

Collective neuronal phenomena: the rise and fall of Directed Percolation in modeling neuronal avalanches

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Neuroscience
ICTP-SAIFR - São Paulo - 2015-10



Outline:

Part I: The rise

Neuronal avalanches in cortical slices, anesthetized or resting animals

The critical brain hypothesis

Directed percolation, branching processes etc

An example of a (theoretical) critical brain at work

Summary of the pros

Part II: The fall

Fingerprints of brain criticality in freely-behaving animals

The undersampling issue

Summary of the cons

What next then?

Undersampling

Long-range time correlations

Concluding remarks



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Acknowledgements

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- ▶ CNPq & CAPES



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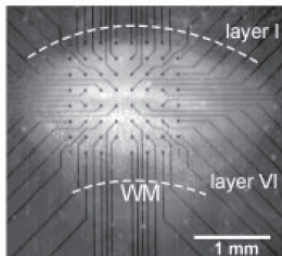
Concluding remarks



Multielectrode recording in slices of rat cortex

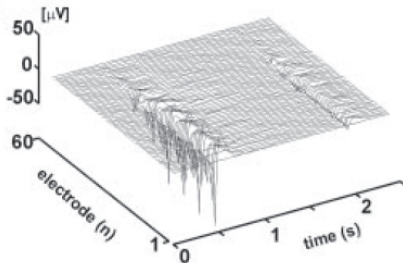
60 electrodes
(in vitro)

A



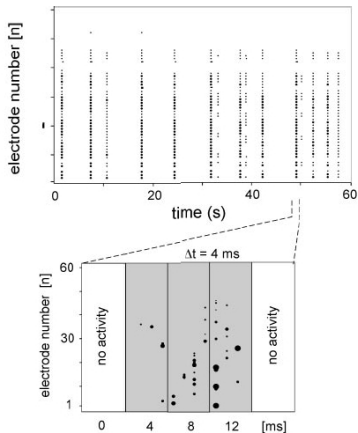
Local Field Potential (LFP)
= “average” electrical activity

B



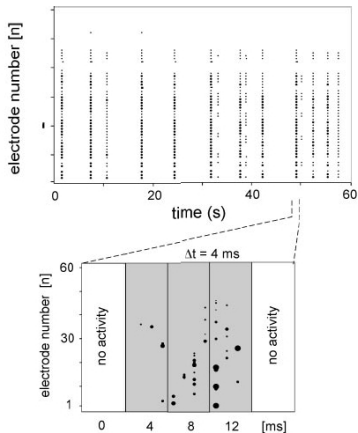
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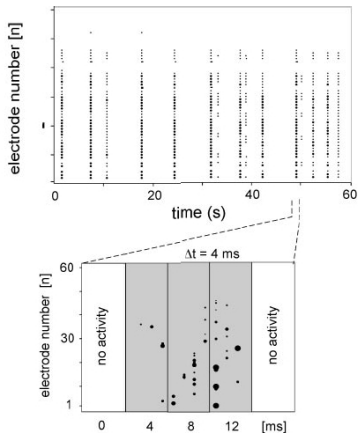
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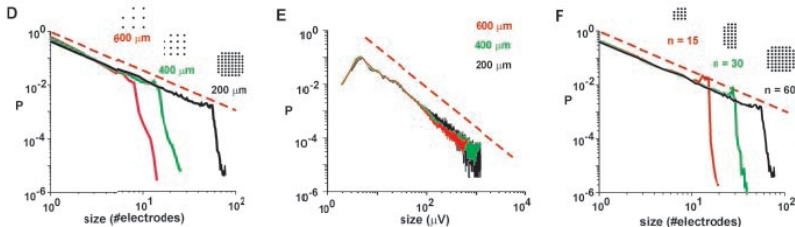
Separation of Time Scales (STS)

(duration $\ll$ recurrence interval)

Beggs and Plenz, *J. Neurosci.*, **23** 11167 (2003)

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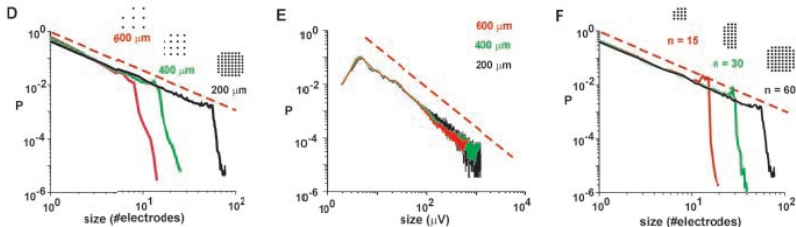
Distribution of avalanche sizes is a **power law**



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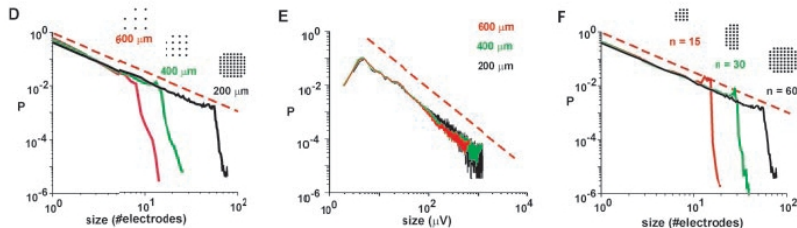


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Multielectrode recording in slices of rat cortex

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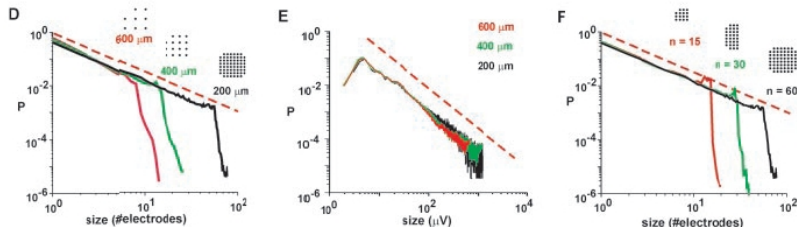
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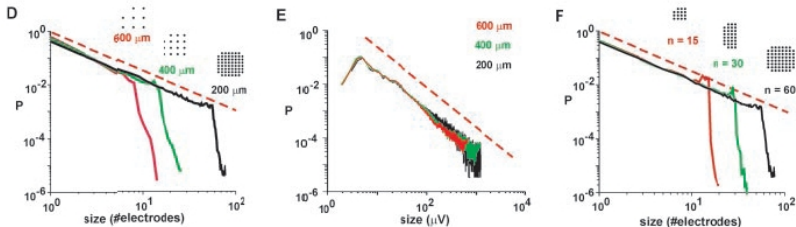
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meaningless



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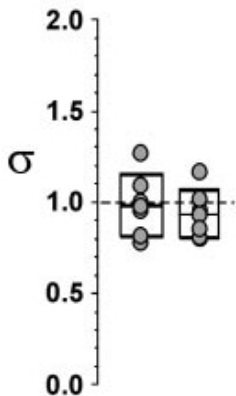
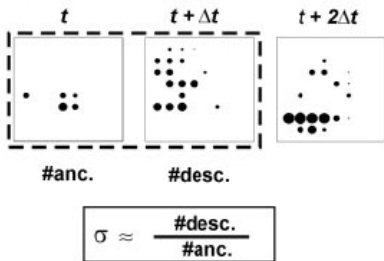
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(non-trivial) scale-invariant statistics

meaningless



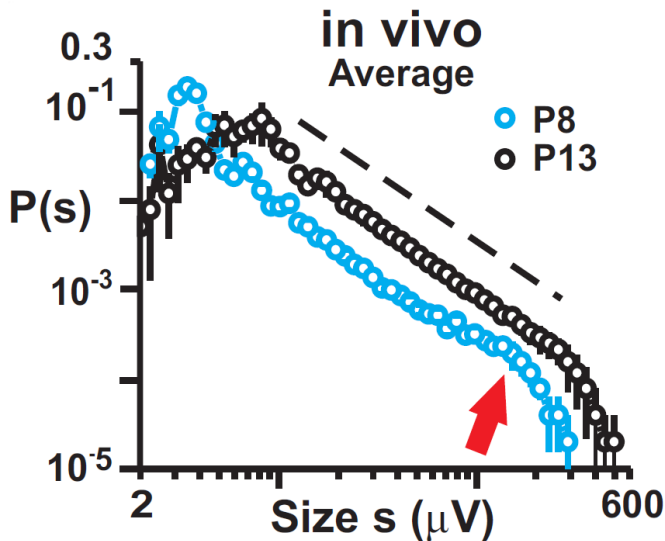
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Branching ratio $\sigma \approx 1$



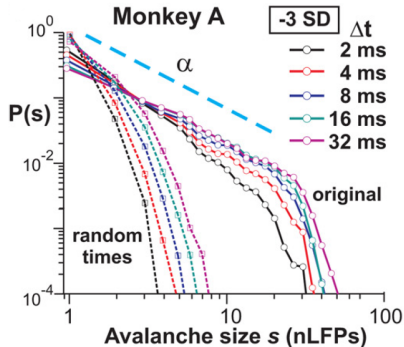
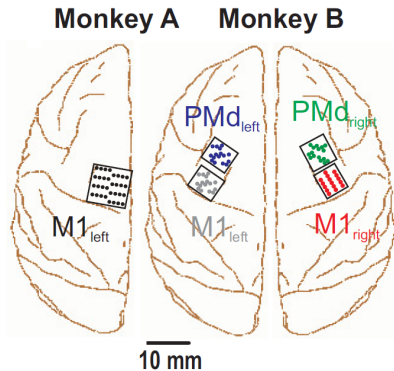
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Anesthetized rats



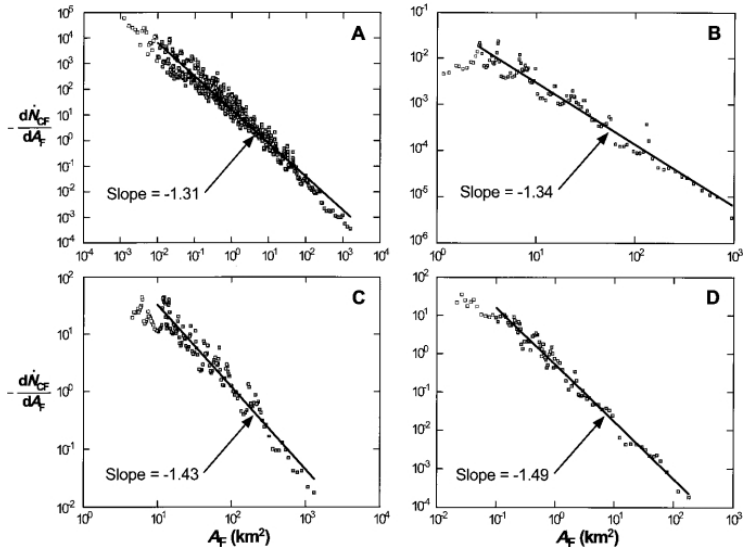
Gireesh and Plenz, *PNAS*, **105** 7576 (2008)

Unanesthetized (but resting) monkeys



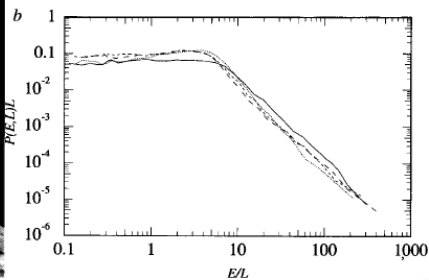
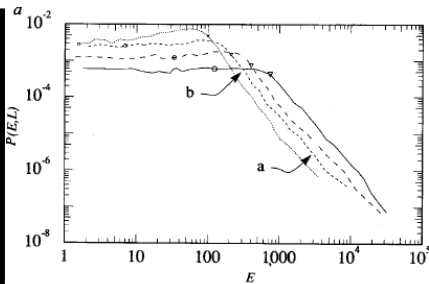
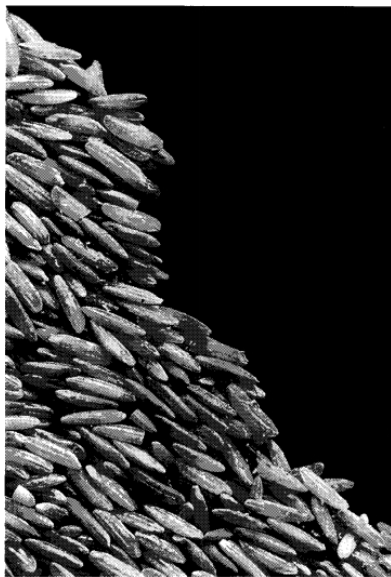
Petermann et al., *PNAS*, **106** 15921 (2009)

Forest fires



Malamud et al., *Science*, **281** 1840 (1998)

Rice piles



Frette et al., *Nature*, **379** 49 (1996)

Solar flares

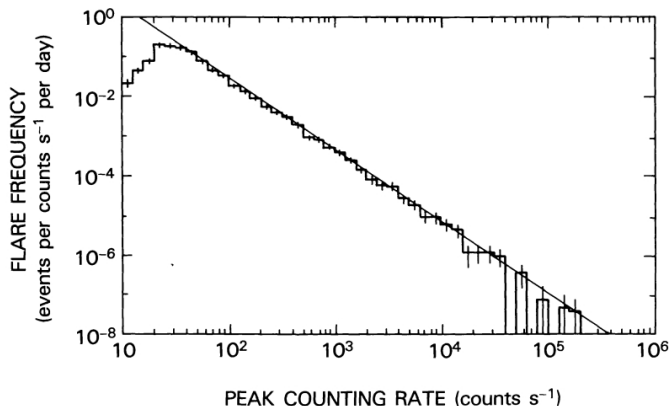


Fig. 3. Peak rate spectrum of all complete events detected with HXRBS from launch to February 1985. The straight line through the data corresponds to the power-law expression given in the text with a spectral index of -1.8 .

Dennis, *Solar Phys.*, **100** 465 (1985)

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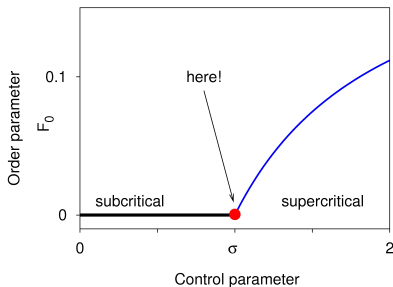


The critical brain hypothesis

The brain (as a dynamical system) operates near a **critical point**



Turing (50's)
Bak & Chialvo (90's)
Chialvo *Nat. Phys.* (2010)
Shew & Plenz *Neuroscientist* (2013)



Second order phase transition

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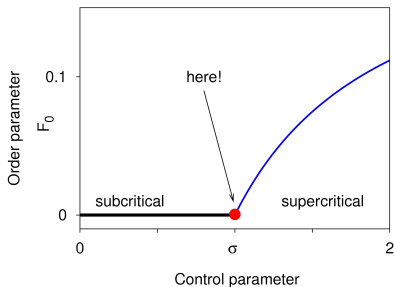


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Second order phase transition

Control parameter(s)?

Order parameter(s)?



Example: ferromagnetism

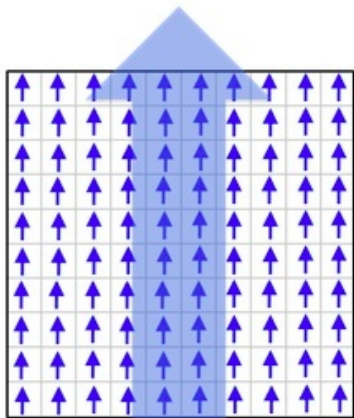
Iron atoms



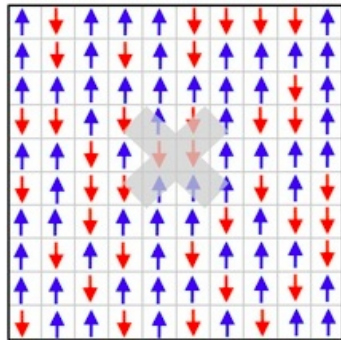
Magnet

Low temperature

High temperature



magnetic



non-magnetic

Example: ferromagnetism

Iron atoms



Magnet

Low temperature



High temperature



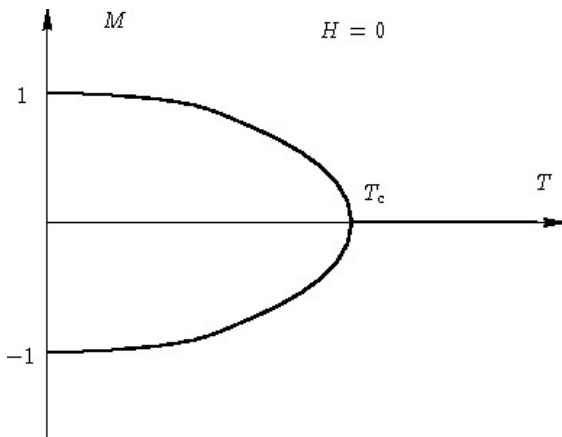
<http://www.youtube.com/watch?v=haVX24hOwQI>

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Phase transition at “critical temperature” T_c

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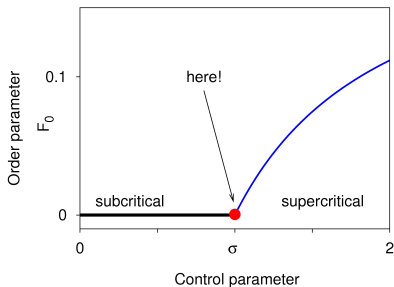


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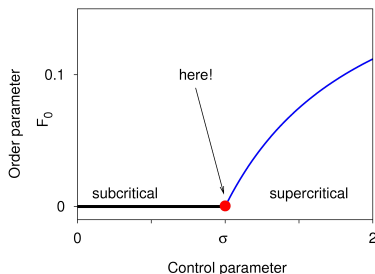
Order parameter(s)?



Two-slide course on phase transitions



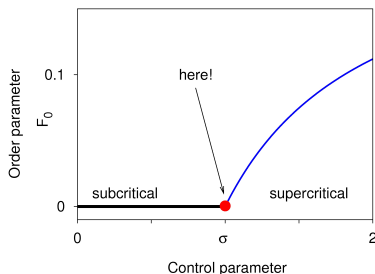
Phase transition at a critical point (CP)



Nontrivial scale-invariant behavior near CP characterized by critical exponents:

- ▶ $F_0 \propto (\sigma - \sigma_c)^\beta$ (order parameter near CP)
- ▶ $F \stackrel{\sigma=\sigma_c}{\propto} h^{1/\delta_h}$ (response to stimulus h at CP)
- ▶ $C(\mathbf{x}) \propto e^{-|\mathbf{x}|/\xi}$, $\xi \propto |\sigma - \sigma_c|^{-\nu_\perp}$ (divergent correlation length)
- ▶ $P(s) \sim s^{-\tau}$ (no characteristic avalanche size)
- ▶ ...

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Let us imagine two different models, A and B:



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Model B:

- ▶ $F_0 \propto (\sigma - \sigma'_c)^{\beta'}$
- ▶ $F \stackrel{\sigma=\sigma'_c}{\propto} h^{1/\delta'_h}$
- ▶ $\xi \propto |\sigma - \sigma'_c|^{-\nu'_\perp}$
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What is universality?

Let us imagine two different models, **A** and **B**:

Model A:

$$\blacktriangleright F_0 \propto (\sigma - \sigma_c)^\beta$$

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$\blacktriangleright \dots$

Model B:

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$\blacktriangleright \dots$

Models **A** and **B** belong to the same **universality class** if their **critical exponents are the same**:

$$\beta = \beta'$$

$$\delta_h = \delta'_h$$

$$\nu_\perp = \nu'_\perp$$

$$\tau = \tau'$$

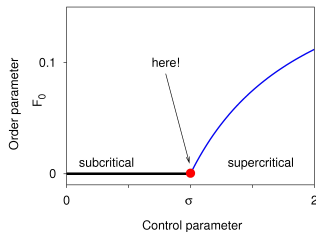
$\vdots$

End of two-slide course on phase transitions



The critical brain hypothesis

The brain (as a dynamical system) operates near a critical point



Why would you expect the hypothesis to be true?

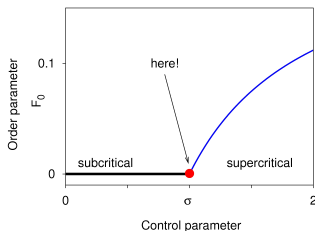
Theory predicts that scale-invariant dynamics provide

- ▶ optimal transmission capacity
- ▶ optimal information processing
- ▶ largest "repertoire" (e.g. for memory storage) etc.
- ▶ optimal sensitivity and dynamic range to incoming stimuli

All highly desirable for a brain! (evolutionary pressure)

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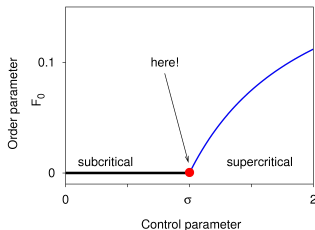
Beggs & Plenz, J. Neurosci. **23**, 11167, 2003

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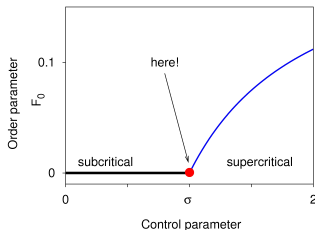
Bertschinger & Natschläger, *Neural Comput.* **16**, 1413, 2004

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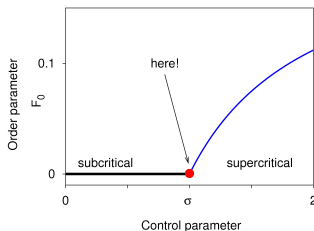
Haldeman & Beggs, *Phys. Rev. Lett.* **94**, 058101, 2005, Shew et al., *J. Neurosci.* **31**, 55, 2011

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Kinouchi & Copelli, Nat. Phys. **2** 348, 2006, Shew et al., J. Neurosci. **29** 15595, 2009

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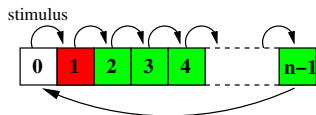
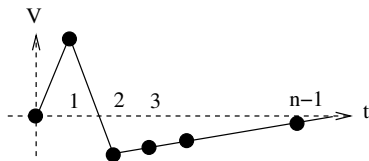
Example of criticality in a (very) simple model of the brain



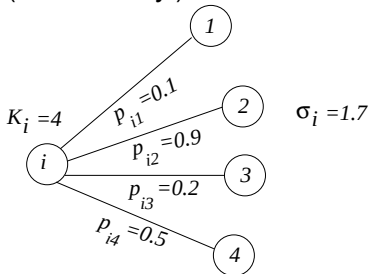
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A network of **excitable** cellular automata



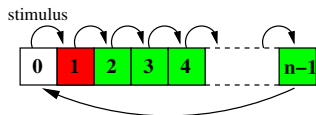
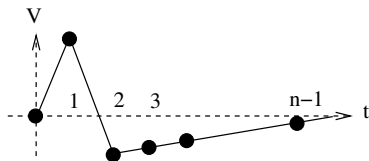
Random graph: $\langle K_i \rangle = K$
(Erdős-Rényi)



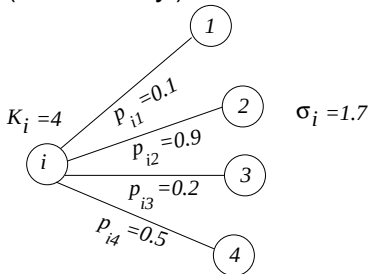
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Control parameter:

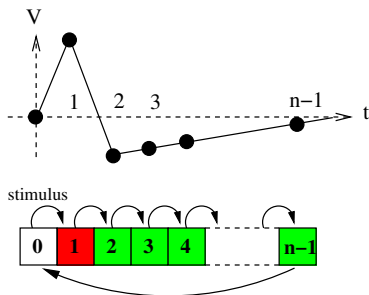
Branching ratio: $\sigma = \langle \sigma_i \rangle$



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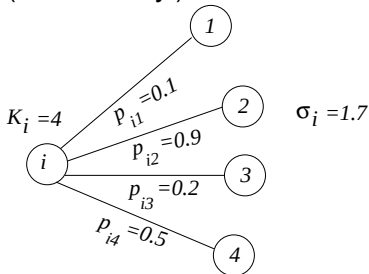
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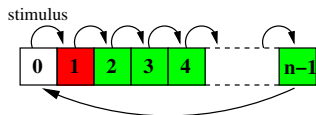
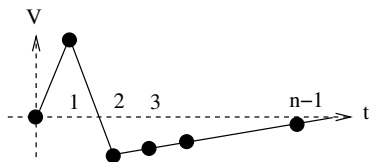
Firing rate

$$F = \langle \rho \rangle = \lim_{t \rightarrow \infty} P_t(1)$$

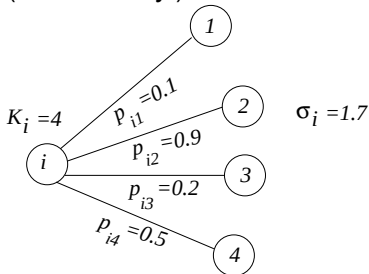
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$$F = \langle \rho \rangle = \lim_{t \rightarrow \infty} P_t(1)$$

Mean-field equation:

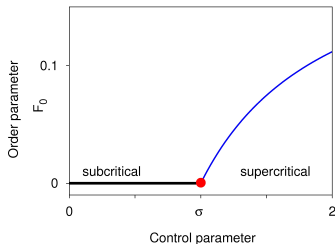
$$F = \overbrace{(1 - (n-1)F)}^{P(\text{quiet})} \times [1 - \underbrace{(1 - \sigma F/K)^K}_{P(! \text{ from neighbors})}] \times \underbrace{(1 - p_h)}_{P(! \text{ external})}$$

Without stimulus:

$$F_0 = F(p_h = 0)$$

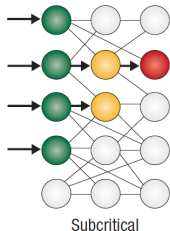
Branching process (Galton-Watson model, 1874)

Contact process (Harris, 1974), Directed percolation (DP) universality class

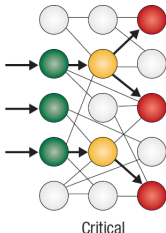


Connected **excitable** neurons:
phase transition at $\sigma = \sigma_c = 1$

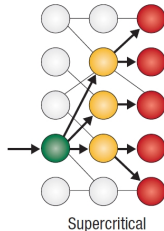
$\sigma < 1$



$\sigma = \sigma_c = 1$

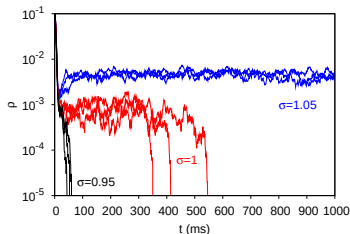


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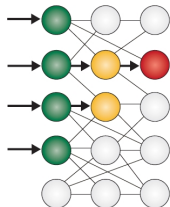


Control parameter: σ = branching ratio

Order parameter: $F = \langle \rho \rangle \sim$ firing rate

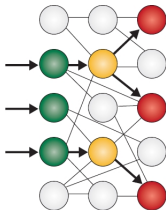
DP phase transition: between
absorbing and **active**
(but otherwise unstructured) phases

$\sigma < 1$



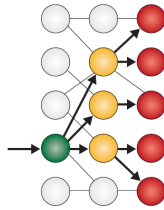
Subcritical

$\sigma = \sigma_c = 1$



Critical

$\sigma > 1$



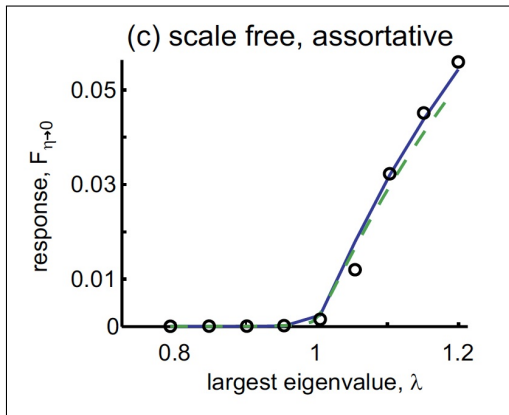
Supercritical

More rigorous criterion for criticality:

E.g. Scale-free networks etc.

$\lambda = \text{largest eigenvalue of } \{p_{ij}\} = 1 \text{ at criticality}$

($\sigma_c = 1 = \lambda$ for uncorrelated random graphs!)



Larremore et al., *Phys. Rev. Lett.* **106** 058101 (2011)

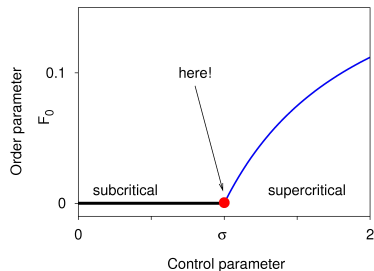
Larremore et al., *Phys. Rev. E* **85** 066131 (2012)

Theory vs. Experiments

(STS = Separation of Time Scales)

Theoretical prediction:

System at **critical point** +
STS by hand=

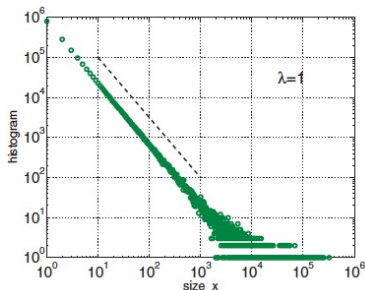


Theory vs. Experiments

(STS = Separation of Time Scales)

Theoretical prediction:

System at **critical point** +
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power-law distributed
avalanches!



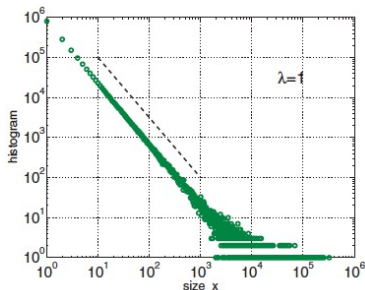
Larremore et al. (2012)

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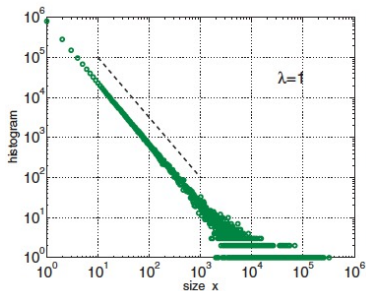
Experimental results:

Theory vs. Experiments

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Theoretical prediction:

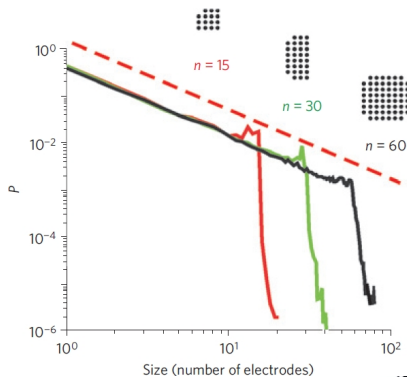
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STS emerges!



Beggs & Plenz (2003)

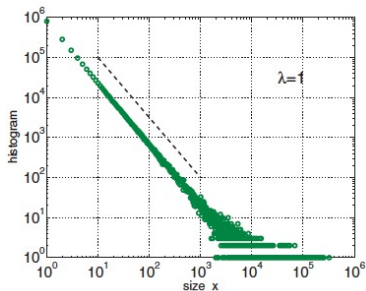


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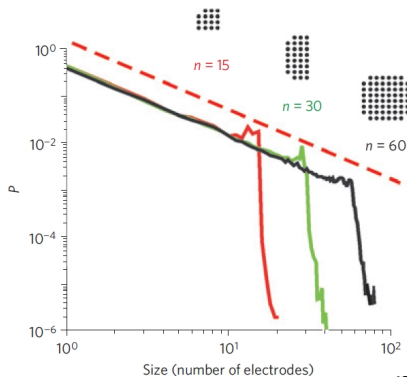
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Neuronal avalanches are compatible with critical dynamics

Critical exponents interdependent via scaling theory

Directed percolation (DP) universality class for $d = 4$: upper critical dimension

Exponent	$d = 1$	$d = 2$	$d = 3$	$d = 4$
$\beta = \beta'$	0.27649(4) ^a	0.583(4) ^b	0.805(10) ^c	1
$1/\delta_h$	0.111(3) ^d	0.285(35) ^d	0.45(2) ^e	1/2
γ'	0.54386(7) ^a	0.35	0.19	0
$\nu_{ }$	1.73383(3) ^a	1.295(6) ^f	1.105(5) ^c	1
$\nu_{\perp}$	1.09684(1) ^a	0.733(4) ^g	0.581(5)	1/2
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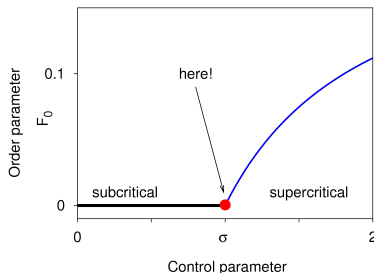
Undersampling

Long-range time correlations

Concluding remarks



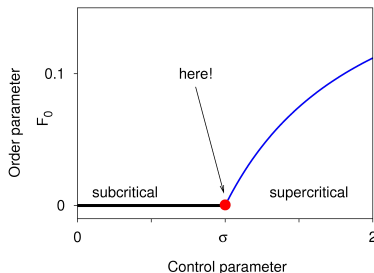
Phase transition at a critical point (CP)



Nontrivial behavior near CP characterized by critical exponents:

- ▶ $F_0 \propto (\sigma - \sigma_c)^\beta$ (order parameter near CP)
- ▶ $F \stackrel{\sigma=\sigma_c}{\propto} h^{1/\delta_h}$ (response to stimulus h at CP)
- ▶ $C(t) \propto e^{-t/\tau}$, $\tau \propto |\sigma - \sigma_c|^{-\nu_\parallel}$ (divergent correlation time)
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Psychophysics

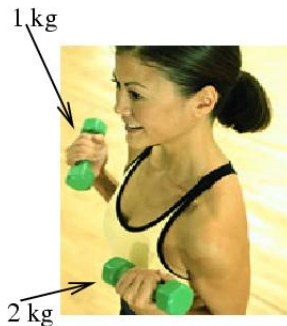
Psychophysics

Founded in 1860: first experimental area of neuropsychology

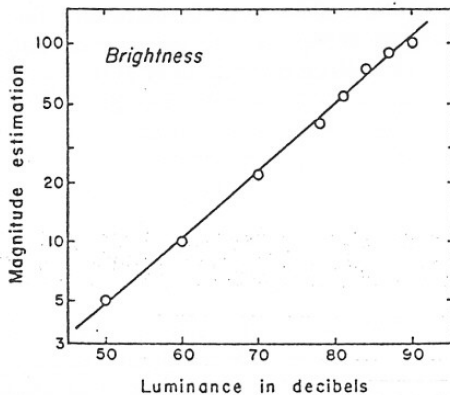
- ▶ What is the **human perception** of the intensity of a **sensory stimulus**?



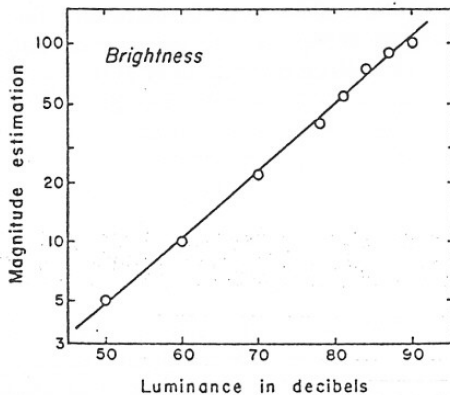
Psychophysics: Nonlinear Response!



Stevens law: perception $\propto$ (stimulus intensity)^m

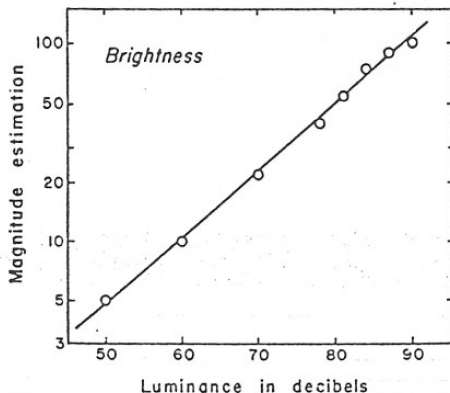


Stevens law: perception $\propto$ (stimulus intensity) m



$$m < 1$$

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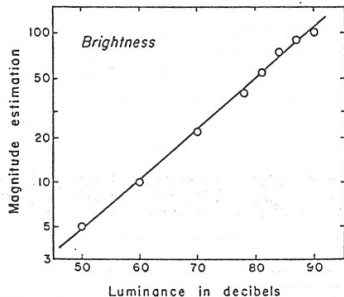


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HOW?

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Psychophysics



Continuum	Measured Exponent	Stimulus condition
Brightness	0.5	Point source
Visual length	1.0	Projected line
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S. S. Stevens, "Psychophysics: Introduction to its Perceptual, Neural and Social Prospects" (1975)

Response to **weak** stimulus (no coupling, $\sigma = 0$)

Response to **stronger** stimulus (no coupling, $\sigma = 0$)



Response to **very strong** stimulus (no coupling, $\sigma = 0$)

Response to stimulus (**weak** coupling, $\sigma \gtrsim 0$)

Response to stimulus (**stronger** coupling, $\sigma \lesssim 1$)

Response to stimulus (**critical** coupling, $\sigma \simeq 1$)



Response to stimulus (**supercritical** coupling, $\sigma > 1$)

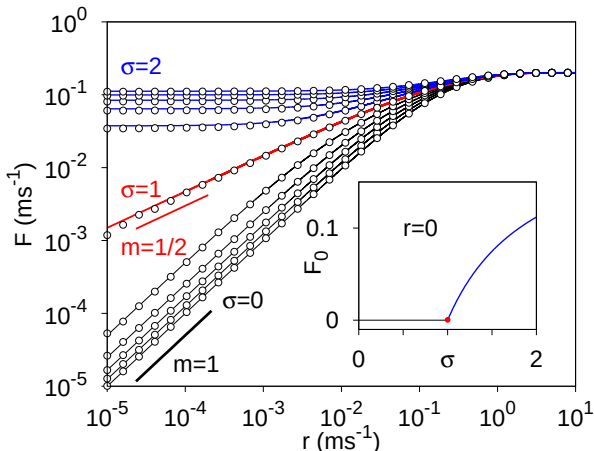


Family of response curves

Random graphs:

critical exponent $m = 1/\delta_h \leq 1/2$ agrees with:

- ▶ psychophysical exponents (Stevens law)
- ▶ olfactory glomerulus responses to odorants

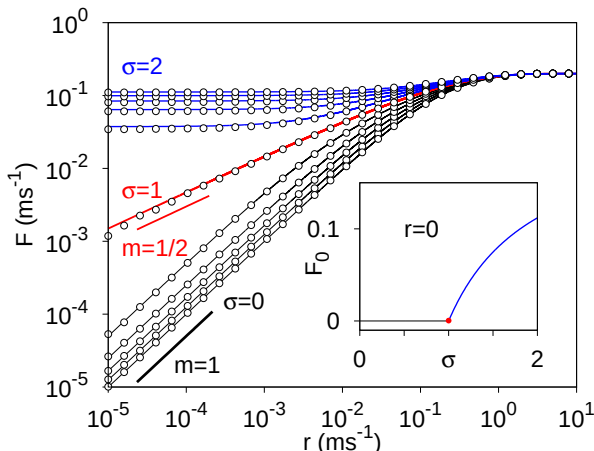


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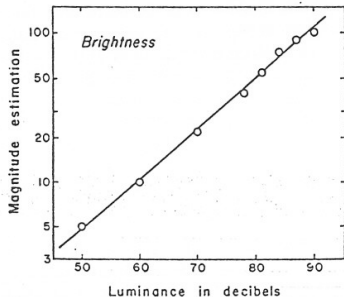
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Psychophysics

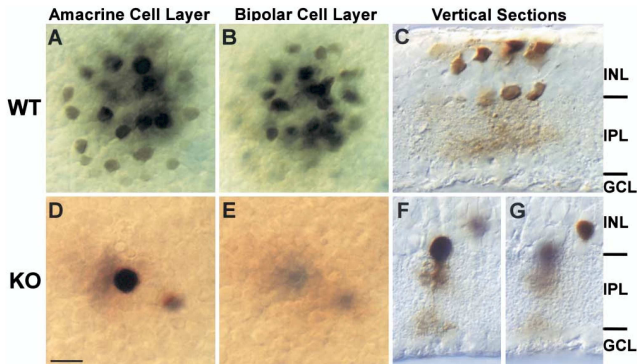


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Supported by experimental data

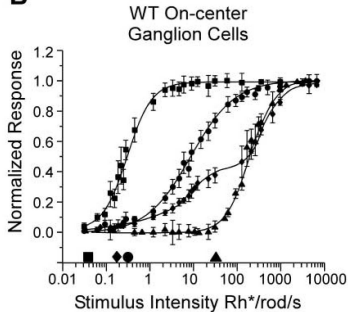
Gap Junctions in the mouse retina



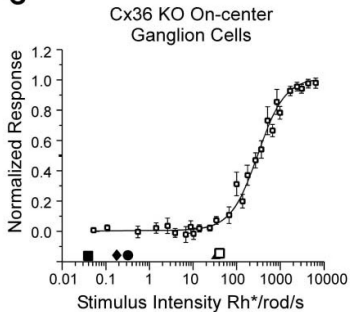
Deans et al., *Neuron*, **36** 703 (2002)

Gap Junctions in the mouse retina

B

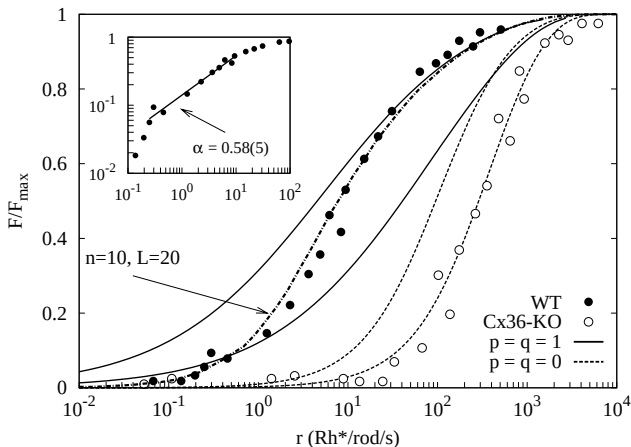


C



Deans et al., *Neuron*, **36** 703 (2002)

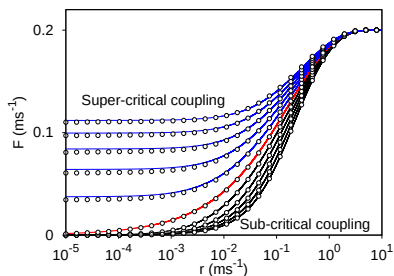
Ganglion cell response has exponent $m \simeq 0.58$!



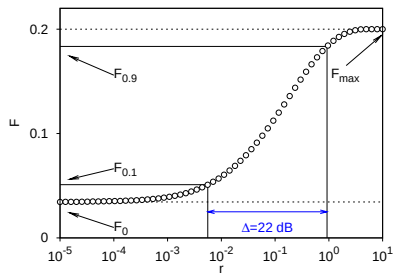
L. S. Furtado & M. Copelli, *Phys. Rev. E*, **73** 011907 (2006)

Family of response curves

Firing rate vs stimulus



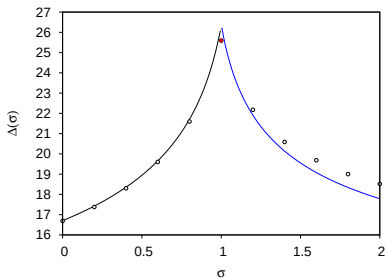
Dynamic range definition:



Dynamic range is optimal at criticality

General property of excitable media

Theory



Kinouchi & Copelli
Nat. Phys. **2** 348 (2006)

Dynamic range is optimal at criticality

General property of excitable media

Supported by very different models:

- ▶ Probabilistic excitable models
 - ▶ random graphs: Kinouchi & MC (*Nat. Phys.* 06)
 - ▶ hypercubic lattices: Furtado & MC (*PRE* 06), Assis & MC (*PRE* 08)
 - ▶ scale-free networks: MC & Campos (*EPJB* 07), Wu et al. (*PRE* 07), Larremore et al. (*PRL* 11)
- ▶ Deterministic excitable models
 - ▶ cellular automata: MC et al (*PRE* 02), MC & Kinouchi (*Phys. A* 05), MC et al (*Neurocomput.* 05)
 - ▶ Morris-Lecar lattices: Ribeiro & MC (*PRE* 08)
- ▶ Detailed conductance-based retina model: Publio et al (*PLoS ONE* 10)
- ▶ Active dendritic trees: Gollo et al (*PLoS CB* 09, *PRE* 12)
- ▶ Disinhibition transition in the antennal lobe: Buckley & Nowotny (*PRL* 11)

Dynamic range is optimal at criticality

General property of excitable media

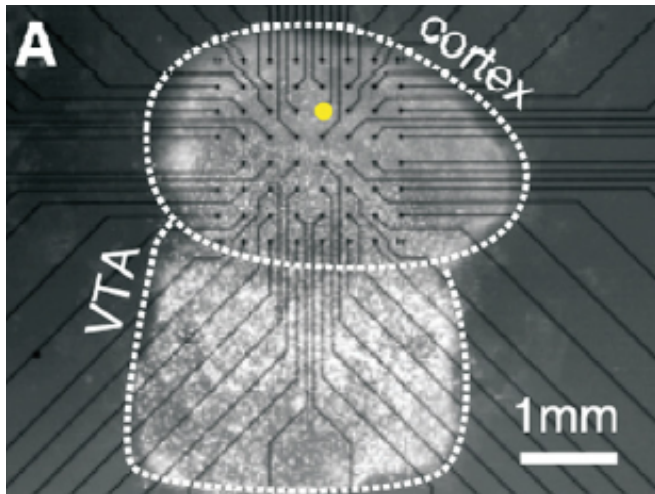
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General property of excitable media

Is it true experimentally?

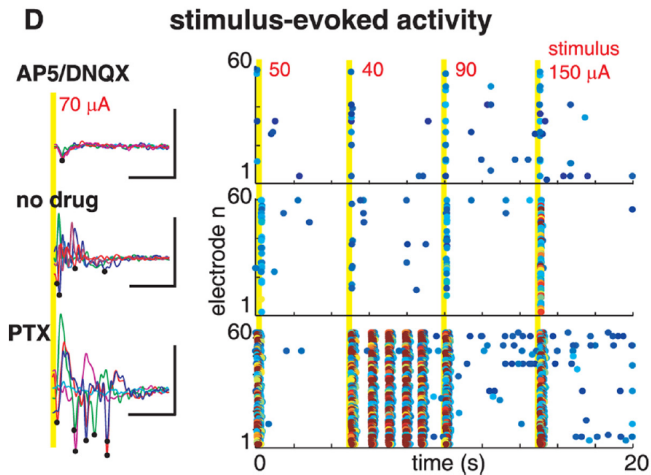


Stimulated slices



Shew et al., *J. Neurosci.* **29** 15595 (2009)

Stimulated slices

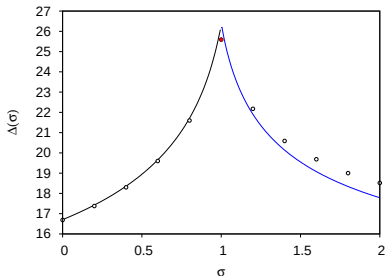


Shew et al., *J. Neurosci.* **29** 15595 (2009)

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General property of excitable media

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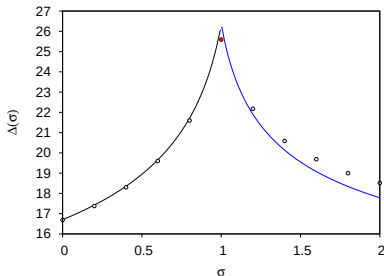


Kinouchi & Copelli
Nat. Phys. **2** 348 (2006)

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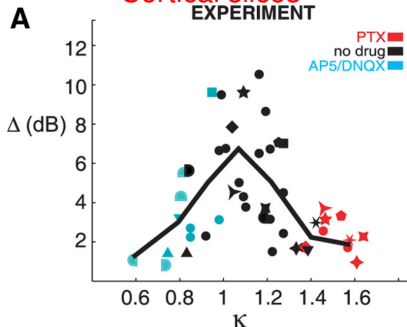
General property of excitable media

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Kinouchi & Copelli
Nat. Phys. **2** 348 (2006)

Cortical slices
EXPERIMENT



Shew et al.
J. Neurosci., **29** 15595 (2009)



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Are critical branching processes satisfactory models of neuronal avalanches?

Branching-process-like models (absorbing vs active phases):

► Pros:

- Neuronal avalanche exponents ($\tau = 3/2$ and $\tau_t = 2$)
- Response exponents ($1/\delta_h = 1/2$: psychophysics, sensory systems, ...) + optimal dynamic range at criticality
- Theoretical workhorse (useful picture) for over a decade

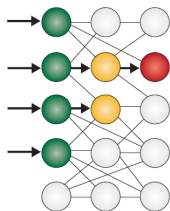
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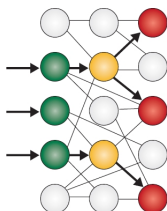
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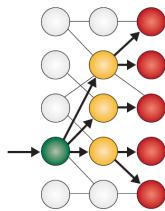
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Acknowledgements

- ▶ **Tiago L. Ribeiro** (NIH, USA)
- ▶ **Leonardo L. Gollo** (Queensland, Australia)
- ▶ **Vladimir R. V. Assis** (UEFS, Brazil)
- ▶ **Lucas S. Furtado** (Petrobras, Brazil)
- ▶ **Leonardo Dalla Porta** (Recife, Brazil)
- ▶ Osame Kinouchi (USP - Ribeirão Preto, Brazil)
- ▶ **Sidarta Ribeiro** (Natal, Brazil)
- ▶ **Fábio Caixeta** (Natal, Brazil)
- ▶ **Hindiael Belchior** (Natal, Brazil)
- ▶ **Dante R. Chialvo** (L.A., USA)

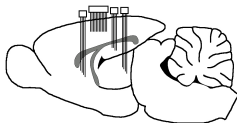
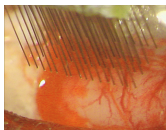
Financial Support:

- ▶ PRONEX (FACEPE/CNPq) Biological Physics
- ▶ PRONEM (FACEPE/CNPq)
- ▶ CNPq & CAPES



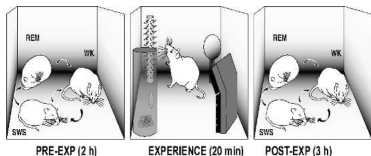
Non-anesthetized and freely-behaving rats

In collaboration with Brain Institute UFRN (Natal)



Main differences:

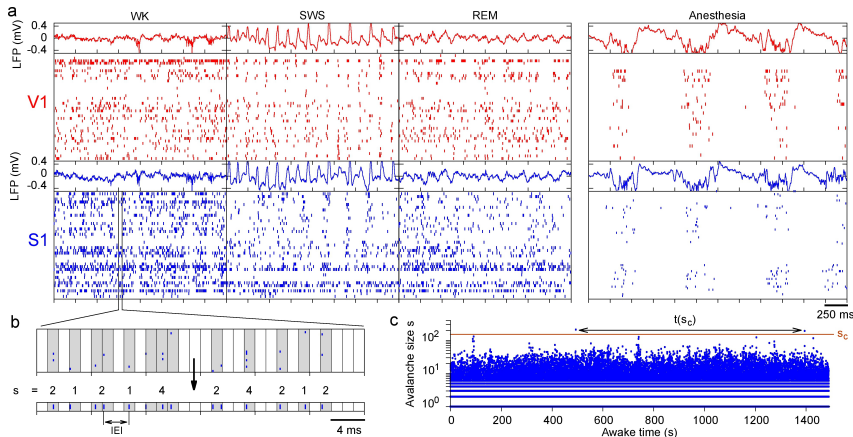
- ▶ Freely-behaving (FB) rats, across the sleep-wake cycle (WK, SWS, REM)
- ▶ 3 brain regions (V1, S1 and HP)
- ▶ Exposure to novel objects (PRE, EXP, POST)
- ▶ Spikes (not LFPs)
- ▶ Anesthesia (AN) as well



Ribeiro et. al, *Frontiers Neurosci.* 1 43 (2007)

Spike avalanches in freely-behaving (FB) rats

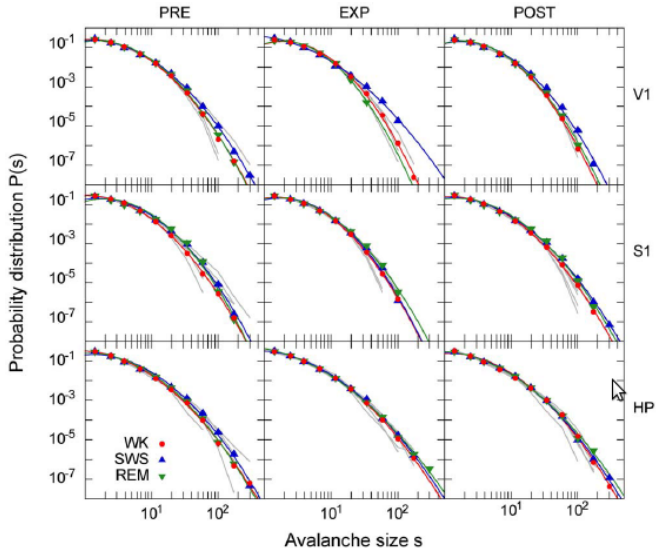
- ▶ Separation of time scales in AN, but not in FB (avalanches?)
- ▶ time bin = $\langle \text{ISI} \rangle$ (calculated for each situation)



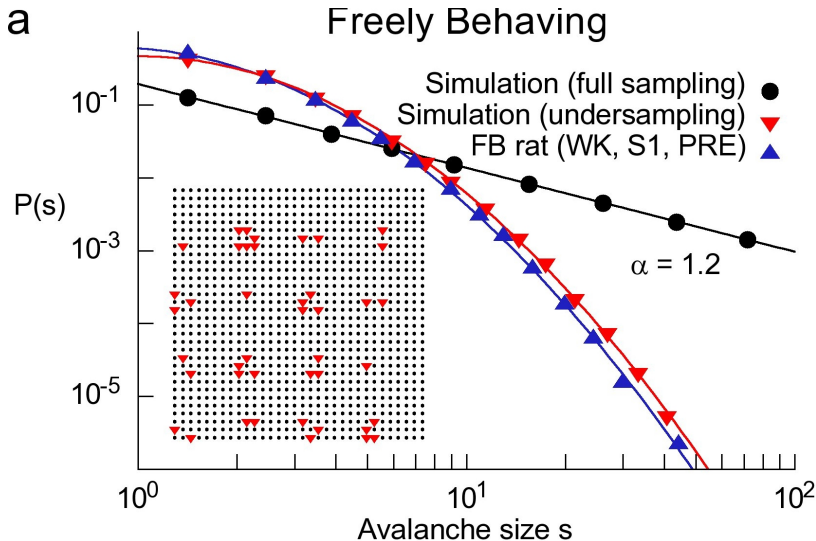
Size distribution of spike avalanches

Not power laws. Well fit by **lognormals**.

Very similar across the sleep-wake cycle and brain regions!



Critical model + undersampling = Lognormal-like size distributions

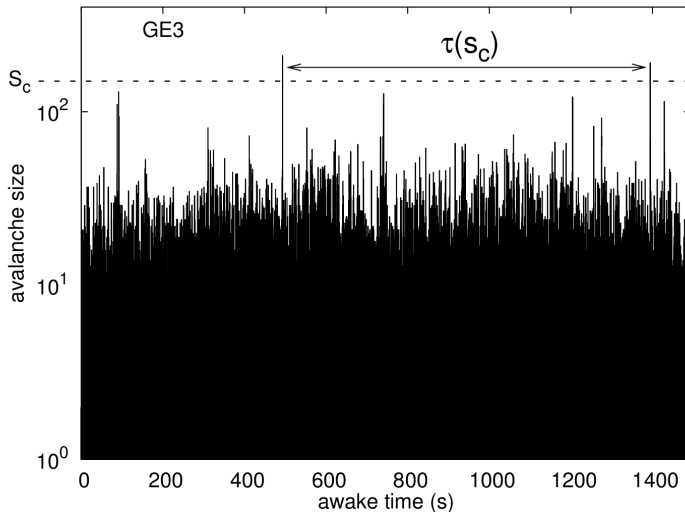


Ribeiro et al., PLoS One **5** e14129 (2010)

Avalanche size time series

Can we obtain fingerprints of criticality from the FB time series?

1) Fourier spectrum & DFA

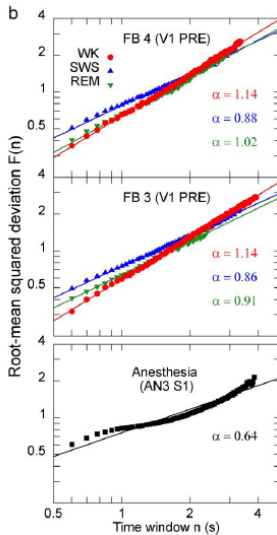
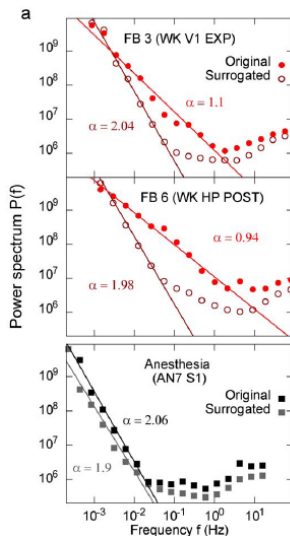


Long-range time correlations

$$C(t) \sim t^{-\gamma} \Rightarrow \gamma = 2 - 2\alpha \quad \text{and} \quad S(f) \sim f^{-\beta} \Rightarrow \beta = 1 - \gamma = 2\alpha - 1$$

1/f spectra

Detrended Fluctuation Analysis



DFA = Detrended Fluctuation Analysis

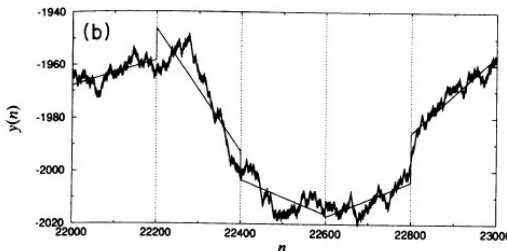
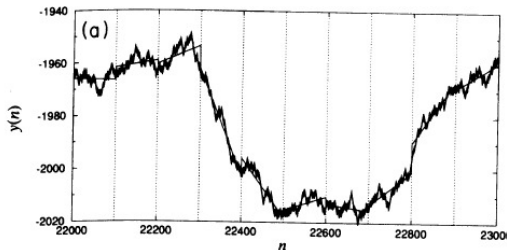
$$F(n) = \sqrt{\frac{1}{N} \sum_{k=1}^N [y(k) - y_n(k)]^2}$$

If $F(n) \sim n^\alpha$, then

$$C(\tau) \sim \tau^{-\gamma}, \text{ with } \gamma = 2 - 2\alpha$$

$$S(f) \sim f^\beta, \text{ with } \beta = 1 - \gamma = 2\alpha - 1$$

- ▶ $0 < \alpha < 0.5 \Rightarrow$ anticorrelations
- ▶ $\alpha = 0.5 \Rightarrow$ white noise
- ▶ $0.5 < \alpha < 1 \Rightarrow$ persistent long-range power-law correlations
- ▶ $\alpha = 1 \Rightarrow 1/f$ noise

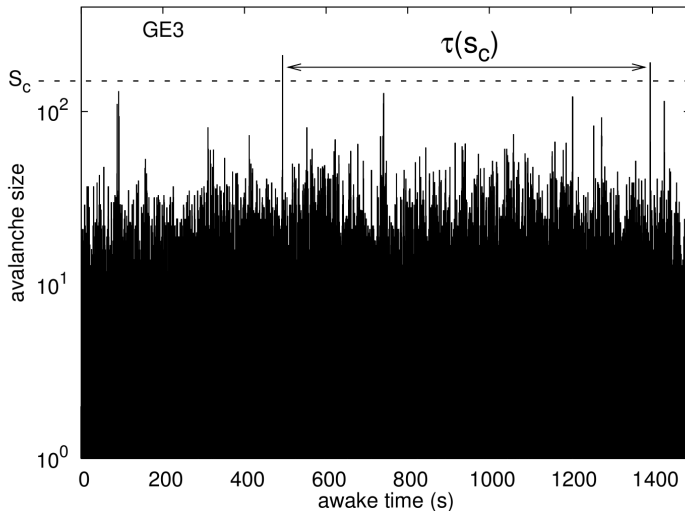


Peng et al., *Phys. Rev. E* **49** 1685 (1994)

Avalanche size time series

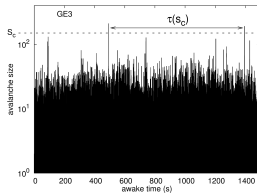
Can we obtain fingerprints of criticality from the FB time series?

2) Waiting-time distribution



Scaling in the distribution of waiting times

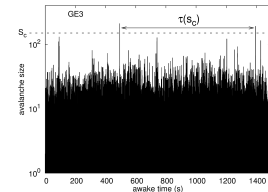
Waiting-time distribution



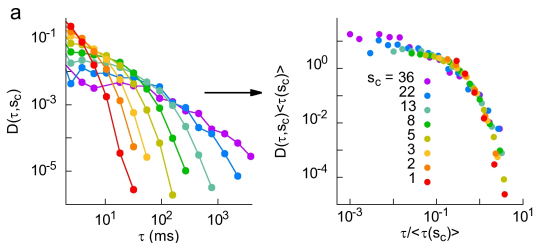
$$D(\tau; s_c)$$

Scaling in the distribution of waiting times

Waiting-time distribution

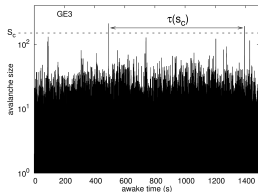


$$D(\tau; s_c)$$

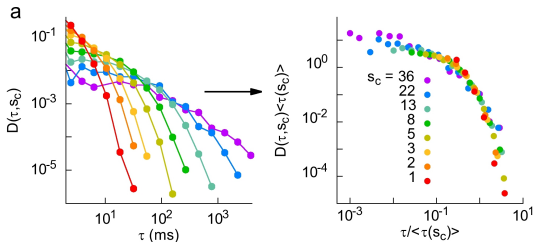


Scaling in the distribution of waiting times

Waiting-time distribution

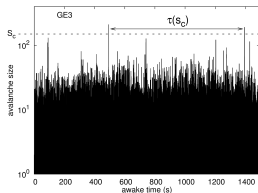


$$D(\tau; s_c) = \langle \tau(s_c) \rangle^{-1} F(\tau / \langle \tau(s_c) \rangle)$$

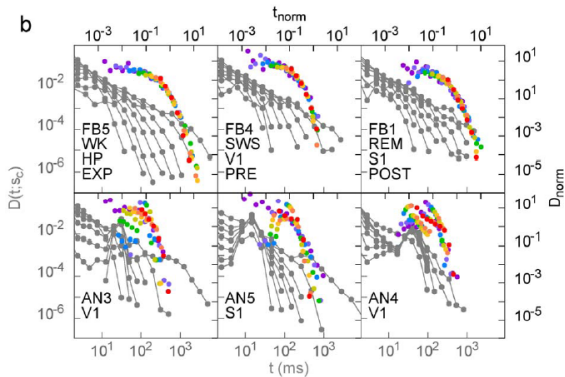


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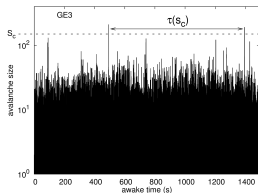


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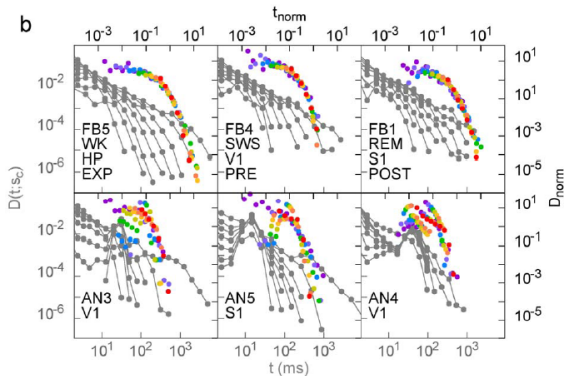
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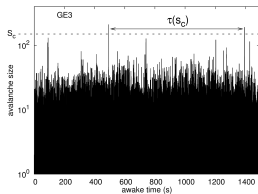
Scaling across brain regions and behavioral states!

(but not anesthesia...)



Scaling in the distribution of waiting times

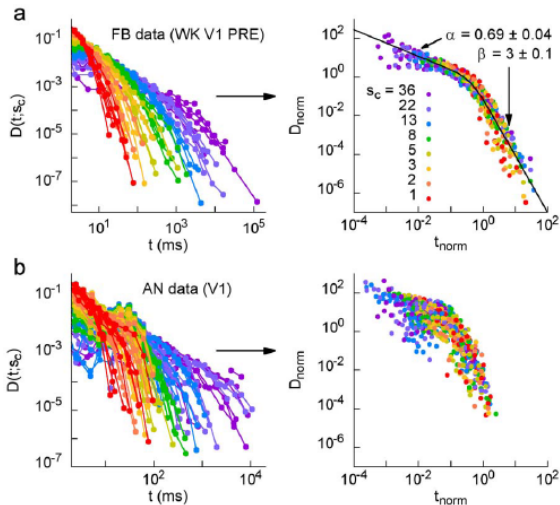
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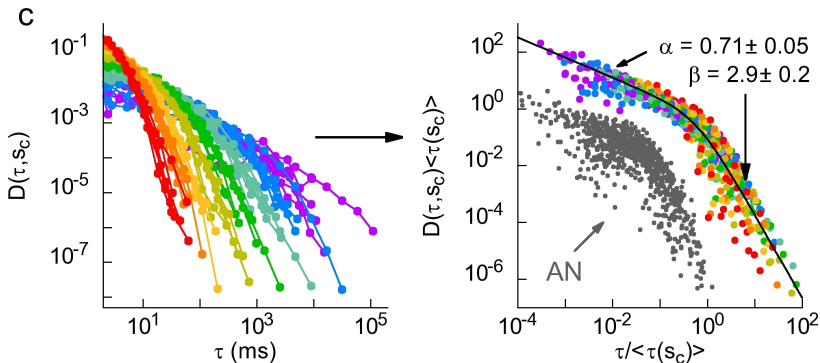
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Scaling in the distribution of waiting times

Universality: Single scaling function for 7 rats and 6 decades...
Well fit by double power law (like earthquakes!)



What is universality?

Let us imagine two different models, **A** and **B**:

Model A:

$$\blacktriangleright F_0 \propto (\sigma - \sigma_c)^\beta$$

$$\blacktriangleright F \stackrel{\sigma=\sigma_c}{\propto} h^{1/\delta_h}$$

$$\blacktriangleright \xi \propto |\sigma - \sigma_c|^{-\nu_\perp}$$

$$\blacktriangleright P(s) \sim s^{-\tau}$$

$\blacktriangleright \dots$

Model B:

$$\blacktriangleright F_0 \propto (\sigma - \sigma'_c)^{\beta'}$$

$$\blacktriangleright F \stackrel{\sigma=\sigma'_c}{\propto} h^{1/\delta'_h}$$

$$\blacktriangleright \xi \propto |\sigma - \sigma'_c|^{-\nu'_\perp}$$

$$\blacktriangleright P(s) \sim s^{-\tau'}$$

$\blacktriangleright \dots$

Models **A** and **B** belong to the same **universality class** if their **critical exponents are the same**:

$$\beta = \beta'$$

$$\delta_h = \delta'_h$$

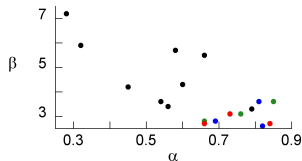
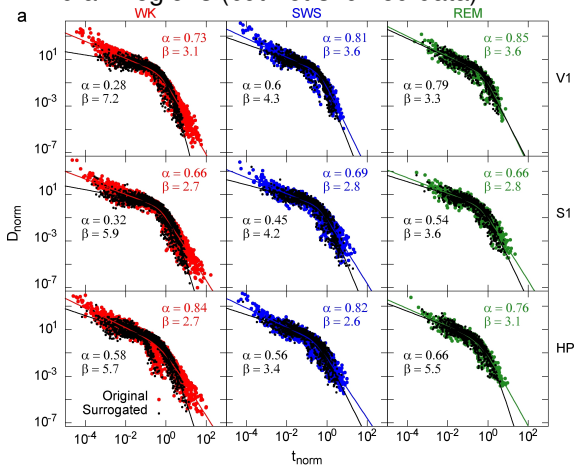
$$\nu_\perp = \nu'_\perp$$

$$\tau = \tau'$$

$\vdots$

Scaling in the distribution of waiting times

Universality: very similar exponents across the sleep-wake cycle and brain regions (but not shuffled data)



Scaling in the distribution of waiting times

Universality + Long-range time correlations in experiments!



Scaling in the distribution of waiting times

Universality + Long-range time correlations in experiments!
But what about models?



Scaling in the distribution of waiting times

STS = separation of time scales

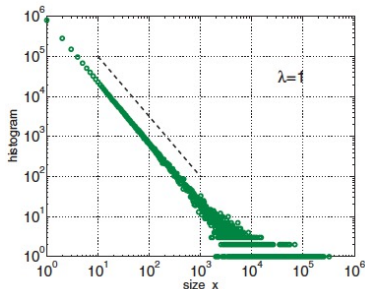
Universality + Long-range time correlations in experiments!

But what about models?

Theoretical prediction:

System at **critical point** +

STS *by hand* =
power-law distributed
avalanches!



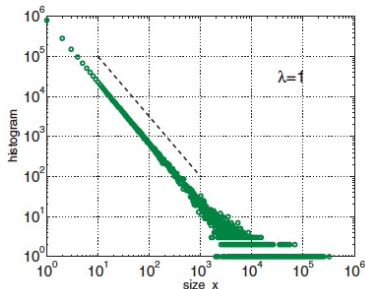
Larremore et al. (2012)

Scaling in the distribution of waiting times

Universality + Long-range time correlations in experiments!
But what about models?

Theoretical prediction:

System at **critical point** +
STS *by hand* =
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avalanches!



Larremore et al. (2012)

If STS has to be imposed by hand in a model, by construction it does not have long-range time correlations.

Waiting-time distributions are also arbitrarily chosen in the simulation...

Pros and cons so far...



Are critical branching processes satisfactory models of neuronal avalanches?

Branching-process-like models (absorbing vs active phases):

► Pros:

- Neuronal avalanche exponents ($\tau = 3/2$ and $\tau_t = 2$)
- Response exponents ($1/\delta_h = 1/2$: psychophysics, sensory systems, ...) + optimal dynamic range at criticality
- Theoretical workhorse (useful picture) for over a decade

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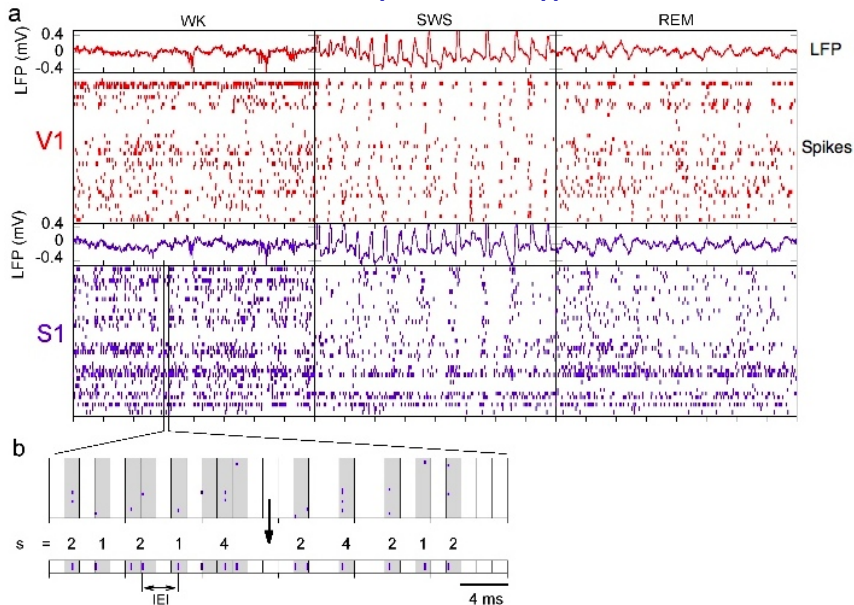
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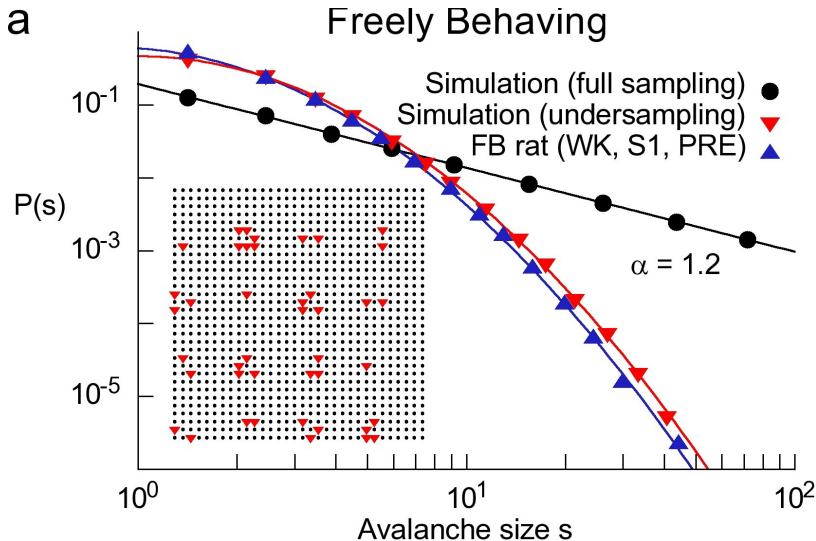
Concluding remarks



Spike avalanches in freely-behaving rats



Critical model + undersampling = Lognormal-like size distributions



Ribeiro et al., PLoS One 5 e14129 (2010)

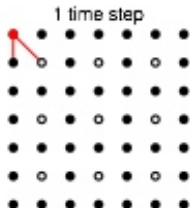
Undersampling changes avalanche statistics!

- ▶ Priesemann et al., BMC Neurosci. **10** 40 (2009)
- ▶ Ribeiro et al., PLoS One **5** e14129 (2010)
- ▶ Priesemann et al., PLoS Comput. Biol. **9** e1002985 (2013)
- ▶ Priesemann et al., Front. Syst. Neurosci. **8** 108 (2014)



Undersampling changes avalanche statistics!

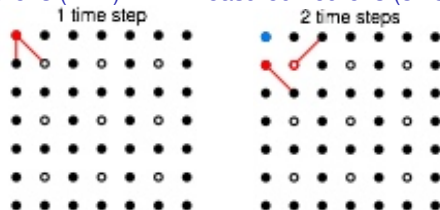
●=neurons (7×7) ○=measured neurons (3×3) red=active



- avalanche size measured with full sampling = 1
- avalanche size measured with undersampling = 0

Undersampling changes avalanche statistics!

●=neurons (7×7) ○=measured neurons (3×3) red=active



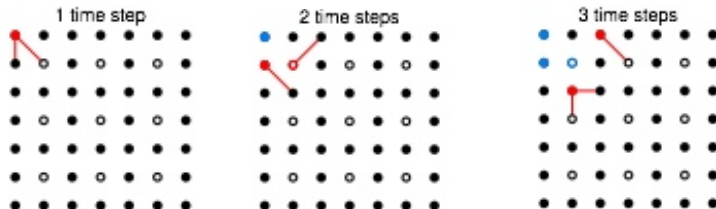
- avalanche size measured with full sampling = 3
- avalanche size measured with undersampling = 1 (finished!)

Undersampling changes avalanche statistics!

●=neurons (7×7)

○=measured neurons (3×3)

red=active



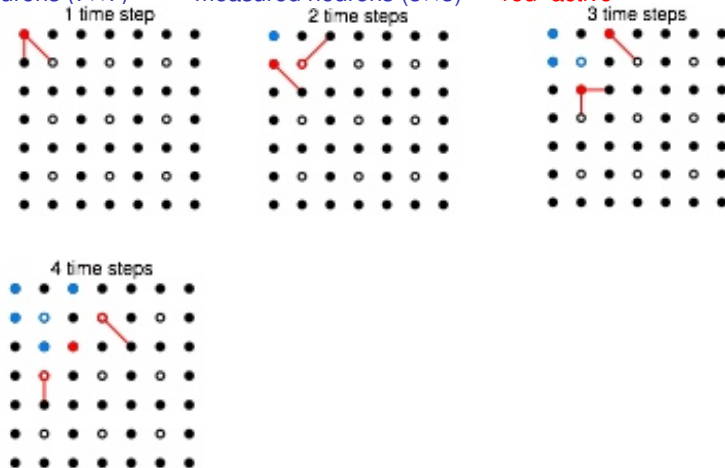
- avalanche size measured with full sampling = 5
- avalanche size measured with undersampling = 0

[1]

Undersampling changes avalanche statistics!

●=neurons (7×7) ○=measured neurons (3×3)

red=active



- avalanche size measured with full sampling = 8
- avalanche size measured with undersampling = 2 (finished!)

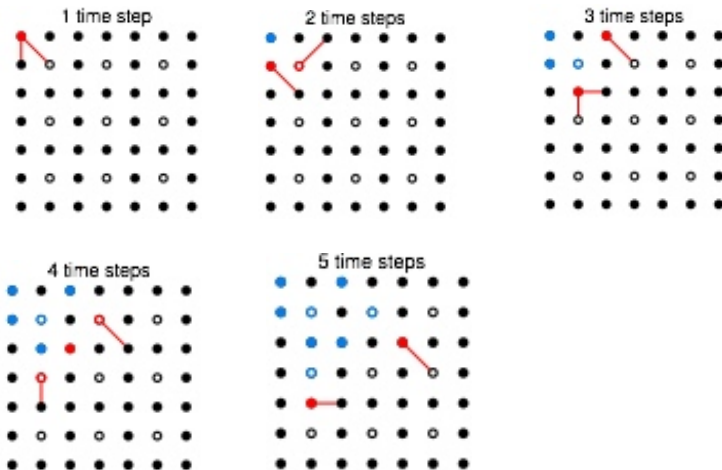
[1]

Undersampling changes avalanche statistics!

●=neurons (7×7)

○=measured neurons (3×3)

red=active



- avalanche size measured with full sampling = 10
- avalanche size measured with undersampling = 0

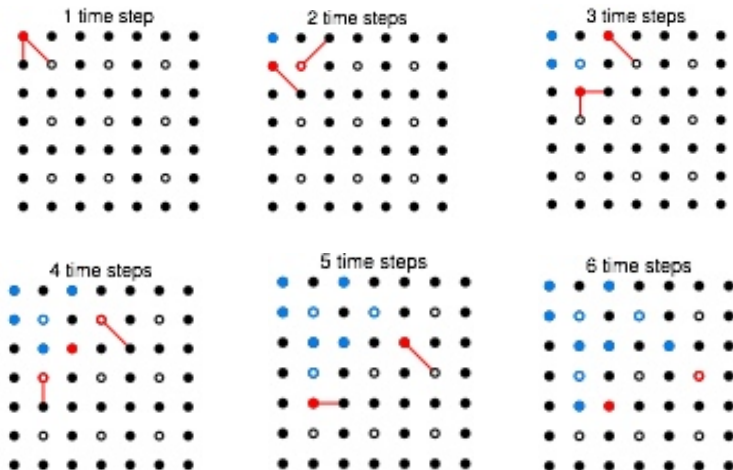
[1, 2]

Undersampling changes avalanche statistics!

●=neurons (7×7)

○=measured neurons (3×3)

red=active



- avalanche size measured with full sampling = 12
- avalanche size measured with undersampling = 1

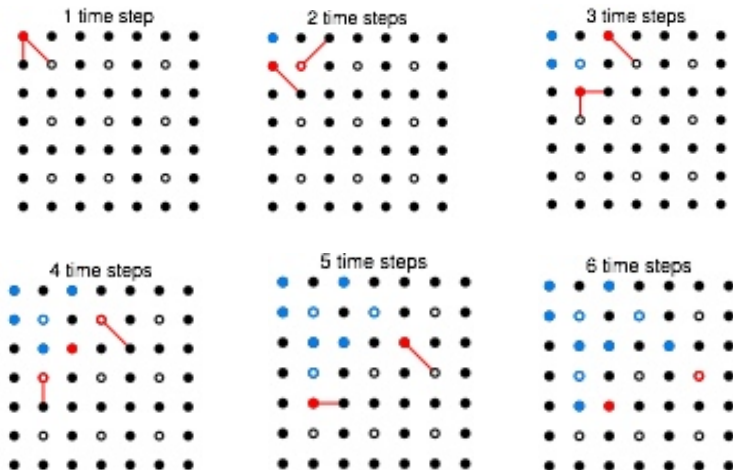
[1, 2]

Undersampling changes avalanche statistics!

●=neurons (7×7)

○=measured neurons (3×3)

red=active



- avalanche size measured with full sampling = 12
- avalanche sizes measured with undersampling = 1, 2, 1

Undersampling solves one problem, but creates another...



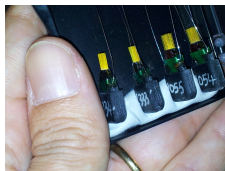
Undersampling solves one problem, but creates another...

Anesthetized rats with the **the same undersampling**:

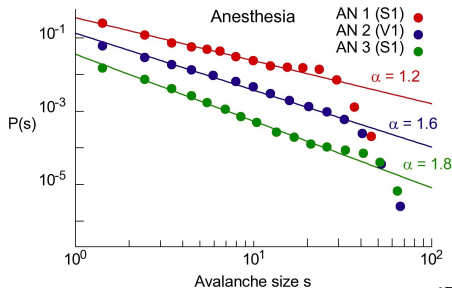
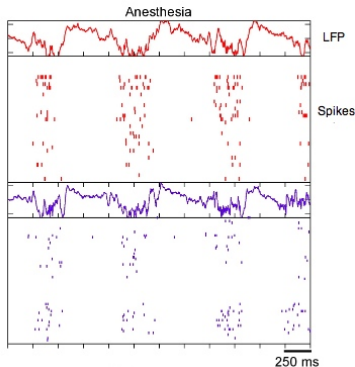
⇒ power-law size distributions!



Spike avalanches in anesthetized rats

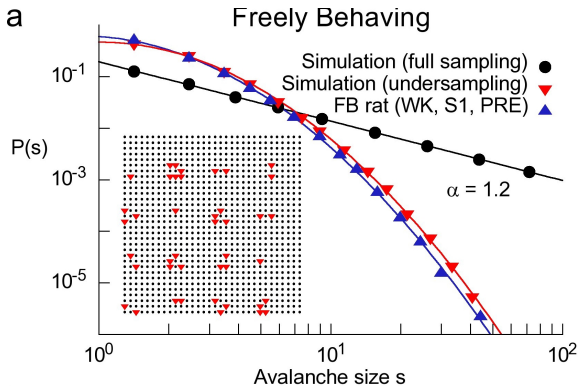


- ▶ 150 μm -500 μm electrode spacing
- ▶ Anesthesia: ketamine-xylazine (8 mg/kg)



Ribeiro et al., PLoS One 5 e14129 (2010)

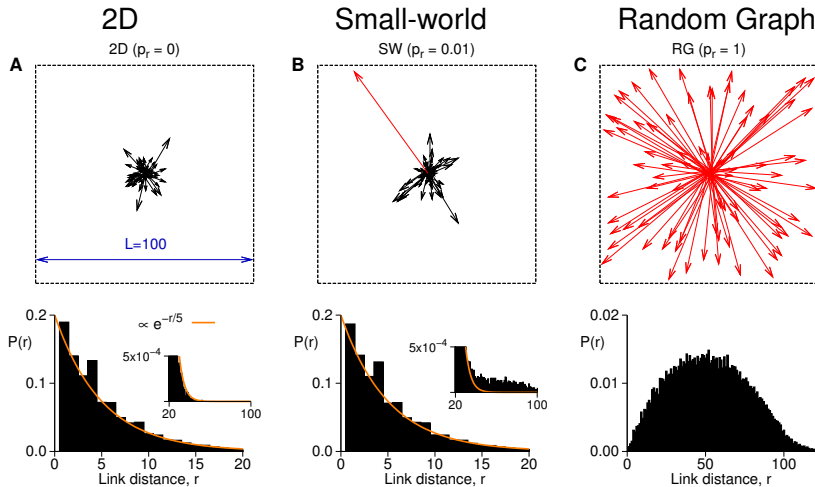




If the model is undersampled (LIKE THE DATA IS!), can it still yield power-law size distributions?

Investigating undersampling in 3 topologies

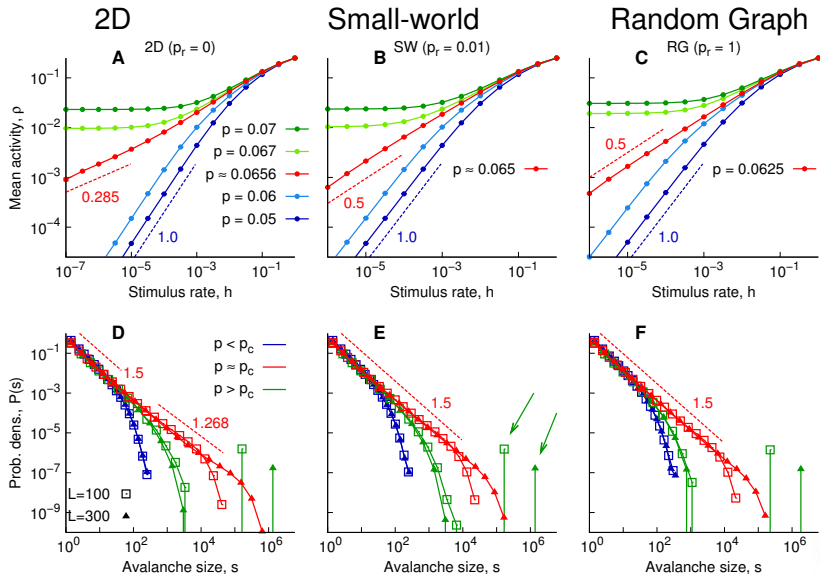
p_r =rewiring probability; $K_{out} = 16$



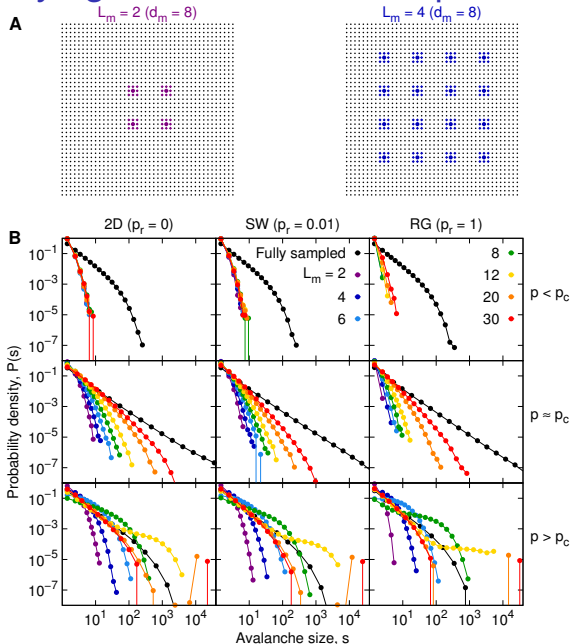
Ribeiro et al. PLoS ONE 9 e94992 (2014)

Determining the critical point

p =transmission probability per link



Varying the number of sampled sites (fixed spacing)



► 0 – 3 neurons per electrode

► $d_m = 8 \Rightarrow$
 $\sim 250 \mu\text{m}$ among electrodes

► $L_m = 8$ (64 electrodes):
 typical arrays

► $L_m = 30$ (900 electrodes):
 state of the art!

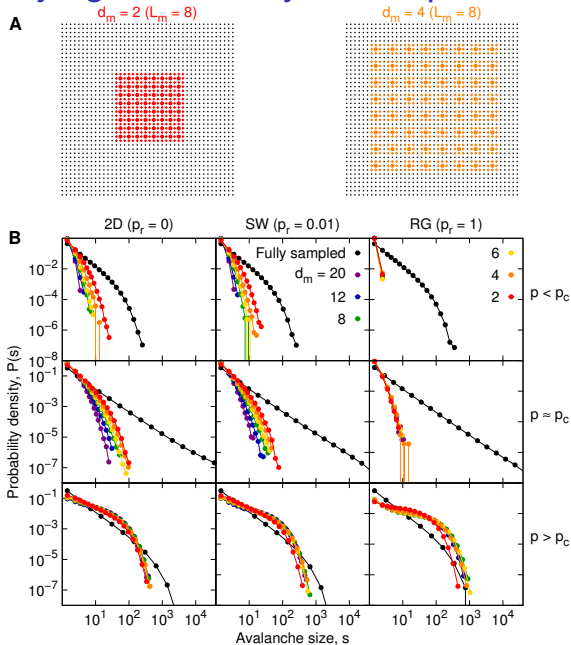
► Subcritical case:
 nothing changes

► Supercritical case:
 bumps for large
 enough matrices

► Critical case:
 power laws gone!

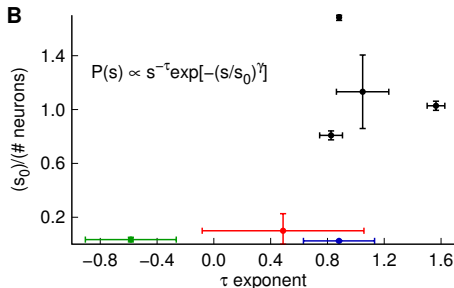
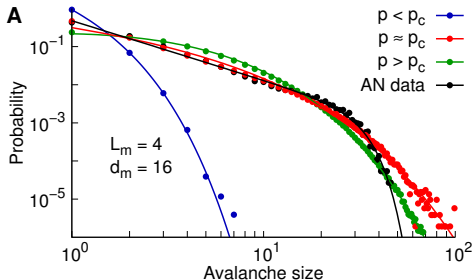


Varying the density of sampled sites (fixed number)



- ▶ 0 – 3 neurons per electrode
- ▶ $L_m = 8$: typical arrays
- ▶ All cases: little changes. Power laws gone!

Last resort: choosing a rescaled time bin



- ▶ $\Delta t^* = \langle |SI| \rangle$
- ▶ SW model and data (4 rats):
 $L_m = 4$ and
 $d_m = 16$ (500 μm)
- ▶ Fit is better, but not much (e.g. wrong cutoff)

Ribeiro et al. PLoS ONE 9 e94992 (2014)

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Branching-process-like models (absorbing vs active phases):

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- ▶ Theoretical workhorse (useful picture) for over a decade



Are critical branching processes satisfactory models of neuronal avalanches? **Only if fully sampled!**

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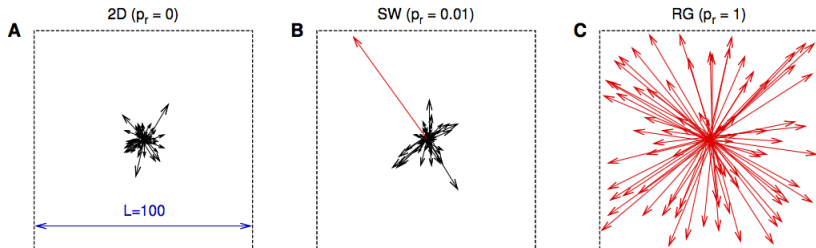
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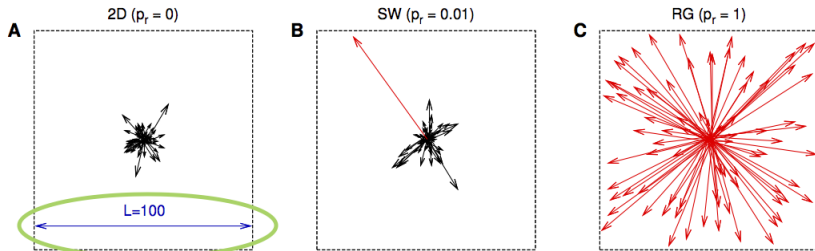
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Undersampling: no power laws in 3 topologies

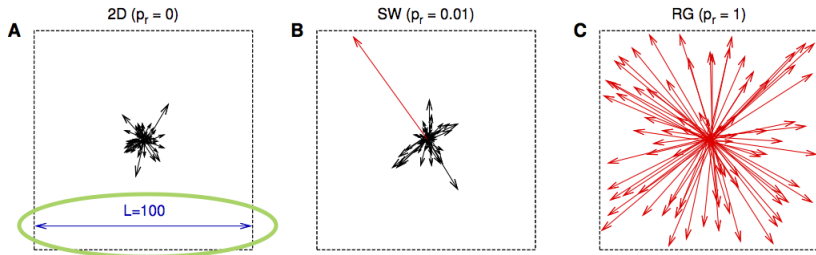


Undersampling: no power laws in 3 topologies



$$\langle K \rangle = 16$$

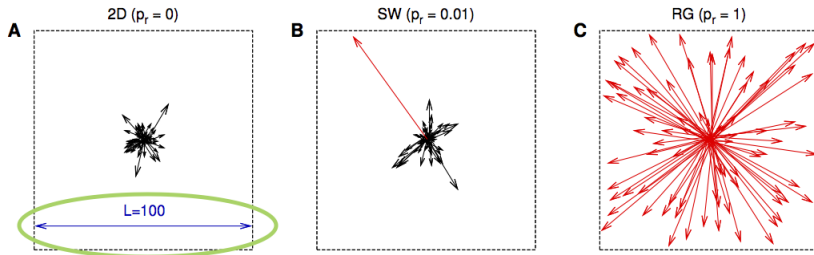
Undersampling: no power laws in 3 topologies



$$\langle K \rangle = 16$$

But if the brain has $\sim 10^9$ neurons, and each cortical neuron receives $\sim 10^4$ synapses...

Undersampling: no power laws in 3 topologies



$$\langle K \rangle = 16$$

But if the brain has $\sim 10^9$ neurons, and
each cortical neuron receives $\sim 10^4$ synapses...
... then maybe a better model would have $\langle K \rangle = \sqrt{N}$

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Are critical branching processes satisfactory models of neuronal avalanches? **Only if fully sampled!**

Branching-process-like models (absorbing vs active phases):

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Is undersampling overcome with a different topology?



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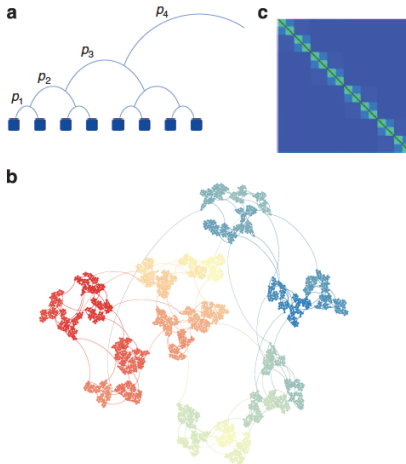
Griffiths phases and the stretching of criticality
in brain networks Paolo Moretti¹ & Miguel A. Muñoz¹ [Nat. Commun. 4 2521 \(2013\)](#)



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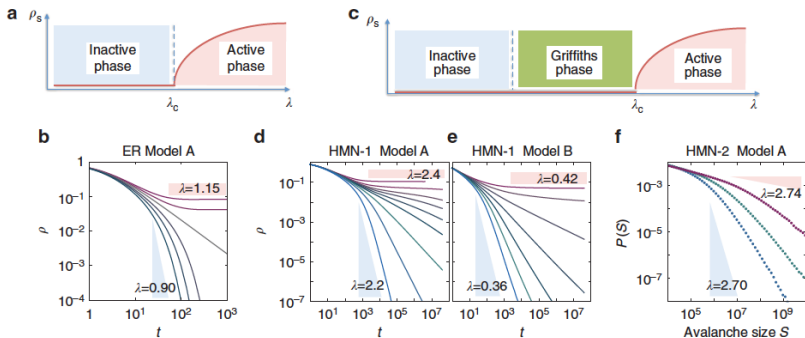
Griffiths phases and the stretching of criticality in brain networks

Paolo Moretti¹ & Miguel A. Muñoz¹ *Nat. Commun.* 4 2521 (2013)



Hierarchical-modular networks

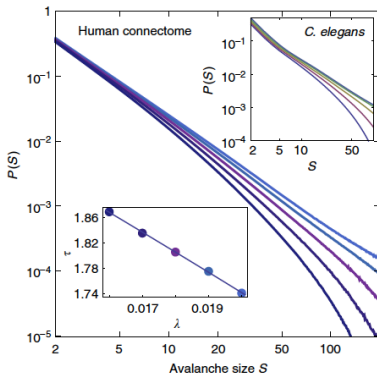
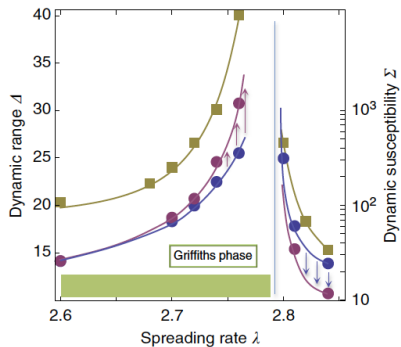
Is undersampling overcome with a different topology?



Disorder $\Rightarrow$ **Rare active regions** $\Rightarrow$ Generic power laws!

Moretti & Muñoz, Nat. Commun. **4** 2521 (2013)

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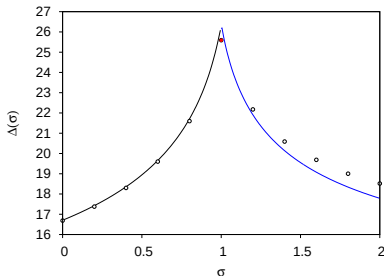
Long-range time correlations

Concluding remarks



Dynamic range is optimal at criticality

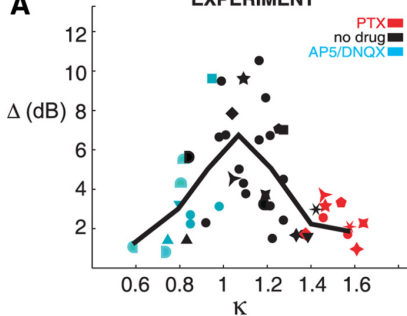
Theory



Kinouchi & Copelli
Nat. Phys. **2** 348 (2006)

A

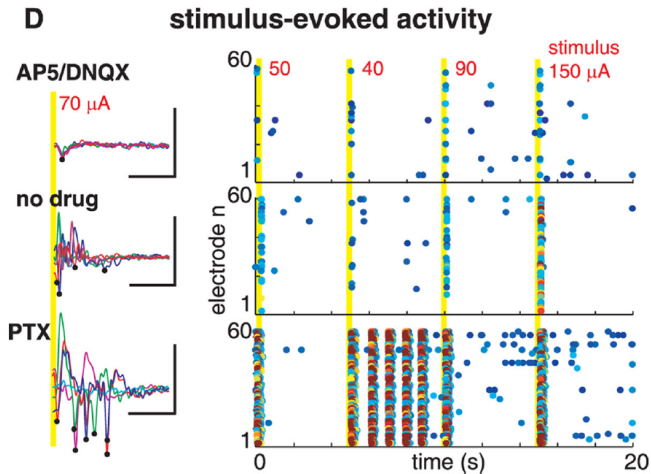
Cortical slices
EXPERIMENT



Shew et al.
J. Neurosci., **29** 15595 (2009)



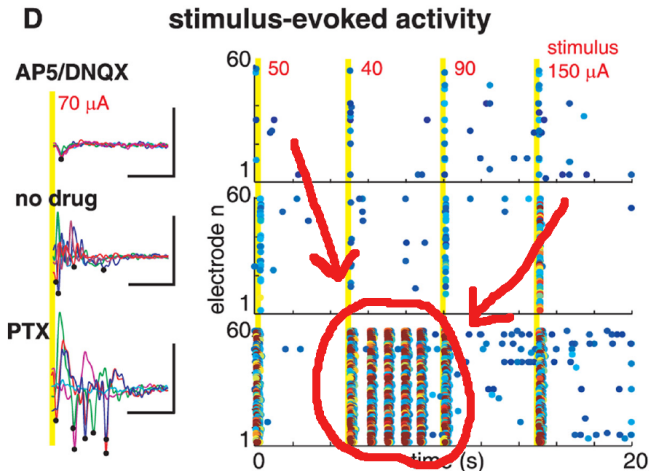
Supercritical slices



Shew et al., *J. Neurosci.* **29** 15595 (2009)

Supercritical slices: collective oscillations?

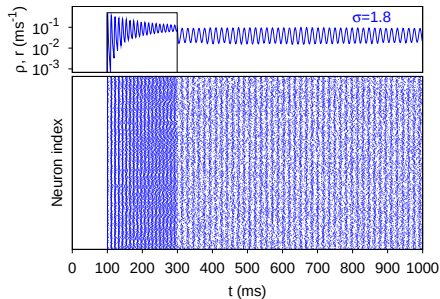
Directed percolation (DP) not **fully** compatible with “supercritical” collective oscillations (coexistence?) in experiment



Shew et al., *J. Neurosci.* **29** 15595 (2009)

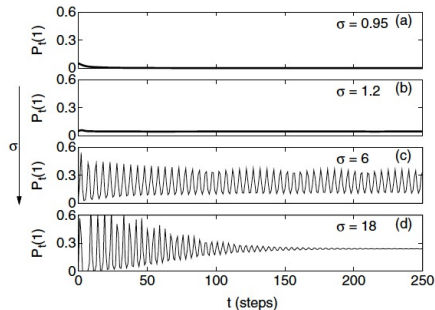
Oscillations in DP-like models, but avalanches?...

Increasing σ even more...



Kinouchi & Copelli
Nat. Phys. **2** 348 (2006)

Reentrance, coexistence etc



Rozenblit & Copelli
JSTAT **P01012** (2011)

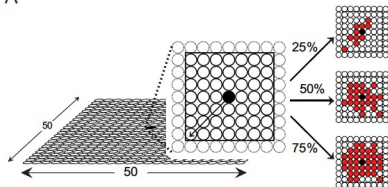


A model with excitatory and inhibitory neurons

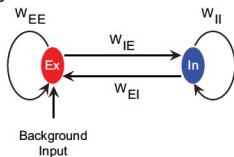
Spiking neurons, **not excitable!**

50 × 50 grid
(interaction square: 7 × 7)

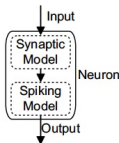
A



B



C

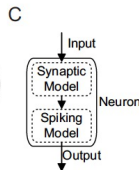
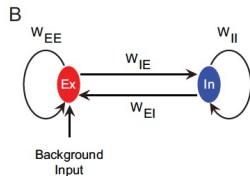
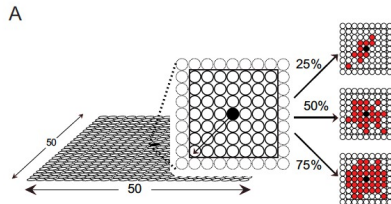


Poil et al., *J. Neurosci.* **32** 9817 (2012)

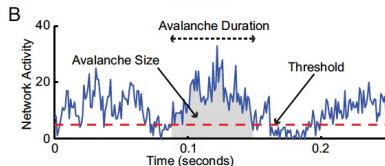
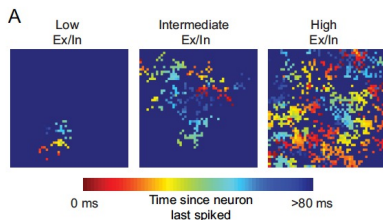
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Avalanche: above threshold

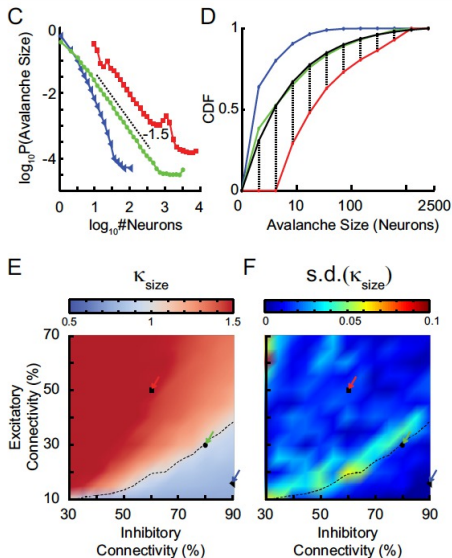


Poil et al., *J. Neurosci.* **32** 9817 (2012)

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Spiking neurons, **not excitable!**

- ▶ Avalanches with power-law size distribution!
- ▶ But only along a transition line in parameter space
- ▶ What kind of transition is this?

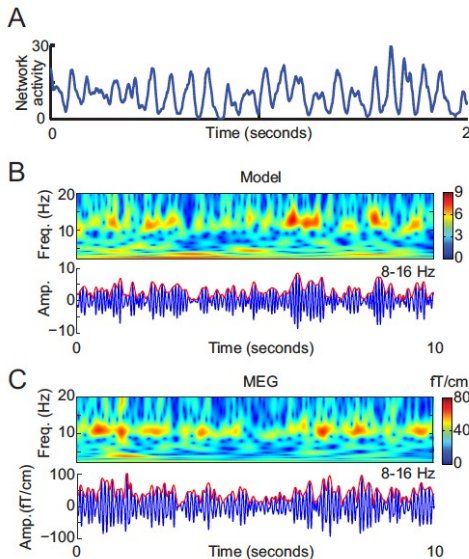


Poil et al., *J. Neurosci.* **32** 9817 (2012)

A model with excitatory and inhibitory neurons

Spiking neurons, **not excitable!**

- ▶ Oscillations emerge!
(8-16 Hz)
- ▶ Similar to MEG data

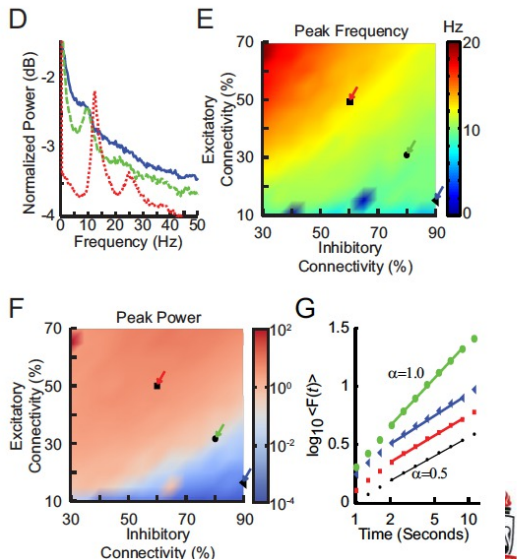


Poil et al., *J. Neurosci.* **32** 9817 (2012)

A model with excitatory and inhibitory neurons

Spiking neurons, **not excitable!**

- ▶ Peak frequency increases with excitatory connectivity
- ▶ Transition from low to high peak power
- ▶ DFA exponent $\alpha \simeq 1$ at the transition



Poil et al., *J. Neurosci.* **32** 9817 (2012)

DFA = Detrended Fluctuation Analysis

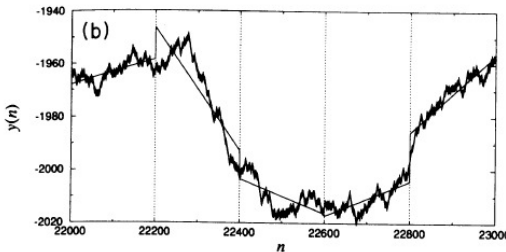
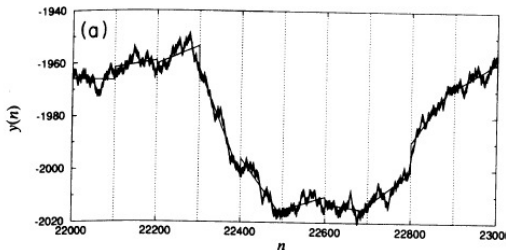
$$F(n) = \sqrt{\frac{1}{N} \sum_{k=1}^N [y(k) - y_n(k)]^2}$$

If $F(n) \sim n^\alpha$, then

$$C(\tau) \sim \tau^{-\gamma}, \text{ with } \gamma = 2 - 2\alpha$$

$$S(f) \sim f^\beta, \text{ with } \beta = 1 - \gamma = 2\alpha - 1$$

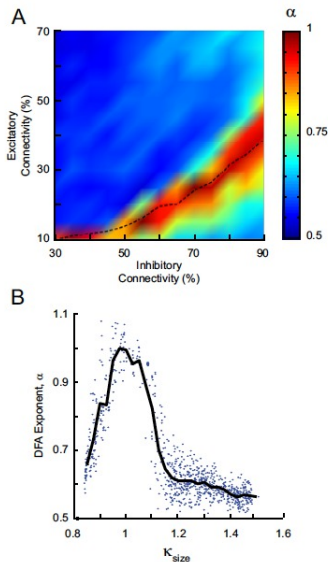
- ▶ $0 < \alpha < 0.5 \Rightarrow$ anticorrelations
- ▶ $\alpha = 0.5 \Rightarrow$ white noise
- ▶ $0.5 < \alpha < 1 \Rightarrow$ persistent long-range power-law correlations
- ▶ $\alpha = 1 \Rightarrow 1/f$ noise



Peng et al., *Phys. Rev. E* **49** 1685 (1994)

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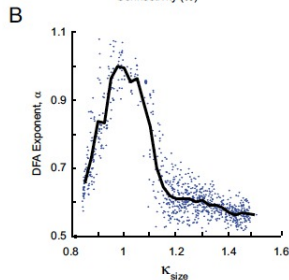
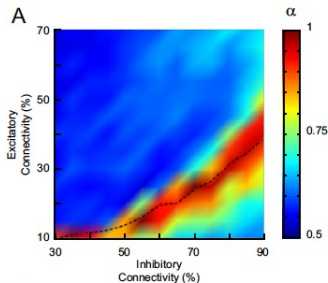


Poil et al., *J. Neurosci.* **32** 9817 (2012)

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Poil et al., *J. Neurosci.* **32** 9817 (2012)

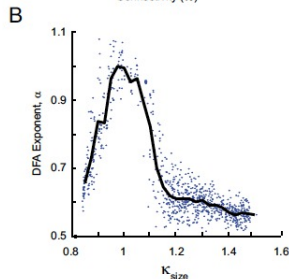
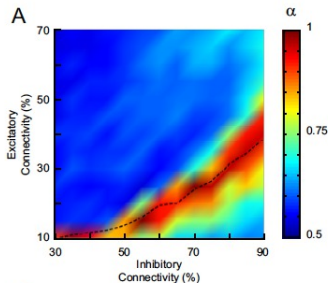
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neuronal avalanches + **long-range power law correlations**



Poil et al., *J. Neurosci.* **32** 9817 (2012)

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But ...

- ▶ How robust is the model if system size is increased (beyond $50 \times 50 \dots$)?
- ▶ Should the "interaction square" be scaled up as well?
- ▶ If so, is the system really two-dimensional? (Remember, $\tau = 3/2$ is a **mean-field** exponent!)
- ▶ What would be an appropriate order parameter to characterize the transition?
- ▶ Does $P(s) \sim s^{-3/2}$ survive undersampling?

Poil et al., *J. Neurosci.* **32** 9817 (2012)



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 - Hierarchical networks (critical point or Griffiths phase??) [Moretti & Muñoz, Nat. Comm. (2013)]
- Avalanches as collective excitations near oscillatory behavior? [Poil et al, J. Neurosci. (2012)]

Thank you!



New neuroelectrophysiology lab: students and post-docs wanted!

Come visit us in Recife!

