Assignment

The time values are denoted as $t(1), t(2), ..t(n), ...$, where $t(1) = 0,\ t(2) = h,\ t(3) = 2*h, ..t(n) = (n-1)*h$; and where h is a small time increment, whose value is at your disposal. The corresponding angle θ values are denoted as $f(1), f(2), ...f(n), ...$, where $f(1) = \theta_0$, and where the other values of $f(n)$ have to be calculated using the algorithm indicated below.

1. Using for $f(1) = \theta_0$, and $f\ '(1) = 0$, from the equations above show that

$$f(2) = f(1) - \sin(f(1))h^2/2 + \mathcal{O}(h^4) \tag{1}$$

$$f(3) = f(2) - 2[\sin^2(\theta_0/2) - \sin^2(f(2)/2)]^{1/2}h - \sin(f(2))h^2/2 + \mathcal{O}(h^3), \tag{2}$$

and in general, for $n \geq 2$ show that

$$f(n+1) = f(n) \pm 2[\sin^2(\theta_0/2) - \sin^2(f(n)/2)]^{1/2}h - \sin(f(n))h^2/2 + \mathcal{O}(h^3), \tag{3}$$

2. A different algorithm, with a better truncation error of $\mathcal{O}(h^4)$ and which does not suffer from the $\pm$ complication is

$$f(3) = -\ f(1) + 2f(2) - \sin(f(2))h^2 + \mathcal{O}(h^4) \tag{4}$$

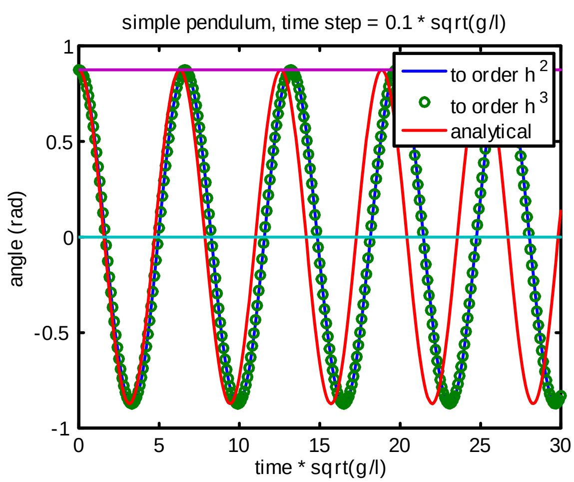
and in general, for $n \geq 2$

$$f(n+1) = -\ f(n-1) + 2f(n) - \sin(f(n))h^2 + \mathcal{O}(h^4) \tag{5}$$

Show the validity of this algorithm. Hint: compare the Taylor series for $f(n+1)$ and $f(n-1)$

3. Using Eq. (1) and making a for-loop using Eq. (3), numerically find the values of $f(n)$, $n = 1, 2, 3, ...n_{\max}$ and plot $\theta = f(n)$ as a function of the time over a time interval of three pendulum periods. Use for $\theta_0 = 50$ *degrees* (transform into radians). In the same graph, also plot the expression $\theta = \theta_0 \cos(\ \bar{t})$, which is based on the approximation $\sin(\theta) = \theta$. Try various values of h, .

4. Make a for loop using algorithm (5). Compare the difference of that result with the result of part 3. A good method is to obtain the absolute value of that difference and plot it on a logy plot as a function of θ. The command for absolute value is *abs*


simple pendulum, time step = 0.1 * sqrt(g/l)
to order h 2
to order h 3
analytical
angle (rad)
time * sqrt(g/l)
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30