

Euler method for solving a differential Eq.
Example:

$$\frac{d^2\theta}{dt^2} = -\sin(\theta) \quad (1)$$

Define $v(t) = d\theta/dt$
 $dv/dt = -\sin(\theta)$

Taylor series $f_{n+1} = f_n + hf'_n + h^2 f''_n/2 + \mathcal{O}(h^3)$
 $d\theta/dt = [\theta(t+h) - \theta(t)]/h + \mathcal{O}(h) = v(n)$
 $dv/dt = [v(n+1) - v(n)]/h + \mathcal{O}(h) = -\sin(\theta)$

$$\theta_{n+1} = \theta_n + h\theta'_n + \mathcal{O}(h^2)$$

$$v_{n+1} = v_n - h\sin(\theta_n) + \mathcal{O}(h^2)$$

Iterate: $\theta_1 = 50 \times \frac{\pi}{180}$; $v_1 = 0$ start from rest at a large angle
 $\theta_2 = \theta_1 + hv_1 + \mathcal{O}(h^2) = \theta_1 + \mathcal{O}(h^2)$
 $v_2 = v_1 + h[-\sin(\theta_1)] + \mathcal{O}(h^2)$

$$\theta_3 = \theta_2 + hv_2 + \mathcal{O}(h^2)$$

$$v_3 = v_2 + h[-\sin(\theta_2)] + \mathcal{O}(h^2)$$

etc

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 $v_{n+1} = v_n + h[-\sin(\theta_n)] + \mathcal{O}(h^2)$

$$\theta_{n+1} = \theta_n + hv_n + \mathcal{O}(h^2), \text{replace}$$

$$v_n \rightarrow (v_{n+1} + v_n)/2 = (v_n + h[-\sin(\theta_n)] + v_n)/2 + \mathcal{O}(h^2) = v_n + h[-\sin(\theta_n)]/2 + \mathcal{O}(h^2)$$

Get: $v_{n+1} = v_n + h[-\sin(\theta_n)] + \mathcal{O}(h^2)$
 $\theta_{n+1} = \theta_n + hv_n + h^2(-\sin(\theta_n)) + \mathcal{O}(h^3)$; gain one additional order of magnitude accuracy