Multiplex networks: Structure

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http://www.maths.qmul.ac.uk/∼vnicosia/

What is a Complex System ?

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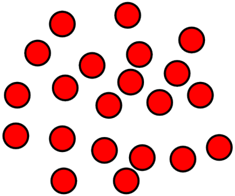
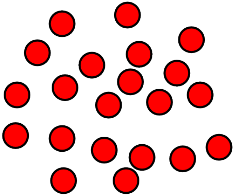


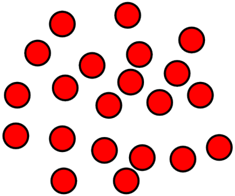
What is a Complex System ?



$$\frac{\partial}{\partial t} x_i = f(X)$$

DYNAMICS

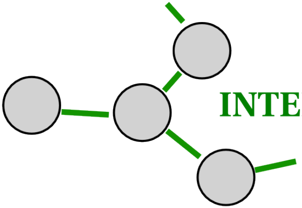
What is a Complex System ?



UNITS

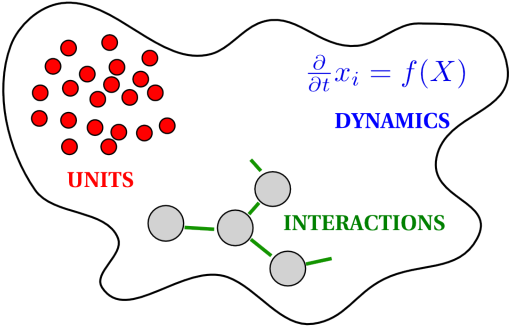
$$\frac{\partial}{\partial t} x_i = f(X)$$

DYNAMICS

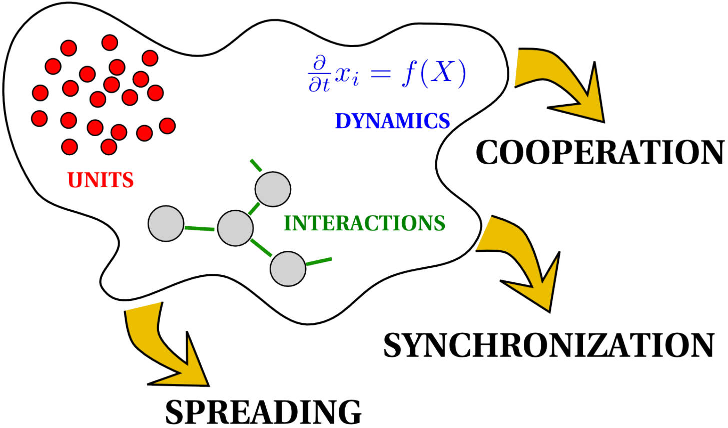


INTERACTIONS

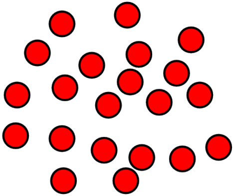
What is a Complex System ?



What is a Complex System ?



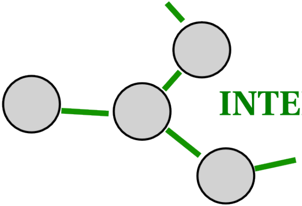
The Complex Network Hypothesys



UNITS

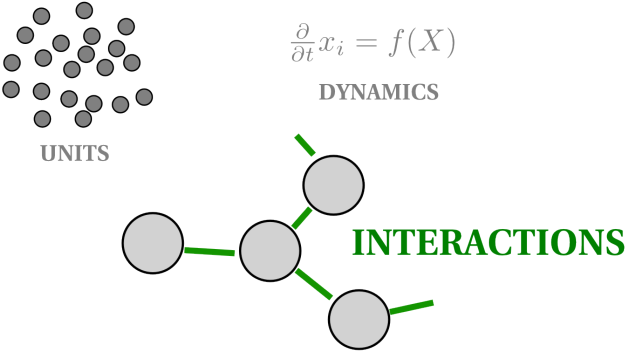
$$\frac{\partial}{\partial t} x_i = f(X)$$

DYNAMICS

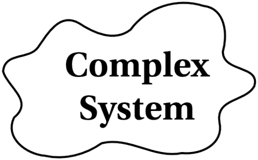


INTERACTIONS

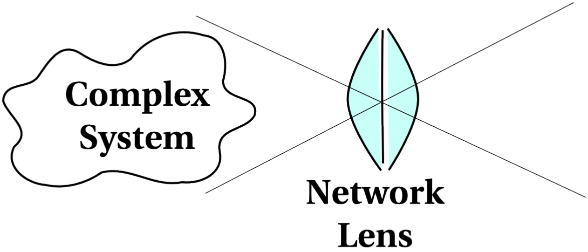
The Complex Network Hypothesys



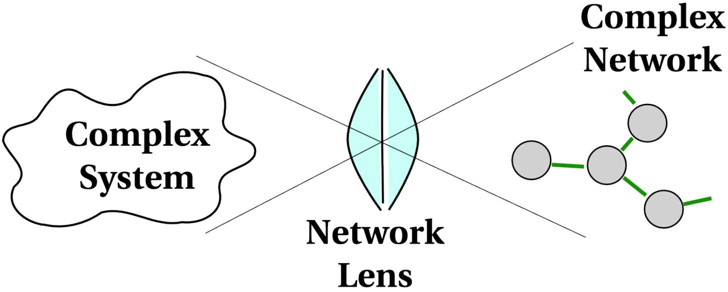
A Minimalist approach



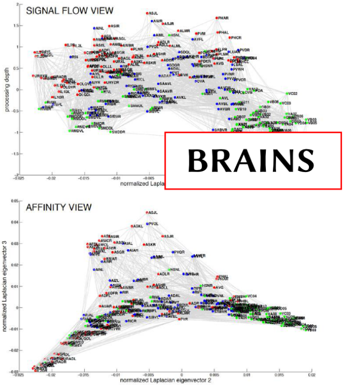
A Minimalist approach

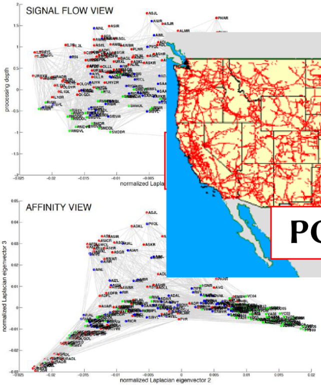


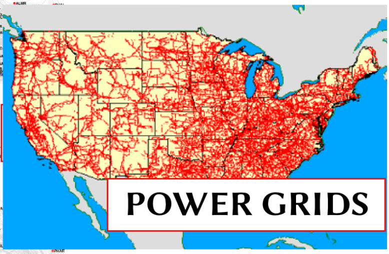
A Minimalist approach

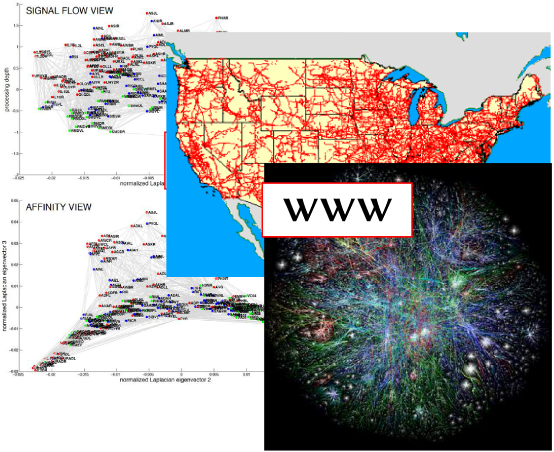


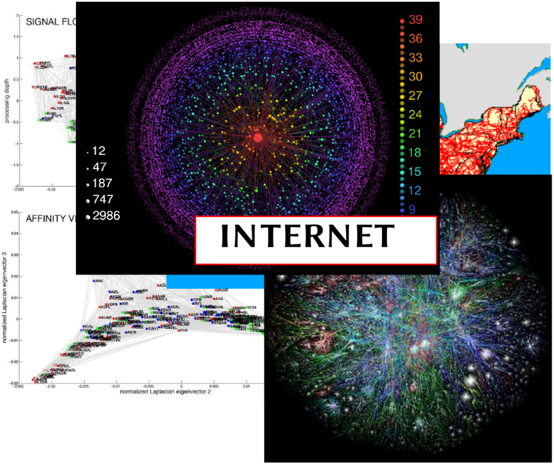
(although non-reductionist!)

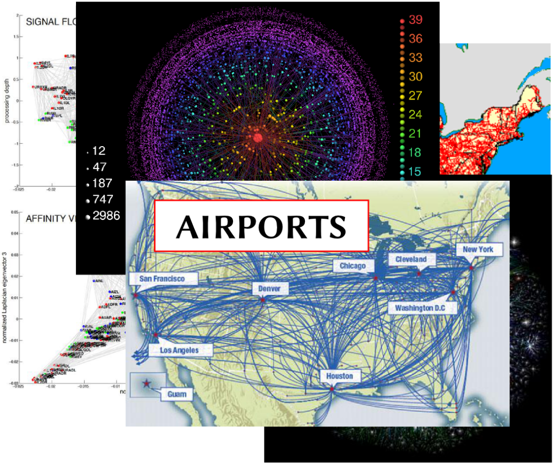

SIGNAL FLOW VIEW
processing depth
BRAINS
AFFINITY VIEW
normalized Laplacian eigenvector 3
normalized Laplacian eigenvector 2

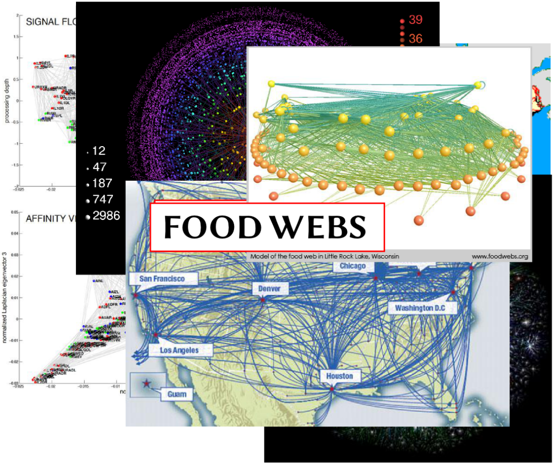

SIGNAL FLOW VIEW
AFFINITY VIEW
normalized Laplacian eigenvector 2
normalized Laplacian eigenvector 3
processing depth

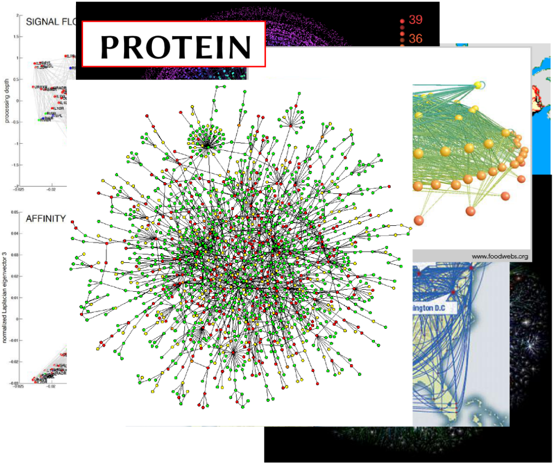

POWER GRIDS


SIGNAL FLOW VIEW
processing depth
AFFINITY VIEW
normalized Laplacian eigenvector 3
normalized Laplacian eigenvector 2
WWW

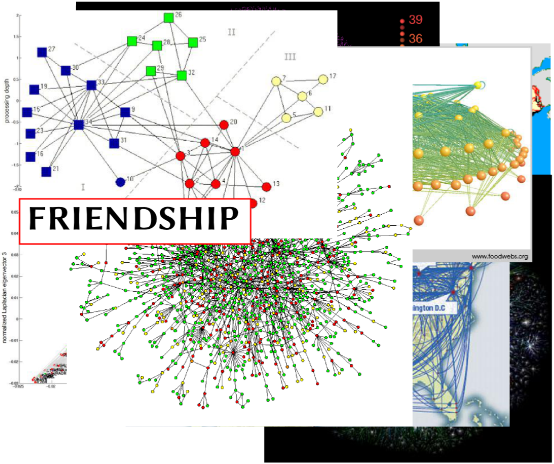

SIGNAL FL
AFFINITY V
processing depth
normalized Laplacian eigenvector 3
normalized Laplacian eigenvector 2
39
36
33
30
27
24
21
18
15
12
9
12
47
187
747
2986
INTERNET

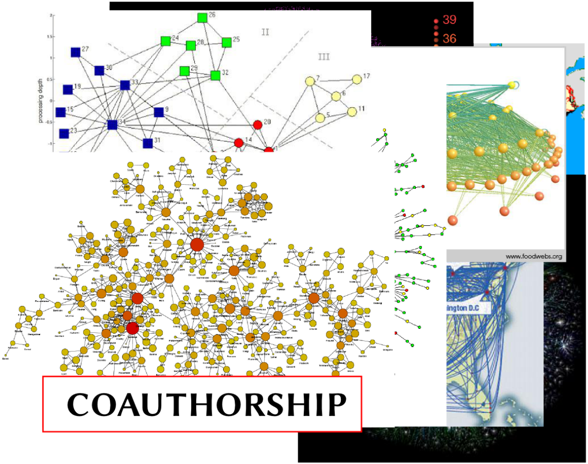

SIGNAL FL
AFFINITY V
39
36
33
30
27
24
21
18
15
12
47
187
747
2986
AIRPORTS
San Francisco
Denver
Chicago
Cleveland
New York
Washington D.C
Los Angeles
Houston
Guam

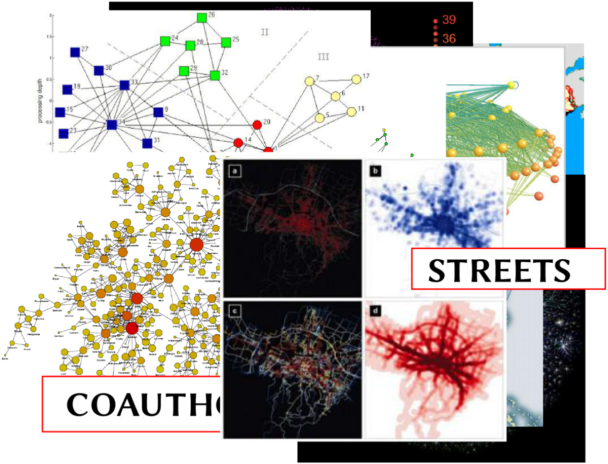

SIGNAL FL
AFFINITY V
39
36
12
47
187
747
2986
FOOD WEBS
Model of the food web in Little Rock Lake, Wisconsin
www.foodwebs.org
San Francisco
Denver
Chicago
Washington D.C
Los Angeles
Houston
Guam



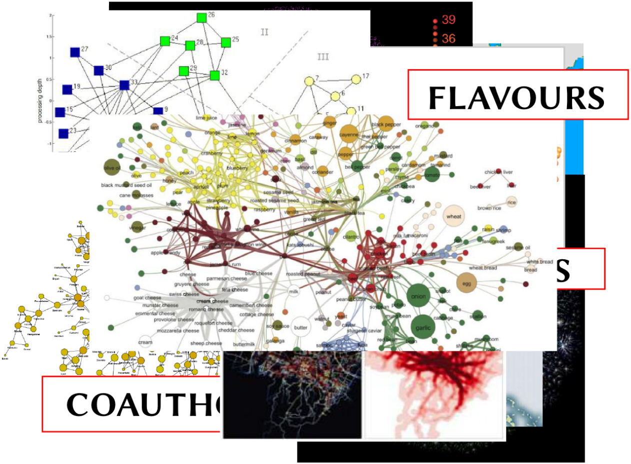
SIGNAL FL
AFFINITY
processing depth
normalized Laplacian eigenvector 3
PROTEIN
39
36
www.foodwebs.org
ington D.C

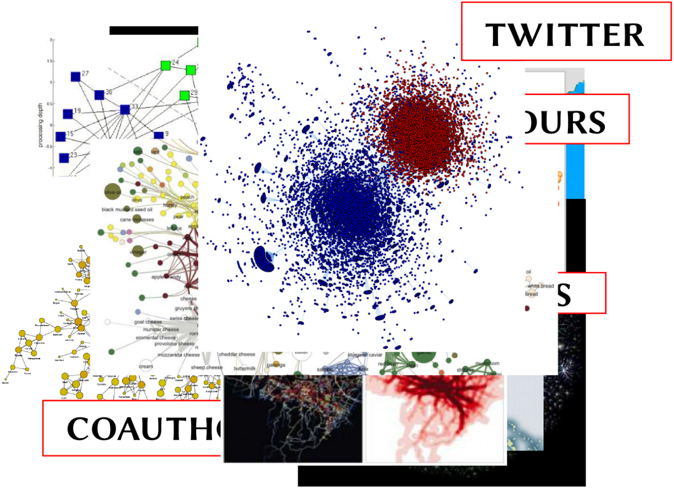

FRIENDSHIP
39
36
II
III
I
processing depth
normalized Laplacian eigenvector 3
www.foodwebs.org
ington D.C

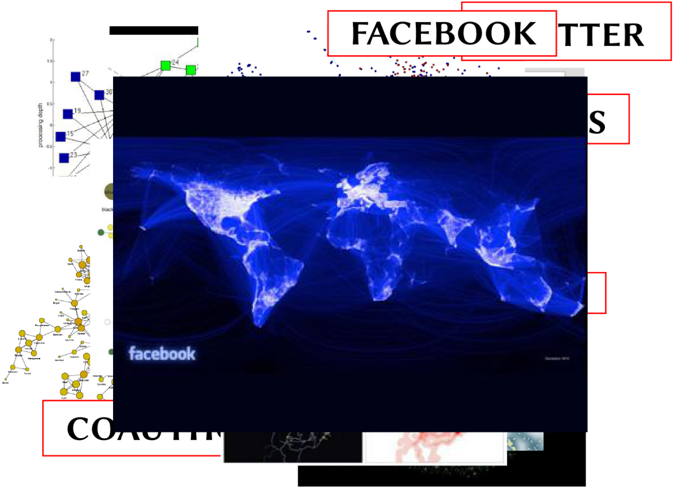

39
36
II
III
processing depth
www.foodwebs.org
ington D.C
COAUTHORSHIP



STREETS
COAUTH
39
36


FLAVOURS


TWITTER
)URS
S
COAUTH(
olive oil
black mustard seed oil
cane molasses
vinegar
peach
honey
pear
lettuce
cheese
goat cheese
munster cheese
emmental cheese
provolone cheese
mozzarella cheese
cream
sheep cheese
cheddar cheese
buttermilk
sturgeon caviar
white bread
bread
processing depth


FACEBOOK
TTER
S
CO
facebook

SCALE-FREE DEGREE DISTRIBUTIONS

SMALL-WORLD EFFECT

ABUNDANCE OF MOTIFS

NON-TRIVIAL DEGREE CORRELATIONS

TIGHT COMMUNITIES

EXTREME FRAGILITY TO ATTACKS

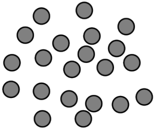
EFFICIENT INFORMATION/DISEASE DIFFUSION

OPINION FORMATION

SYNCHRONIZATION

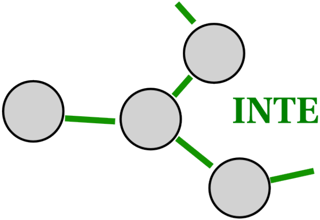
COOPERATION

The Multiplex metaphor

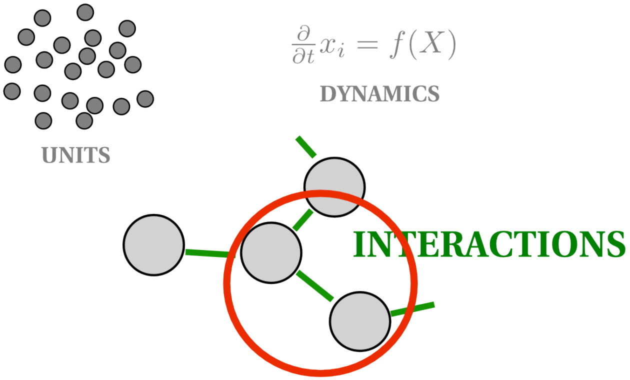


$$\frac{\partial}{\partial t} x_i = f(X)$$

DYNAMICS

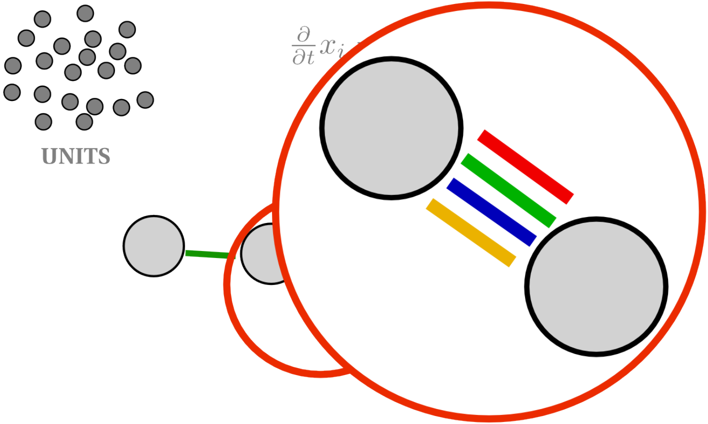


The Multiplex metaphor



$$\frac{\partial}{\partial t} x_i = f(X)$$

The Multiplex metaphor



Multi-level systems

Multi-level systems

FRIENDSHIP

"FRIENDSHIP"

FAMILY/KINS

ECONOMIC/ FINANCIAL

COMMUNICATION

COLLABORATION

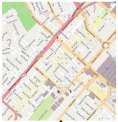
CO-LOCATION

Multi-level systems

Multi-level systems

LASAGNE
Cutting through complexity

Roads



Trains

Bus routes

Tube

Riverboats

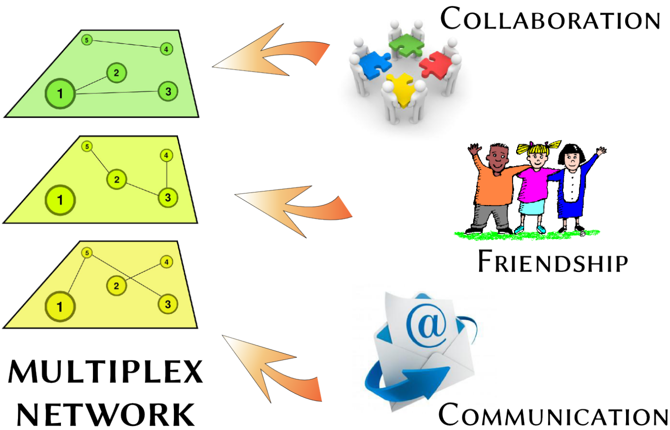
The Multiplex metaphor

Collaboration

Friendship

Communication

The Multiplex metaphor



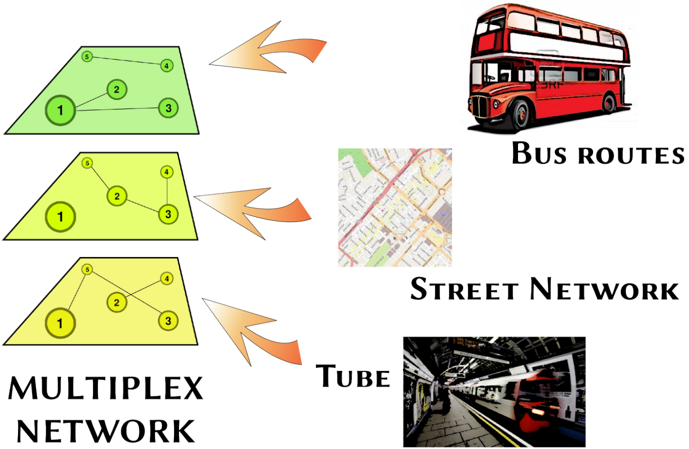
The Multiplex metaphor

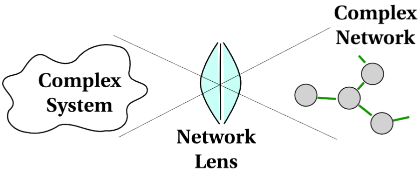
Bus routes

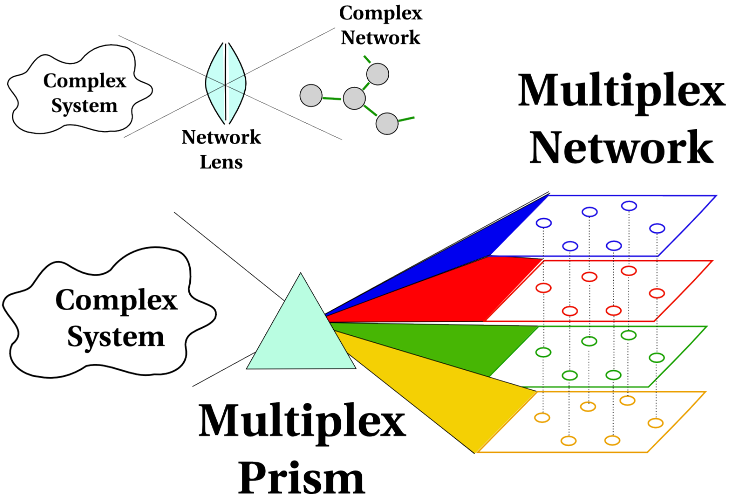
Street Network

Tube

The Multiplex metaphor




Complex
System
Network
Lens
Complex
Network


Complex System
Network Lens
Complex Network
Multiplex Network
Complex System
Multiplex Prism

Examples of "new" multiplex systems

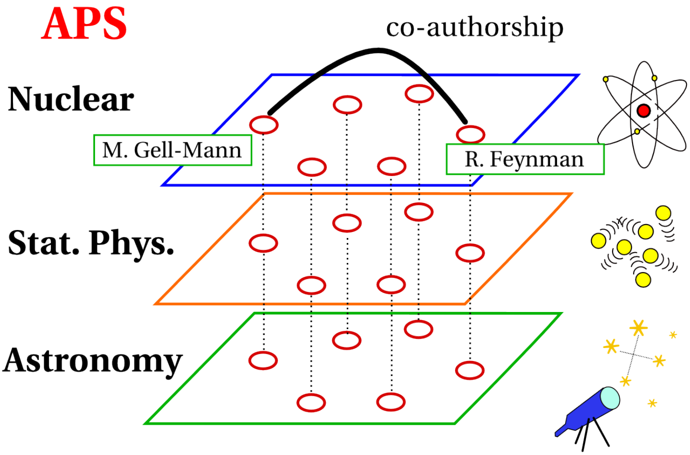
Co-authorship networks (APS)

Single-layer

- Nodes: authors
- Edges: co-authorship

Multiplex

- Nodes: authors
- Edges: co-authorship
- Layers: physics topics (e.g. top-level PACS codes)


APS
co-authorship
Nuclear
M. Gell-Mann
R. Feynman
Stat. Phys.
Astronomy

Co-starring networks (IMDB)

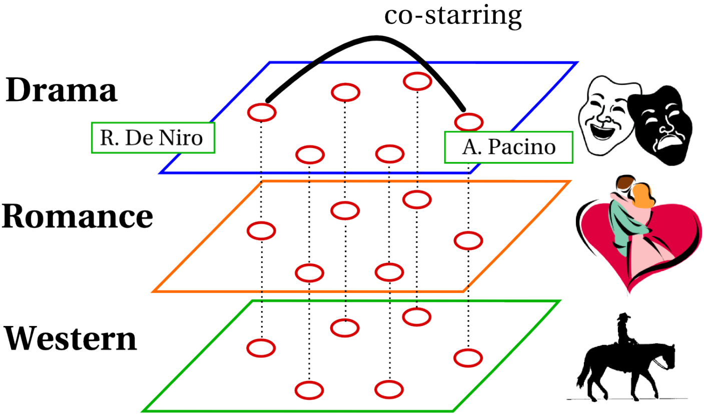
Single-layer

- Nodes: actors
- Edges: co-starring

Multiplex

- Nodes: actors
- Edges: co-starring
- Layers: genres (e.g. Drama, Western, Adult...)

IMDB



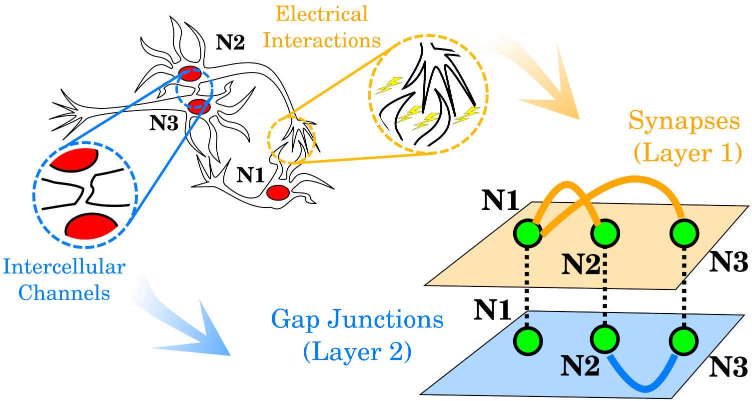
Neural networks (C.Elegans)

Single-layer

- Nodes: neurons
- Edges: communication channels

Multiplex

- Nodes: neurons
- Edges: communication channels
- Layers: synapses, gap-junctions



Electrical
Interactions
N2
N3
N1
Intercellular
Channels
Synapses
(Layer 1)
N1
N2
N3
N1
N2
N3
Gap Junctions
(Layer 2)

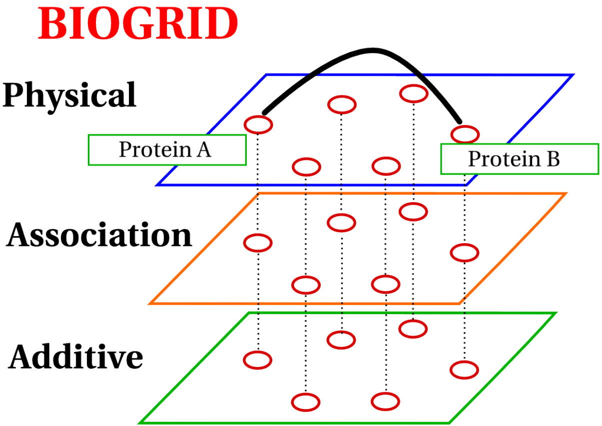
Protein networks (BioGRID)

Single-layer

- Nodes: proteins/genes
- Edges: genetic/mechanic interactions

Multiplex

- Nodes: proteins/genes
- Edges: genetic/mechanic interactions
- Layers: genetic suppression, genetic activation, physical association etc...


BIOGRID
Physical
Protein A
Protein B
Association
Additive

Airport networks (OpenFlight)

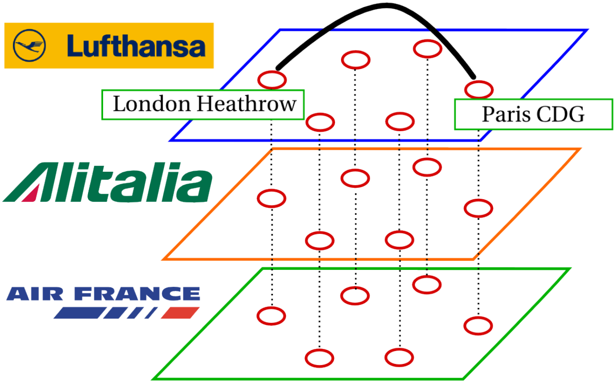
Single-layer

- Nodes: airports
- Edges: routes

Multiplex

- Nodes: airports
- Edges: routes
- Layers: airlines

AIRPORTS



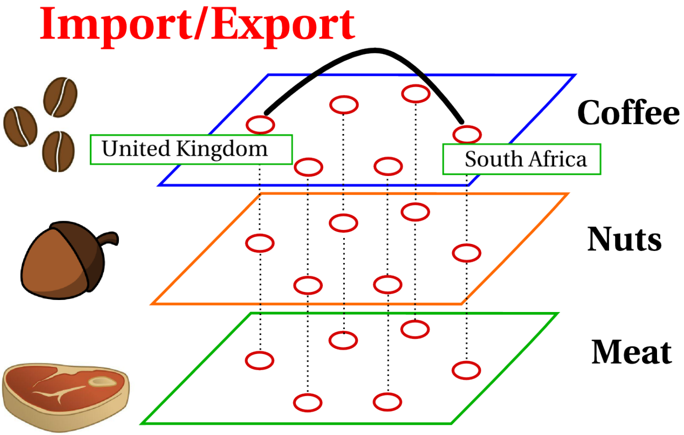
Economical Networks (FAO)

Single-layer

- Nodes: countries
- Edges: import/export

Multiplex

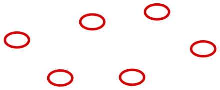
- Nodes: countries
- Edges: import/export
- Layers: food products

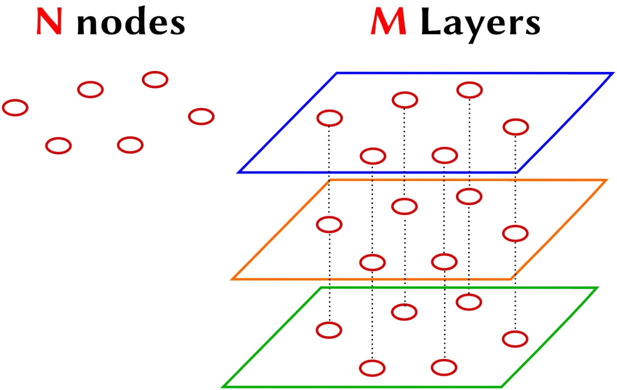

Import/Export
United Kingdom
South Africa
Coffee
Nuts
Meat

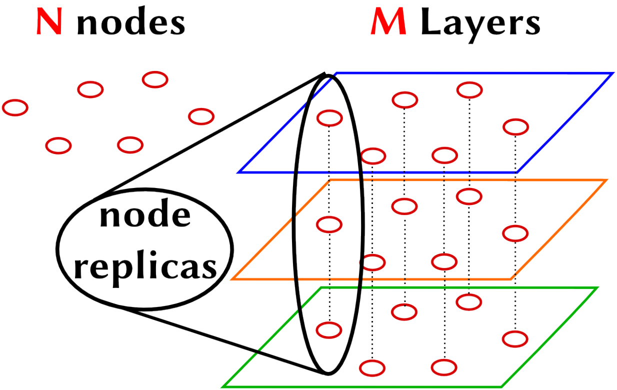
STRUCTURE

(HOW DOES A MULTIPLEX NETWORK LOOK LIKE?)

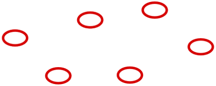
N nodes



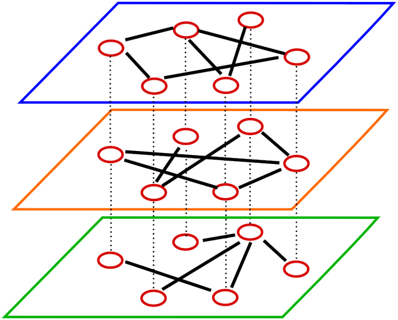

N nodes
M Layers

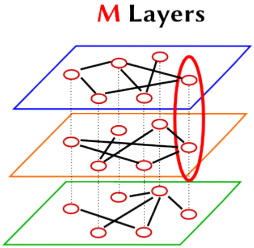

N nodes
M Layers
node replicas

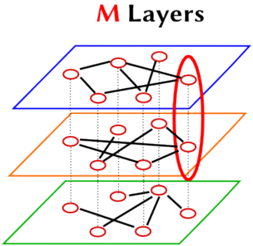
N nodes



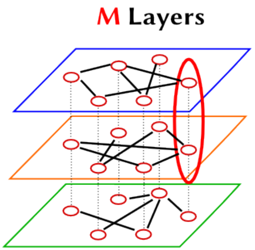
M Layers




M Layers



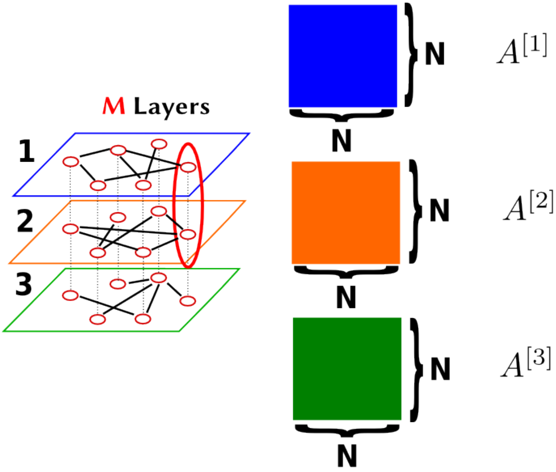
1) NODE IDENTITY

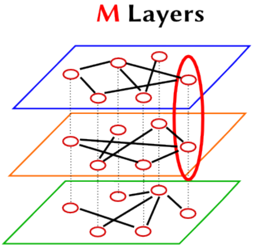


1) NODE IDENTITY

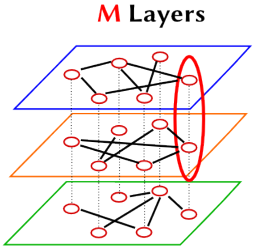
$$\mathcal{M} = \{A^{[1]}, A^{[2]}, \ldots, A^{[M]}\}$$

M-dimensional array of NxN matrices


M Layers
1
2
3
N
N
N
N
N
N
$A^{[1]}$
$A^{[2]}$
$A^{[3]}$



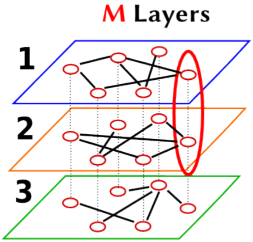
2) INTERNAL FLOW



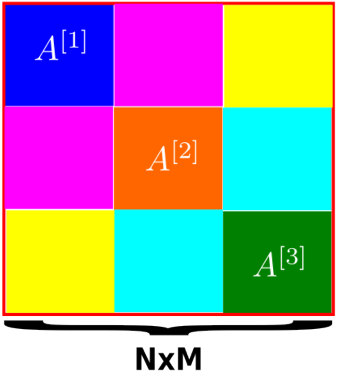
2) INTERNAL FLOW

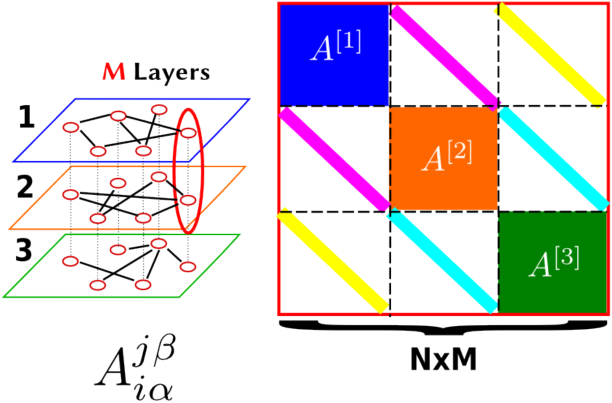
$$A_{i\alpha}^{j\beta}$$

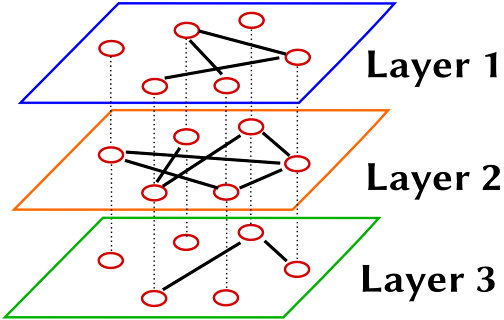
NxM x NxM TENSOR

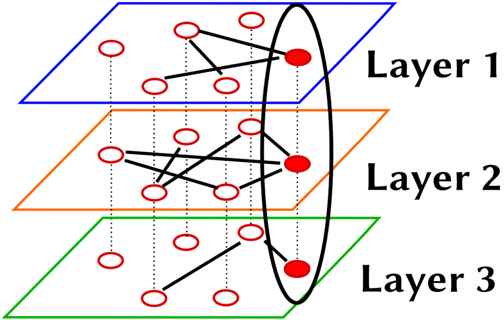


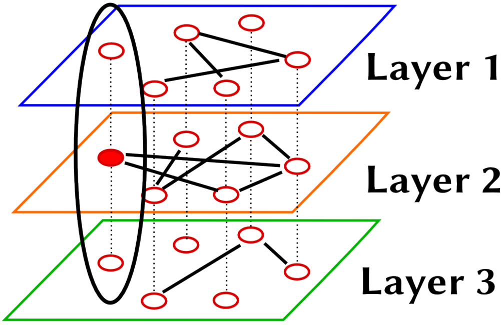
$$A_{i\alpha}^{j\beta}$$

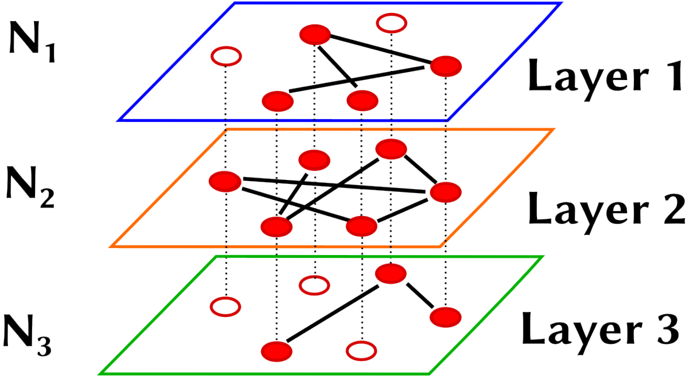



M Layers
1
2
3
$A_{i\alpha}^{j\beta}$
$A^{[1]}$
$A^{[2]}$
$A^{[3]}$
NxM

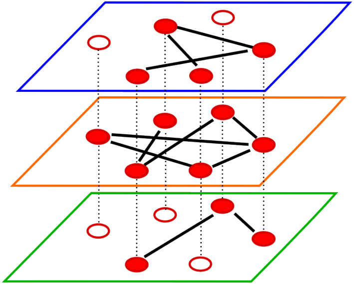

Layer 1
Layer 2
Layer 3


Layer 1
Layer 2
Layer 3


Layer 1
Layer 2
Layer 3

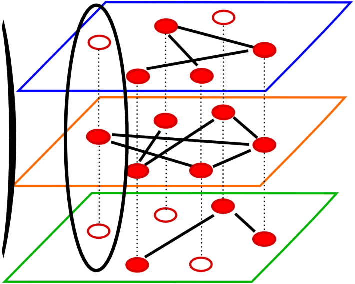

N1
N2
N3
Layer 1
Layer 2
Layer 3

Node activity vectors



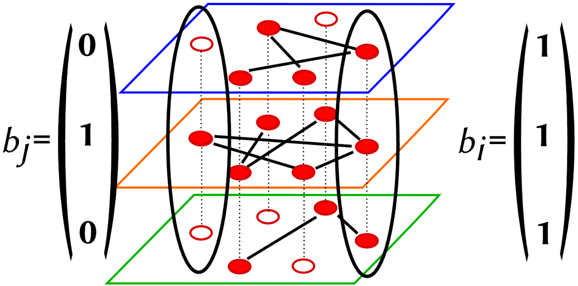
Node activity vectors

$$b_j = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

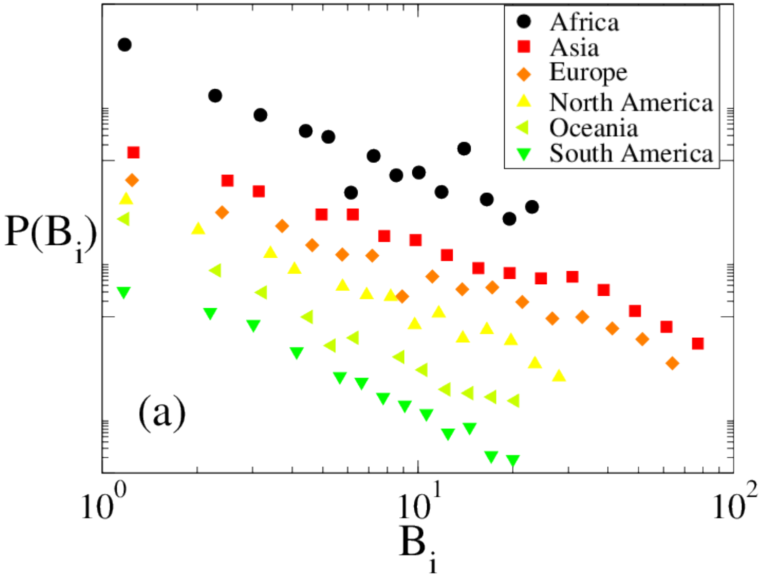


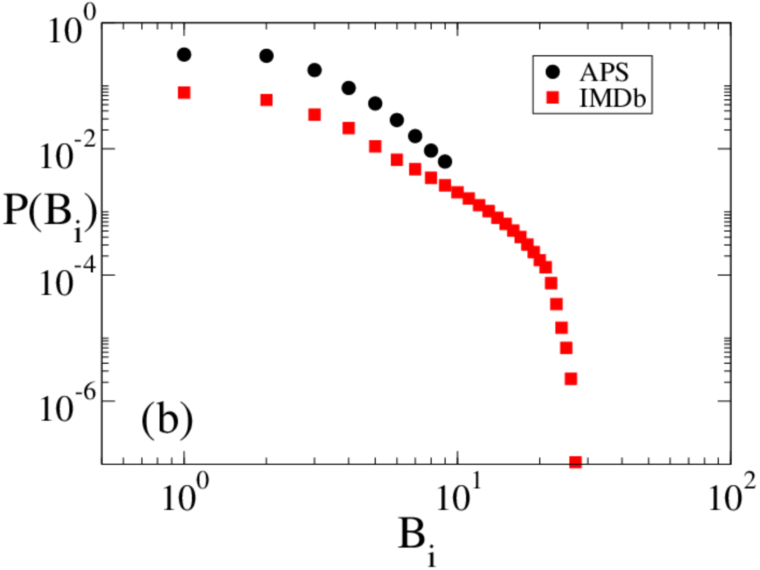
$$\mathrm{B_j} = \sum_\alpha b_j^{[\alpha]} = 1$$

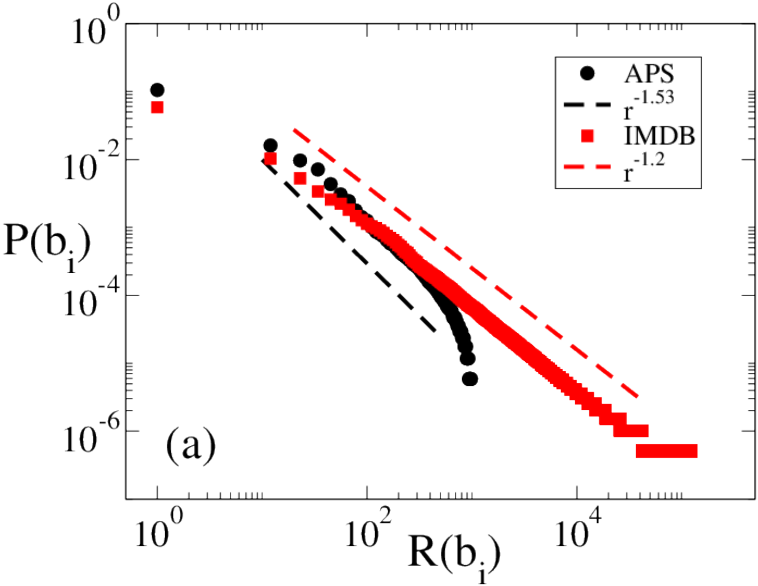
Node activity vectors



$$B_j = \sum_\alpha b_j^{[\alpha]} = 1 \qquad B_i = \sum_\alpha b_i^{[\alpha]} = 3$$

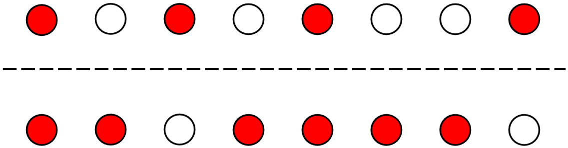

Africa
Asia
Europe
North America
Oceania
South America
P(B_i)
(a)
10^0
10^1
10^2
B_i


10^0
10^{-2}
10^{-4}
10^{-6}
$P(B_i)$
APS
IMDb
(b)
10^0
10^1
10^2
B_i



10^0
10^{-2}
10^{-4}
10^{-6}
$P(b_i)$
APS
$r^{-1.53}$
IMDB
$r^{-1.2}$
(a)
10^0
10^2
10^4
$R(b_i)$

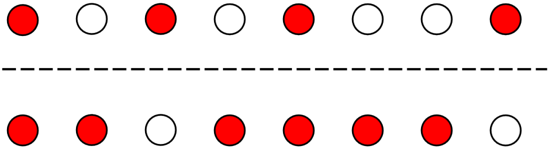
Layer α



Layer β

Layer α

$$d^{[\alpha]} = \{1, 0, 1, 0, 1, 0, 0, 1\}$$

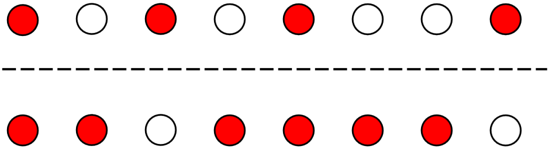


$$d^{[\beta]} = \{1, 1, 0, 1, 1, 1, 1, 0\}$$

Layer β

Layer α

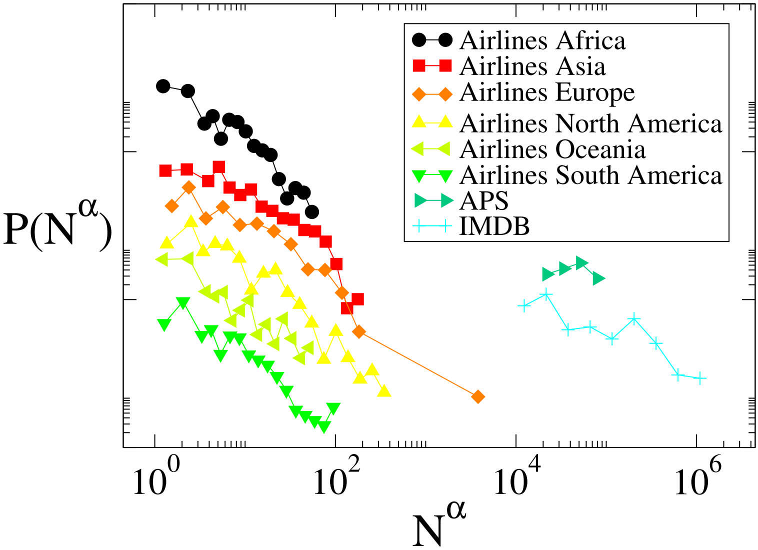
$d^{[\alpha]} = \{1, 0, 1, 0, 1, 0, 0, 1\}$ $\quad N^{[\alpha]} = 4$



$d^{[\beta]} = \{1, 1, 0, 1, 1, 1, 1, 0\}$ $\quad N^{[\beta]} = 6$

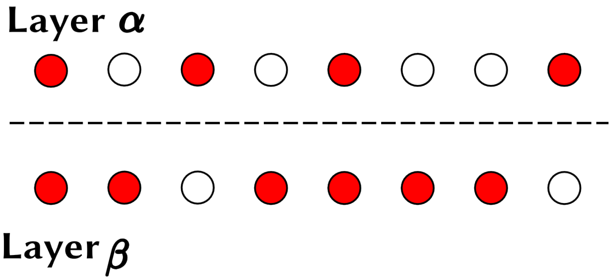
Layer β

Layer activity

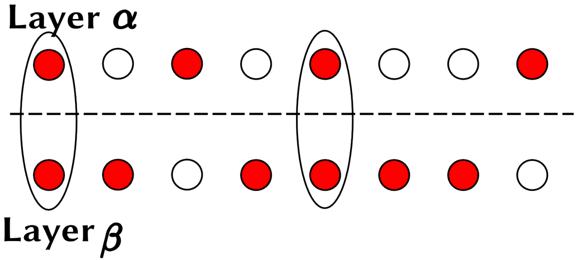


Airlines Africa
Airlines Asia
Airlines Europe
Airlines North America
Airlines Oceania
Airlines South America
APS
IMDB
P(N^α)
N^α
10^0
10^2
10^4
10^6

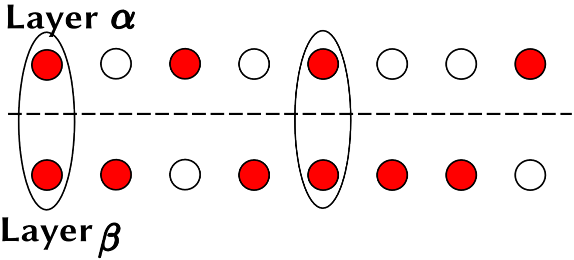
Multiplexity and Hamming distance



Multiplexity and Hamming distance

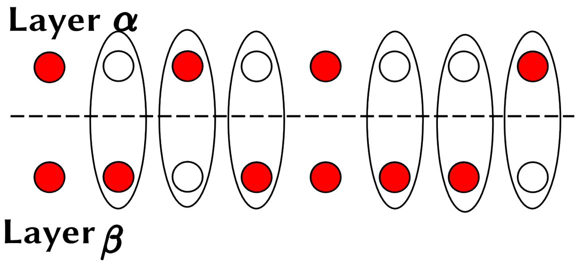


Multiplexity and Hamming distance

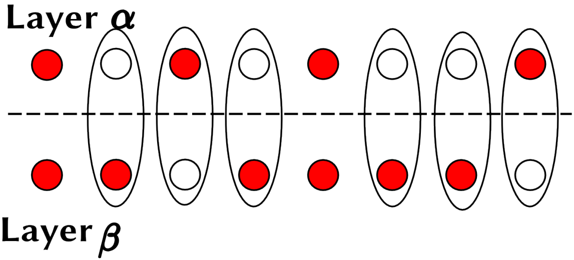


MULTIPLEXITY

$$Q_{\alpha,\beta} = \frac{\sum_i b_i^{[\alpha]} b_i^{[\beta]}}{N} = \frac{2}{8}$$

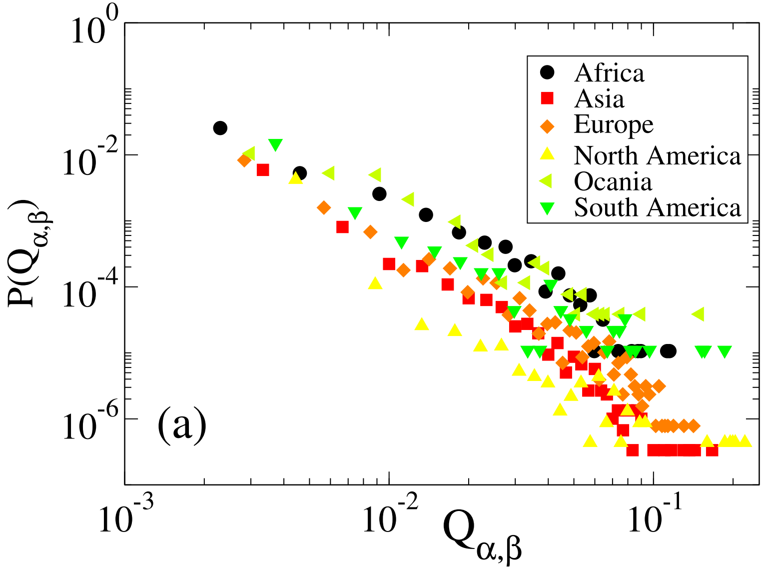

Layer α
Layer β

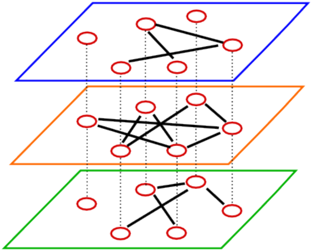
Multiplexity and Hamming distance

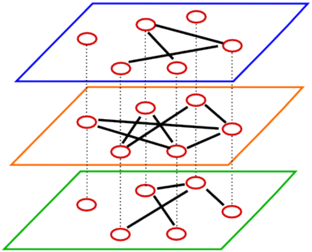


Hamming Distance

$$H_{\alpha,\beta} = \frac{\sum_i b_i^{[\alpha]}(1-b_i^{[\alpha]})+b_i^{[\beta]}(1-b_i^{[\beta]})}{min(N,N^{[\alpha]}+N^{[\beta]}} = \frac{6}{8}$$

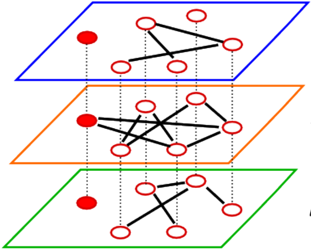

10^0
10^-2
10^-4
10^-6
P(Q_{α,β})
10^-3
10^-2
10^-1
Q_{α,β}
(a)
Africa
Asia
Europe
North America
Ocania
South America





TOTAL (OVERLAPPING) DEGREE

$$o_i = \sum_\alpha k_i^{[\alpha]}$$



$k_i^{[1]} = 0$

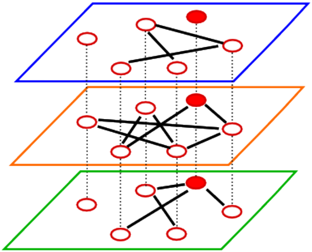
$k_i^{[2]} = 2$

$k_i^{[3]} = 0$

$o_i = 2$

TOTAL (OVERLAPPING) DEGREE

$$o_i = \sum_\alpha k_i^{[\alpha]}$$



$k_i^{[1]} = 0$

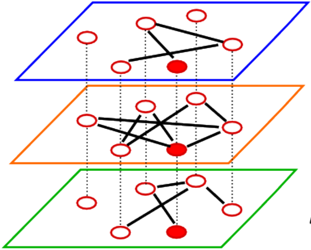
$k_i^{[2]} = 2$

$k_i^{[3]} = 3$

$o_i = 5$

TOTAL (OVERLAPPING) DEGREE

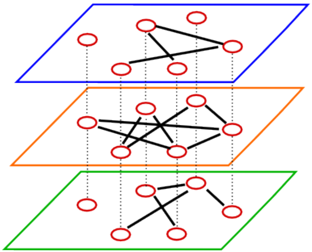
$$o_i = \sum_\alpha k_i^{[\alpha]}$$

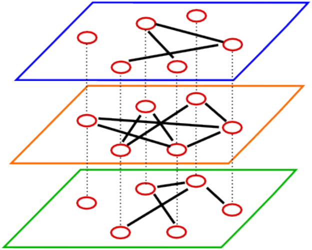


$$o_i = 5$$

TOTAL (OVERLAPPING) DEGREE

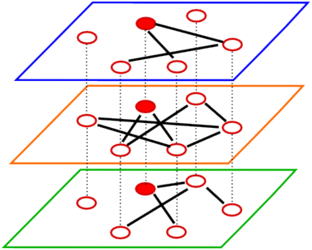
$$o_i = \sum_\alpha k_i^{[\alpha]}$$





MULTIPLEX PARTICIPATION COEFFICIENT

$$P_i = \frac{M}{M-1}\left[1 - \sum_\alpha \left(\frac{k_i^{[\alpha]}}{\sum_\beta k_i^{[\beta]}}\right)^2\right]$$



$$k_i^{[1]} = 2$$

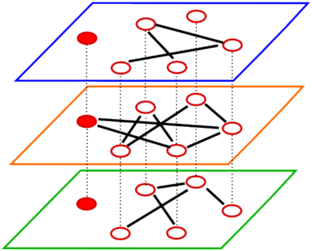
$$k_i^{[2]} = 2$$

$$k_i^{[3]} = 2$$

$$P_i = 1.0$$

MULTIPLEX PARTICIPATION COEFFICIENT

$$P_i = \frac{M}{M-1}\left[1 - \sum_\alpha \left(\frac{k_i^{[\alpha]}}{\sum_\beta k_i^{[\beta]}}\right)^2\right]$$



$$k_i^{[1]} = 0$$

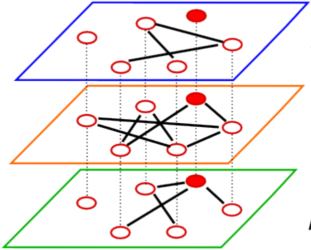
$$k_i^{[2]} = 2$$

$$k_i^{[3]} = 0$$

$$P_i = 0.0$$

MULTIPLEX PARTICIPATION COEFFICIENT

$$P_i = \frac{M}{M-1}\left[1 - \sum_\alpha \left(\frac{k_i^{[\alpha]}}{\sum_\beta k_i^{[\beta]}}\right)^2\right]$$



$$k_i^{[1]} = 0$$

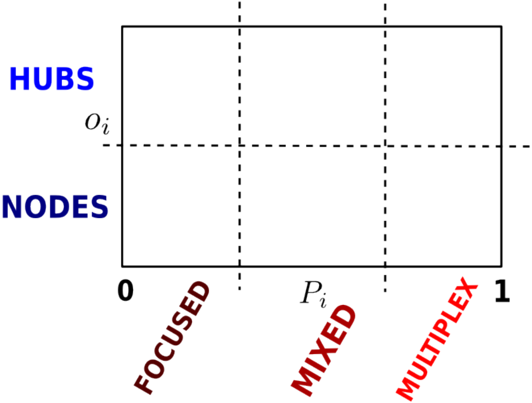
$$k_i^{[2]} = 2$$

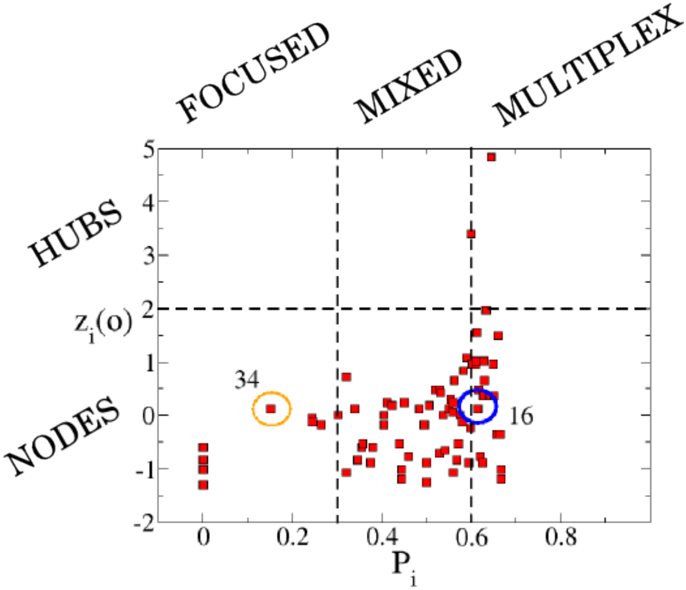
$$k_i^{[3]} = 3$$

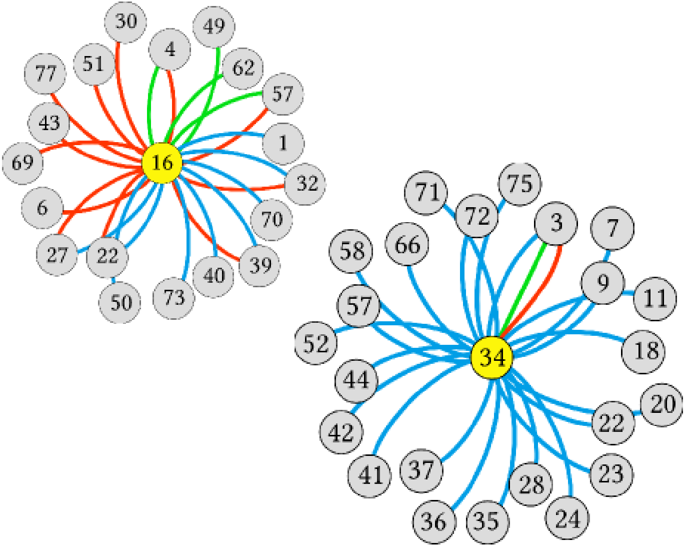
$$P_i = 0.72$$

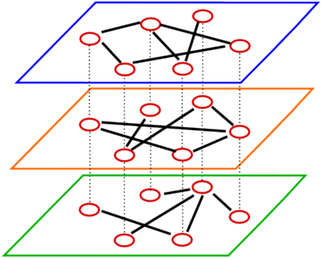
MULTIPLEX PARTICIPATION COEFFICIENT

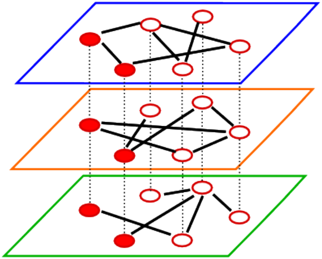
$$P_i = \frac{M}{M-1}\left[1 - \sum_\alpha \left(\frac{k_i^{[\alpha]}}{\sum_\beta k_i^{[\beta]}}\right)^2\right]$$

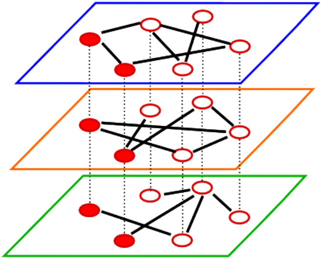

HUBS
o_i
NODES
0
P_i
1
FOCUSED
MIXED
MULTIPLEX


FOCUSED
MIXED
MULTIPLEX
HUBS
NODES
$z_i(o)$
P_i
34
16
5
4
3
2
1
0
-1
-2
0
0.2
0.4
0.6
0.8


30
49
4
77
51
62
57
43
1
69
16
32
6
70
27
22
39
40
50
73
71
75
72
3
7
58
66
9
11
57
52
34
18
44
20
22
42
23
41
37
28
36
35
24

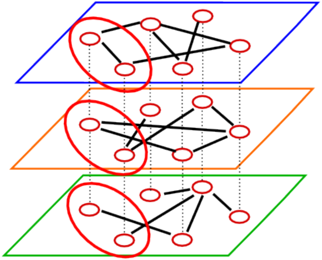






EDGE OVERLAP

$$o_{ij} = \frac{1}{M} \sum_\alpha a_{ij}^{[\alpha]}$$



$$a_{ij}^{[1]} = 1$$

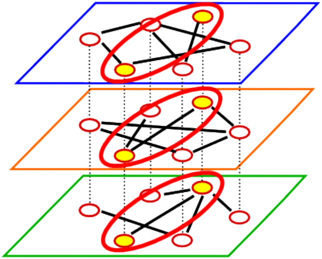
$$a_{ij}^{[2]} = 0$$

$$a_{ij}^{[3]} = 0$$

$$o_{ij} = \frac{1}{3}$$

EDGE OVERLAP

$$o_{ij} = \frac{1}{M} \sum_{\alpha} a_{ij}^{[\alpha]}$$



$$a_{ij}^{[1]} = 0$$

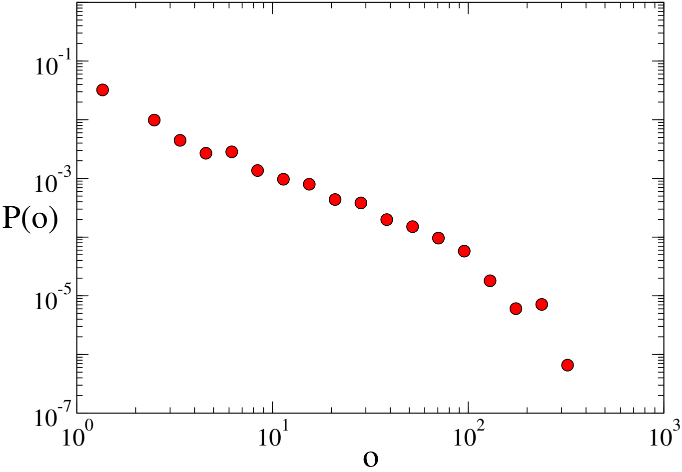
$$a_{ij}^{[2]} = 1$$

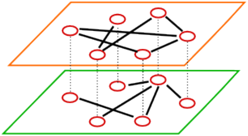
$$a_{ij}^{[3]} = 1$$

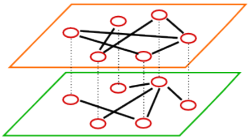
$$o_{ij} = \frac{2}{3}$$

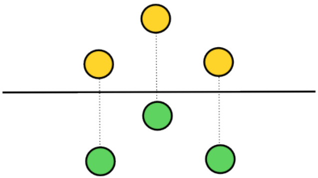
EDGE OVERLAP

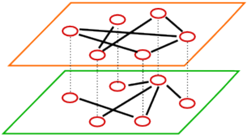
$$o_{ij} = \frac{1}{M} \sum_{\alpha} a_{ij}^{[\alpha]}$$

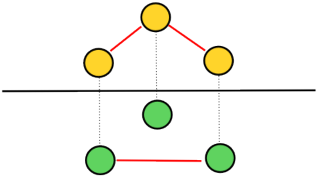

P(o)
o
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{0}
10^{1}
10^{2}
10^{3}

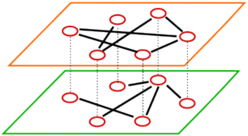


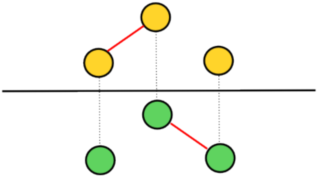


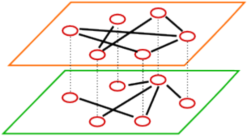


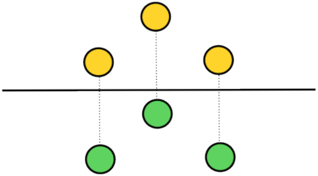




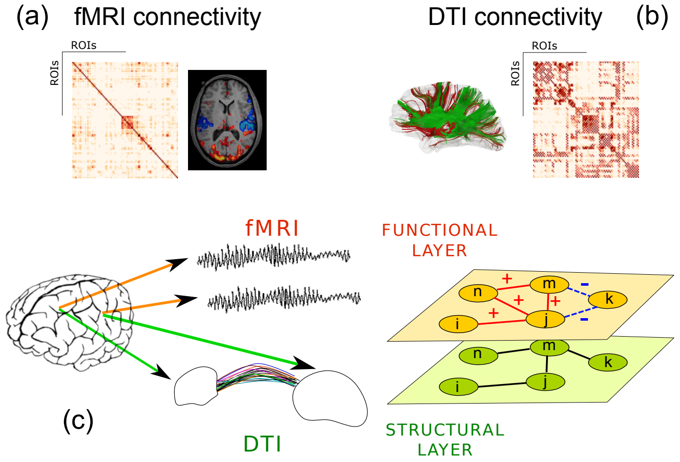




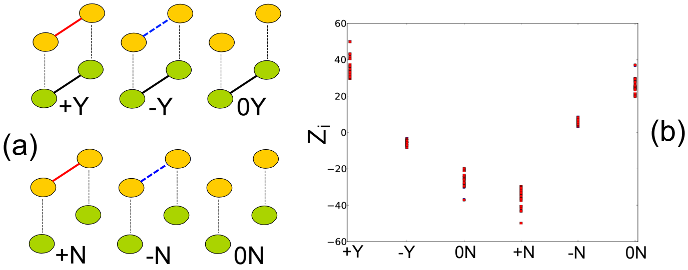




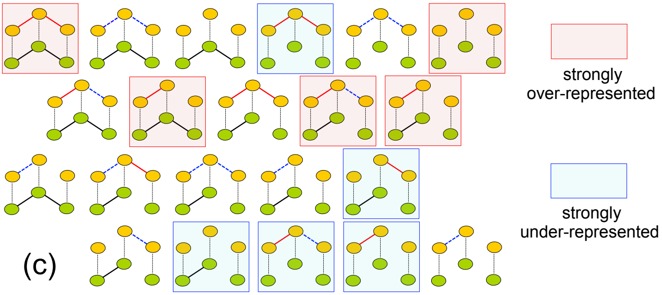
How "important" are small subgraphs?


(a) fMRI connectivity
ROIs
ROIs
DTI connectivity (b)
ROIs
ROIs
fMRI
FUNCTIONAL LAYER
+
+
+
+
-
-
n
m
k
i
j
n
m
k
i
j
(c)
DTI
STRUCTURAL LAYER

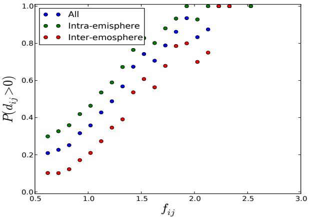
Edges



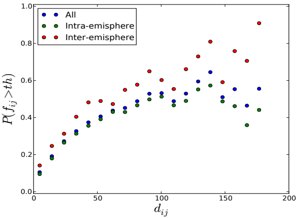
Triads



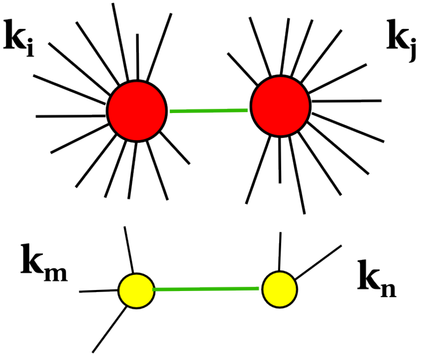
(a) DTI given fMRI



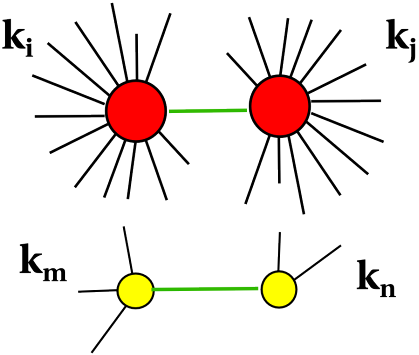
fMRI given DTI (b)



(Intra-layer) Degree correlations

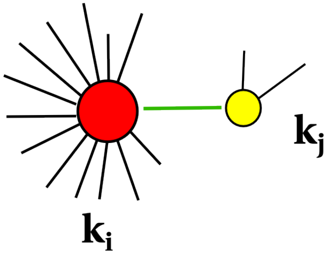


(Intra-layer) Degree correlations

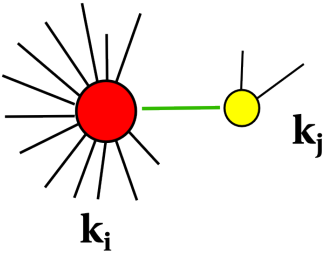


ASSORTATIVE (POSITIVE)

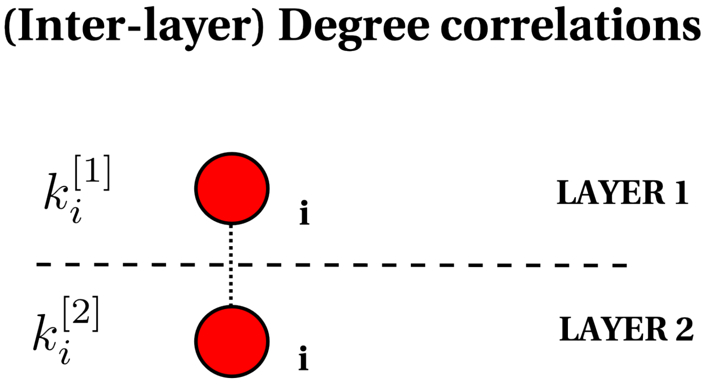
(Intra-layer) Degree correlations



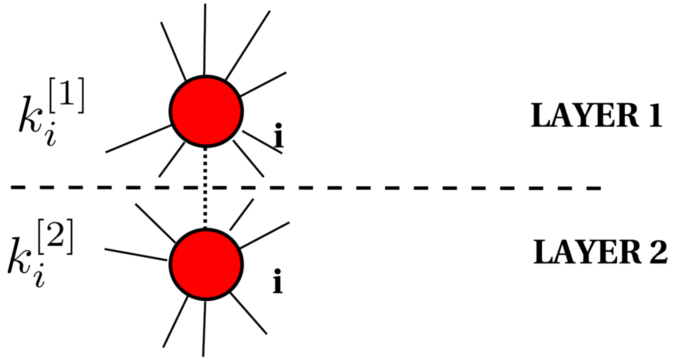
(Intra-layer) Degree correlations



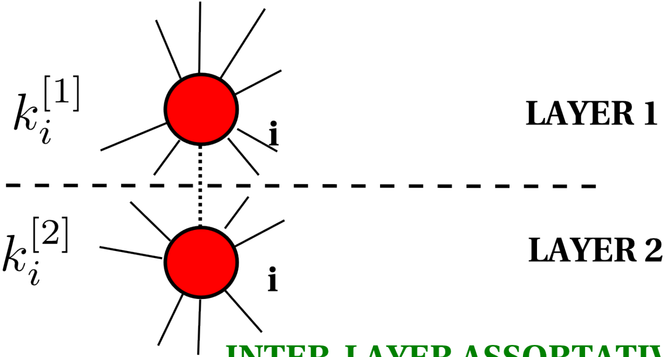
DISASSORTATIVE (NEGATIVE)


(Inter-layer) Degree correlations
$k_i^{[1]}$
i
LAYER 1
$k_i^{[2]}$
i
LAYER 2

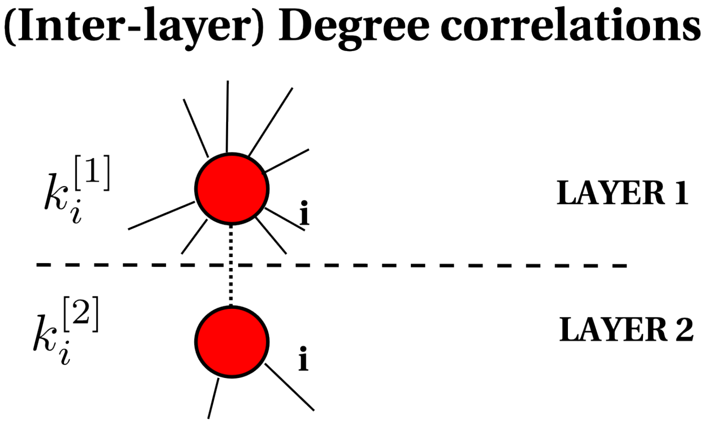
(Inter-layer) Degree correlations

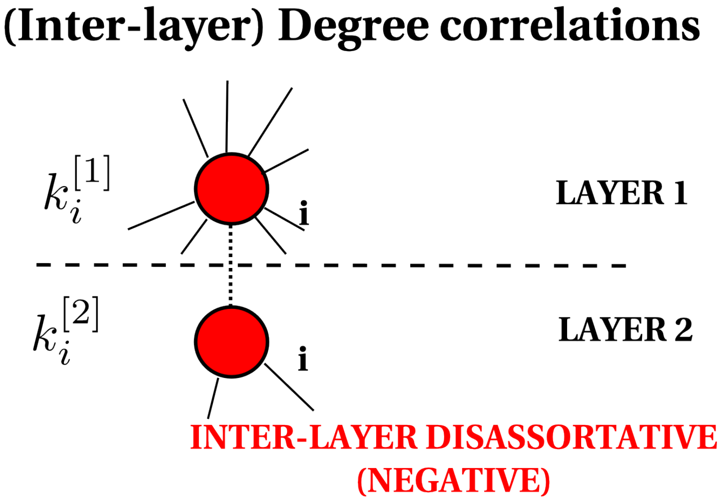


(Inter-layer) Degree correlations



INTER-LAYER ASSORTATIVE (POSITIVE)


(Inter-layer) Degree correlations
$k_i^{[1]}$
i
LAYER 1
$k_i^{[2]}$
i
LAYER 2


(Inter-layer) Degree correlations
$k_i^{[1]}$
i
LAYER 1
$k_i^{[2]}$
i
LAYER 2
INTER-LAYER DISASSORTATIVE
(NEGATIVE)

Inter-layer degree correlations

Node degree:

$$\boldsymbol{k}_i = \left\{ k_i^{[1]}, k_i^{[2]}, \ldots, k_i^{[M]} \right\}$$

Inter-layer degree correlations

Node degree:

$$\boldsymbol{k}_i = \left\{ k_i^{[1]}, k_i^{[2]}, \ldots, k_i^{[M]} \right\}$$

$\Longrightarrow$ inter-layer degree distributions

$$P(k^{[\alpha]}, k^{[\beta]}) \quad \text{and} \quad P(k^{[\beta]} | k^{[\alpha]})$$

Inter-layer degree correlations

Node degree:

$$\boldsymbol{k}_i = \left\{ k_i^{[1]}, k_i^{[2]}, \ldots, k_i^{[M]} \right\}$$

$\Longrightarrow$ inter-layer degree distributions

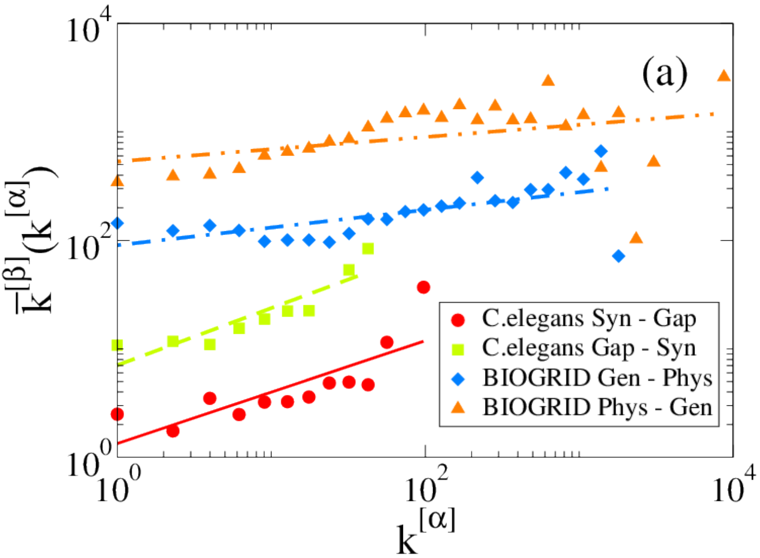
$$P(k^{[\alpha]}, k^{[\beta]}) \quad \text{and} \quad P(k^{[\beta]} | k^{[\alpha]})$$

$\Longrightarrow$ inter-layer degree correlation function

$$\overline{k^{[\beta]}}(k^{[\alpha]}) = \sum_{k^{[\beta]}} k^{[\beta]} P(k^{[\beta]} | k^{[\alpha]})$$

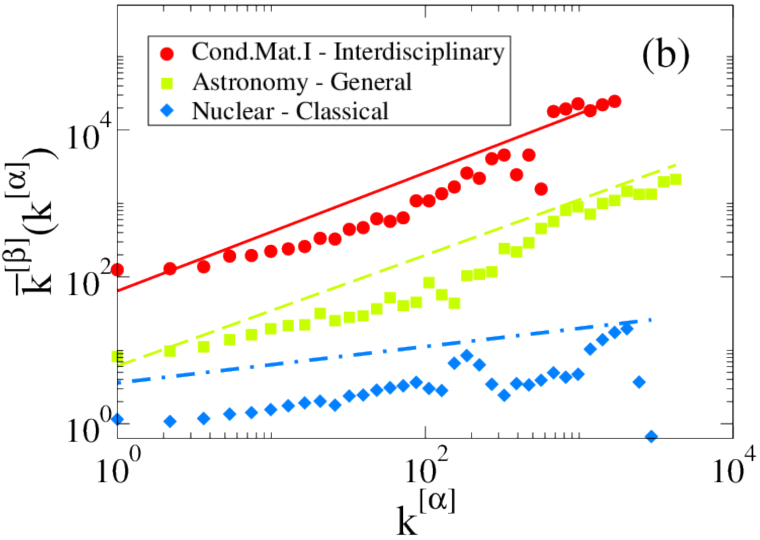
V. Nicosia, V. Latora, "Measuring and modelling correlations in multiplex networks", arxiv:1403.1546 (2014)

Inter-layer correlation function



V. Nicosia, V. Latora, "Measuring and modelling correlations in multiplex networks", arxiv:1403.1546 (2014)

Inter-layer correlation function



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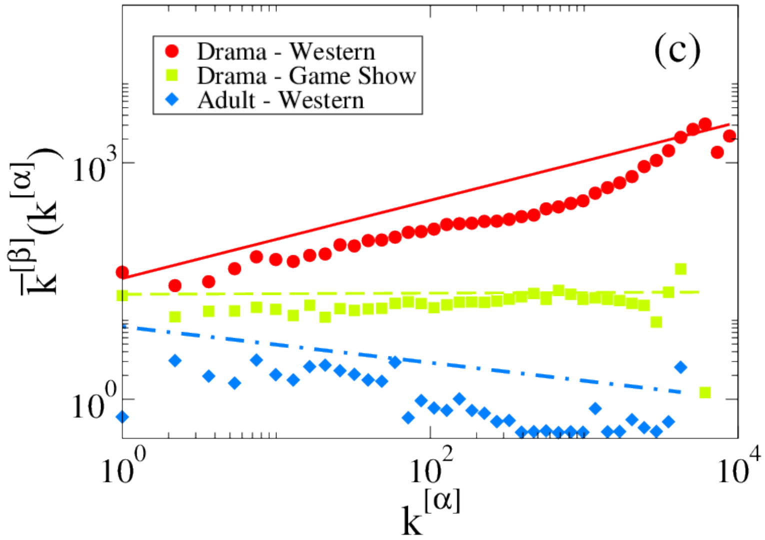
Inter-layer correlation function



V. Nicosia, V. Latora, "Measuring and modelling correlations in multiplex networks", arxiv:1403.1546 (2014)

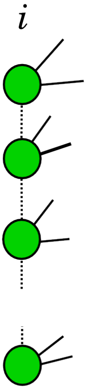
MODELLING

(HOW MULTIPLEX NETWORKS MIGHT HAVE FORMED?)

New node i

m **edge stubs**

ON EACH LAYER



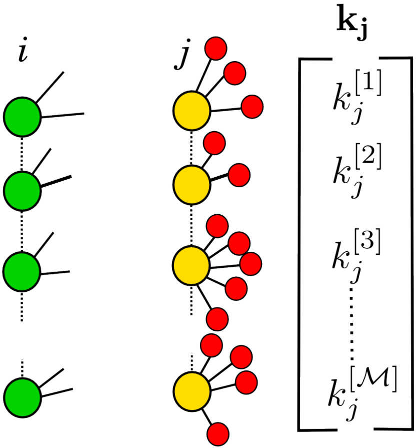
V. Nicosia, G. Bianconi, V. Latora, M. Barthélemy, *Phys. Rev. Lett.* 111, 058701 (2013)

Multiplex linear preferential attachment

New node i

m **edge stubs**

ON EACH LAYER



V. Nicosia, G. Bianconi, V. Latora, M. Barthélemy, *Phys. Rev. Lett.* 111, 058701 (2013)

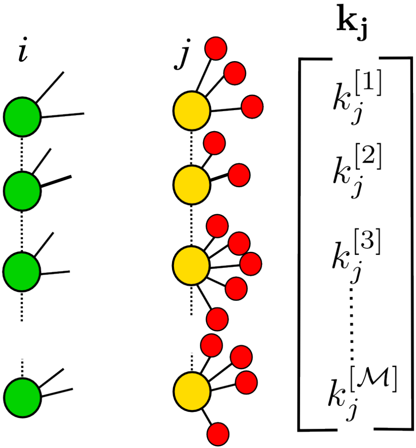
Multiplex linear preferential attachment

New node i

m edge stubs

on each layer

$$\Pi_{i\to j}^{[\alpha]} \propto F^{[\alpha]}(\mathbf{k_j})$$



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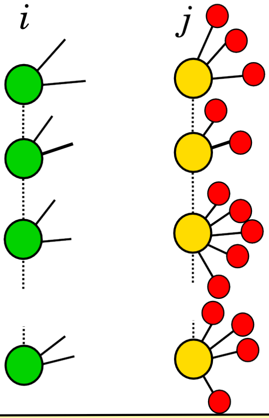


$$\Pi_{i\to j}^{[\alpha]} \propto F^{[\alpha]}(\mathbf{k_j})$$

$$\mathbf{k_j} = \begin{bmatrix} k_j^{[1]} \\ k_j^{[2]} \\ k_j^{[3]} \\ \vdots \\ k_j^{[\mathcal{M}]} \end{bmatrix}$$

The attachment probability can depend on the degrees at all layers

V. Nicosia, G. Bianconi, V. Latora, M. Barthélemy, *Phys. Rev. Lett.* 111, 058701 (2013)

LINEAR ATTACHMENT

$$f(k_j) = \sum_\alpha c_\alpha k_j^{[\alpha]}$$

INTER-LAYER ASSORTATIVITY

V. Nicosia, G. Bianconi, V. Latora, M. Barthélemy, *Phys. Rev. Lett.* 111, 058701 (2013)

Non-linear preferential attachment

In general:

$$\Pi_{i\to j}^{[\ell]} \propto f^{[\ell]}\left(k_j^{[1]}, k_j^{[2]}, \ldots, k_j^{[M]}\right)$$

Non-linear preferential attachment

In general:

$$\Pi_{i\to j}^{[\ell]} \propto f^{[\ell]}\left(k_j^{[1]}, k_j^{[2]}, \ldots, k_j^{[M]}\right)$$

Two layers: $k_i \equiv k_i^{[1]}$ and $q_i = k_i^{[2]}$:

$$\Pi_{i\to j}^{[1]} \propto f(k_j, q_j), \quad \text{and} \quad \Pi_{i\to j}^{[2]} \propto f(q_j, k_j).$$

In general:

$$\Pi_{i\to j}^{[\ell]} \propto f^{[\ell]}\left(k_j^{[1]}, k_j^{[2]}, \ldots, k_j^{[M]}\right)$$

Two layers: $k_i \equiv k_i^{[1]}$ and $q_i = k_i^{[2]}$:

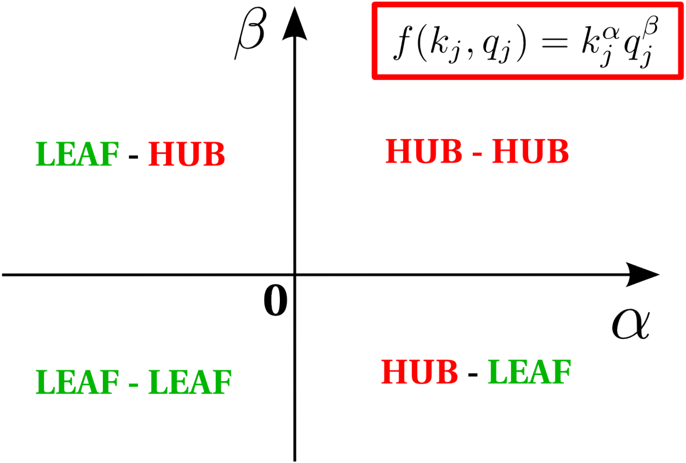
$$\Pi_{i\to j}^{[1]} \propto f(k_j, q_j), \quad \text{and} \quad \Pi_{i\to j}^{[2]} \propto f(q_j, k_j).$$

where we set:

$$f(k_j, q_j) = k_j^{\alpha} q_j^{\beta}$$

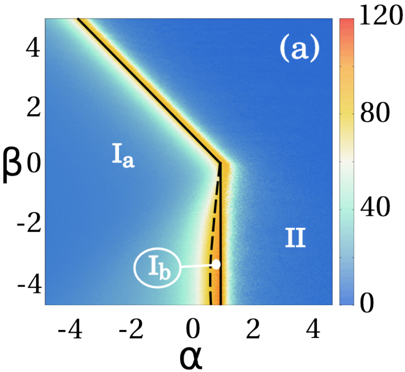
and $\alpha, \beta \in \mathbb{R}$ free parameters

V. Nicosia, G. Bianconi, V. Latora, M. Barthélemy, accepted in *Phys. Rev. E* (2014)



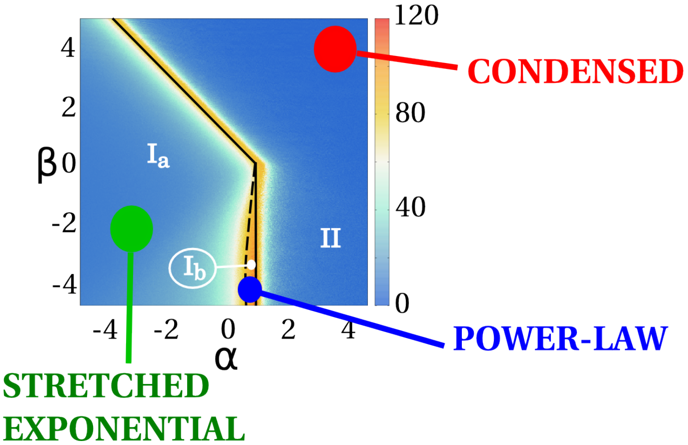
V. Nicosia, G. Bianconi, V. Latora, M. Barthélemy, accepted in *Phys. Rev. E* (2014)

Layer Degree Distribution P(k)



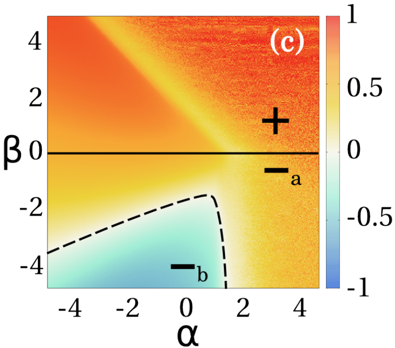
V. Nicosia, G. Bianconi, V. Latora, M. Barthélemy, accepted in *Phys. Rev. E* (2014)

Layer Degree Distribution P(k)



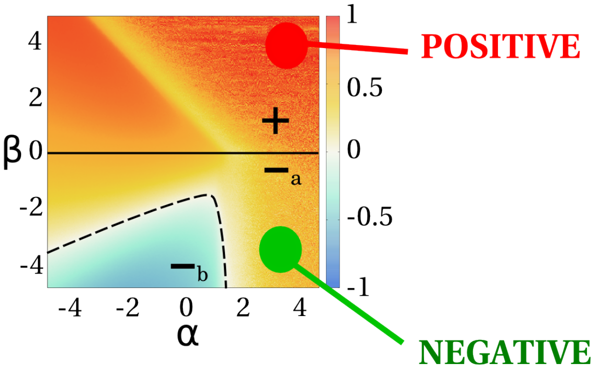
V. Nicosia, G. Bianconi, V. Latora, M. Barthélemy, accepted in *Phys. Rev. E* (2014)

Inter-layer Degree Correlations



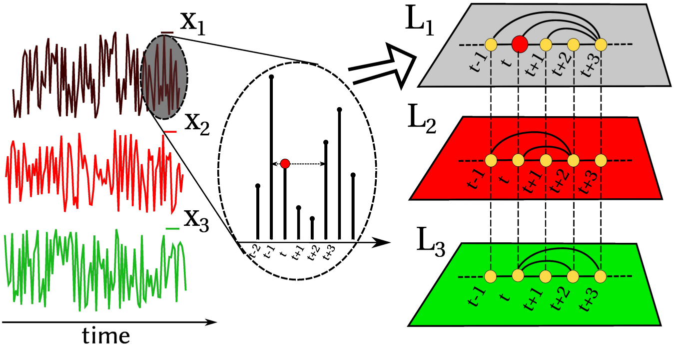
V. Nicosia, G. Bianconi, V. Latora, M. Barthélemy, accepted in *Phys. Rev. E* (2014)

Inter-layer Degree Correlations

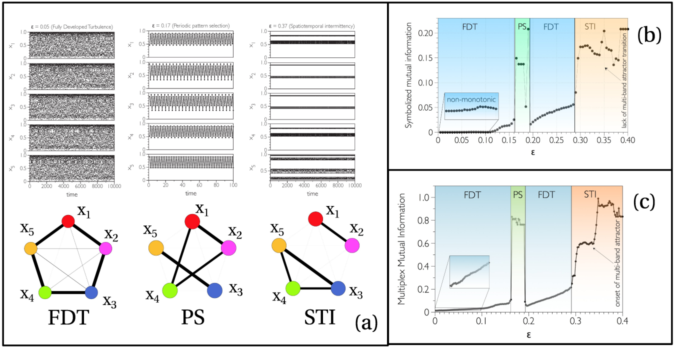


V. Nicosia, G. Bianconi, V. Latora, M. Barthélemy, accepted in *Phys. Rev. E* (2014)

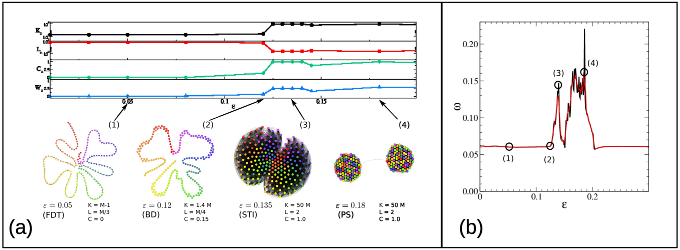

EXOTIC APPLICATIONS

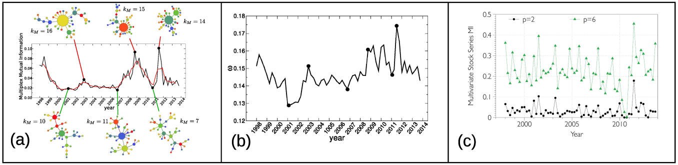


x_1
x_2
x_3
time
t-2
t-1
t
t+1
t+2
t+3
L_1
L_2
L_3



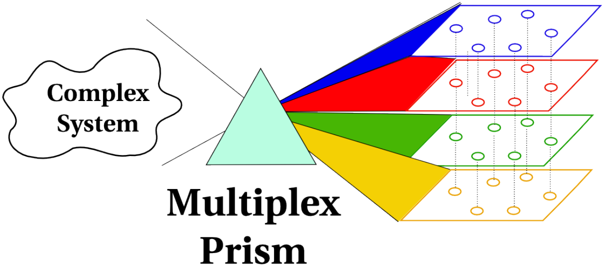
FDT
PS
STI
(a)
(b)
(c)
Symbolized mutual information
non-monotonic
lack of multi-band attractor transition
Multiplex Mutual Information
onset of multi-band attractor
ε

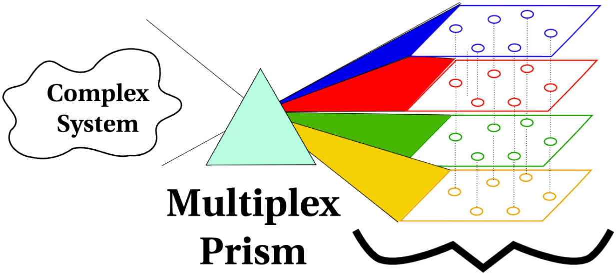

(1)
(2)
(3)
(4)
$\varepsilon = 0.05$
(FDT)
K = M-1
L = M/3
C = 0
$\varepsilon = 0.12$
(BD)
K = 1.4 M
L = M/4
C = 0.15
$\varepsilon = 0.135$
(STI)
K = 50 M
L = 2
C = 1.0
$\varepsilon = 0.18$
(PS)
K = 50 M
L = 2
C = 1.0
(a)
ω
ε
0.20
0.15
0.10
0.05
0.00
0
0.1
0.2
(b)


(a)
(b)
(c)
year
Year
Multivariate Stock Series MI
p=2
p=6

USEFULNESS

(DO WE REALLY NEED ALL THIS COMPLEXITY?)


Complex
System
Multiplex
Prism

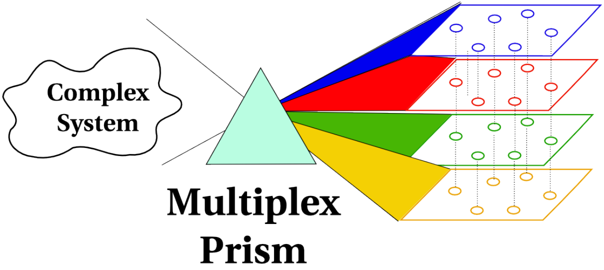


Is this representation "minimal"?

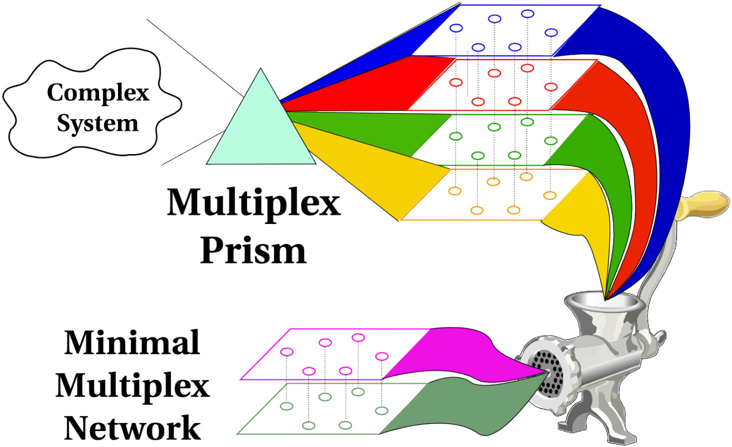
Smaller number of layers
"without information loss?"

The Multiplex Mincer Machine (a.k.a. MMM or M^3)

M. De Domenico, V. Nicosia, A. Arenas, V. Latora, to appear in *Nature Communications* (2015)



M. De Domenico, V. Nicosia, A. Arenas, V. Latora, to appear in *Nature Communications* (2015)



M. De Domenico, V. Nicosia, A. Arenas, V. Latora, to appear in *Nature Communications* (2015)

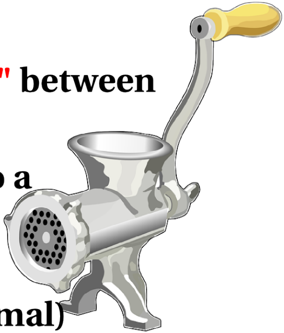
What does MMM do?

1. Quantify the information content carried by each layer

2. Estimate the "difference" between layers

3. Merge similar layers into a single one

4. Hopefully find the best (i.e. least redundant, minimal) representation



M. De Domenico, V. Nicosia, A. Arenas, V. Latora, to appear in *Nature Communications* (2015)

Note:

Graphs

Quantum states

M. De Domenico, V. Nicosia, A. Arenas, V. Latora, to appear in *Nature Communications* (2015)

Note:

Graphs

Laplacian: $\mathcal{L}$

$$Tr(\mathcal{L}) = 1$$

Quantum states

Density operator: ρ

$$Tr(\rho) = 1$$

M. De Domenico, V. Nicosia, A. Arenas, V. Latora, to appear in *Nature Communications* (2015)

Note:

Graphs | **Quantum states**

Laplacian: $\mathcal{L}$

$$Tr(\mathcal{L}) = 1$$

Complexity: $h_{VN}(\mathcal{L})$

Density operator: ρ

$$Tr(\rho) = 1$$

Mixedness: $h_{VN}(\rho)$

M. De Domenico, V. Nicosia, A. Arenas, V. Latora, to appear in *Nature Communications* (2015)

Note:

Graphs

Laplacian: $\mathcal{L}$

$$Tr(\mathcal{L}) = 1$$

Complexity: $h_{VN}(\mathcal{L})$

$\mathcal{L}_1 \;\; \mathcal{L}_2$

Diff. btw graphs:

$$D_{JS}(\mathcal{L}_1, \mathcal{L}_2)$$

Quantum states

Density operator: ρ

$$Tr(\rho) = 1$$

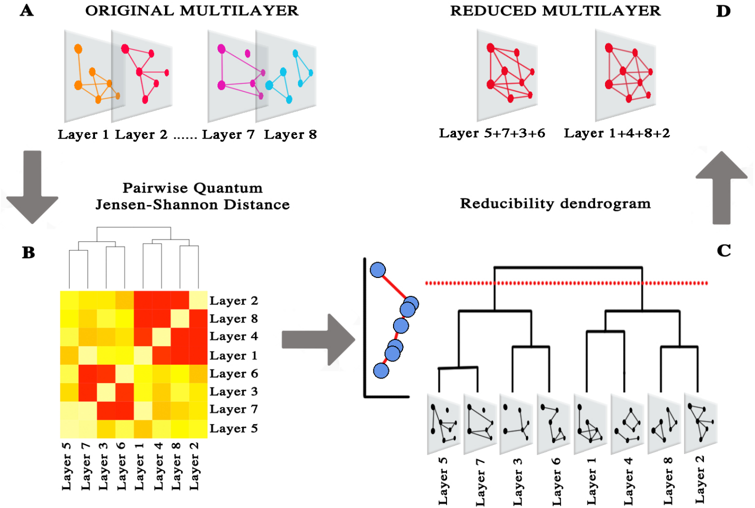
Mixedness: $h_{VN}(\rho)$

$\rho_1 \;\; \rho_2$

Diff. btw states *:**

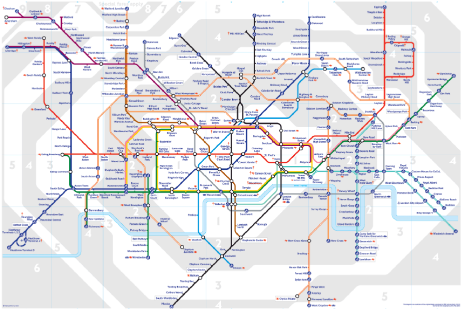
$$D_{JS}(\rho_1, \rho_2)$$

M. De Domenico, V. Nicosia, A. Arenas, V. Latora, to appear in *Nature Communications* (2015)



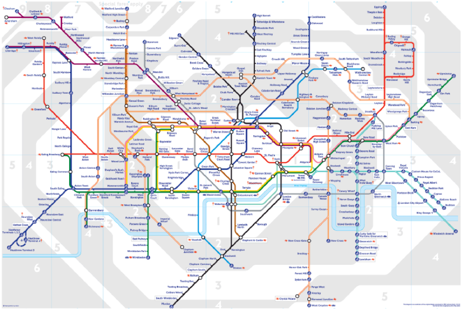
M. De Domenico, V. Nicosia, A. Arenas, V. Latora, to appear in *Nature Communications* (2015)

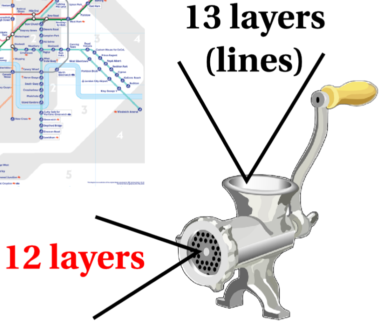
Results: London Tube



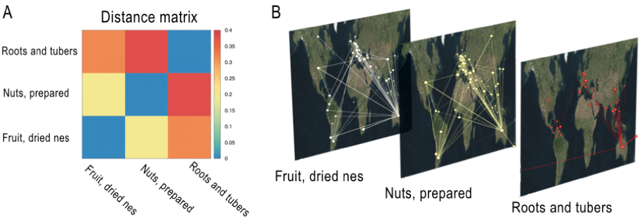
13 layers (lines)

Results: London Tube



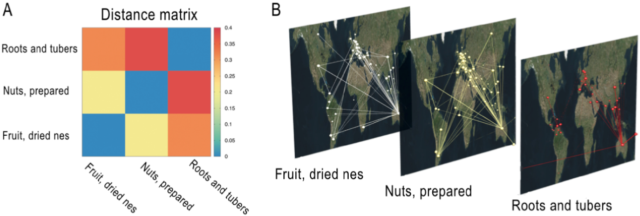


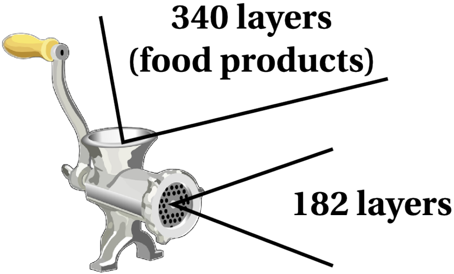
Results: FAO food trading network



340 layers (food products)

Results: FAO food trading network





Conclusions

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Multi-layer networks...

- ...are everywhere

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- ...are everywhere
- ...have many non-trivial structural properties

Conclusions

Multi-layer networks...

- ...are everywhere
- ...have many non-trivial structural properties
- ...are the natural framework for several real-world systems

Conclusions

Multi-layer networks...

- ...are everywhere
- ...have many non-trivial structural properties
- ...are the natural framework for several real-world systems
- ...are interesting (I admit I'm a bit biased on this point...)

References

- S. Boccaletti et al. "The structure and dynamics of multilayer networks", Phys. Rep. 544, 1–122 (2014).
- M. Kivela et al. "Multilayer networks", J. Compl. Networks 2(3): 203-271 (2014).
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