measurement requiring the proton
- And… agrees with NOMAD data's preferred model instead of model preferred by MINERvA μ only CCQE
- Maybe (likely?) this is because of mismodeling of interactions of the proton leaving the nucleus?



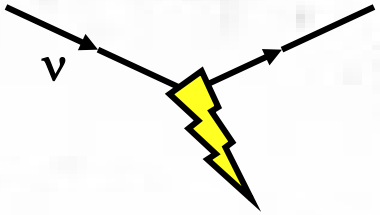
Multi-Nucleon Correlations



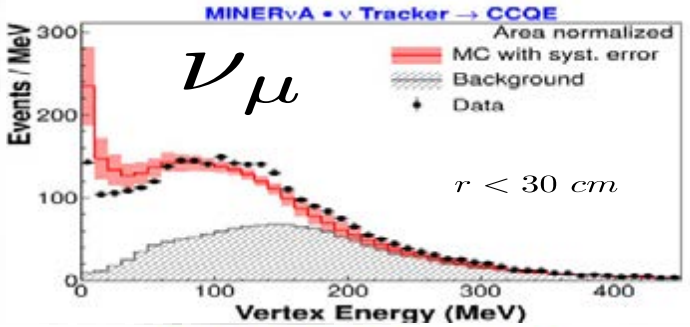
- Inclusion of correlations among nucleons in nucleus would add another quasielastic like process knocking two nucleons from nucleus
  - Could alter kinematics and rate in a way that would make a better fit to the data muon inclusive CCQE data
- How to implement?
  - Microphysical models don't yet give complete final state description
  - "Ad hoc" enhancement scaled from electron scattering data?
    (Carlson & Bodek, Budd, Christy)

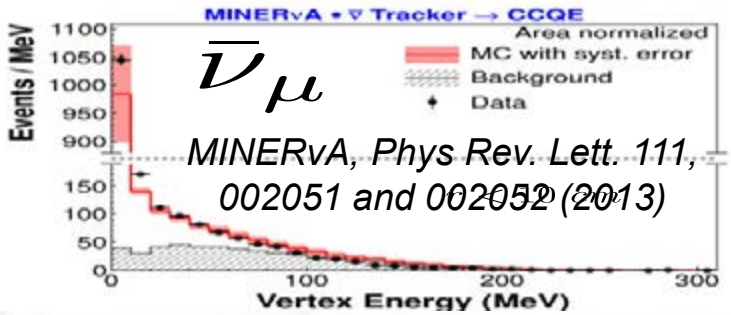


Extra final state protons from multinucleon effects?

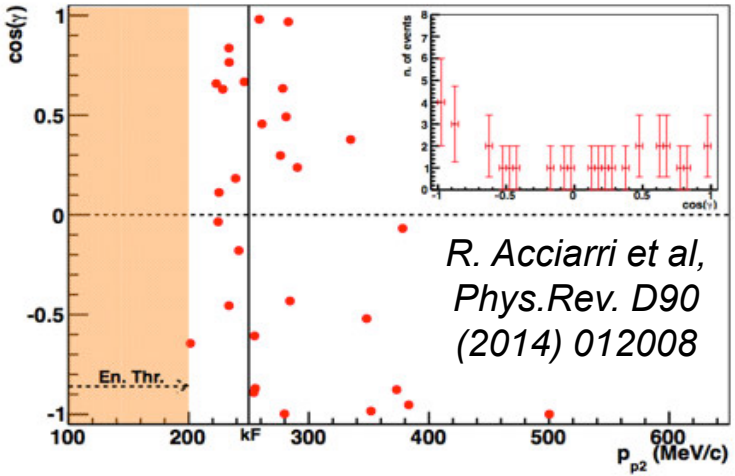


- MINERvA sees evidence of significant pp final states not in simulation from v beam, but no extra np in anti-v beam
- ArgoNeuT finds evidence of back-to-back protons which would be very to explain from final state interactions
- Interesting hints that multi-nucleon processes are present. Need more data!





MINERvA, Phys Rev. Lett. 111, 002051 and 002052 (2013)



R. Acciarri et al, Phys.Rev. D90 (2014) 012008

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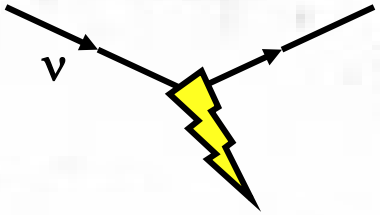

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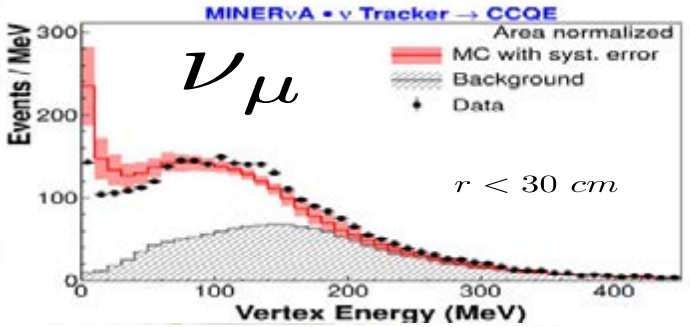
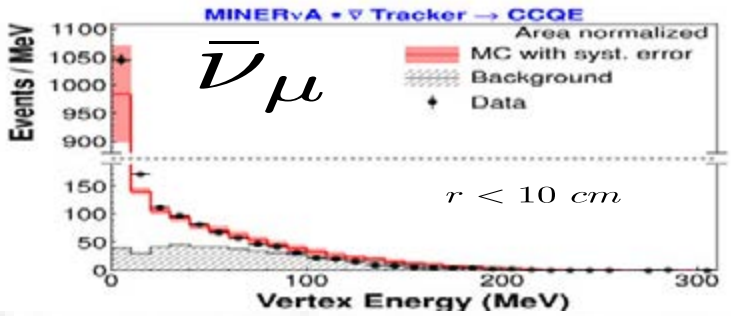
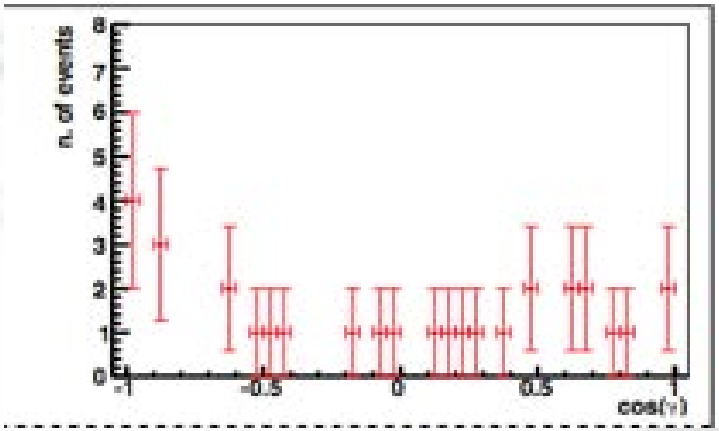
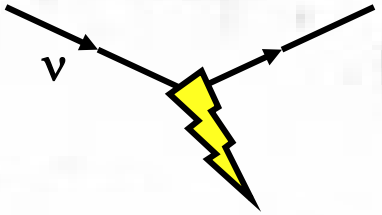


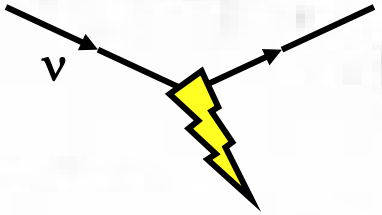




Summary of CCQE in Nuclear Targets

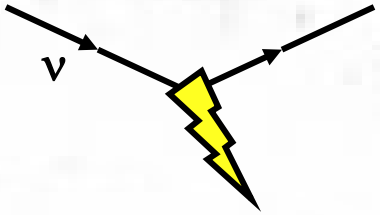


- There is evidence for nuclear modification of quasielastic neutrino-nucleon reactions
  - Kinematics of nucleons: Fermi motion, Pauli blocking
  - Multi-nucleon processes seem to also be present
- There are other models of nuclear effects
  - More complete nucleon kinematics (spectral function)
  - A suppression is expected at low Q^2 (long probe wavelength) from interactions of probe with multiple nuclei in “random phase approximation” calculations
- Some of these model contain overlapping physics effects! A challenge for the prediction.

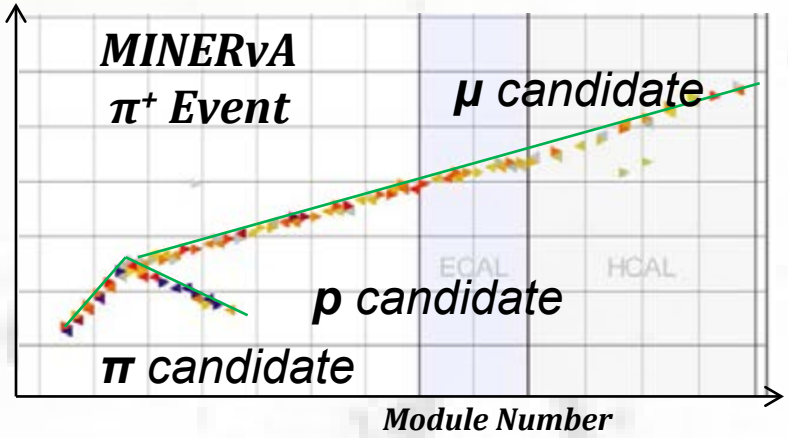


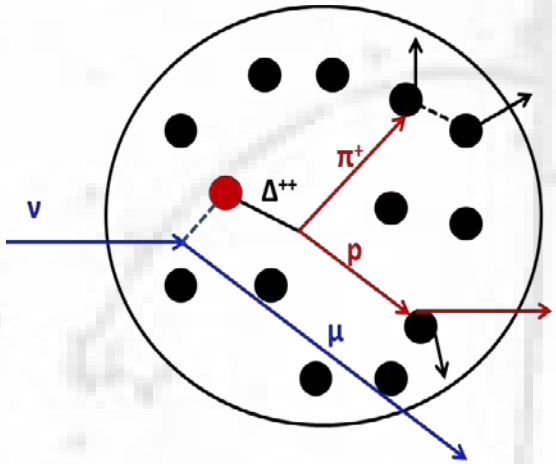
Studying Final State Interactions with Meson Production

Pion Production

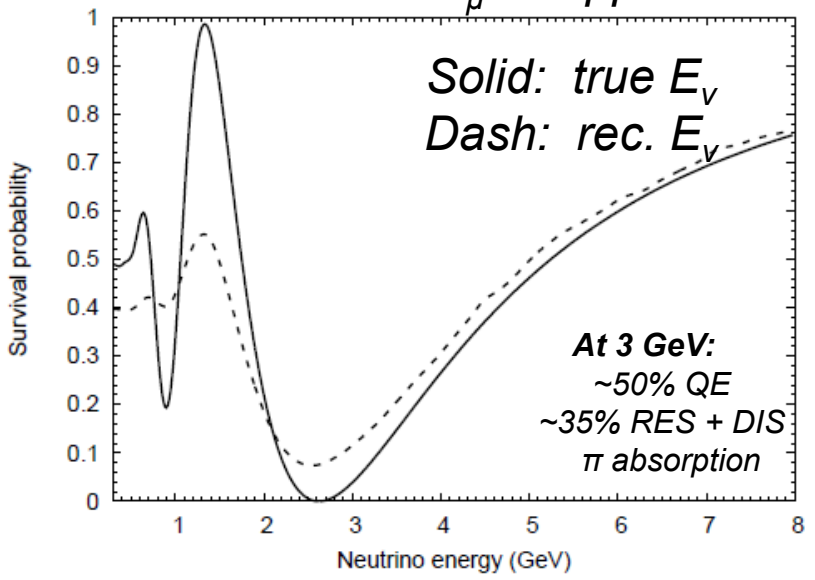


- Most common inelastic interaction at low energies
- Oscillation experiments that don't identify the pion suffer an energy bias
- Nuclear effects are important, both in initial and final state



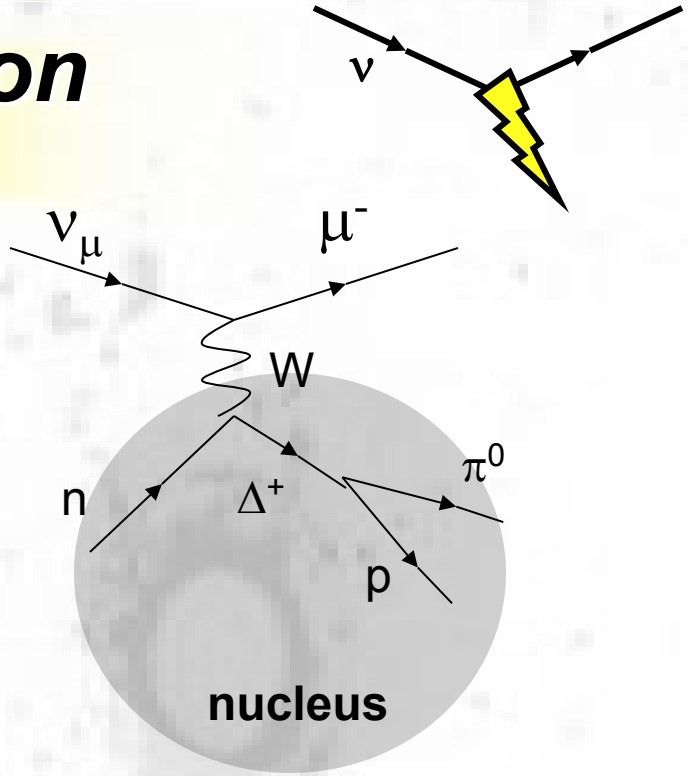


Simulated ELBNF ν_μ disappearance



Mosel et al: arxiv 1311.7288

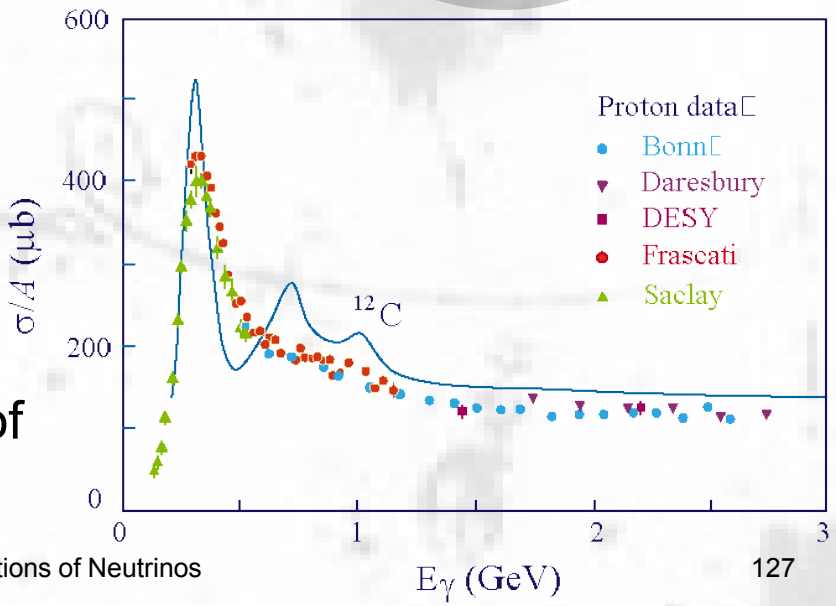
Nuclear Effects in Pion Production



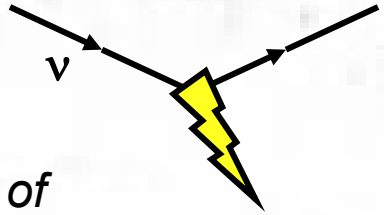
- An important reaction like

$$\nu_\mu n \rightarrow \mu^- p \pi^0$$

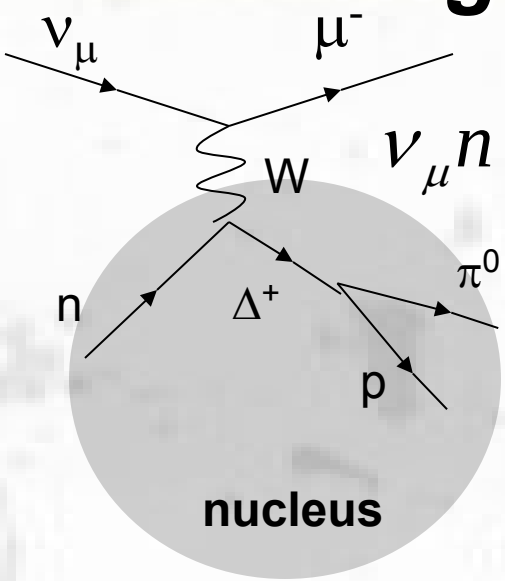
(ν_e background) can be modified in a nucleus
- Production kinematics are modified by nuclear medium
  - at right have photoabsorption showing resonance structure
  - line is proton; data is ^{12}C
  - except for first Δ peak, the structure is washed out
  - Fermi motion and interactions of resonance inside nucleus



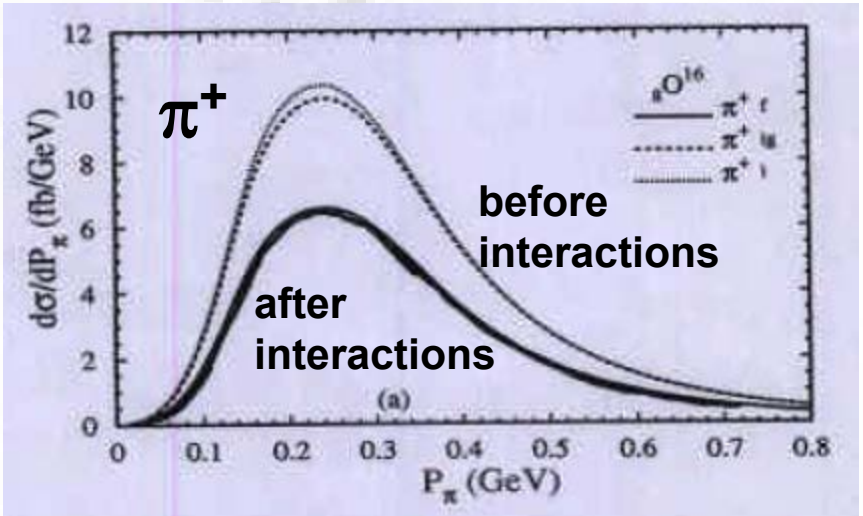
Nuclear Effects in Resonance Region (cont'd)

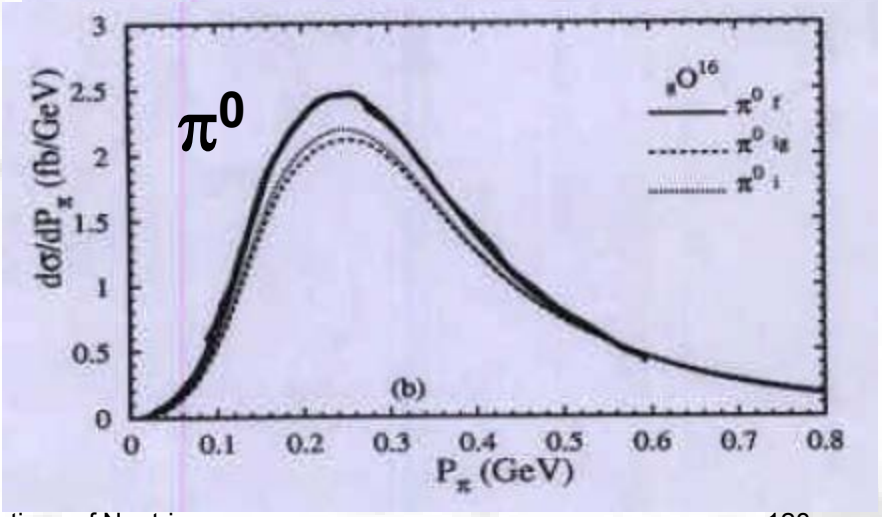


model of
E. Paschos, NUINT04



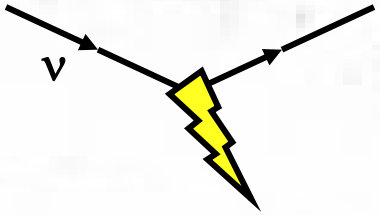
$$\nu_\mu n \rightarrow \mu^- p \pi^0$$





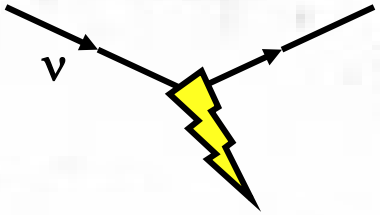
- How does nucleus affect π^0 after production?
- "Final State Interactions": migration of one state to another and pion absorption

Approaches to Final State Interactions



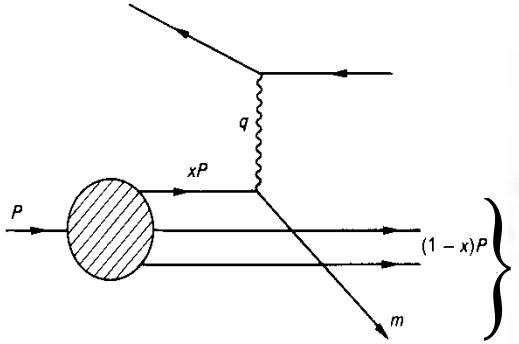
- Propagate final state particles through the nuclear medium with varying degrees of sophistication where they interact according the measured cross-sections or models
- Issues:
  - Are the hadrons modified by the nuclear medium?
  - Are hadrons treated as only on-shell or is off-shell transport allowed?
  - How to cleanly separate the initial state particles from their final state interactions?
  - How to relate scattering of external pions or nucleons from nuclei to scattering of particle created in nucleus?

Lecture Question #7



- **Two questions with (*hint*) related answers…**
  1. ***W*² is…**

$$W^2 = M_P^2 + 2M_P\nu - Q^2$$
$$= M_P^2 + 2M_P\nu(1-x)$$

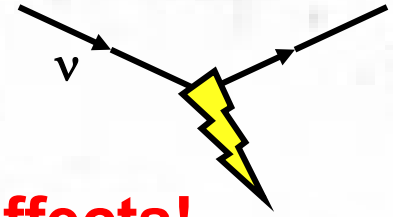


W^2

the square of the invariant mass of the hadronic system. ($\nu=E_\nu-E_\mu$; *x* is the parton fractional momentum)
It can be measured, as you see above with only leptonic quantities (neutrino and muon 4-momentum).
In neutrino scattering on a scintillator target, you observe an event with a recoiling proton and with W reconstructed (perfectly) from leptonic variables <M_p. Explain this event.

  2. **In the same scintillator target, you observe the reaction…** $\nu_\mu\ ^{12}\mathrm{C} \rightarrow \mu^- p\pi^-$ + remnant nucleus
Why might this be puzzling? Explain the process.

Lecture Question #7

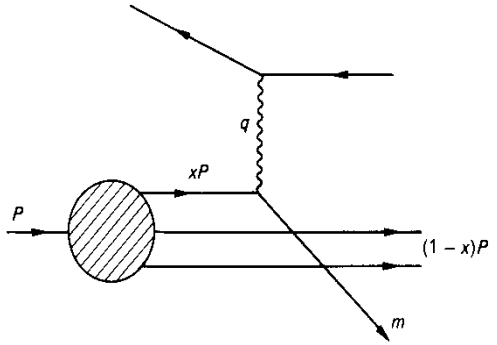


- **Both phenomena occur because of nuclear effects!**

1. $M_P > W^2 = M_P^2 + 2M_P\nu(1-x)$

   can only be true if x>1.

   That means the fractional momentum by the struck target parton is >1! This can only happen for in a nucleon boosted towards the collision in the CM frame by interactions within the nucleus ("Fermi momentum")



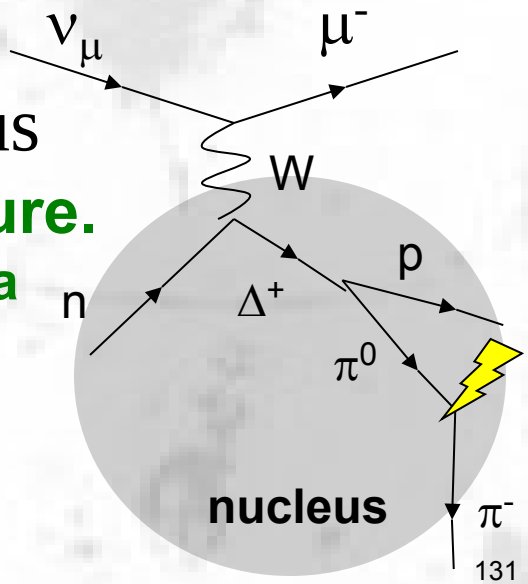
3. $\nu_\mu\ ^{12}\mathrm{C} \rightarrow \mu^- p\pi^-$ + remnant nucleus

   is nonsense in a free nucleon picture.

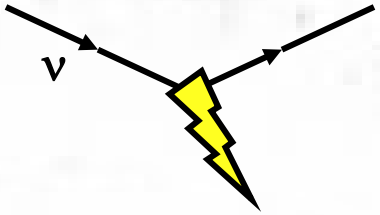
   It is forbidden to occur off of a proton or a neutron target by charge conservation!

   But remember…

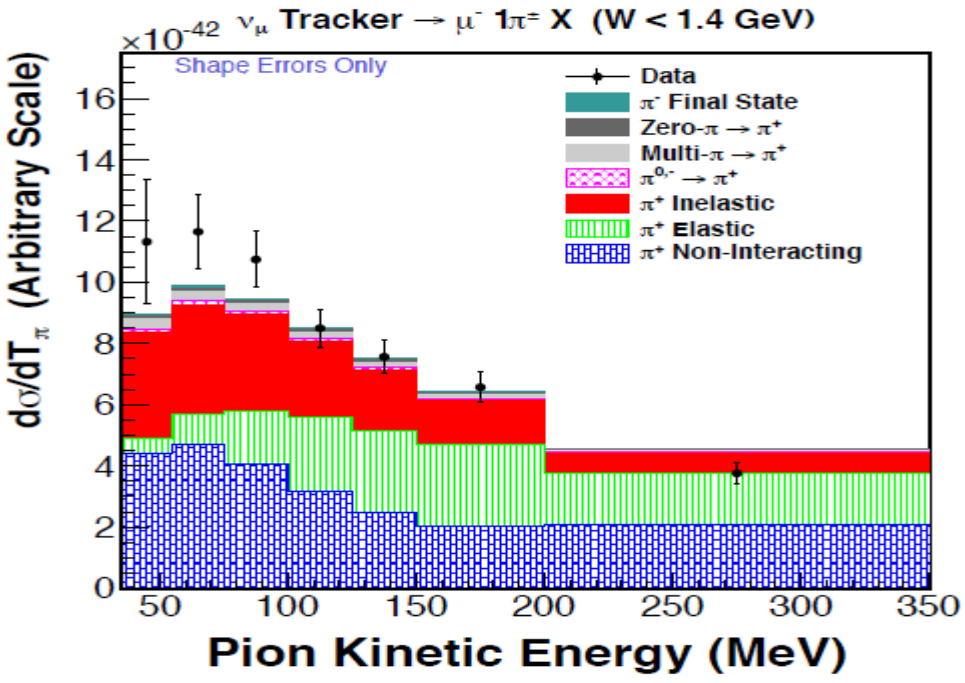
   reinteraction of pions!

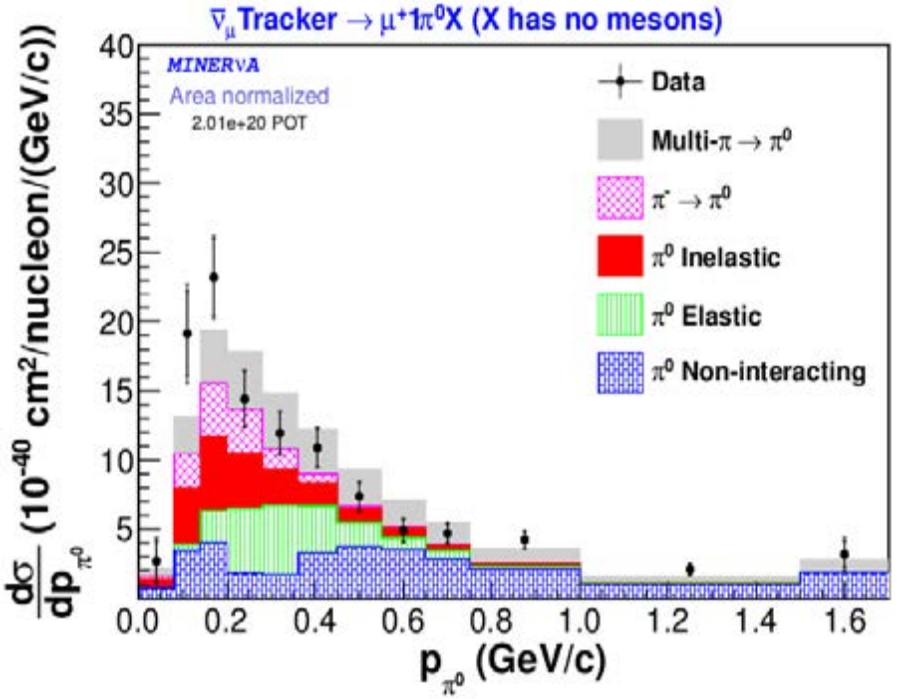


MINERvA: Pion Spectrum as Probe of Final State Effects

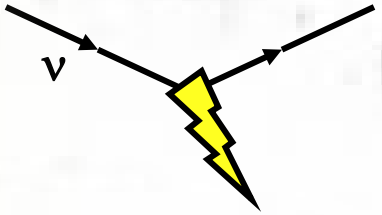


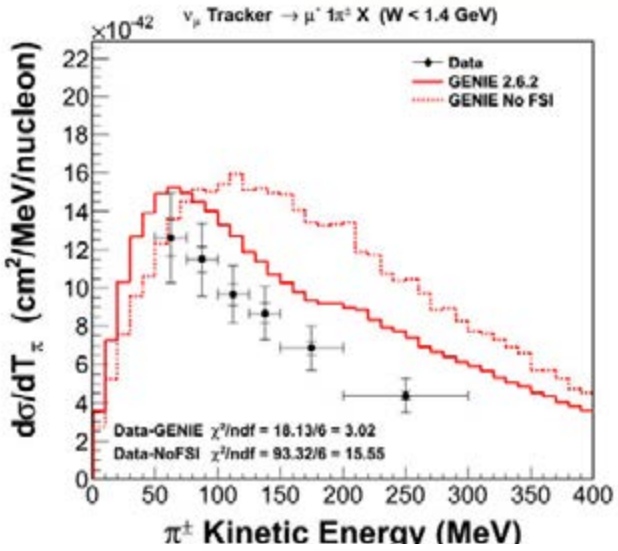
- MINERvA has measured both π^+ and π^0 production. Both prefer slightly softer pions than GENIE's final state cascade model predicts.

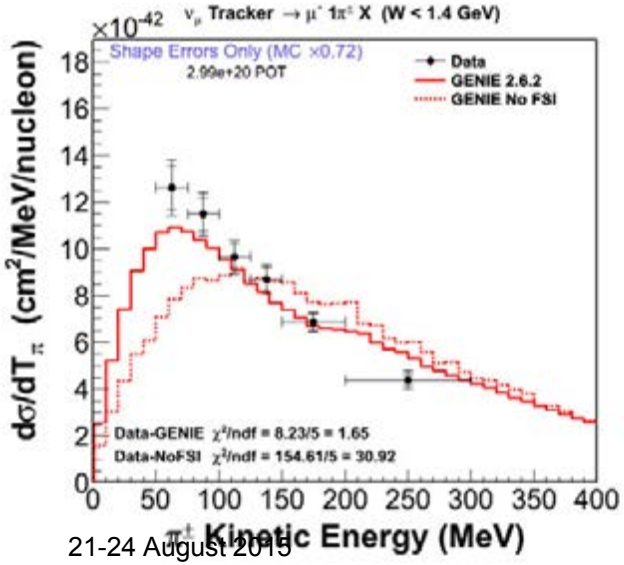


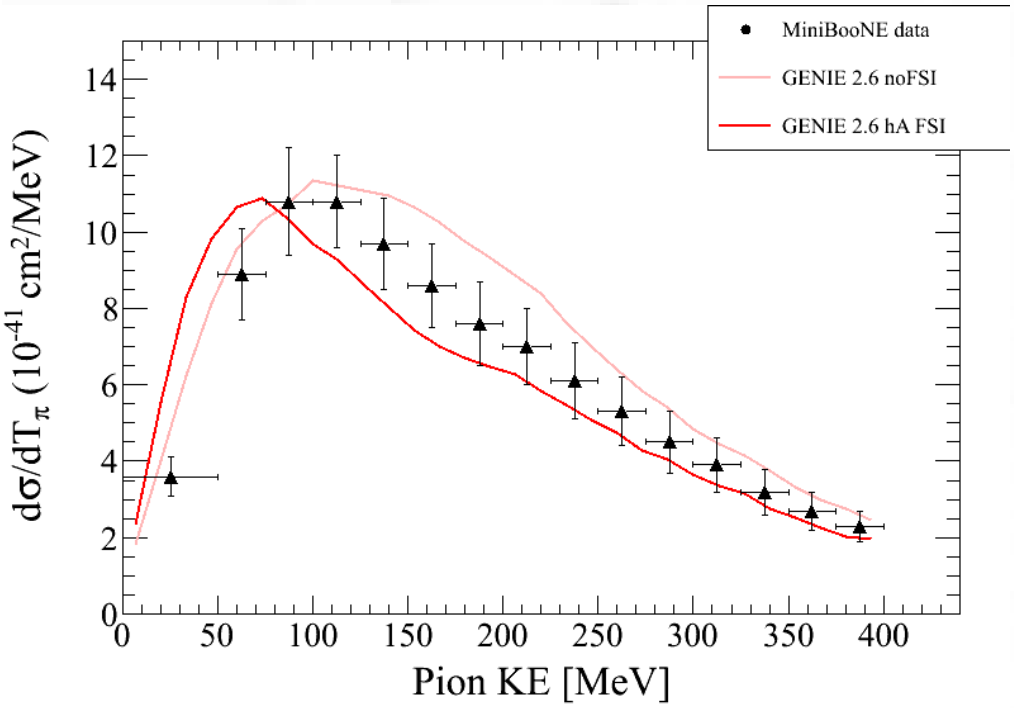


π^+ comparison to MiniBooNE



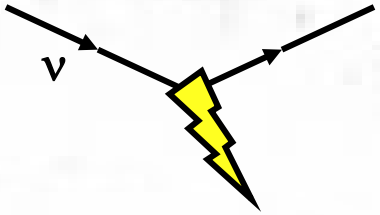






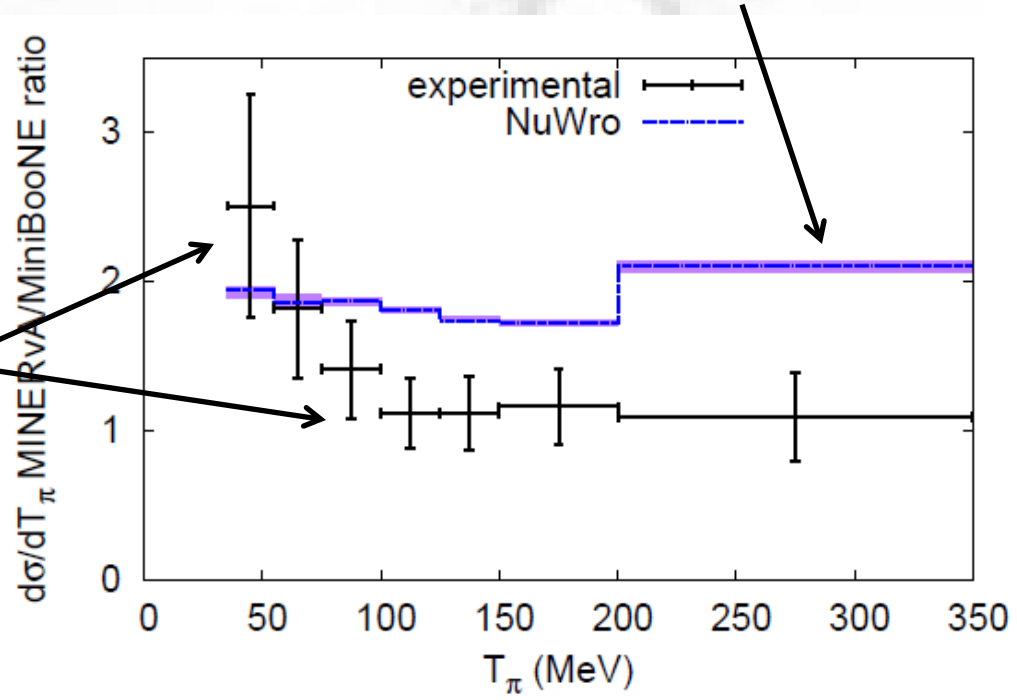
- Even with ~10% flux uncertainties from both experiments, there is ~2σ tension between MINERvA and MiniBooNE
- Shape tension also
- Note, MINERvA π^+ and π^0 are similar in rate and shape

Can Current Models Resolve this Tension?

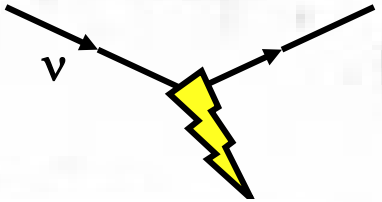


- Interesting study by Sobczyk and Zmuda (arXiV:1410.7788) asks if uncertainties in final state "cascade" models and pion production to explain MiniBooNE-MINERvA difference
- Their conclusion: it cannot. Theory uncertainties on the ratio are very small.

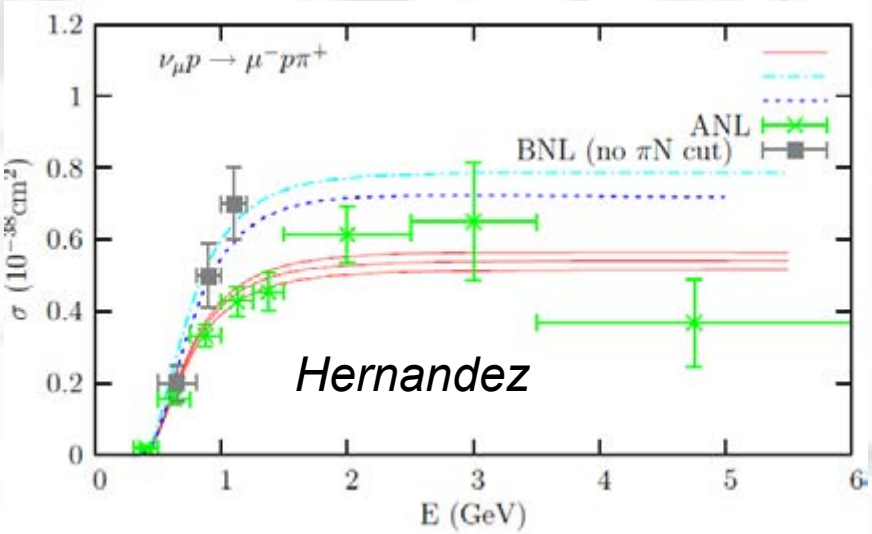
- *Uncertainties in bins are highly correlated, so maybe explains high energy part?*
- *And maybe low energy is a statistical fluctuation?*
- *Unlucky or real?*



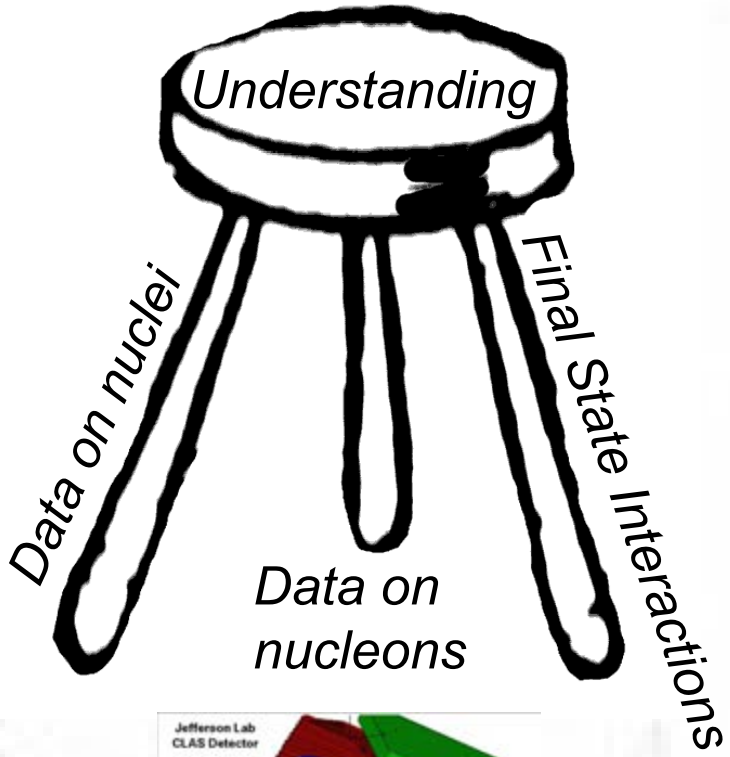
D_2 : Disappointing Data?

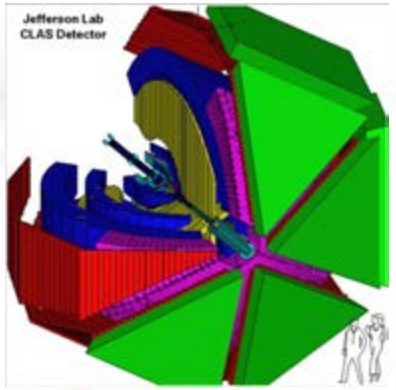


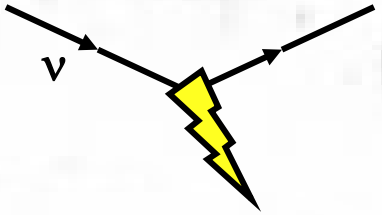
- Ideally to resolve our pion conundrum, we would go to *reliable* nucleon level data
  - Unfortunately, we don't have it.



- eN vs. eA data: our only hope for exclusive states?

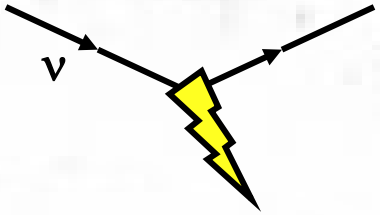




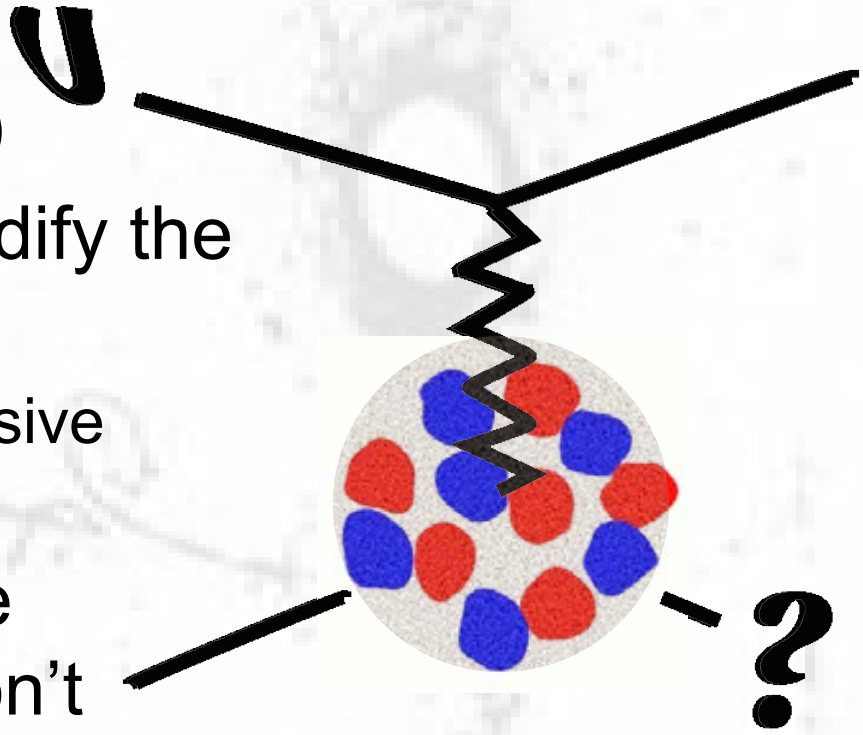


Nuclear Effects in Deep Inelastic Scattering

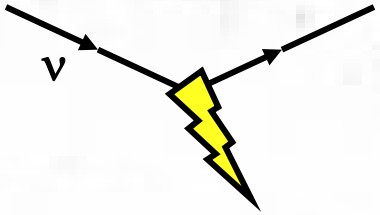
For Inclusive Scattering, Does Nucleus Matter?



- In high energy limit, calculate of strongly coupled system should be “easy”. However…
- Nucleon are not at rest in nucleus (Fermi motion)
- Nuclear medium may modify the structure of free nucleon
  - Evidence of this from inclusive charged lepton scattering
- Less important: final state interactions, since you don’t care about exclusive final states



Is the DIS Limit Simple?



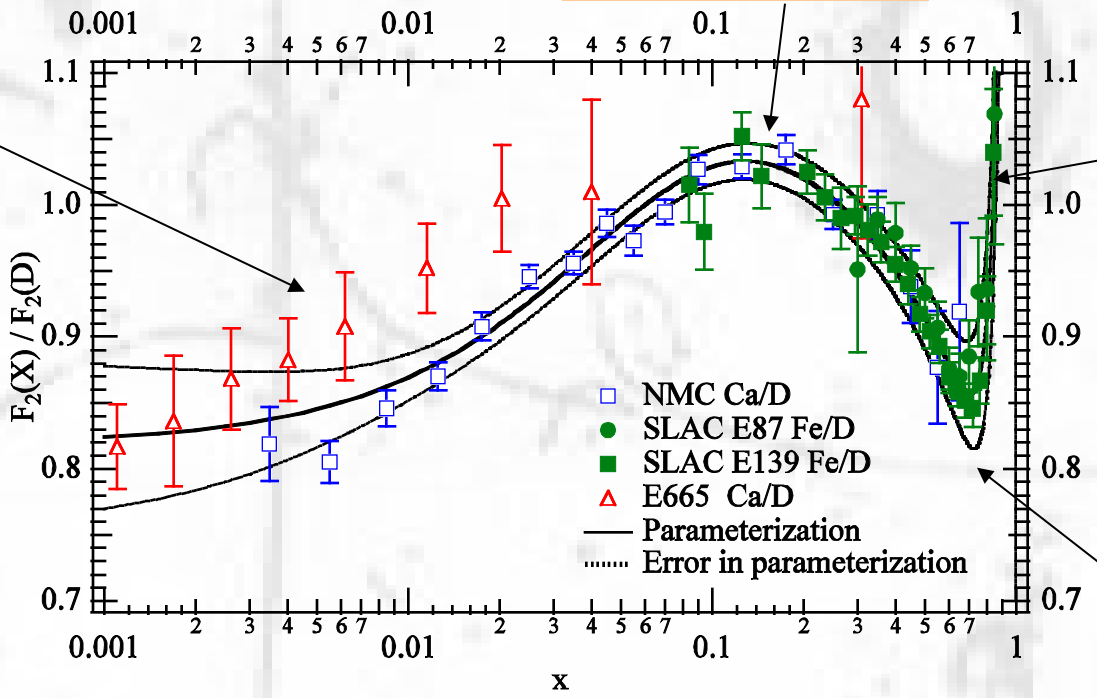
- Well measured effects in charged-lepton DIS
  - Maybe the same for neutrino DIS; maybe not… all precise neutrino data is on Ca or Fe targets!
  - Conjecture: these can be absorbed into effective nucleon PDFs in a nucleus

Anti-shadowing

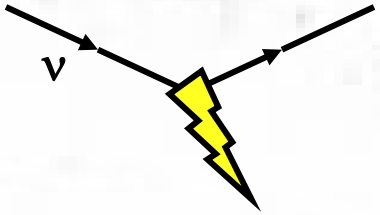
shadowing

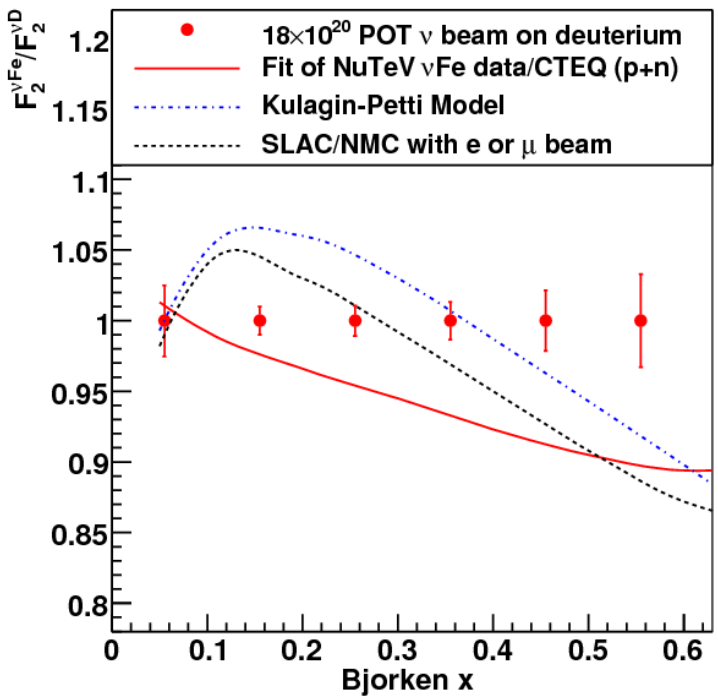
Fermi motion

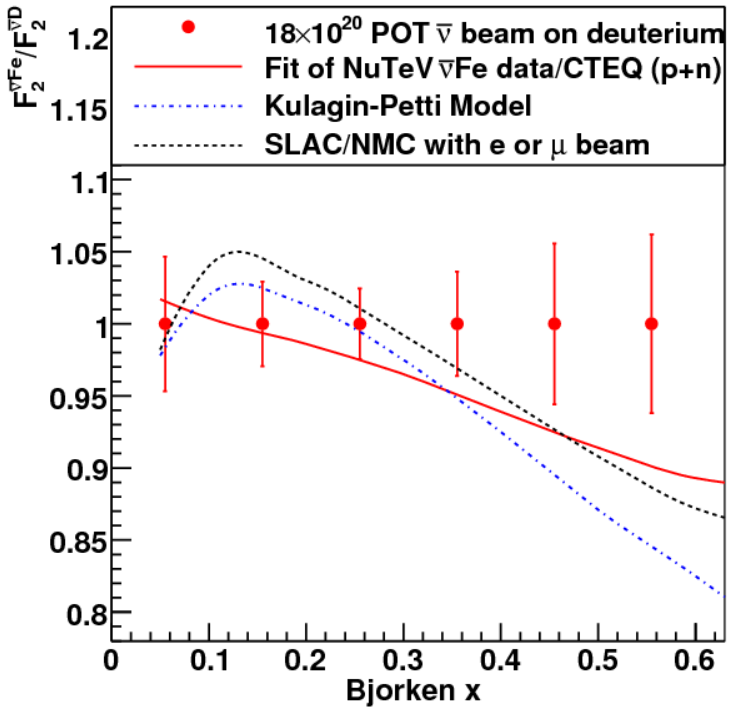
EMC effect



But that conjecture may be wrong…





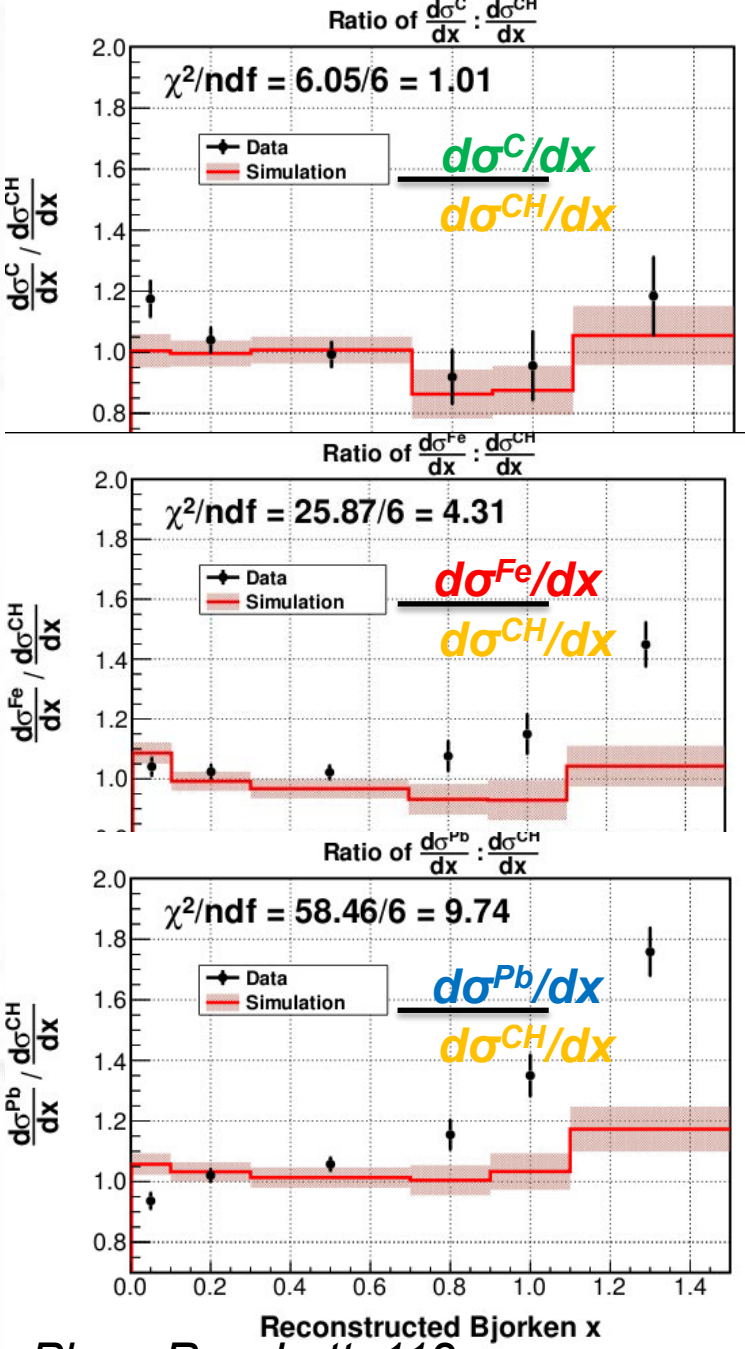


Curves from: Ingo Schienbein et al., Phys.Rev.D80(2009)094004; PRD77(2008)054013

- Only answer is to measure… red points *would have been* precision of MINERvA experiment *if it could have added a deuterium target in the NOvA running of NuMI beamline*.

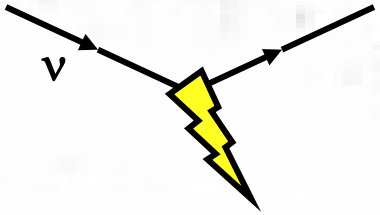
MINERvA Ratios vs x_{Bj}

- GENIE simulation has nuclear effects from N≠Z, Fermi Gas for exclusive processes, and DIS assuming same as "EMC" effects
- Modest disagreement at low x suggests impact of shadowing or anti-shadowing differences?
- High x dramatic disagreement is dominated by elastic or nearly elastic events
  - x>1 is from resolution. Final state energy reconstruction a culprit?
  - Dramatic failure of RFG at high A?

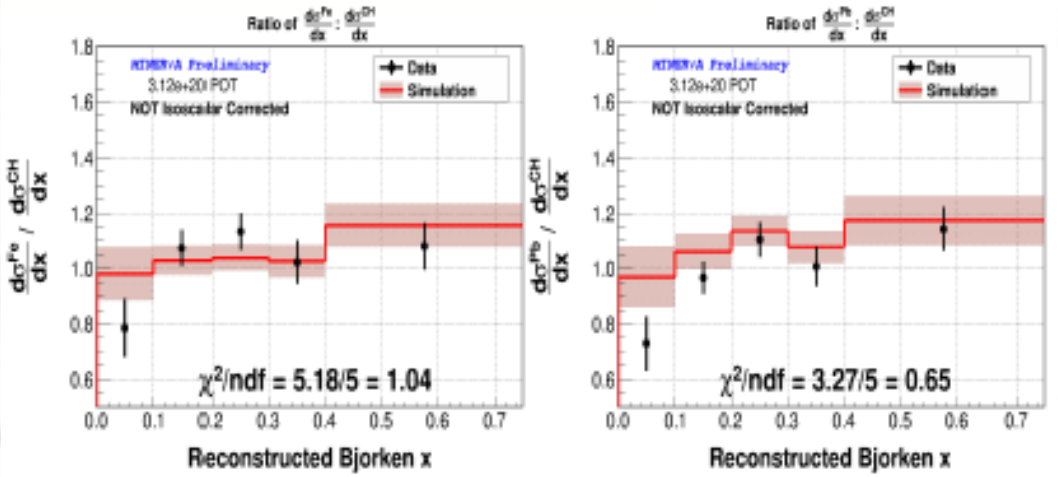


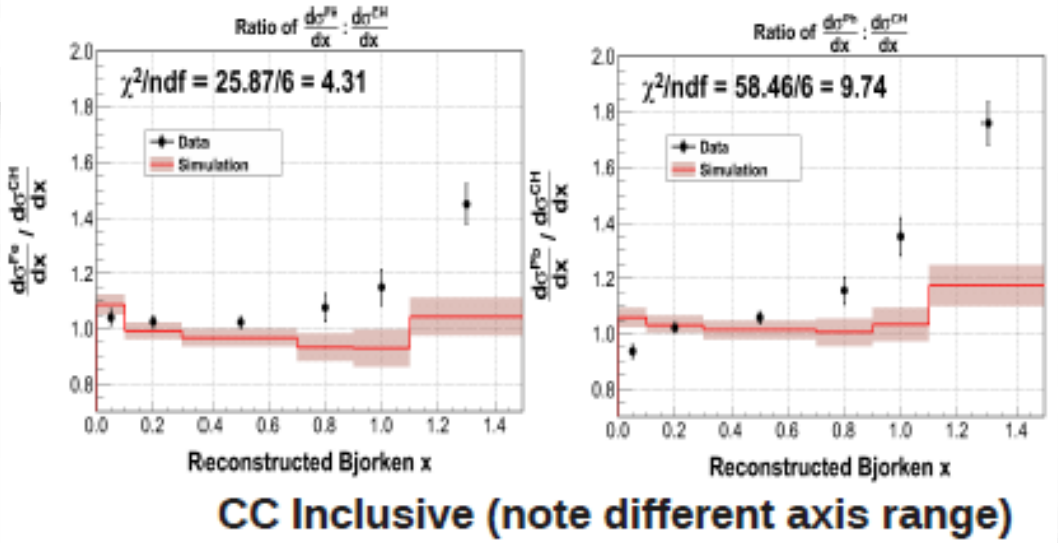
Phys. Rev. Lett. 112, 231801 (2014)

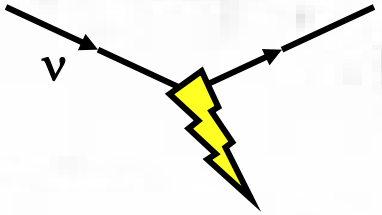
Elastic or Not at high x_{Bj}?



- Events at high x have a large contribution from quasielastic scattering
- If we require inelastic kinematics, maybe effect is gone?
  - But statistics are limited, for now.

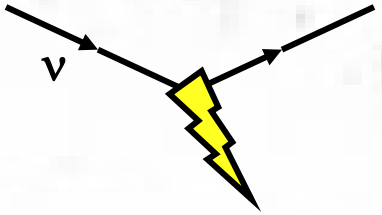


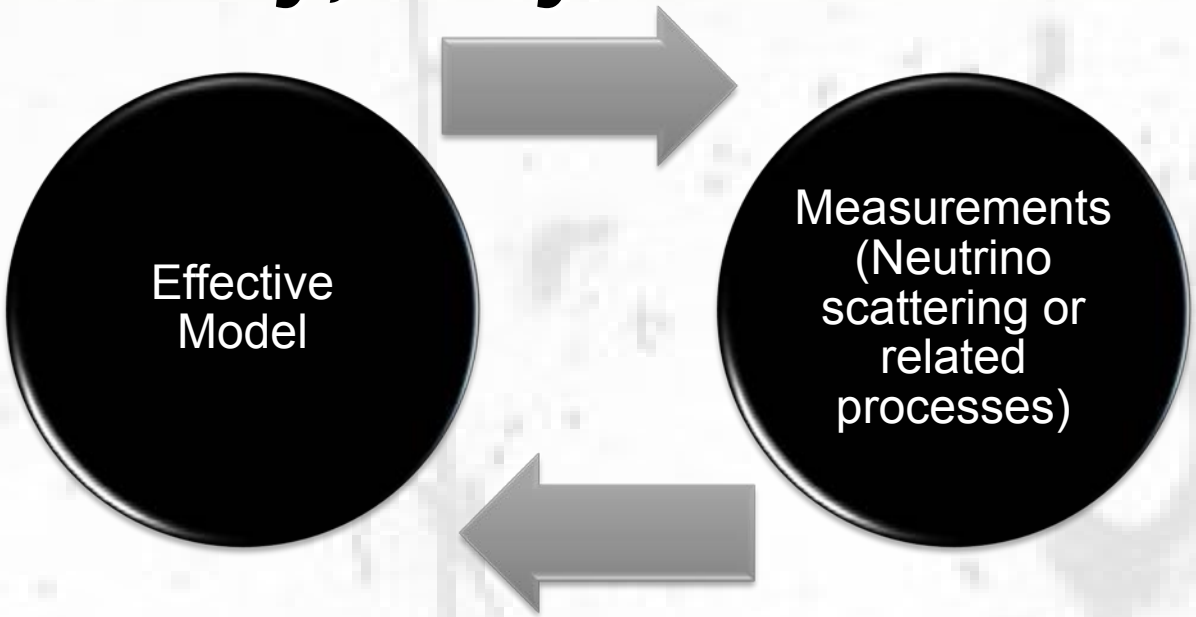




Thoughts on Effective Models and Neutrino Interaction Generators

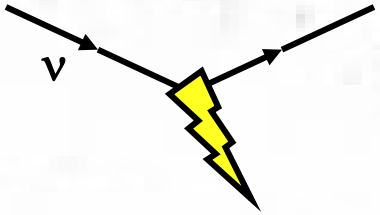
The Problem of the Nucleus is Very, Very Hard



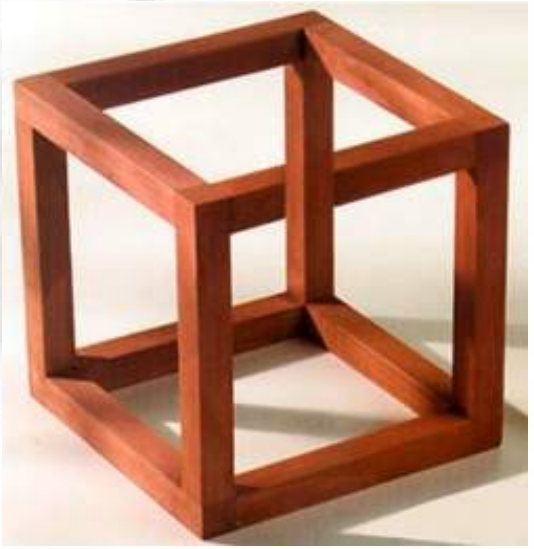


- Our iterative process uses data to improve models
- Our models are effective theories, ranging from pure parameterizations of data to microphysical models with simplifying assumptions.
  - "Effective" has both positive and negative meanings, but in particular here I mean that these are not first-principles calculations from QCD.

The Mosel Paradox



We don't have models which fit (all) the available data, although many models provide valuable insight into features of this data

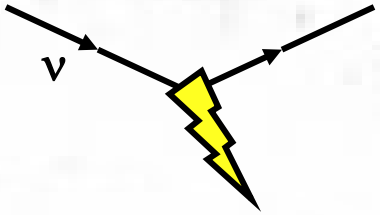


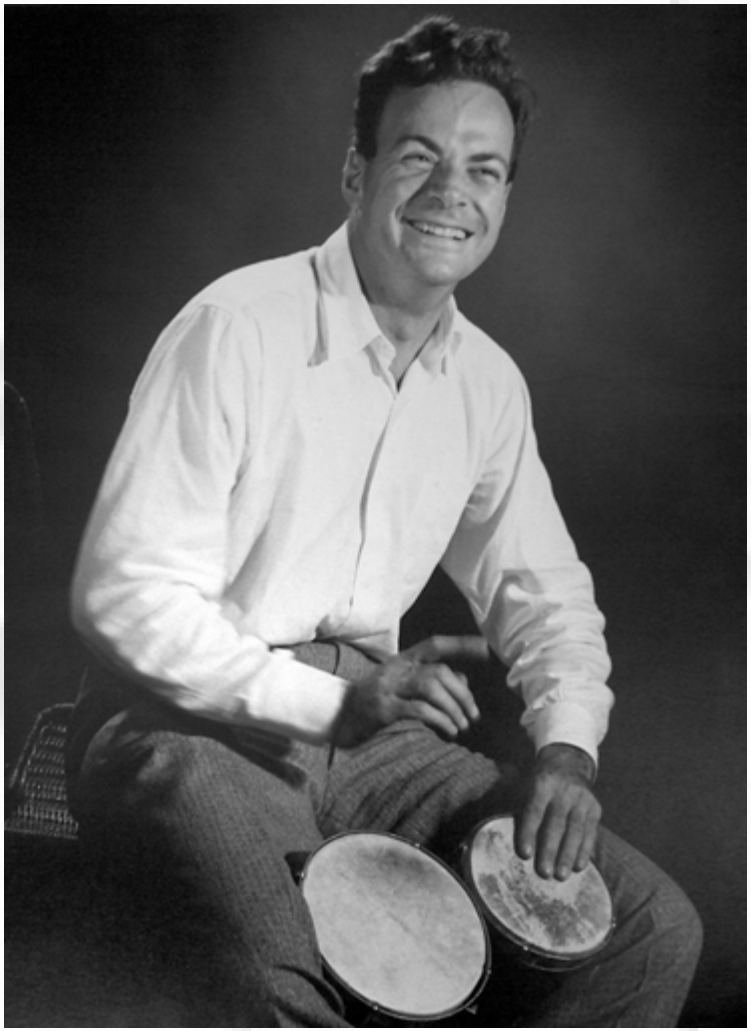
Theorist's paradigm: "A good generator does not have to fit the data, provided [its model] is right"

Experimentalist's paradigm: "A good generator does not have to be right, provided it fits the data"

Ulrich Mosel, first articulated at NuINT11 conference

Feynman Weighs In...





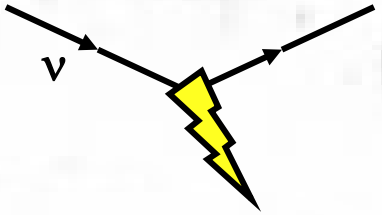
“It doesn't matter how beautiful your theory is; it doesn't matter how smart you are.

If it doesn't agree with experiment, it's wrong.”

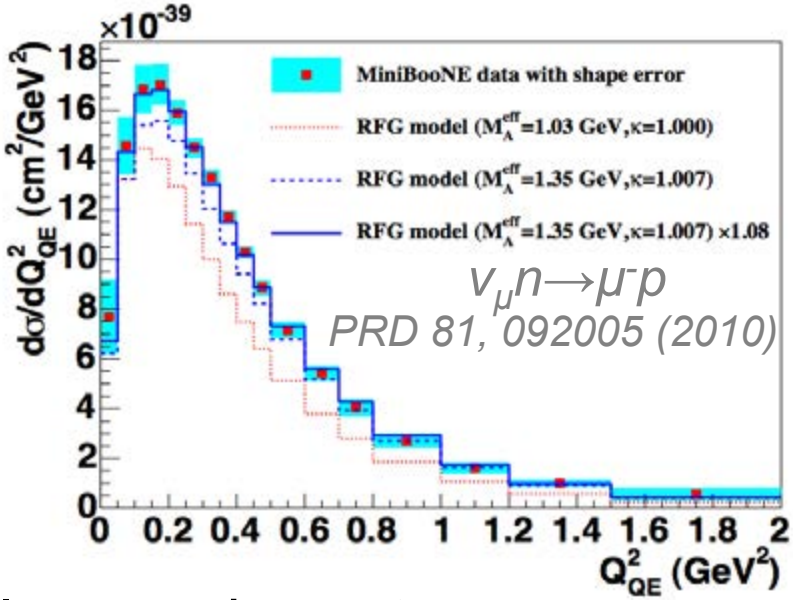
— Richard Feynman

This is surely true, but invalidating one side of an argument doesn’t make the other side correct!

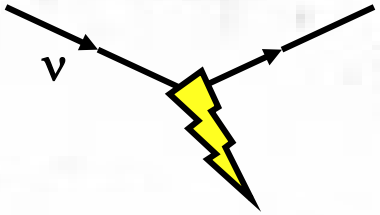
Counter Argument



- *Experimentalists can do (and have done, and will do) shameful things when confronted with data and model disagreements!*
- MiniBooNE oscillation analysis approach:
  - Modify the dipole axial mass and Pauli blocking until model fits data.
  - But there is nothing fundamental backing this approach. It's a mechanical convenience to parameterize the data for the oscillation analysis.

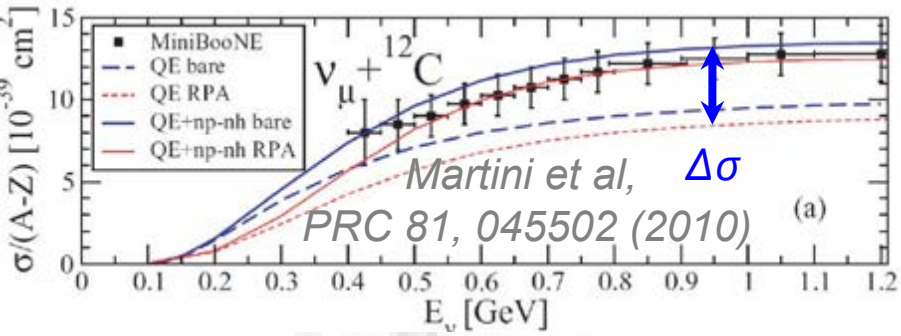


Counter Argument (cont'd)

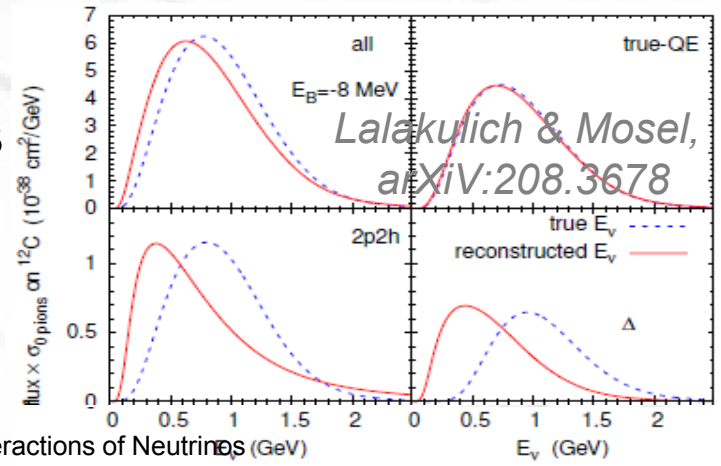


- What we now believe about the MiniBooNE oscillation analysis approach:
  - In a simplistic view, there are neglected contributions from multi-nucleon pairs.
  - Those pairs alter the kinematics.
  - MiniBooNE got its energy reconstruction wrong by picking the wrong physics to modify.
  - OK within uncertainties? If so, only by luck.

$\nu_\mu n \rightarrow \mu^- p$ + $\nu_\mu (np)_{corr.} \rightarrow \mu^- pp$



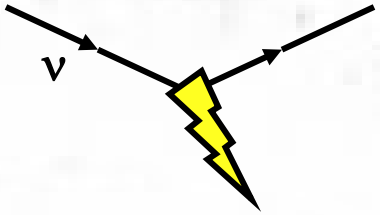
Martini et al, PRC 81, 045502 (2010)

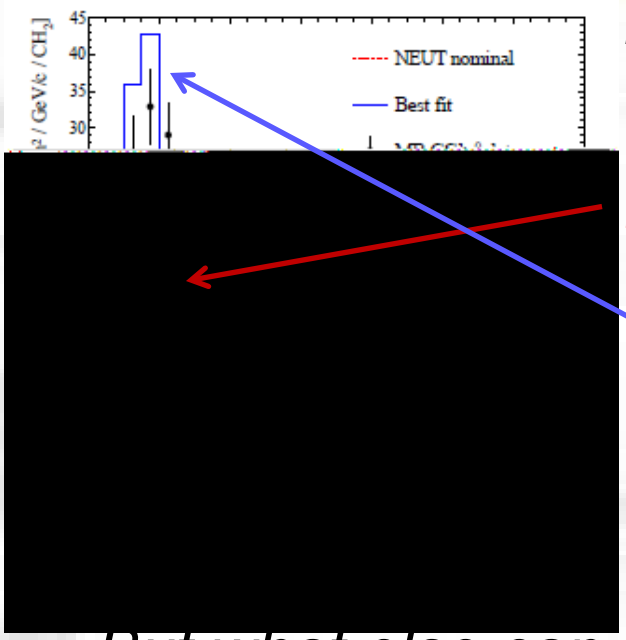


Lalakulich & Mosel, arXiV:208.3678

Also demonstrated by Nieves, Ankowski here at NuINT12

Counter Argument (cont'd)





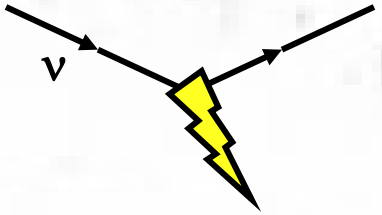
P. Rodrigues, NuFact 2012 and NuINT12

Rein-Sehgal
[Ann. Phys. 133, 79-153 (1981)]
implementation in NEUT

"Tuned" Rein-Sehgal to modify Q^2 distribution, pion spectrum, rate

- *But what else can experimentalists do?* ***Mea*** *culpa.*
- T2K finds poor agreement between Rein-Sehgal and MiniBooNE $\nu_\mu N \rightarrow \mu^- \pi^{(+)0} N^{(')}$ and $\nu_\mu N \rightarrow \nu_\mu \pi^0 N$ data.
- A*d hoc* tuning "breaks" assumptions of underlying model, e.g. CC-NC universality of process and relation among resonances, to force good agreement.

Structure Functions (SFs)



- A model-independent picture of these interactions can also be formed in terms of nucleon "structure functions"
  - All Lorentz-invariant terms included
  - Approximate zero lepton mass (small correction)

$$\frac{d\sigma^{\nu,\bar{\nu}}}{dxdy} \propto \left[y^2 2xF_1(x,Q^2) + \left(2 - 2y - \frac{M_T xy}{E} \right) F_2(x,Q^2) \pm y(2-y)xF_3(x,Q^2) \right]$$

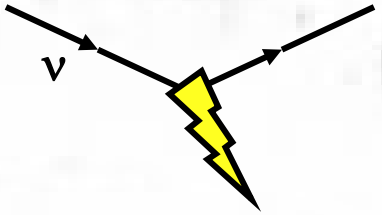
- For massless free spin-1/2 partons, one simplification…
  - Callan-Gross relationship, $2xF_1=F_2$
  - Implies intermediate bosons are completely transverse

Can parameterize longitudinal cross-section by R_L.

Callan-Gross violations result from M_T, NLO pQCD, $g \to qq$

$$R_L = \frac{\sigma_L}{\sigma_T} = \frac{F_2}{2xF_1}\left(1 + \frac{4M_T^2 x^2}{Q^2}\right)$$

SFs to PDFs



- Can relate SFs to PDFs in naïve quark-parton model by matching y dependence
  - Assuming Callan-Gross, massless targets and partons…
  - F_3: $2y-y^2=(1-y)^2-1$, $2xF_1=F_2$: $2-2y+y^2=(1-y)^2+1$

$$2xF_1^{\nu p,CC} = x\left[d_p(x)+\overline{u_p}(x)+s_p(x)+\overline{c_p}(x)\right]$$

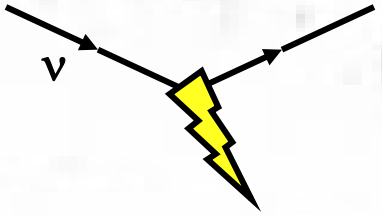
$$xF_3^{\nu p,CC} = x\left[d_p(x)-\overline{u_p}(x)+s_p(x)-\overline{c_p}(x)\right]$$

- In analogy with neutrino-electron scattering, CC only involves left-handed quarks
- However, NC involves both chiralities (V-A and V+A)
  - Also couplings from EW Unification
  - And no selection by quark charge

$$2xF_1^{\nu p,NC} = x\left[(u_L^2+u_R^2)\left(u_p(x)+\overline{u_p}(x)+c_p(x)+\overline{c_p}(x)\right)+(d_L^2+d_R^2)\left(d_p(x)+\overline{d_p}(x)+s_p(x)+\overline{s_p}(x)\right)\right]$$

$$xF_3^{\nu p,NC} = x\left[(u_L^2-u_R^2)\left(u_p(x)-\overline{u_p}(x)+c_p(x)-\overline{c_p}(x)\right)+(d_L^2-d_R^2)\left(d_p(x)-\overline{d_p}(x)+s_p(x)-\overline{s_p}(x)\right)\right]$$

Isoscalar Targets



- Heavy nuclei are roughly neutron-proton isoscalar
- Isospin symmetry implies $u_p = d_n, d_p = u_n$
- Structure Functions have a particularly simple interpretation in quark-parton model for this case…

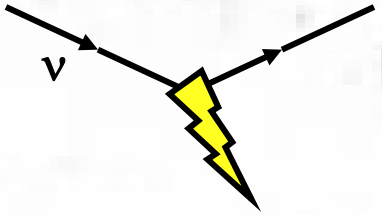
$$\frac{d^2\sigma^{\nu(\bar{\nu})N}}{dxdy} = \frac{G_F^2 s}{2\pi}\left\{ \left(1+(1-y)^2\right) F_2(x) \pm \left(1-(1-y)^2\right) xF_3^{\nu(\bar{\nu})}(x)\right\}$$

$$2xF_1^{\nu(\bar{\nu})N,CC}(x) = x(u(x)+d(x)+\bar{u}(x)+\bar{d}(x)+s(x)+\bar{s}(x)+c(x)+\bar{c}(x) = xq(x)+x\bar{q}(x)$$

$$xF_3^{\nu(\bar{\nu})N,CC}(x) = xu_{Val}(x)+xd_{Val}(x) \pm 2x(s(x)-c(x))$$

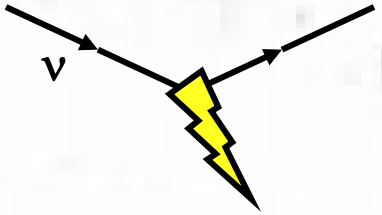
$$\text{where } u_{Val}(x) = u(x)-\bar{u}(x)$$

From SFs to PDFs



- As you all know, there is a large industry in determining Parton Distributions for hadron collider simulations.
  - to the point where some of my colleagues on collider experiments might think of parton distributions as an annoying piece of FORTRAN code in their software package
- The purpose, of course, is to use factorization to predict cross-sections for various processes
  - combining deep inelastic scattering data from various sources together allows us to “measure” parton distributions
  - which then are applied to predict hadron-hadron processes at colliders, and can also be used in predictions for neutrino scattering, as we shall see.

From SFs to PDFs (cont'd)



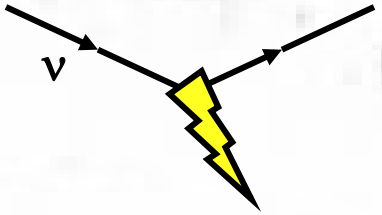
- We just learned that…

$$2xF_1^{\nu(\bar{\nu})N,CC}(x) = xq(x) + x\bar{q}(x)$$
$$xF_3^{\nu(\bar{\nu})N,CC}(x) = xu_{Val}(x) + xd_{Val}(x) \pm 2x(s(x) - c(x))$$
$$\text{where } u_{Val}(x) = u(x) - \bar{u}(x)$$

- In charged-lepton DIS

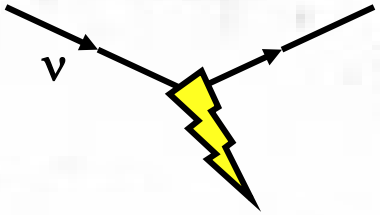
$$2xF_1^{\gamma p}(x) = \left(\tfrac{2}{3}\right)^2 \sum_{\text{up type quarks}} q(x) + \bar{q}(x) + \left(\tfrac{1}{3}\right)^2 \sum_{\text{down type quarks}} q(x) + \bar{q}(x)$$

- So you begin to see how one can combine neutrino and charged lepton DIS and separate
  - the quark sea from valence quarks
  - up quarks from down quarks

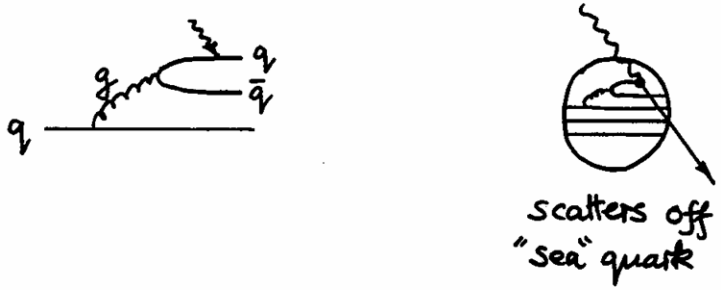


SUPPLEMENT:
Scaling Violations of Partons

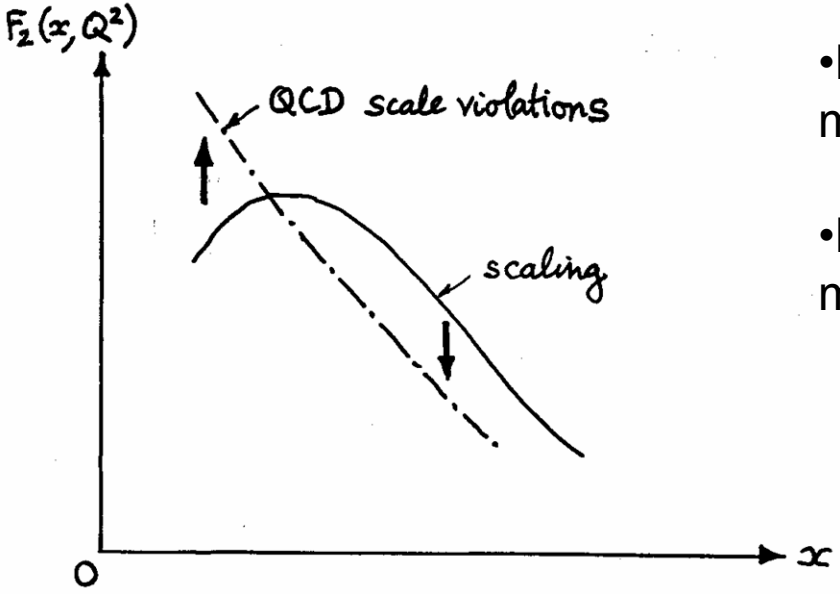
Strong Interactions among Partons



Q^2 Scaling fails due to these interactions



$$\frac{\partial q(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{qq}\left(\frac{x}{y}\right)q(y,Q^2) + P_{qg}\left(\frac{x}{y}\right)g(y,Q^2)\right]$$



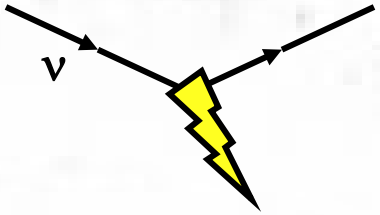
- Pqq(x/y) = probability of finding a quark with momentum x within a quark with momentum y

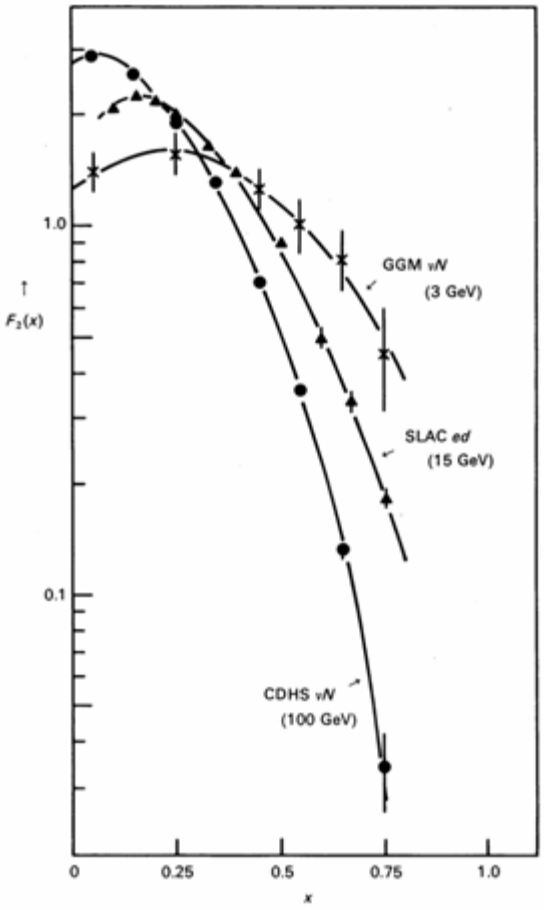
- Pqq(x/y) = probability of finding a q with momentum x within a gluon with momentum y

$$P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{(1-z)} + 2\delta(1-z)$$

$$P_{gq}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

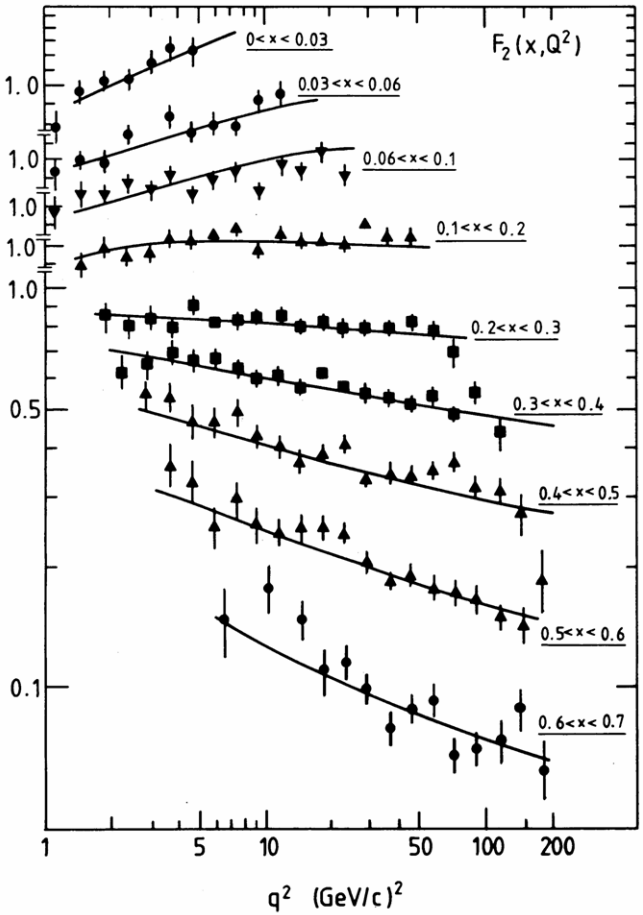
Scaling from QCD

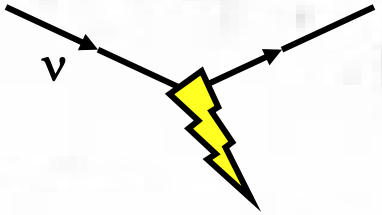




Observed quark distributions vary with Q^2

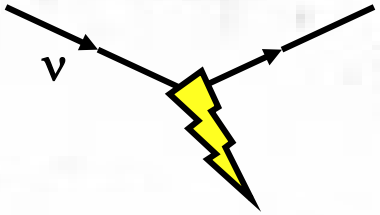
Scaling well modeled by perturbative QCD with a single free parameter (α_s)



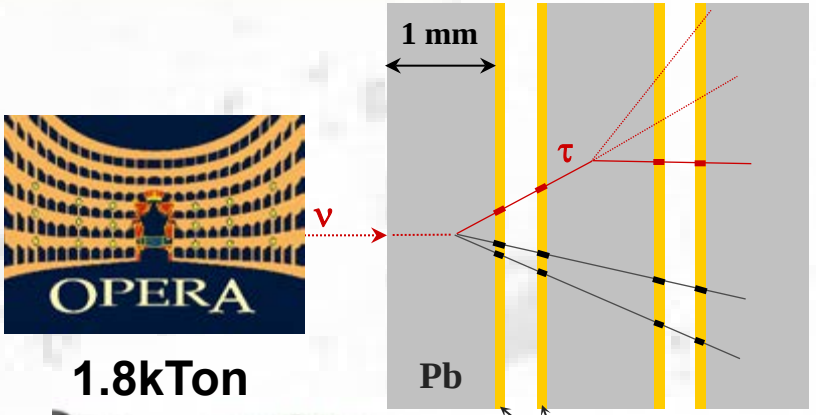


SUPPLEMENT: Massive Leptons (Taus) and Quarks (Charm) in DIS

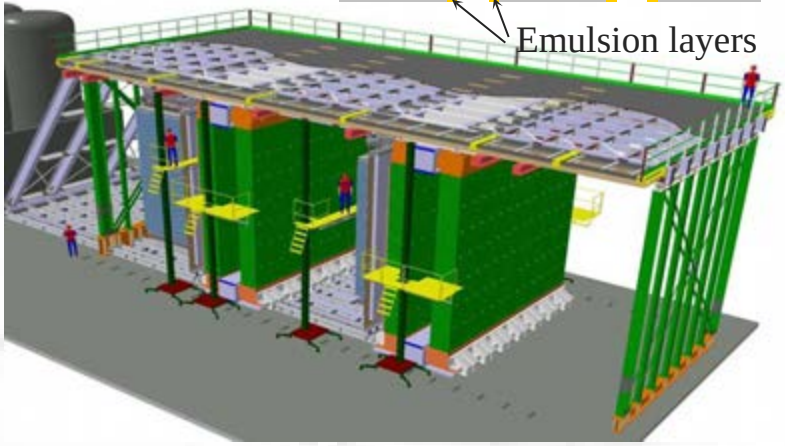
Opera at CNGS



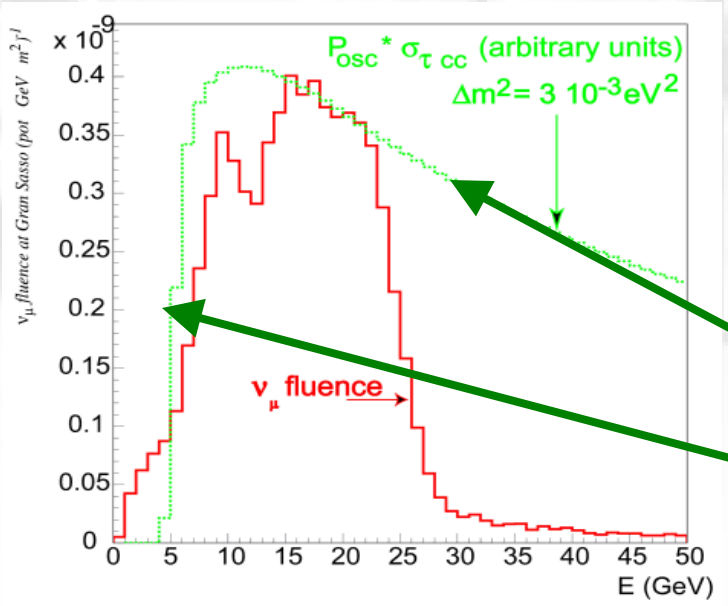
Goal: ν_τ appearance
- 0.15 MWatt source
- high energy ν_μ beam
- 732 km baseline
- handfuls of events/yr



1.8kTon



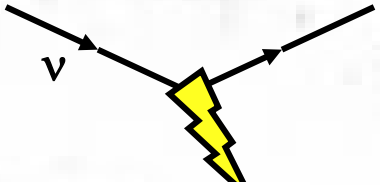
figures courtesy D. Autiero



oscillation probability

but what is this effect?

Lepton Mass Effects in DIS



- Recall that final state mass effects enter as corrections:

$$1-\frac{m^2_{\text{lepton}}}{s_{\text{point-like}}} \rightarrow 1-\frac{m^2_{\text{lepton}}}{xs_{\text{nucleon}}}$$

  - *relevant center-of-mass energy is that of the "point-like" neutrino-parton system*
  - *this is high energy approximation*
- For ν_τ charged-current, there is a threshold of

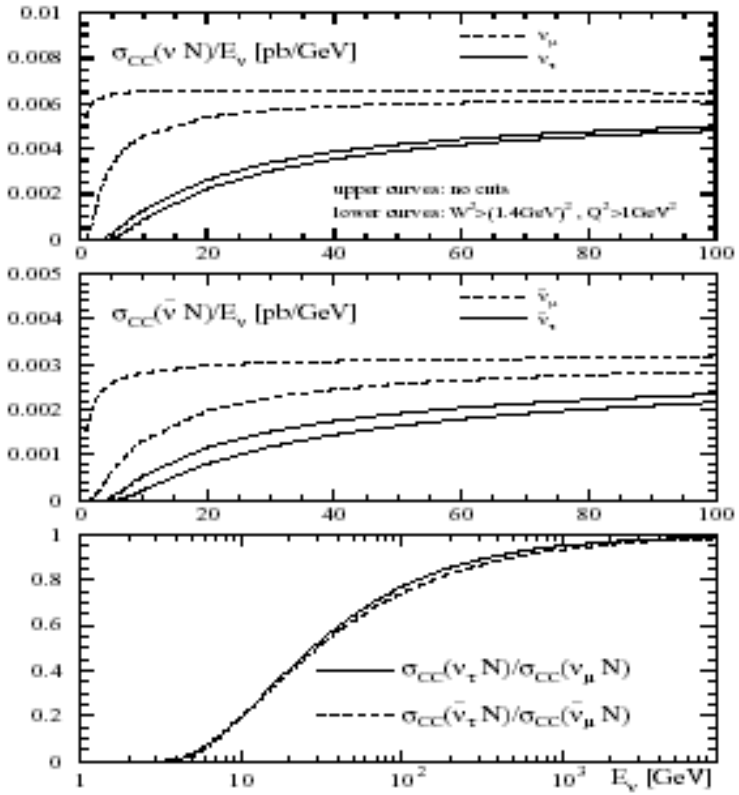
$$s_{\text{min}} = (m_{\text{nucleon}} + m_\tau)^2$$

where

$$s_{initial} = m^2_{\text{nucleon}} + 2E_\nu m_{\text{nucleon}}$$

$$\therefore E_\nu > \frac{m_\tau{}^2 + 2m_\tau m_{\text{nucleon}}}{2m_{\text{nucleon}}} \approx 3.5 \text{ GeV}$$

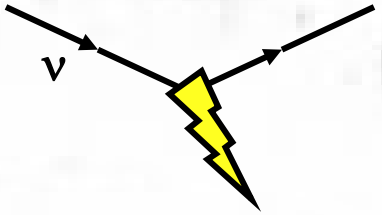
"m_{nucleon}" is M_T elsewhere, but don't want to confuse with m_τ...



(Kretzer and Reno)

- This is threshold for partons with *entire* nucleon momentum
  - effects big at higher E_ν also

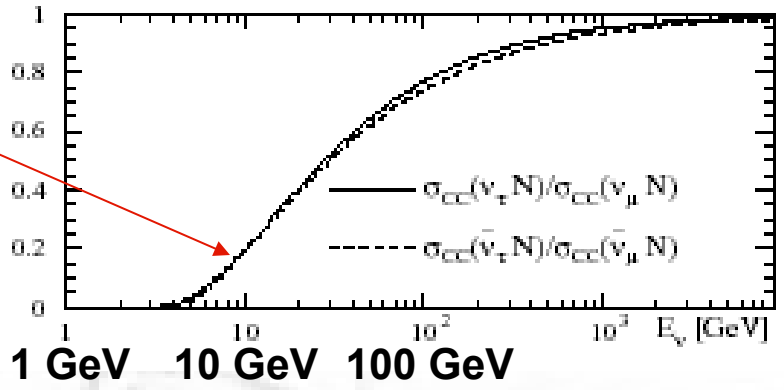
Lecture Question: What if Taus were Lighter?



- Imagine we lived in a universe where the tau mass was not 1.777 GeV, but was 0.888 GeV
- By how much would the tau appearance cross-section for an 8 GeV tau neutrino increase at OPERA?

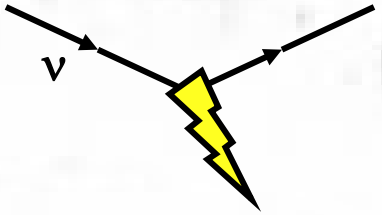
mass suppression: $1-\frac{m^2_{\text{lepton}}}{xs_{\text{nucleon}}}$

$$s_{\text{nucleon}} = m^2_{\text{nucleon}} + 2E_\nu m_{\text{nucleon}}$$



(a) $\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 1.4$ **(b)** $\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 2$ **(c)** $\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 3$

Lecture Question: What if Taus were Lighter?



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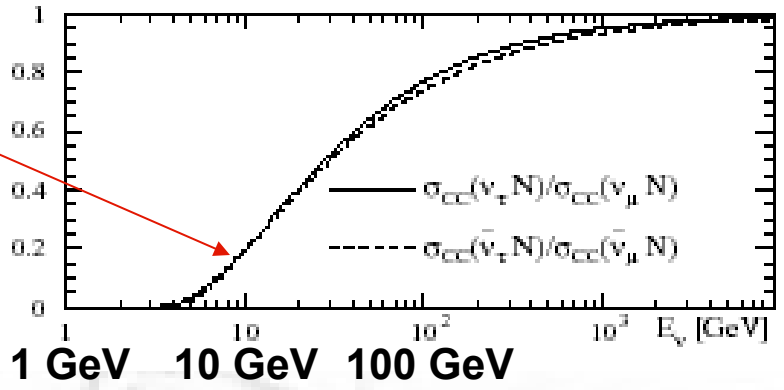

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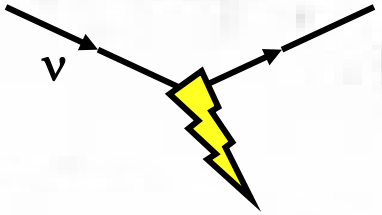
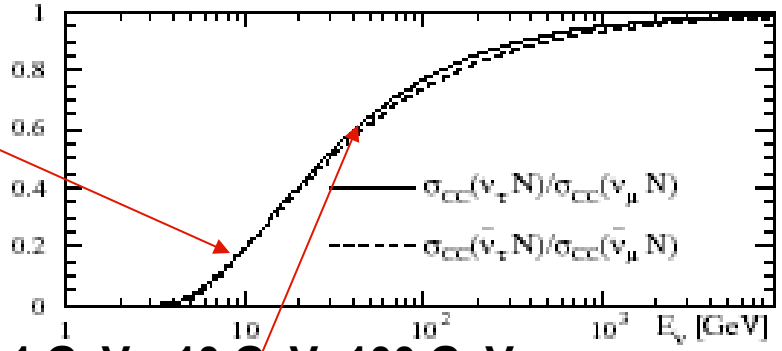


- By how much would the tau appearance cross-section for an 8 GeV tau neutrino increase at OPERA?

mass suppression: $1 - \frac{m^2_{\text{lepton}}}{xs_{\text{nucleon}}}$



1 GeV 10 GeV 100 GeV

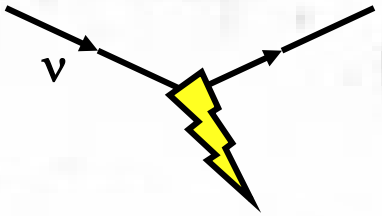
Numerator goes down by factor of four. Equivalent to denominator increasing by factor of four and tau mass unchanged…

$$s_{\text{nucleon}} = m^2_{\text{nucleon}} + 2E_\nu m_{\text{nucleon}}$$

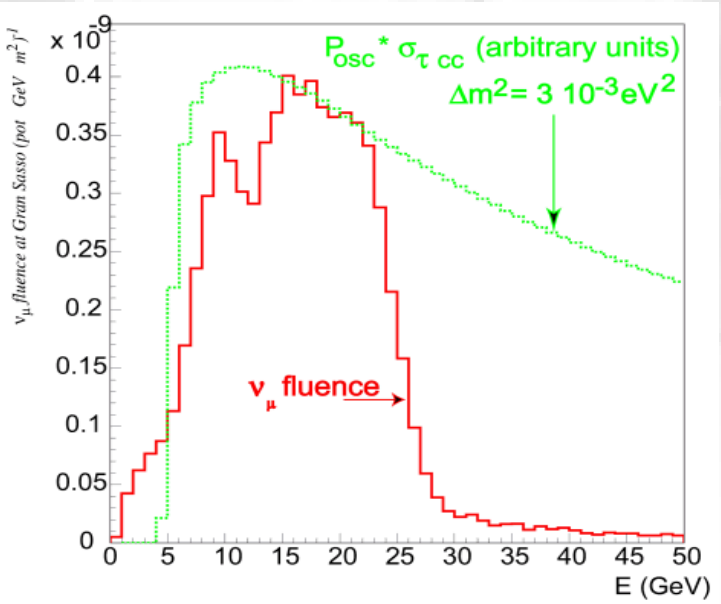
energy term dominates… so set energy a factor of four higher

$$\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 3$$

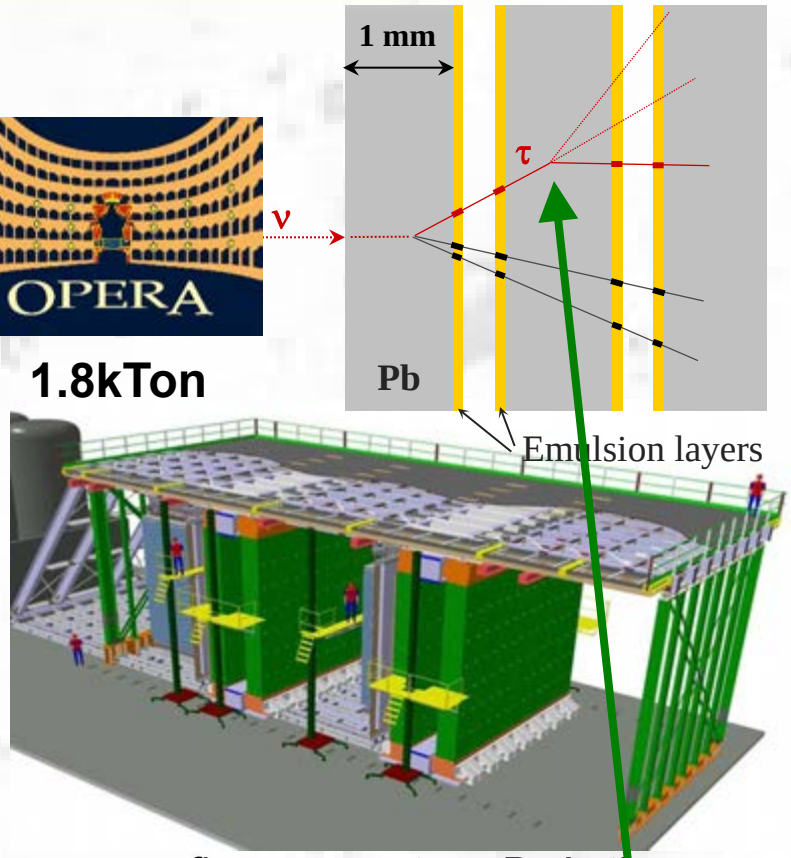
Opera at CNGS



Goal: ν_τ appearance
- 0.15 MWatt source
- high energy ν_μ beam
- 732 km baseline
- handfuls of events/yr



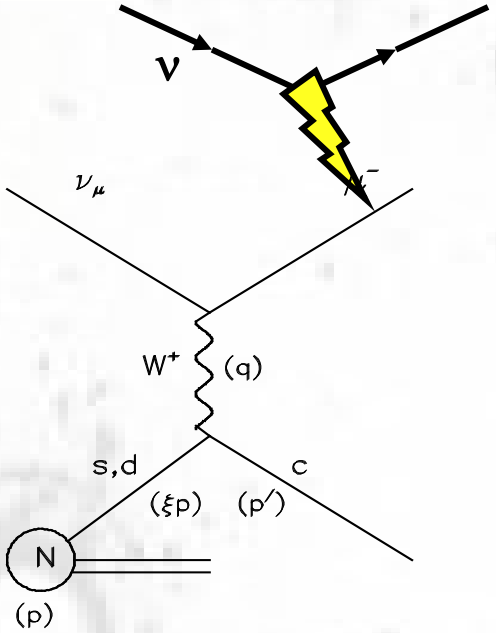
1.8kTon



figures courtesy D. Autiero

what else is copiously produced in neutrino interactions with $c\tau \sim 100\mu m$ and decays to hadrons?

Heavy Quark Production



- Production of heavy quarks modifies kinematics of our earlier definition of x.
  - Charm is heavier than proton; hints that its mass is not a negligible effect…

$$(q+\zeta p)^2 = p'^2 = m_c^{\,2}$$

$$q^2 + 2\zeta p \bullet q + \zeta^2 M^2 = m_c^{\,2}$$

Therefore $$\zeta \cong \frac{-q^2 + m_c^{\,2}}{2p \bullet q}$$

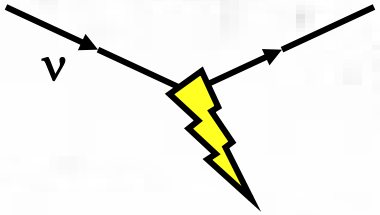
Note different definition of fractional momentum

$$\zeta \cong \frac{Q^2 + m_c^{\,2}}{2M\upsilon} = \frac{Q^2 + m_c^{\,2}}{Q^2/x}$$

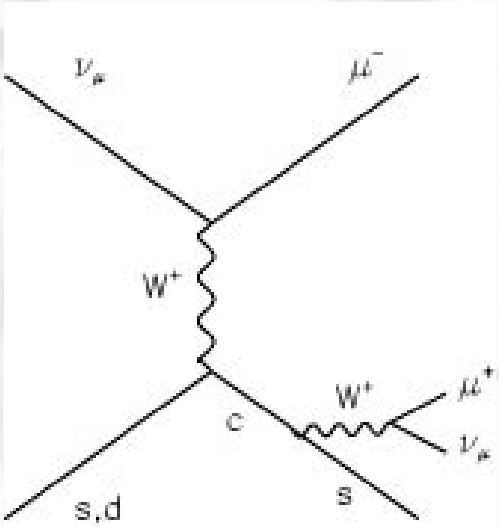
$$\zeta \cong x\left(1 + \frac{m_c^{\,2}}{Q^2}\right)$$

"slow rescaling" leads to kinematic suppression of charm production

Neutrino Dilepton Events

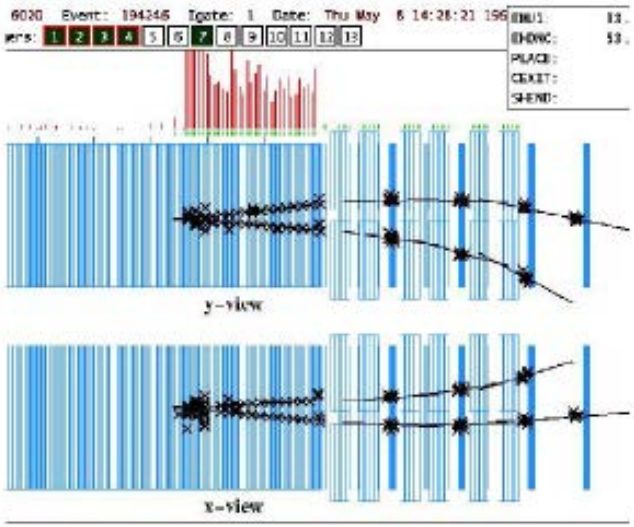


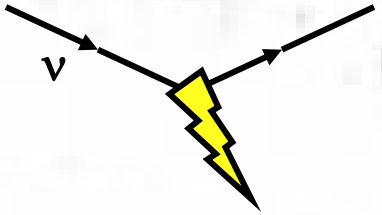
- Neutrino induced charm production has been extensively studied
  - Emulsion/Bubble Chambers (low statistics, 10s of events). Reconstruct the charm final state, but limited by target mass.
  - "Dimuon events" (high statistics, 1000s of events)



$$\nu_\mu + \begin{pmatrix} d \\ s \end{pmatrix} \rightarrow \mu^- + c + X, \qquad c \rightarrow \mu^+ + \nu_\mu + X'$$

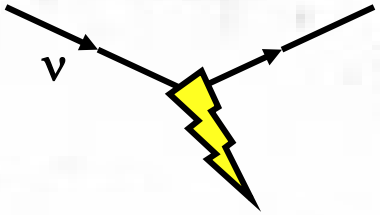
$$\bar{\nu}_\mu + \begin{pmatrix} \bar{d} \\ \bar{s} \end{pmatrix} \rightarrow \mu^+ + \bar{c} + X, \qquad \bar{c} \rightarrow \mu^- + \bar{\nu}_\mu + X'$$





SUPPLEMENT: NuTeV Measurement of Strange Sea

Neutrino Dilepton Events



- Rate depends on:
  - d, s quark distributions, $|V_{cd}|$
  - Semi-leptonic branching ratios of charm
  - Kinematic suppression and fragmentation

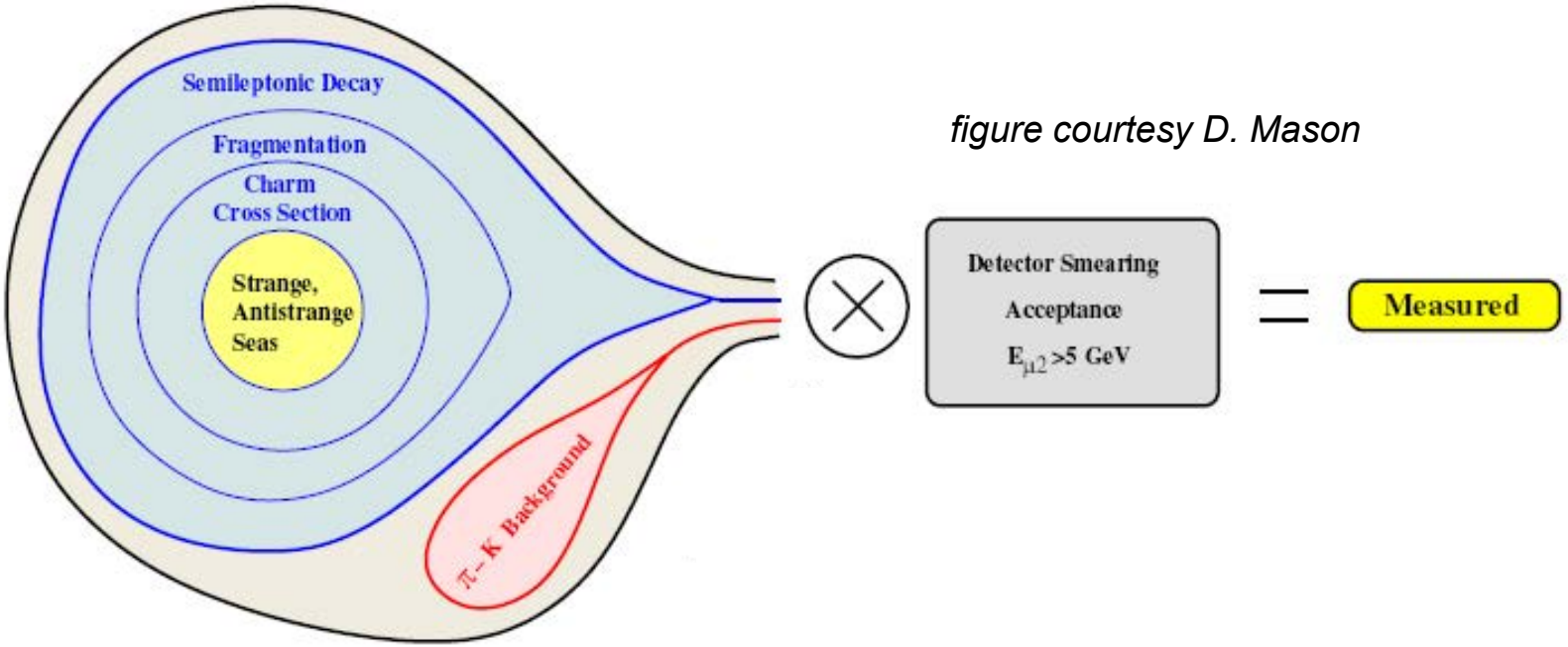
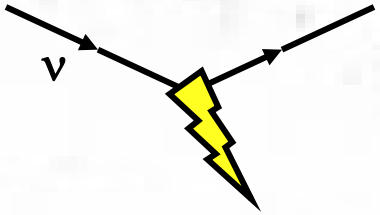
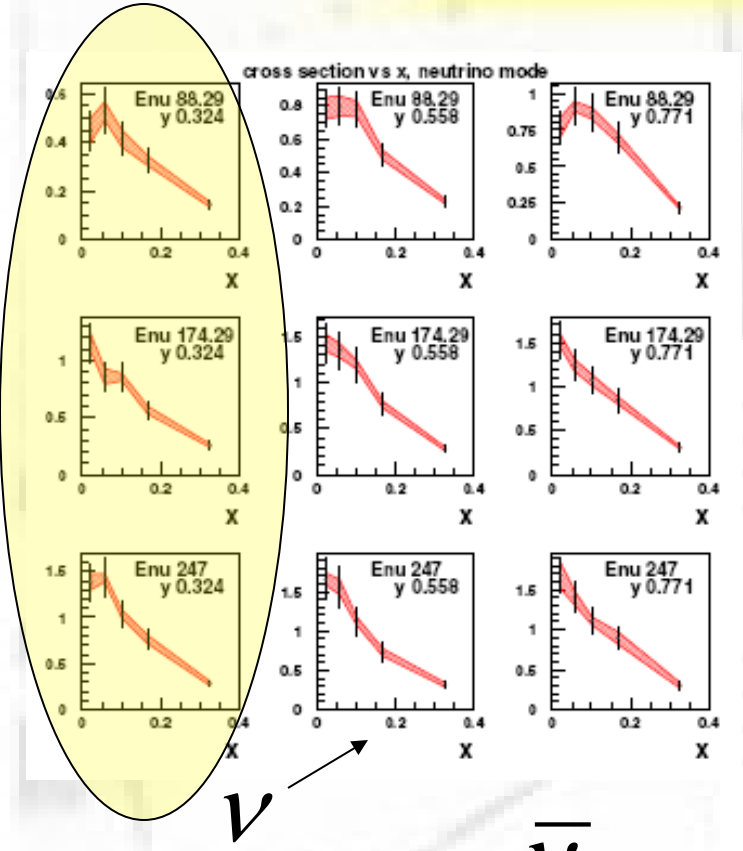


figure courtesy D. Mason

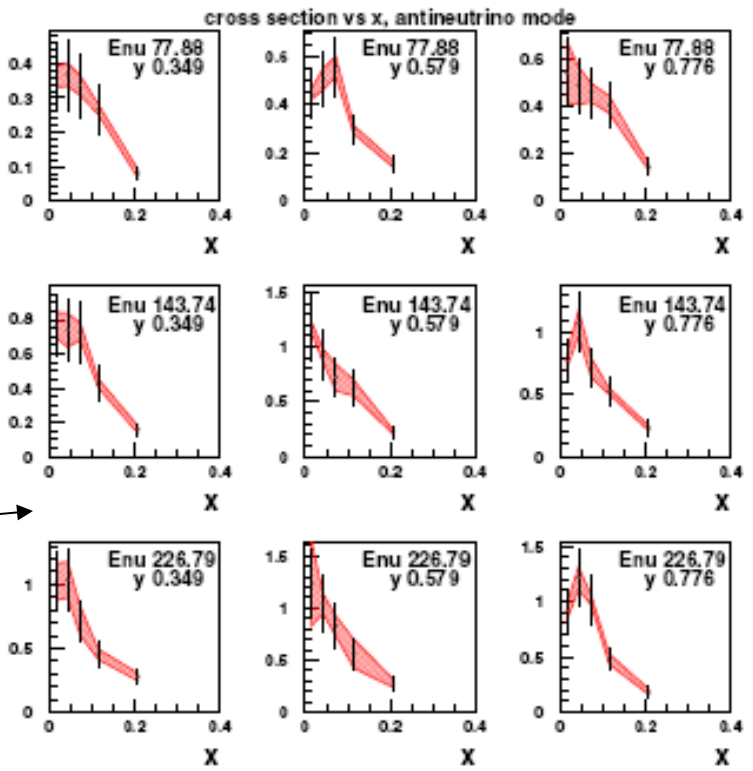
NuTeV Dimuon Sample



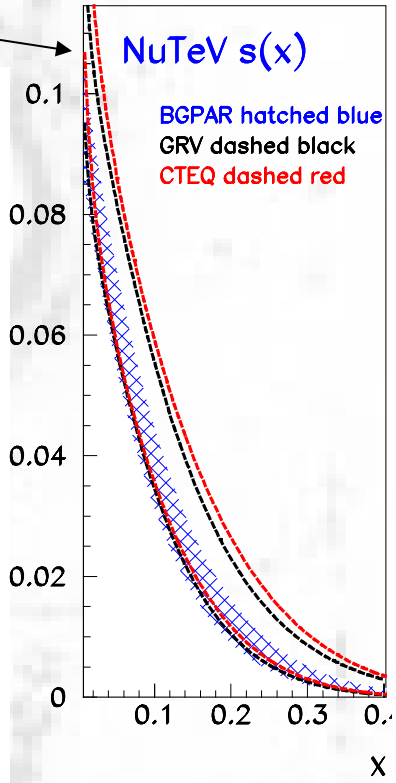


ν

$\overline{\nu}$

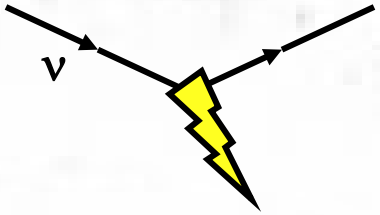


- Lots of data!
- Separate data in energy, x and y (inelasticity)
  - Energy important for charm threshold, m_c
  - x important for s(x)

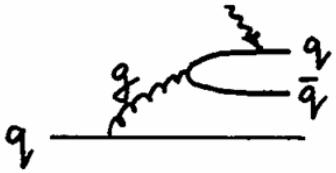


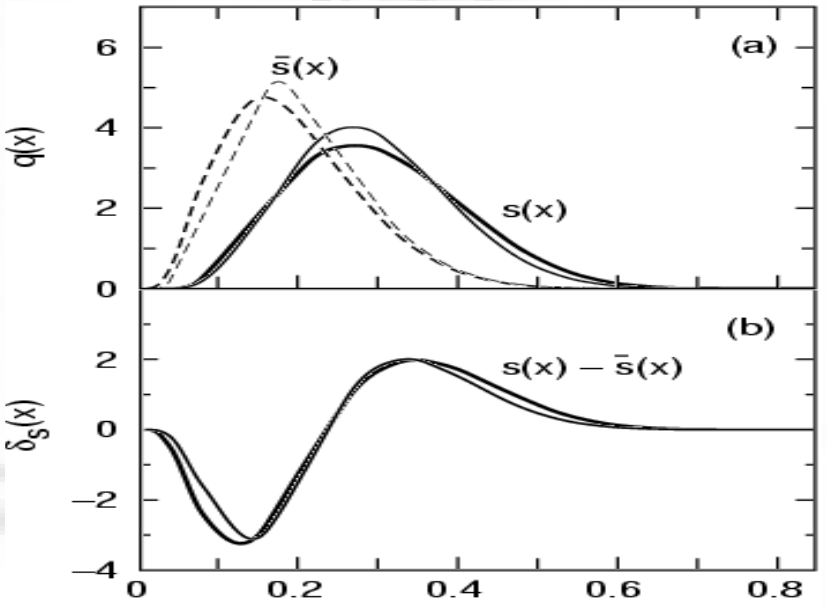
$$\pi \times \frac{d^2\sigma(\nu N \to \mu\mu X)}{dxdy} \Big/ G_F^2 M_N E_\nu$$

QCD at Work: Strange Asymmetry?



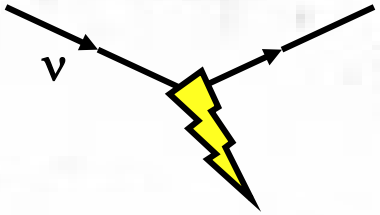
- An interesting aside…
  - The strange sea can be generated perturbatively from g→s+sbar.
  - BUT, in perturbative generation the momenta of strange and anti-strange quarks is equal
    - well, in the leading order splitting at least. At higher order get a vanishingly small difference.
  - SO s & sbar difference probe non-perturbative ("intrinsic") strangeness
    - Models: Signal&Thomas, Brodsky&Ma, etc.





(Brodsky & Ma, s-sbar)

NuTeV's Strange Sea



- NuTeV has tested this
  - NB: very dependent on what is assumed about non-strange sea
  - Why? Recall CKM mixing…

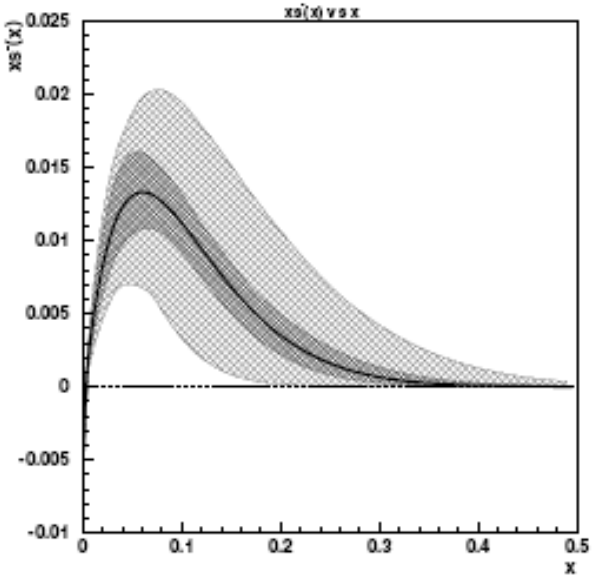
$$V_{cd}\,d(x) + V_{cs}\,s(x) \rightarrow s'(x)$$
$$V_{cd}\,\bar{d}(x) + V_{cs}\,\bar{s}(x) \rightarrow \bar{s}'(x)$$

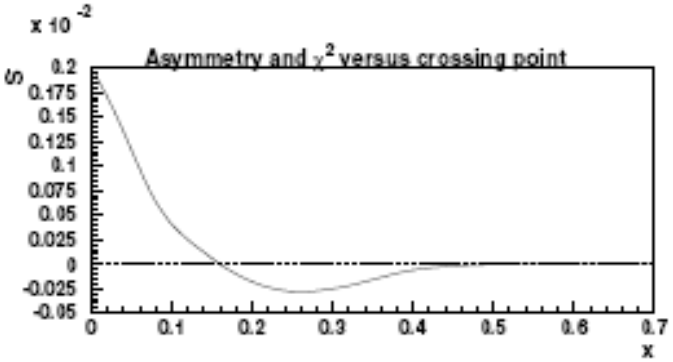
small *big*

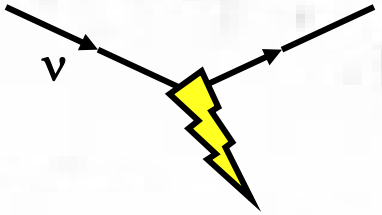
  - Using CTEQ6 PDFs…

$$\int dx\left[x\left(s-\bar{s}\right)\right] = 0.0019 \pm 0.0005 \pm 0.0014$$

$$\text{c.f.,}\ \int dx\left[x\left(s+\bar{s}\right)\right] \approx 0.02$$

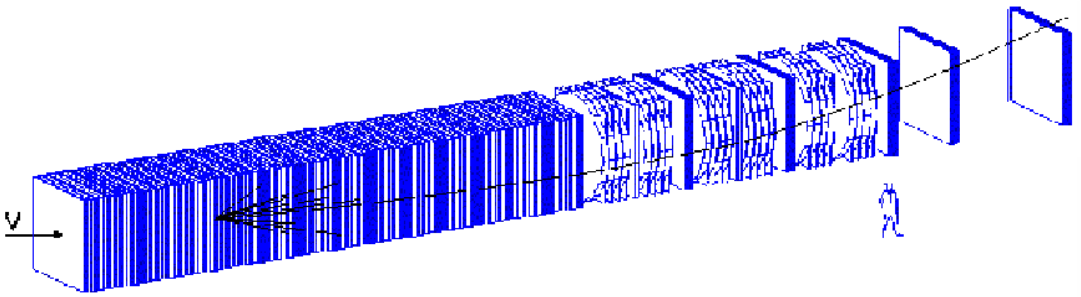


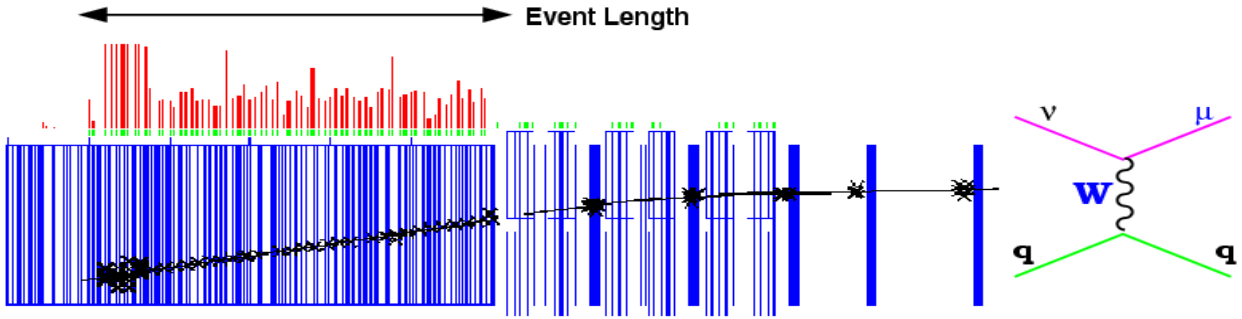


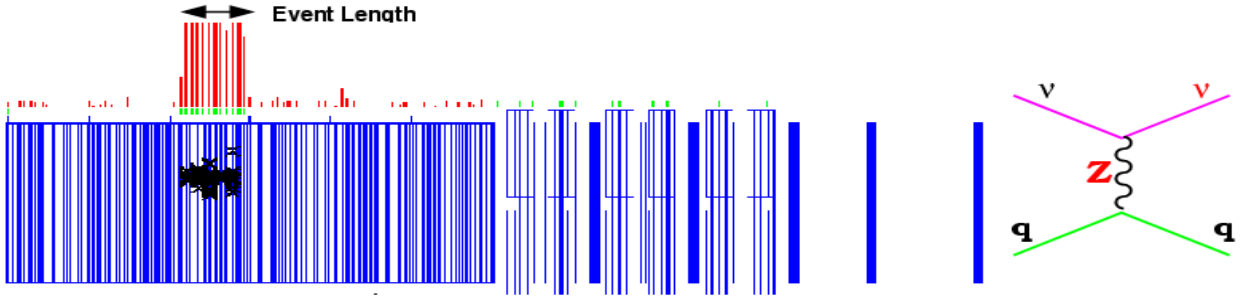


SUPPLEMENT: NuTeV $\sin^2\theta_W$

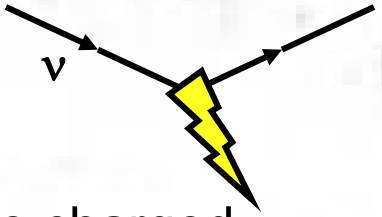
NuTeV at Work…



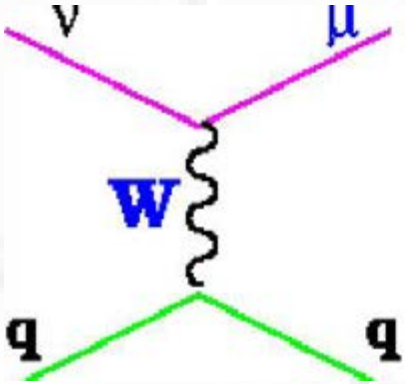




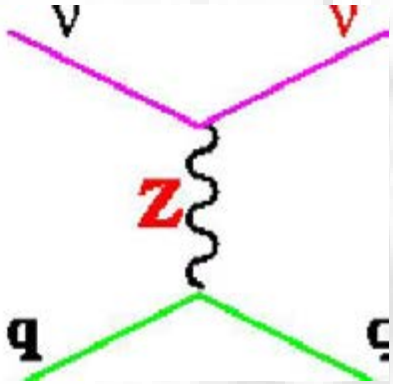
DIS NC/CC Ratio



- Experimentally, it's "simple" to measure ratios of neutral to charged current cross-sections on an isoscalar target to extract NC couplings



W-q coupling is I_3



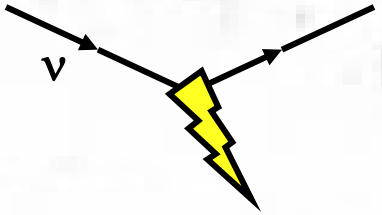
Z-q coupling is I_3-$Q\sin^2\theta_W$

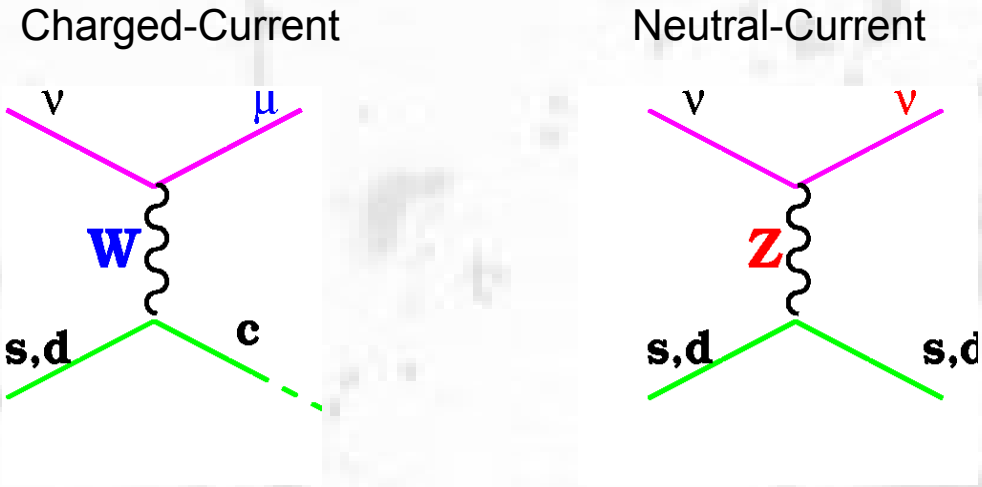
Llewellyn Smith Formulae

$$R^{\nu(\bar{\nu})} = \frac{\sigma_{NC}^{\nu(\bar{\nu})}}{\sigma_{CC}^{\nu(\bar{\nu})}} = \left(\left(u_L^2 + d_L^2\right) + \frac{\sigma_{CC}^{\bar{\nu}(\nu)}}{\sigma_{CC}^{\nu(\nu)}} \left(u_R^2 + d_R^2\right) \right)$$

- Holds for isoscalar targets of u and d quarks only
  - Heavy quarks, differences between u and d distributions are corrections
- Isospin symmetry causes PDFs to drop out, even outside of naïve quark-parton model

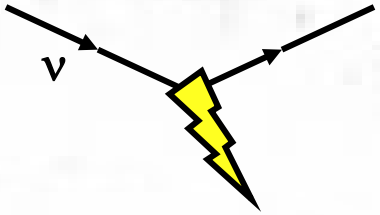
Lecture Question: Paschos-Wolfenstein Relation



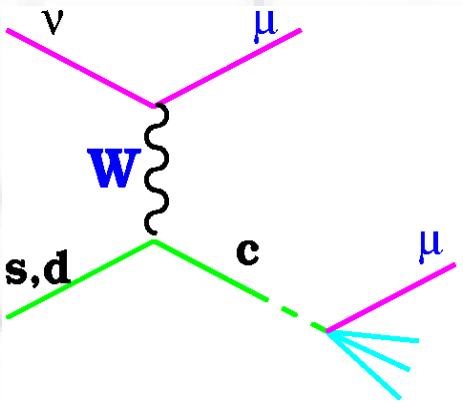


- If we want to measure electroweak parameters from the ratio of charged to neutral current cross-sections, what problem will we encounter from these processes?

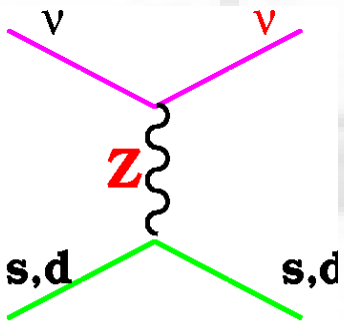
Lecture Question: Paschos-Wolfenstein Relation



Charged-Current

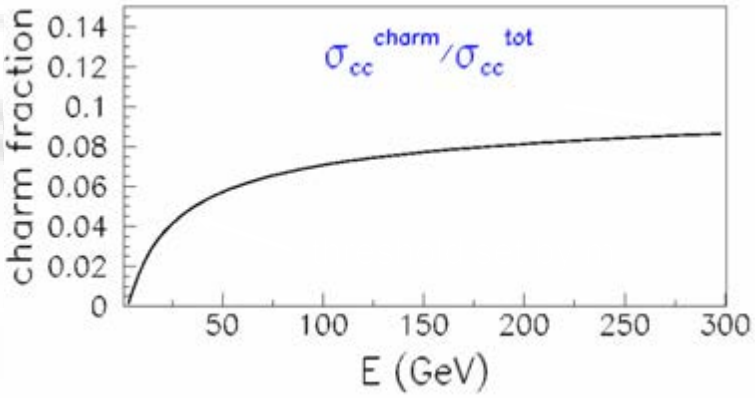


Neutral-Current

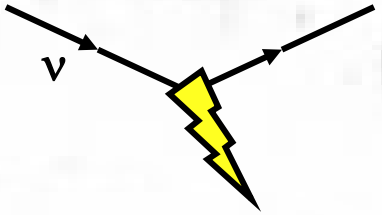


- CC is suppressed due to final state charm quark
  - $\Rightarrow$ Need strange sea and m_c
    - Remember heavy quark mass effect:

$$x \rightarrow \xi = x\left(1 + m_c^2/Q^2\right)$$



Lecture Question: Paschos-Wolfenstein Relation



- The NuTeV experiment employed a complicated design to measure

Paschos - Wolfenstein Relation

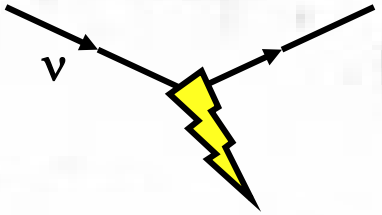
$$R^- = \frac{\sigma^{\nu}_{NC} - \sigma^{\bar{\nu}}_{NC}}{\sigma^{\nu}_{CC} - \sigma^{\bar{\nu}}_{CC}} = \rho^2 \left(\tfrac{1}{2} - \sin^2 \theta_W \right)$$

- How did this help with the heavy quark problem of the previous question?

Hint: what to you know about the relationship of:

$$\sigma(\nu q) \text{ and } \sigma(\bar{\nu}\bar{q})$$

Lecture Question: Paschos-Wolfenstein Relation



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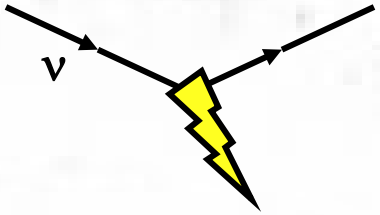
- How did this help with the heavy quark problem of the previous question?

$$\sigma(\nu q) = \sigma(\bar{\nu}\bar{q})$$
$$\sigma(\nu \bar{q}) = \sigma(\bar{\nu} q)$$

$$\therefore \sigma(\nu q) - \sigma(\bar{\nu}\bar{q}) = 0$$

So any quark-antiquark symmetric part is not in difference, e.g., strange sea.

NuTeV Fit to R^{ν} and $R^{\bar{\nu}bar}$



- NuTeV result:

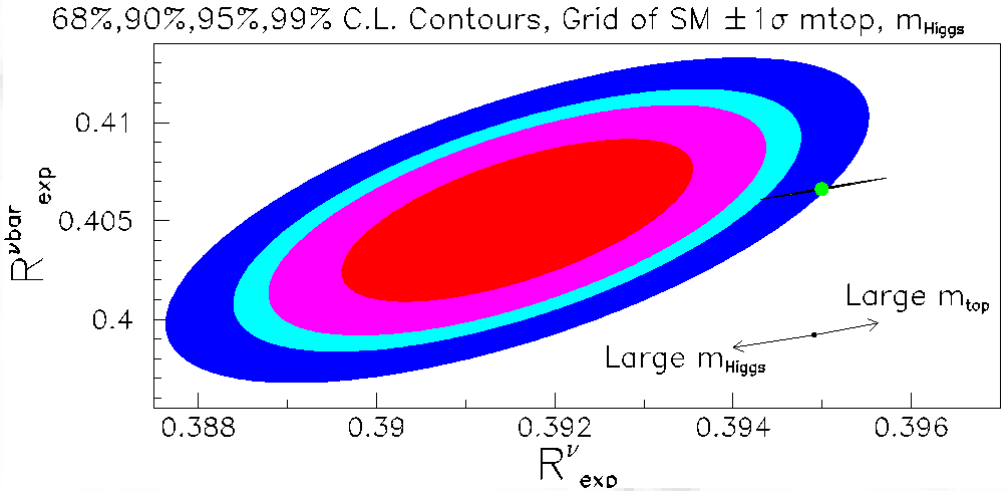
$$\sin^2\theta_W^{(on-shell)} = 0.2277 \pm \pm 0.0013(stat.) \pm 0.0009(syst.)$$
$$= 0.2277 \pm 0.0016$$

(Previous neutrino measurements gave 0.2277 ± 0.0036)

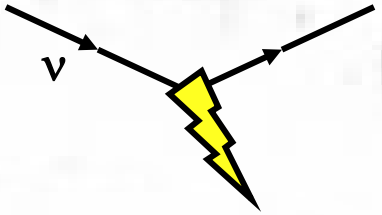
- Standard model fit (LEPEWWG): 0.2227 ± 0.00037

A 3σ discrepancy…

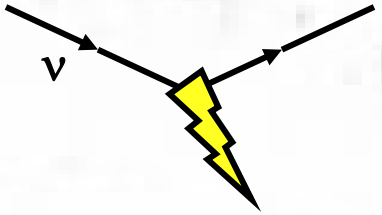
$$R^{\nu}_{\exp} = 0.3916 \pm 0.0013$$
$$(SM: 0.3950) \Leftarrow 3\sigma \text{ difference}$$
$$R^{\bar{\nu}}_{\exp} = 0.4050 \pm 0.0027$$
$$(SM: 0.4066) \Leftarrow \text{Good agreement}$$



NuTeV Electroweak: What does it Mean?

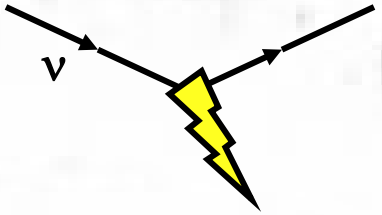


- If I knew, I'd tell you.
- It could be BSM physics. Certainly there is no exclusive of a Z' that could cause this. But why?
- It could be the asymmetry of the strange sea…
  - it would contribute because the strange sea would not cancel in
  - but it's been measured; not anywhere near big enough
- It could be very large isospin violation
  - if $d_p(x) \neq u_n(x)$ at the 5% level… it would shift charge current (normalizing) cross-sections enough.
  - no data to forbid it. any reason to expect it?

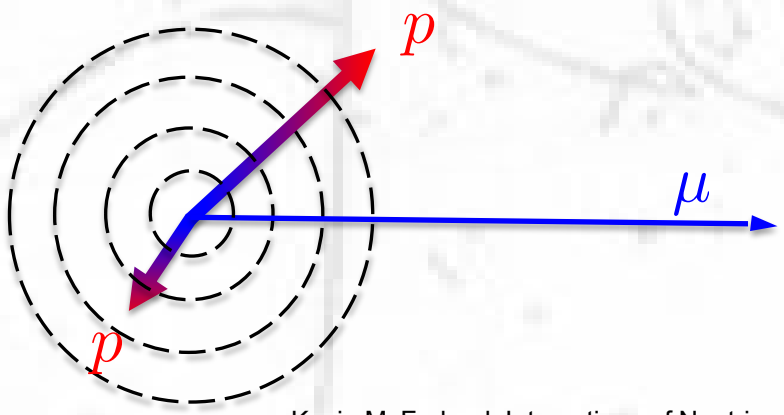


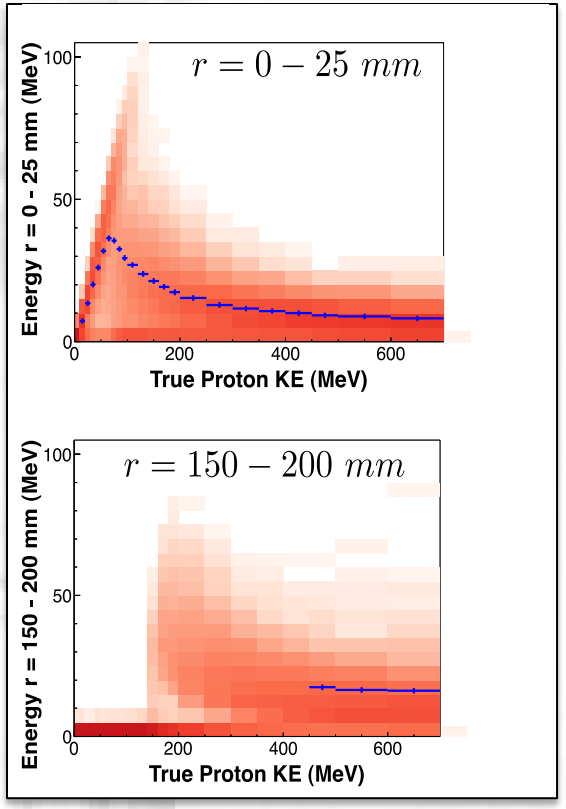
SUPPLEMENT: MINERvA Quasielastic Vertex Energy Measurement, Multinucleons?

Vertex Region Energy

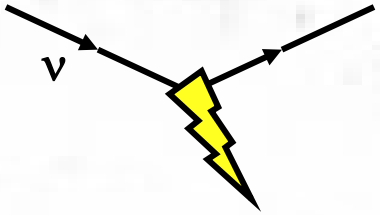


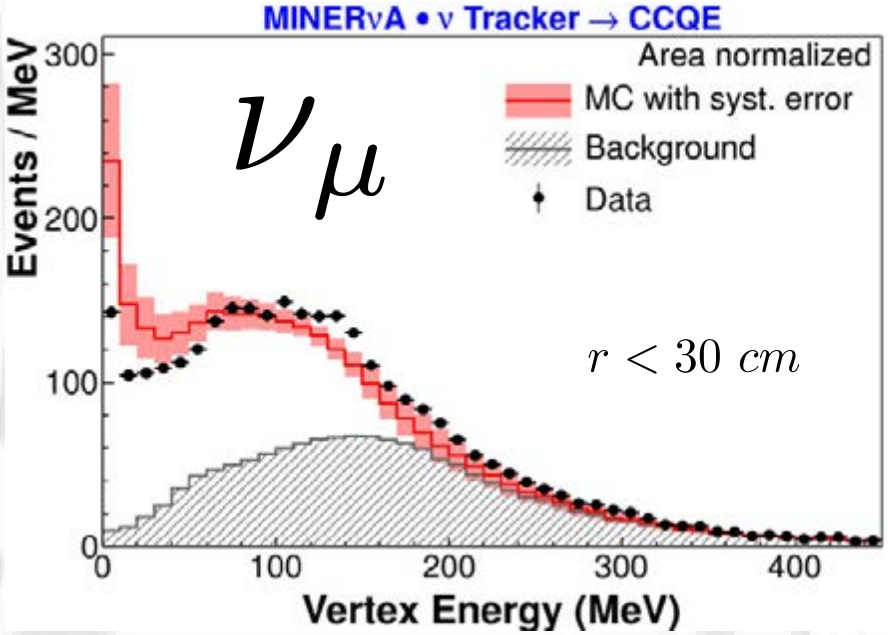
- Vertex region ignored in MINERvA recoil cut
  - Therefore selection is mostly insensitive to low energy nucleons in the final state
- Study energy near vertex
  - Vertex is precisely located, so distance of energy from vertex is sensitive to range of extra protons

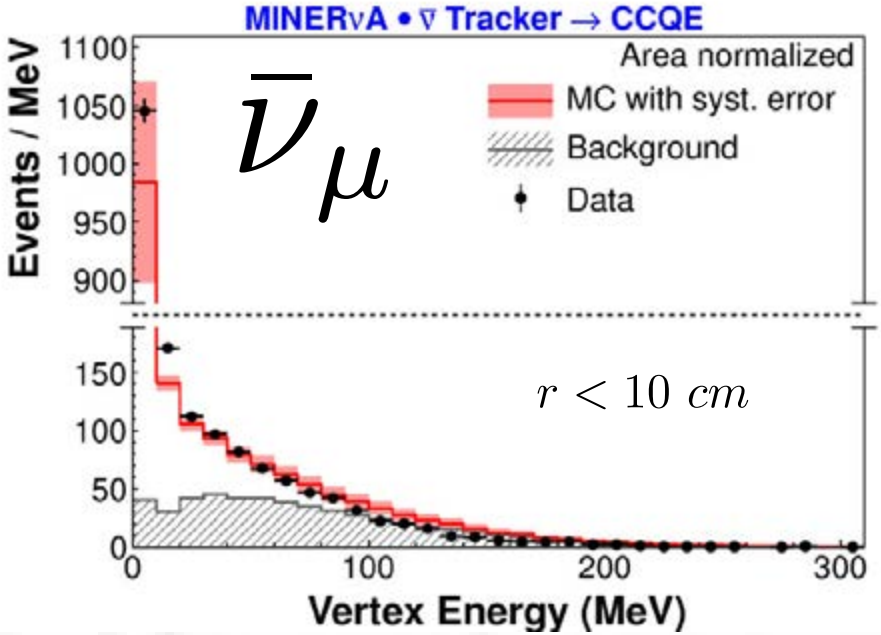




MINERvA: Vertex Energy

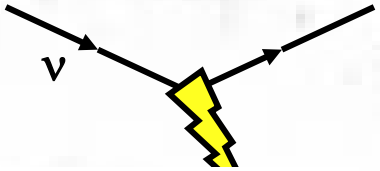


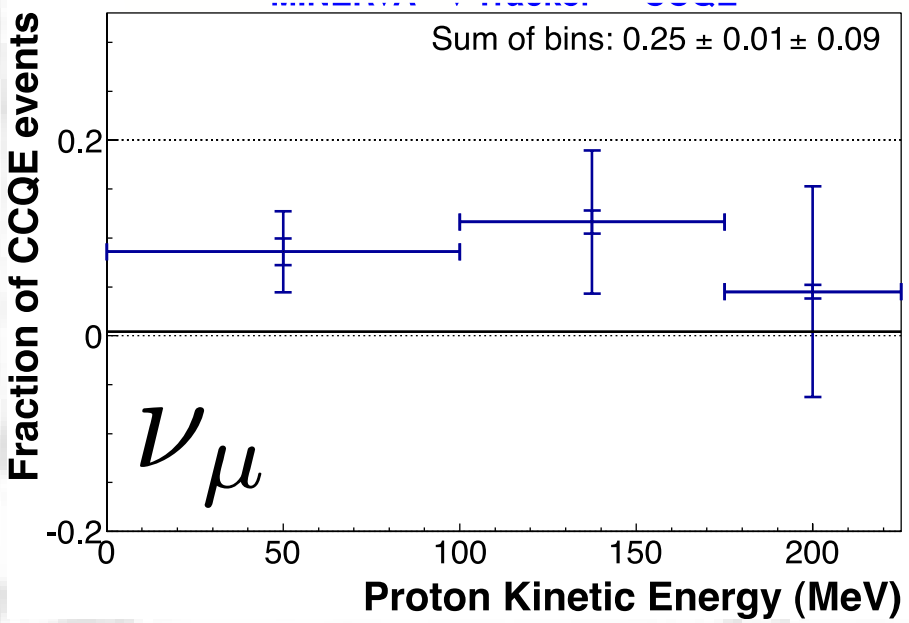


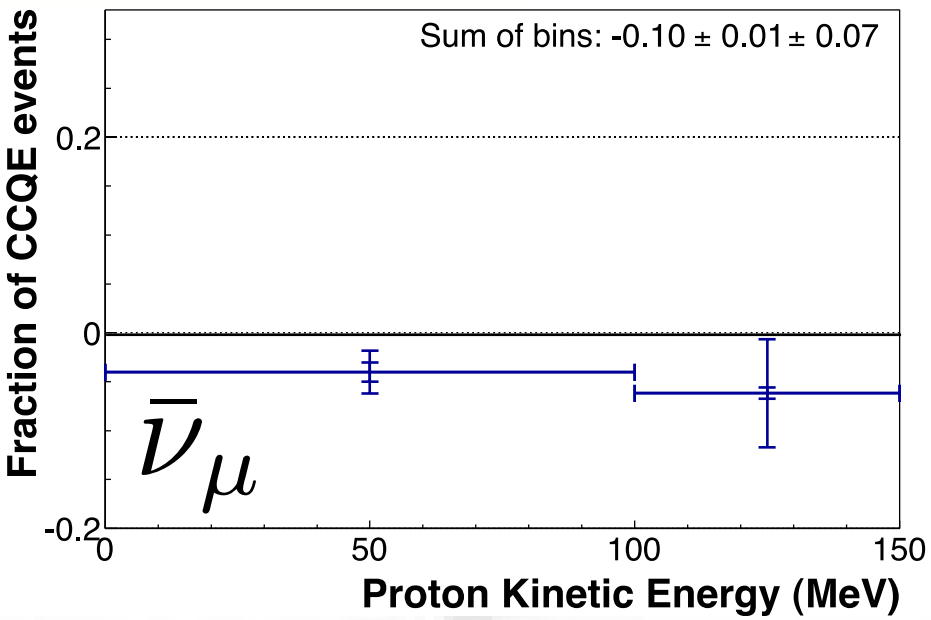


- A trend toward higher vertex energy is observed in the neutrino data, but not in anti-neutrino data
- Red band represents uncertainties on energy reconstruction and final state interactions
- Assume extra energy is due to additional protons

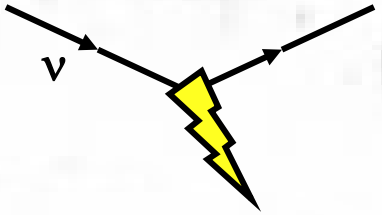
Extra Protons in MINERvA?





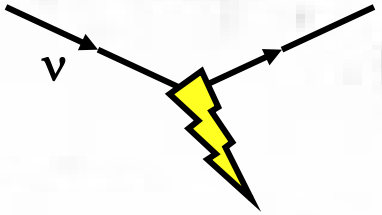


- Data wants to add low energy protons in 25±9% of neutrino events, but prefers 10±7% fewer protons in anti-neutrino
- Suggests correlated pairs are dominantly n+p in initial state, and therefore p+p or n+n in CCQE



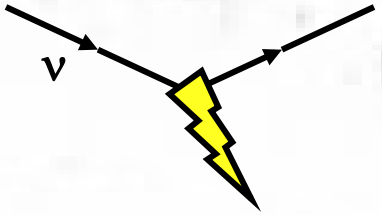
SUPPLEMENT: More on Inclusive Scattering on Heavy Targets

Measuring Inclusive Interactions



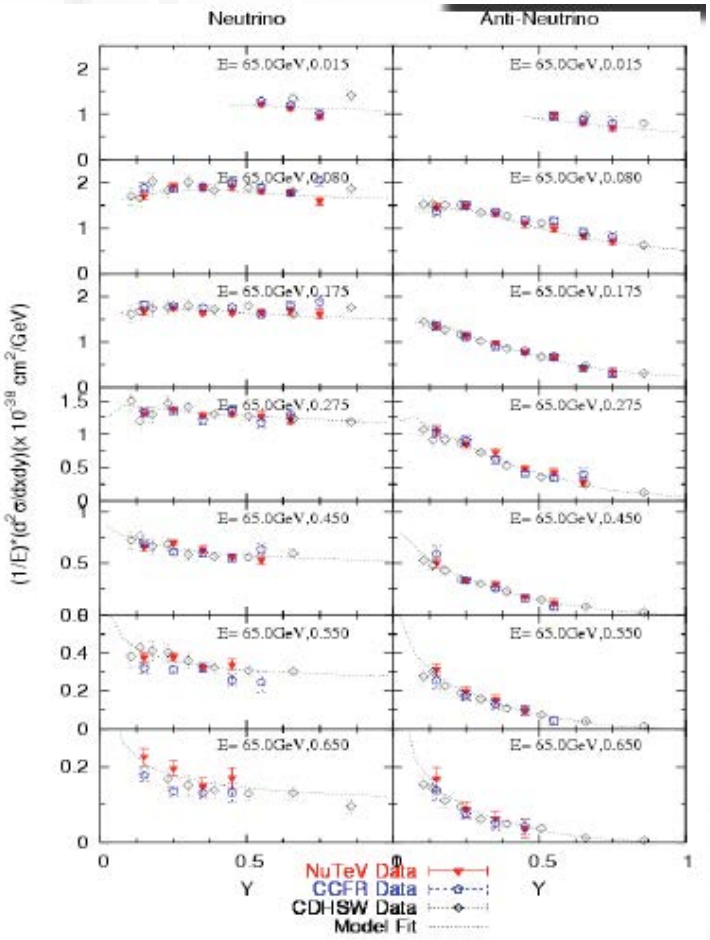
- Much of the data we have is at high energies
  - Neutrino flux is usually poorly known. Common wideband technique is "low recoil" method which uses the observation that $\lim_{\nu \to 0} \frac{d\sigma}{d\nu}$ is independent of E_ν
  - Cross-section normalized from narrow band expt's which counted secondary particles to measure flux
- Typical goal is to extract structure functions $2xF_1(x,Q^2)$, $F_2(x,Q^2)$, $xF_3(x,Q^2)$ from dependence in y and E_ν.
- Most recently, NuTeV, CHORUS, NOMAD, MINOS

NuTeV CC Differential Cross-Sections

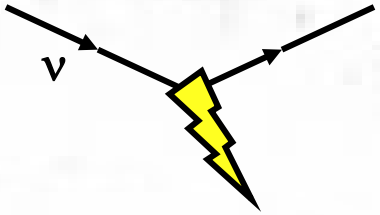


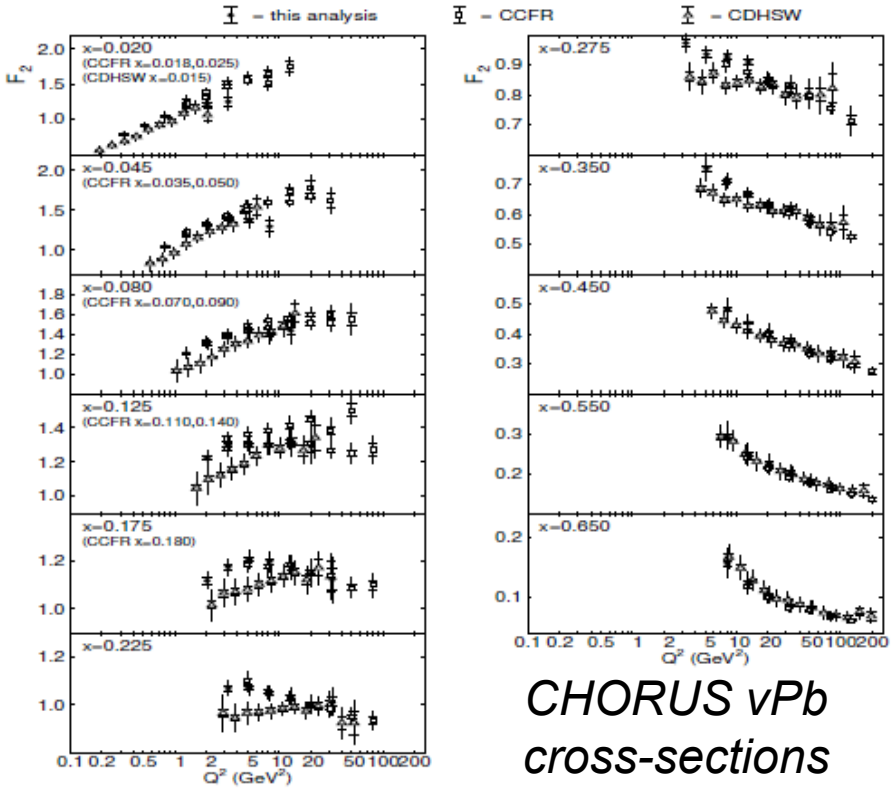
Phys.Rev.D74:012008,2006

- NuTeV has a very large data sample on iron
  - High energies, precision calibration from testbeam
- Uses:
  - pQCD fits for Λ_{QCD}
  - Extract structure functions for comparisons with other experiments



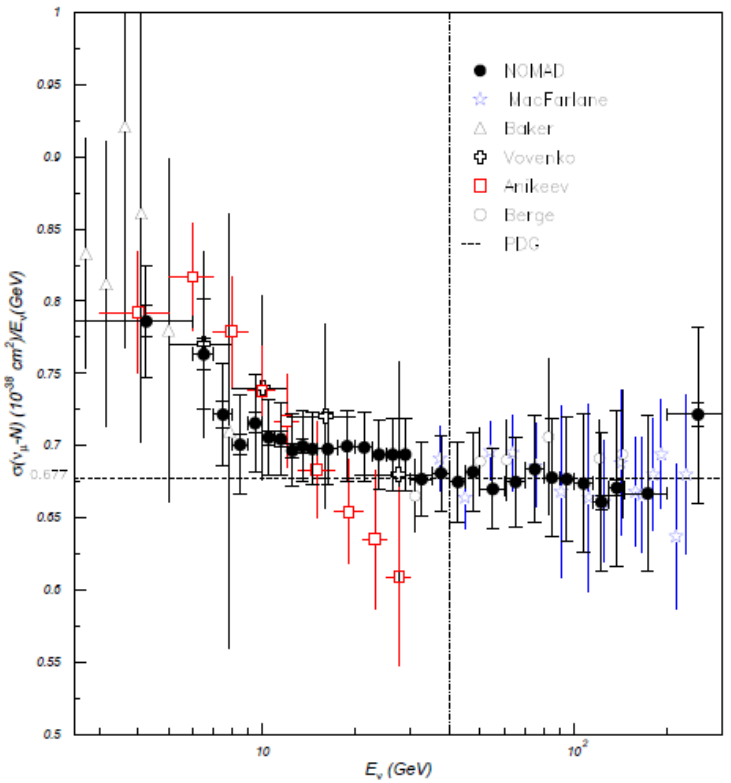
CHORUS and NOMAD





CHORUS vPb cross-sections

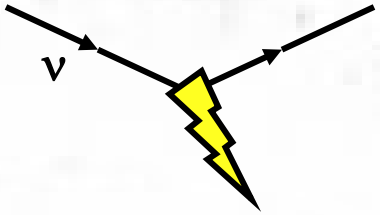
Phys.Lett..632(2006) 65



NOMAD vC CC total cross-sections

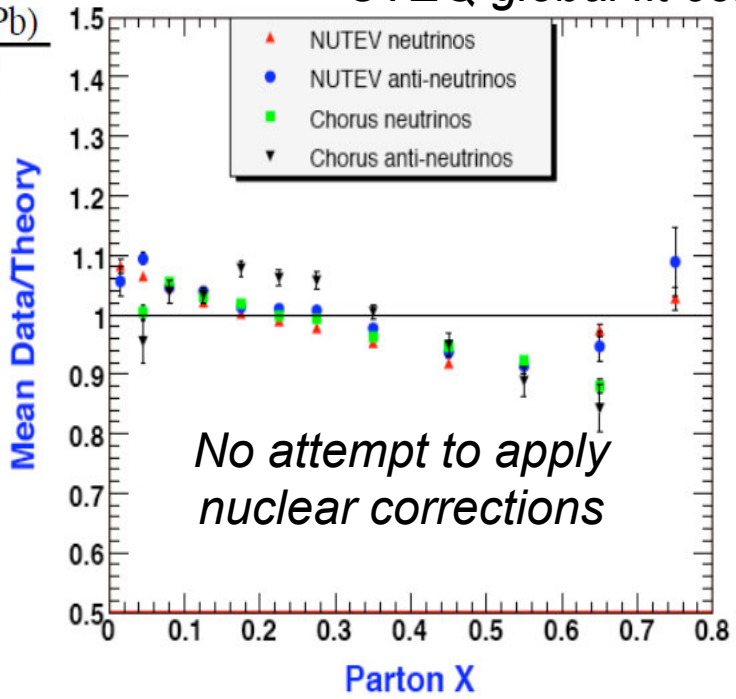
Phys.Lett.B660:19-25,2008

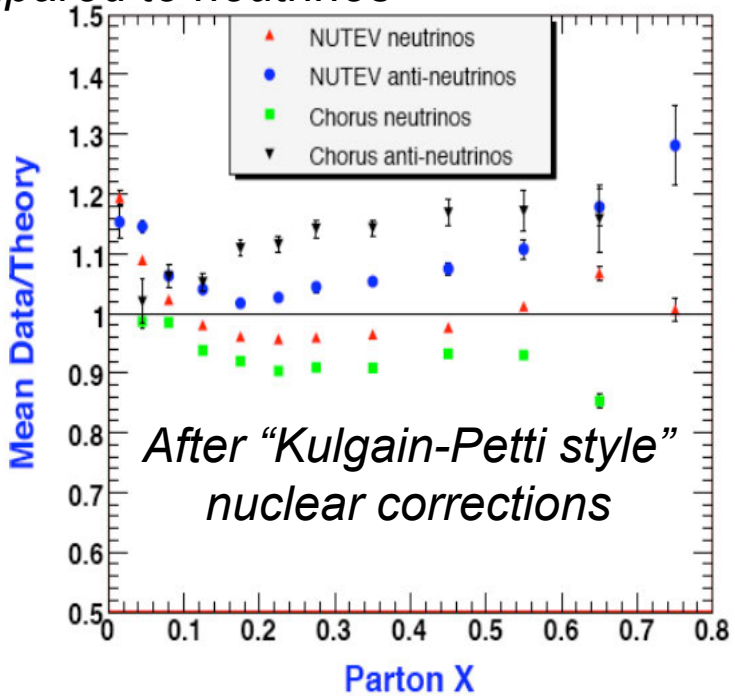
Nuclear Corrections and High-x PDFs



CTEQ global fit compared to neutrinos

$\frac{\sigma(\nu Fe \text{ or } \nu Pb)}{\sigma(\nu \text{"}D_2\text{"})}$

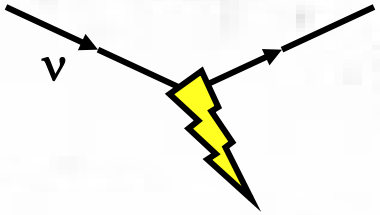




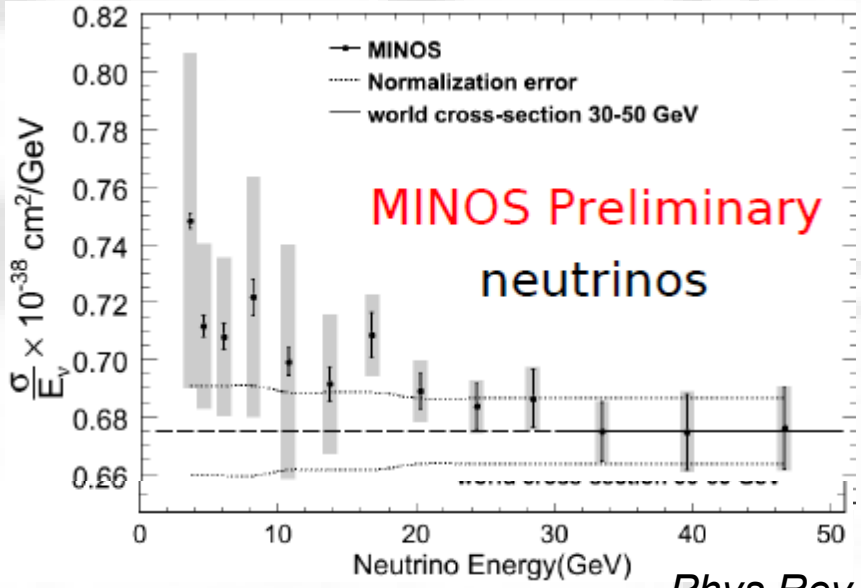
Figures courtesy J. Morfín

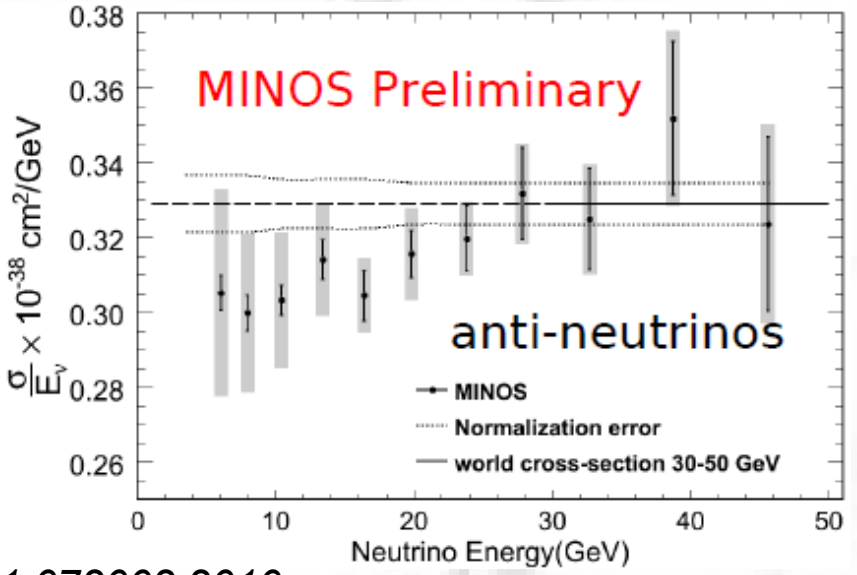
- There are two confusing aspect of these comparisons
  - We observed problems before in nuclear corrections from models
  - Also, some strange behavior at high x… difficult to incorporate both data sets in one model

MINOS Total Cross-Section

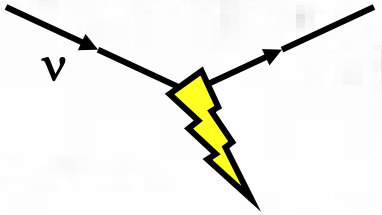


- Attempt to bravely extend low recoil technique to very low energies
  - "Low recoil" sample is visible hadronic energy below 1 GeV, so a fair fraction of the cross-section at the lowest energy (3 GeV)



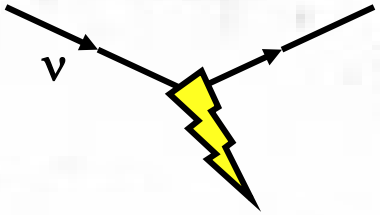


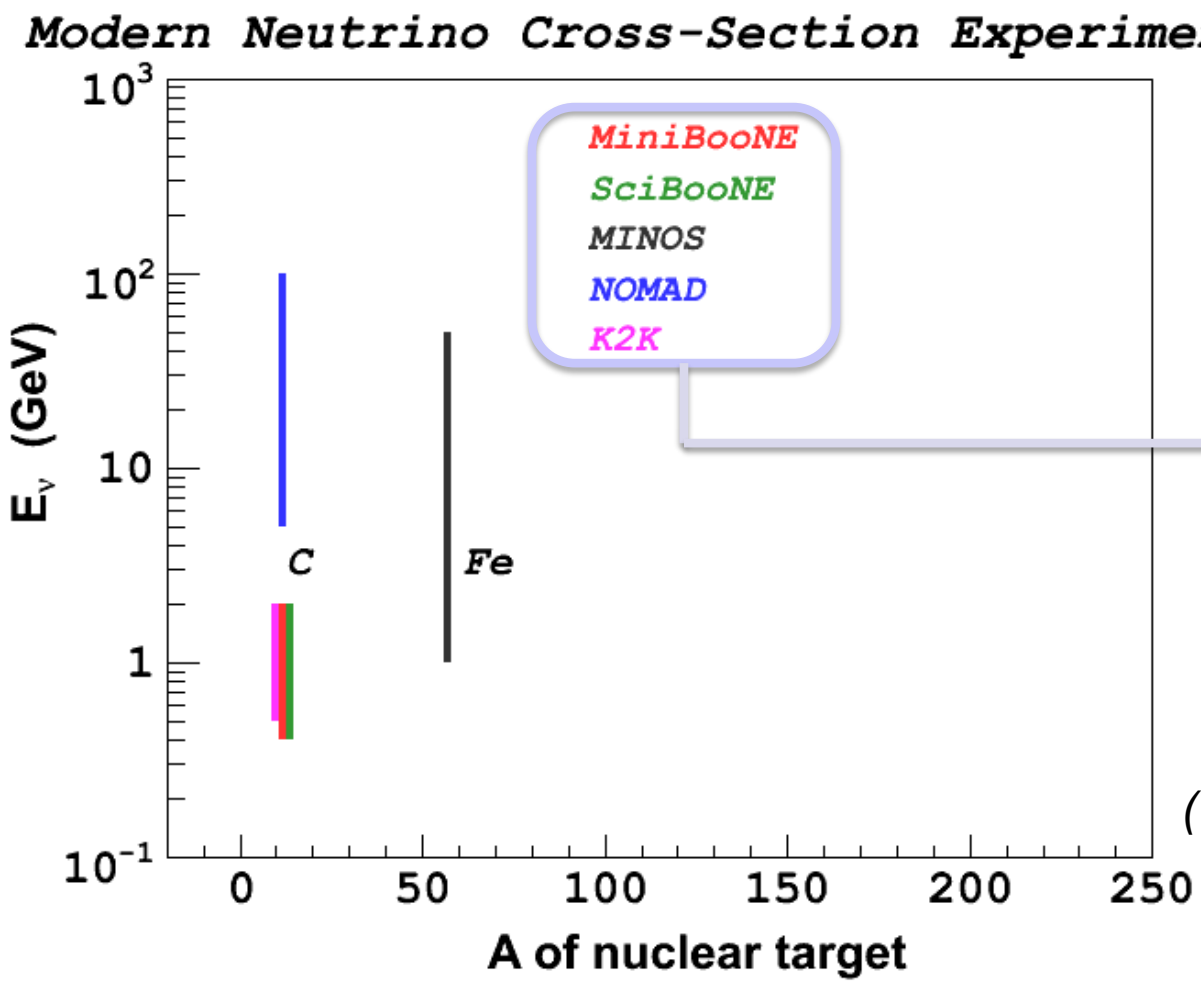
Phys.Rev.D81:072002,2010



SUPPLEMENT: Experiments to Measure GeV Cross-Sections

Energies and Targets of Cross-Section Measurements

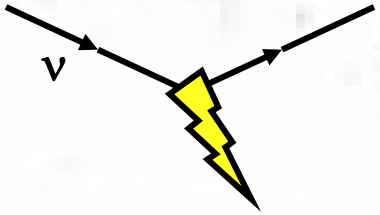


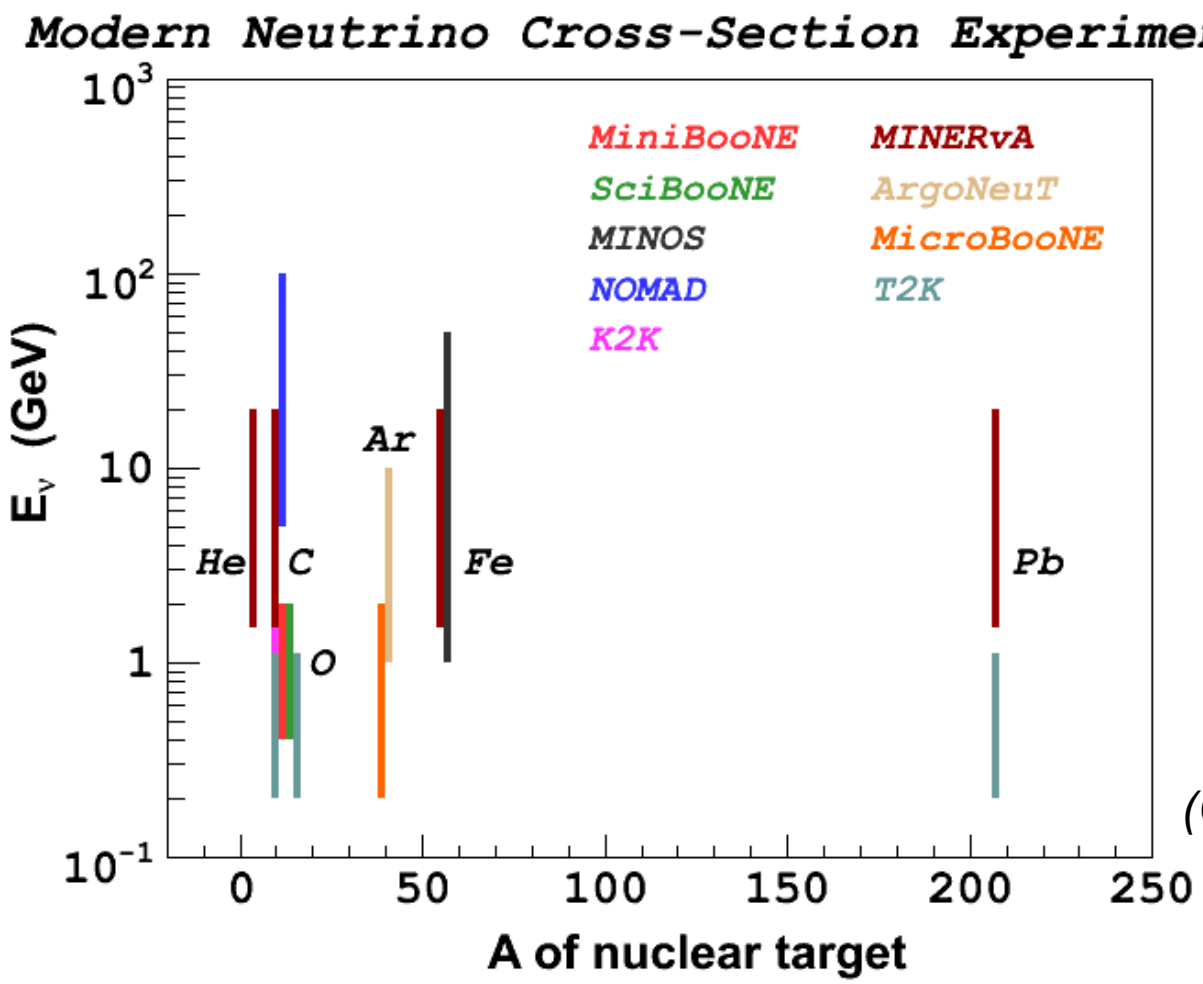


recent results and/or currently analyzing and publishing new cross-section data

(Compilation from D. Schmitz)

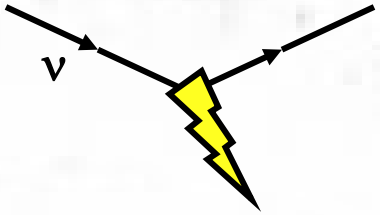
Energies and Targets of Cross-Section Measurements





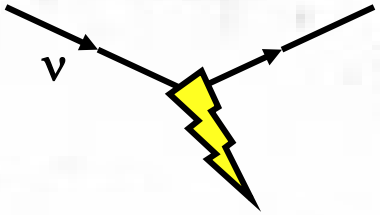
(Compilation from D. Schmitz)

Technologies of "Old" Experiments

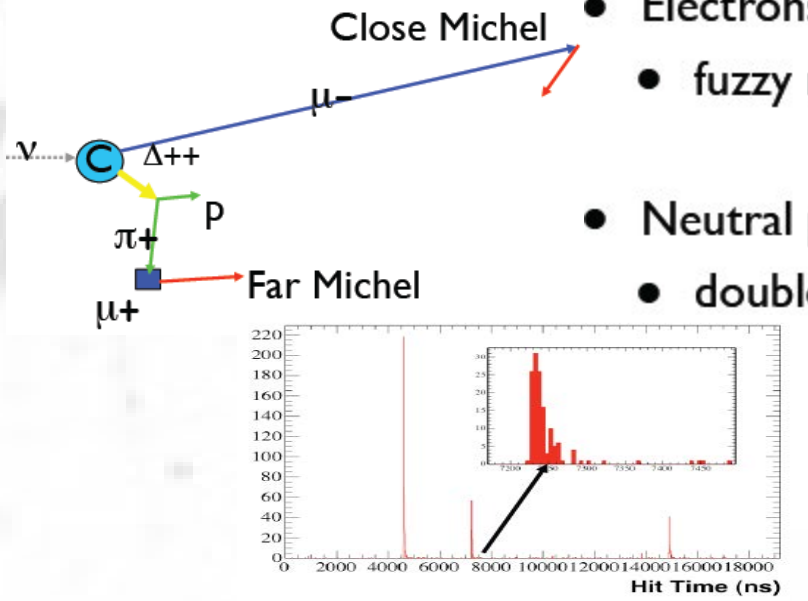


- BooNE and K2K: both have Cerenkov and Scintillator Bar detectors for measuring neutrino interactions
  - Cerenkov detectors have uniform acceptance, but high thresholds for massive particles
  - Scintillator bar detectors usually have a directional bias, typically smaller and may not contain interaction, but thresholds are lower than Cerenkov and particles can be identified by dE/dx
- NOMAD: drift chambers in an analyzing magnet
  - Good momentum measurement and possibly better particle identification by dE/dx, but diffuse material makes photon reconstruction difficult
- MINOS: coarse sampling iron detector
  - Difficult to distinguish particles other than muons, but very high rate

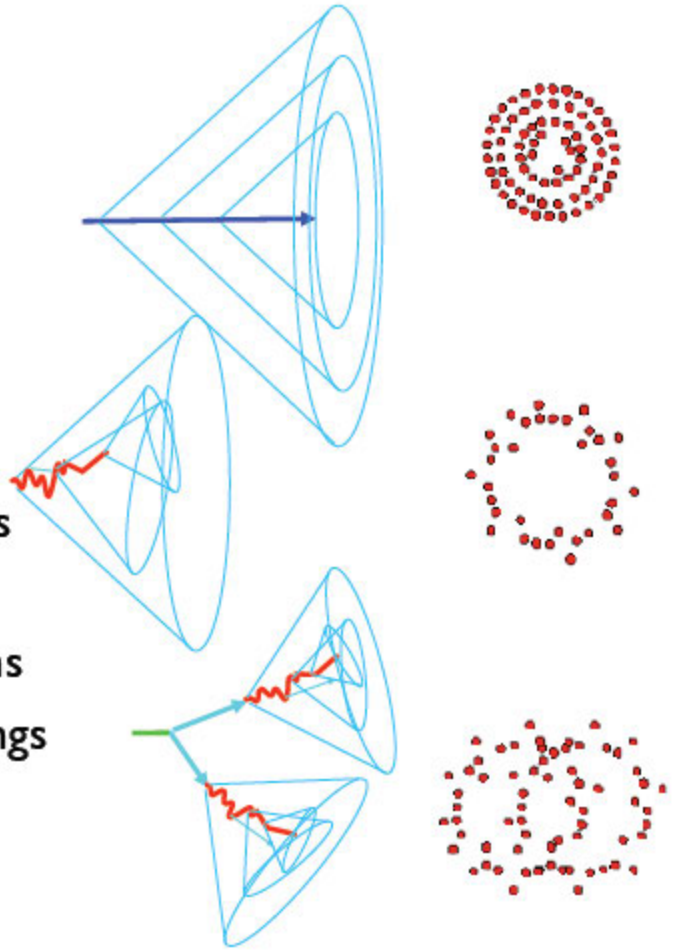
Technologies: Cerenkov Detectors

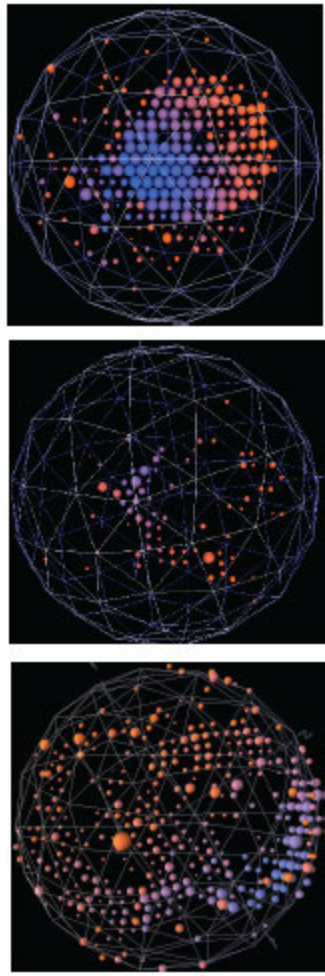


- Cerenkov gives efficient muon or e/γ identification
- Also, tag soft pions by decay



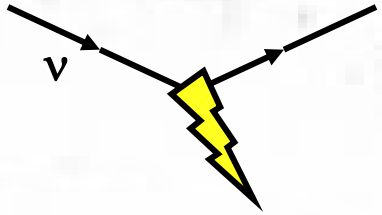
- Muons
  - full rings
- Electrons
  - fuzzy rings
- Neutral pions
  - double rings



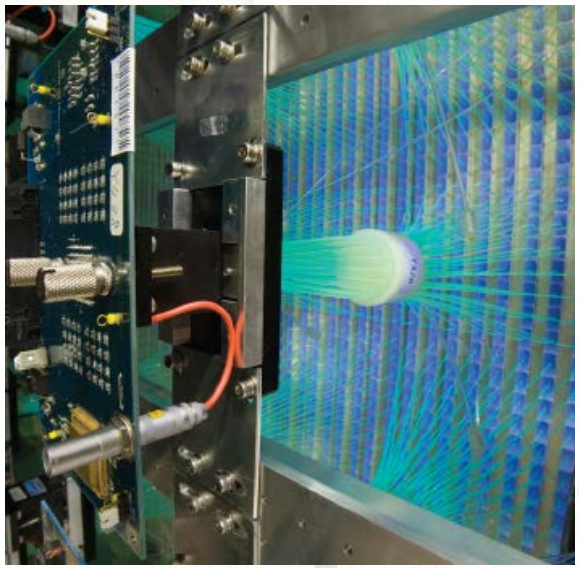


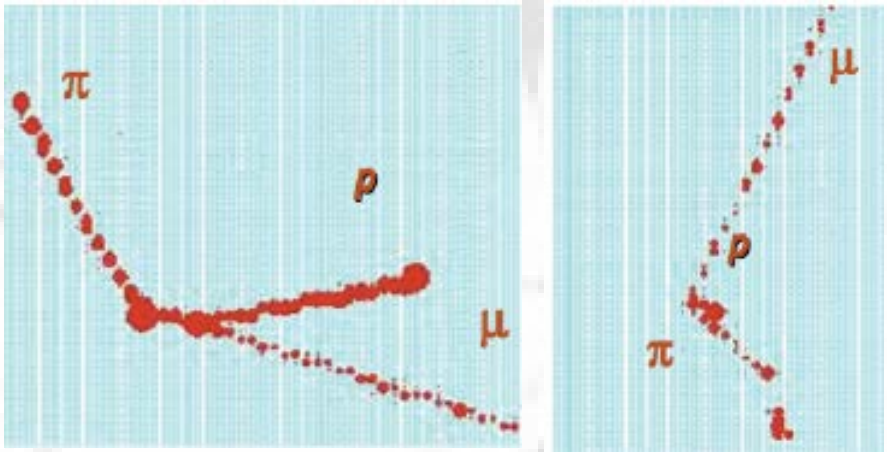
Figures from M. Wascko

Technologies: Segmented Scintillator



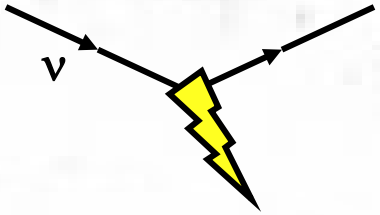
- Lower thresholds, particle ID by dE/dx, calorimetric energy reconstruction
  - i.e., vertex activity
- But detectors must be smaller (cost), so escaping particles
- Reconstruction not uniform in angle





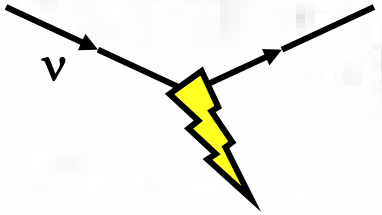
Figures from M. Wascko

Current and Near Future Experiments

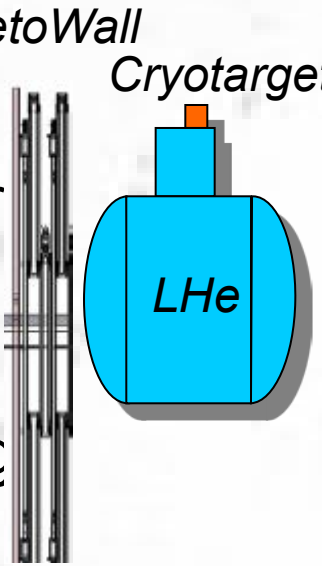


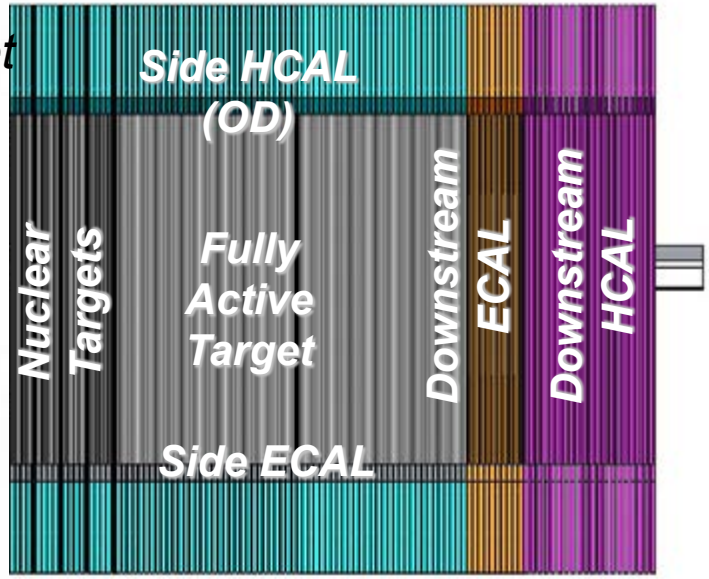
- MINERνA: in NuMI at Fermilab
  - Fine-grained scintillator detector
  - Nuclear targets of He, C, H_2O, Fe, Pb
- T2K 280m Near Detector at J-PARC
  - Fine-grained scintillator, water, and TPC's in a magnetic field
- NOνA near detector: to run in 2014
  - Segmented Liquid scintillator in off-axis beam
- MicroBooNE: to run in 2014
  - Liquid Argon TPC in FNAL Booster Beam
  - Some data from ArgoNeuT, a test in NuMI

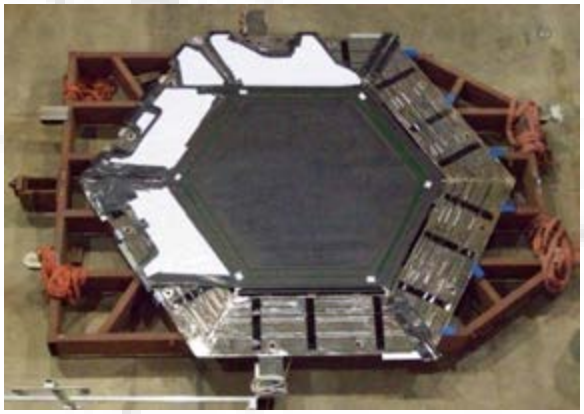
MINERνA Detector



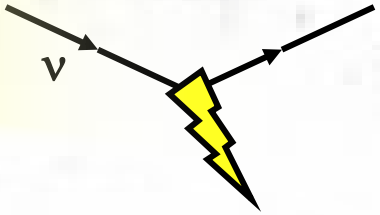
- 120 modules
  - Finely segmented scintillator planes read out by WLS fiber
  - Side calorimetry
- Signals to 64-anode PMT's
- Front End Electronics using Trip-t chips (thanks to D0)
- Side and downstream EM and hadron calorimetry
- MINOS Detector gives muon momentum and charge







ν Events in MINERνA



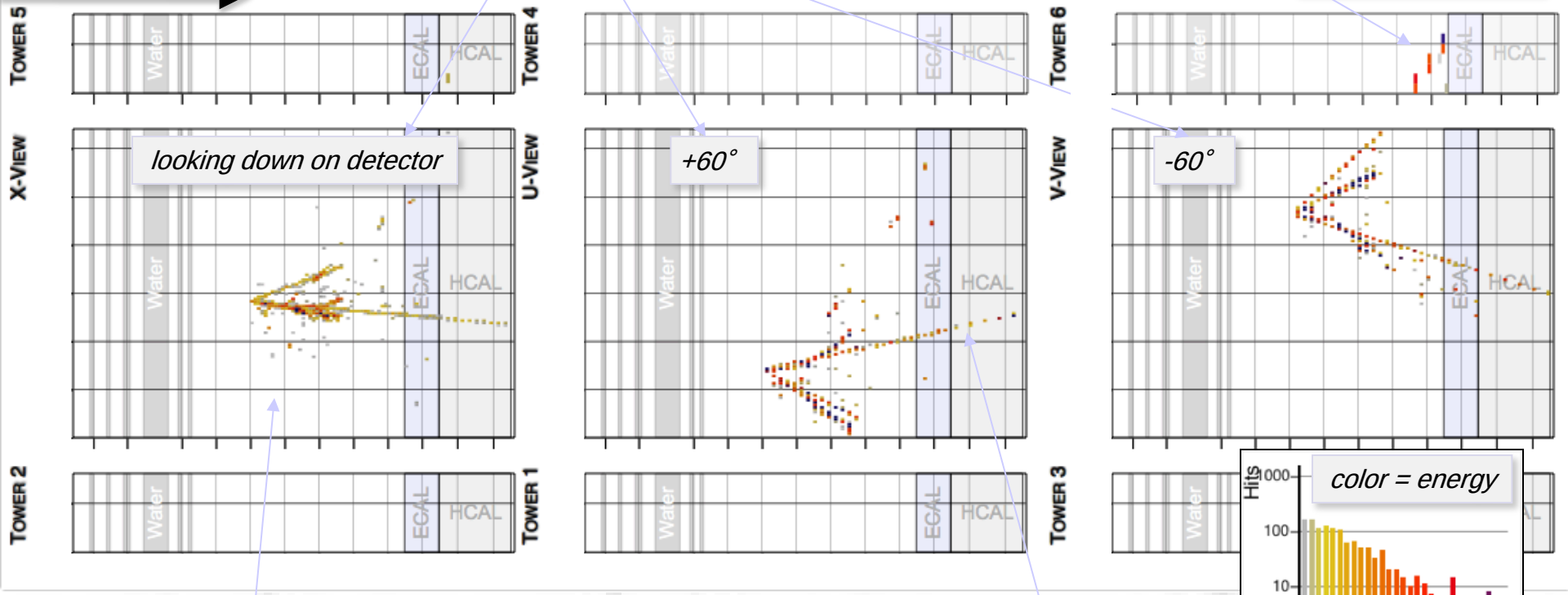
- So what does an event look like in MINERνA…

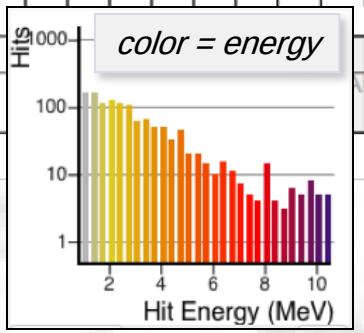
3 stereo views, X—U—V , shown separately

DATA

Particle leaves the inner detector, and stops in outer iron calorimeter

beam direction

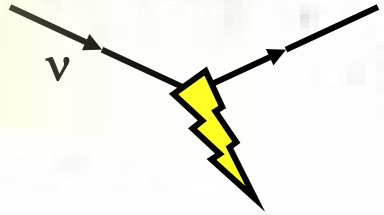




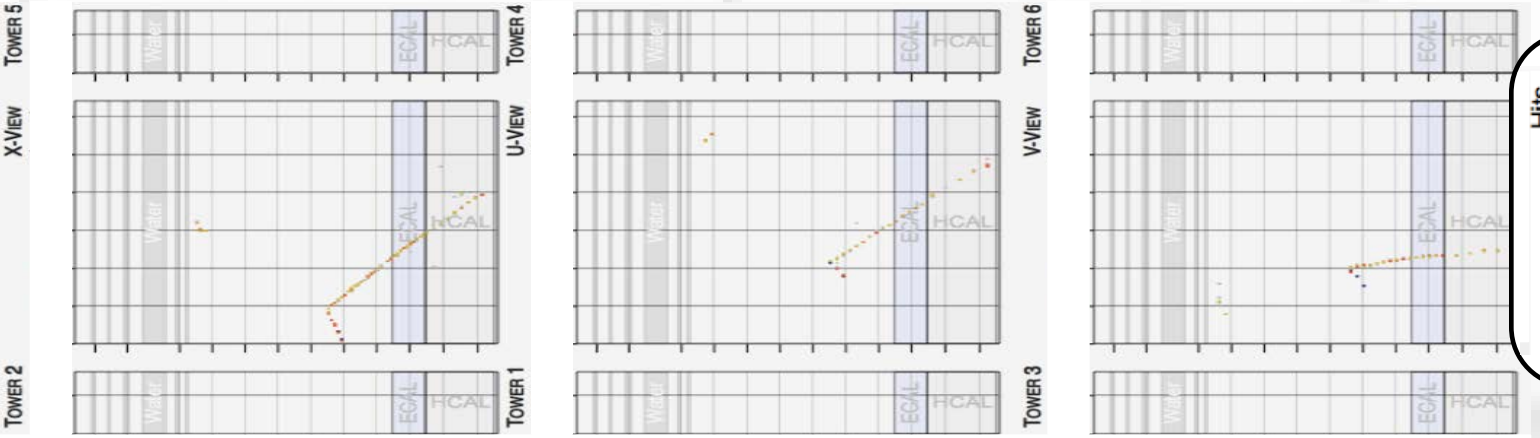
X views twice as dense, UX,VX,UX,VX,…

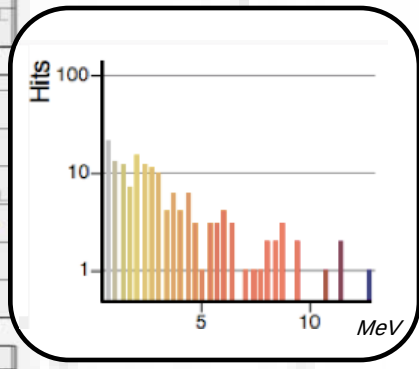
Muon leaves the back of the detector headed toward MINOS

ν Events in MINERνA



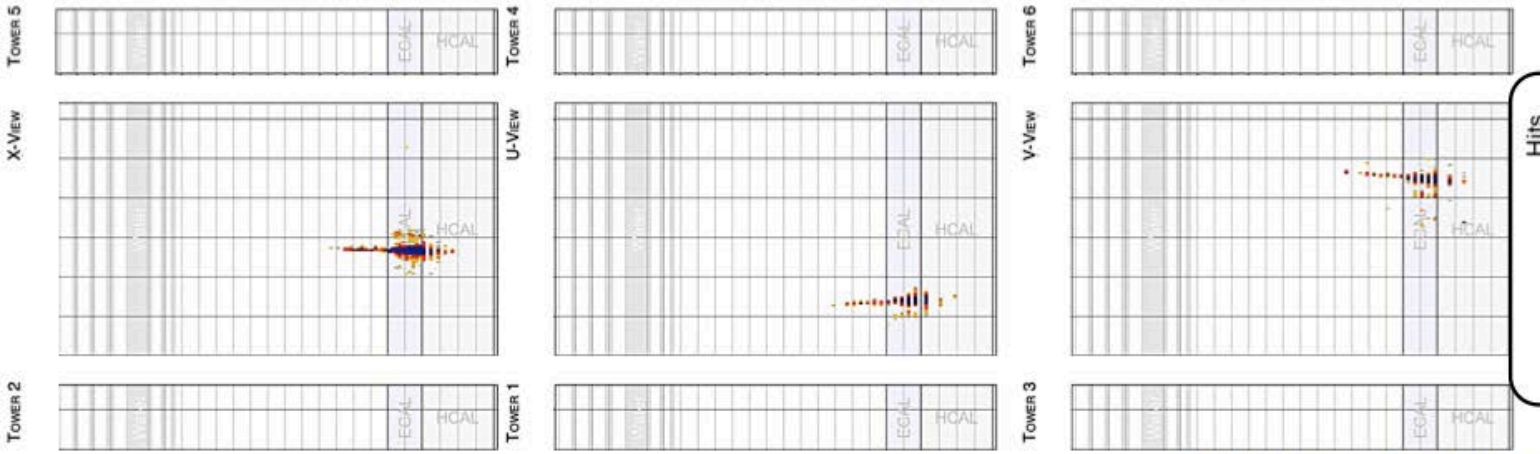
- Charged-current Quasi-elastic candidate

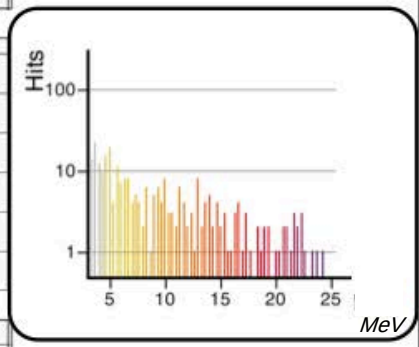




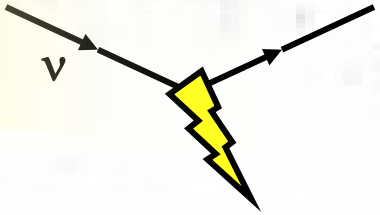
DATA

- Single Electron Candidate

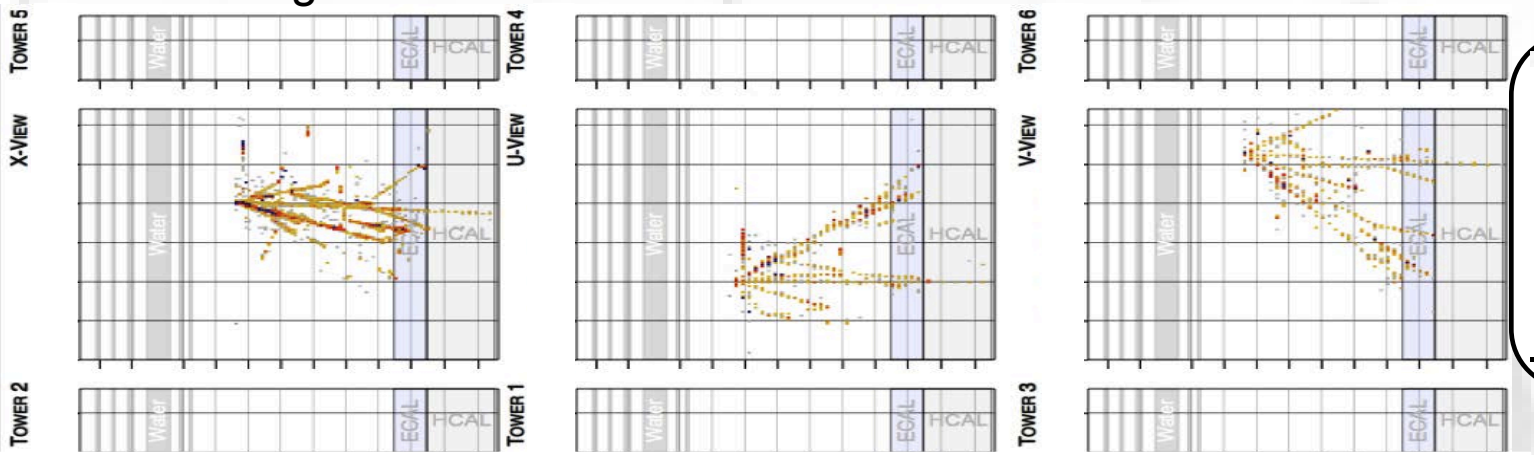


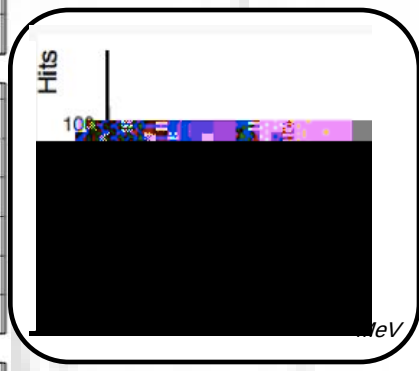


ν Events in MINERνA



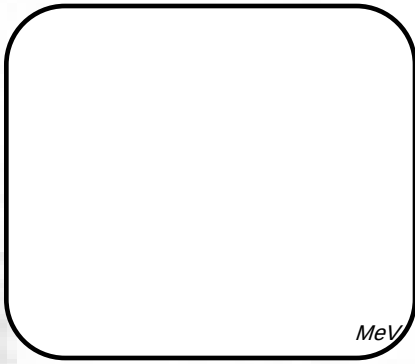
- Charged-current DIS candidate



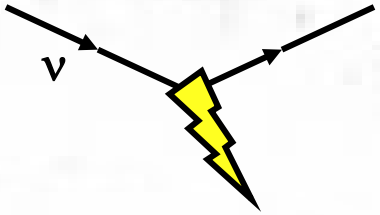


DATA

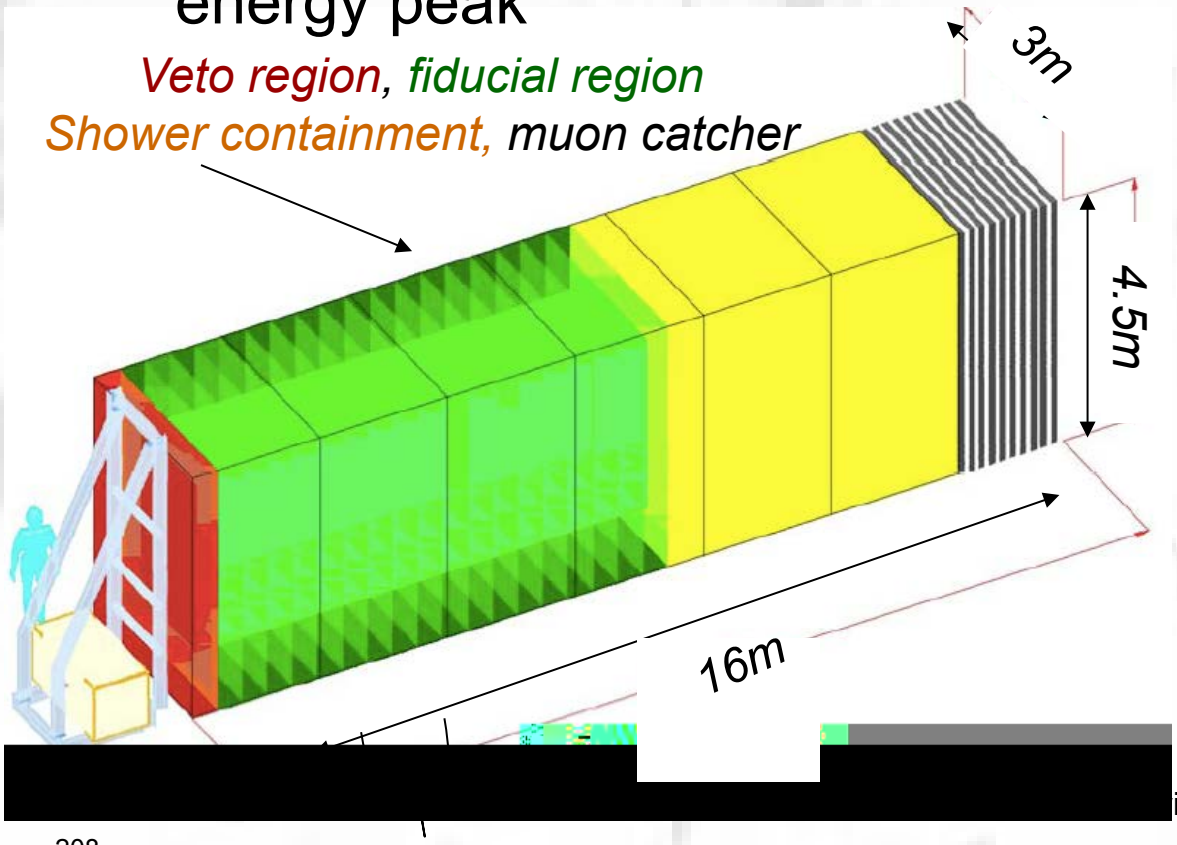
- Charged-current DIS candidate



NOvA Near Detector



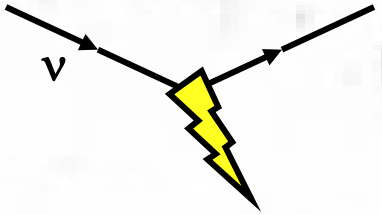
- Scintillator extrusion cross section of 3.87cm x 6cm , but with added muon range stack to see 2 GeV energy peak

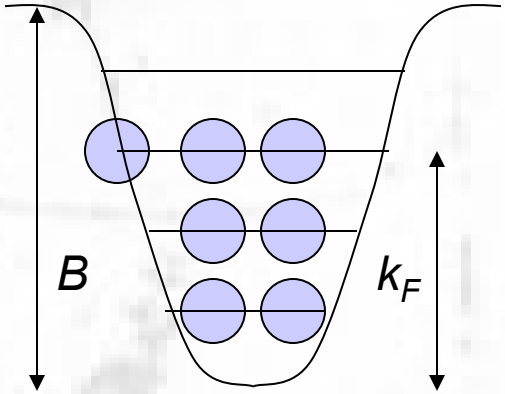


•Range stack: 1.7 meters long, steel interspersed with 10 active planes of liquid scintillator

•First located on the surface, then moved to final underground location

Nuclear Effects in Elastic Scattering



- Several effects:
  - In a nucleus, target nucleon has some initial momentum which modifies the observed scattering
    - Simple model is a "Fermi Gas" model of nucleons filling available states up to some initial state Fermi momentum, k_F

  - The nucleon is bound in the nucleus, so it take energy to remove it
  - Pauli blocking for nucleons not escaping nucleus… states are already filled with identical nucleon
  - Outgoing nucleon can interact with the target