Project 1b: Numerical truncation errors for calculating Derivatives

Assignment:

a) Using the approximation

$$f_n{}' = \frac{f_{n+1} - f_n}{(\Delta\theta)} + \mathcal{O}(\Delta\theta) \tag{1}$$

for the derivative $df(\theta)/d\theta = f\,'(\theta)$ of a function $f(\theta)$, where in this case $f(\theta) = \sin(\theta)$, examine the accuracy of Eq.(1), i.e., compare $f_n{}' = \frac{f_{n+1}-f_n}{(\Delta\theta)}$ with the exact result $d(\sin\theta)/d\theta = \cos(\theta)$, and check whether the error is proportional to the first power of $\Delta\theta$ (which it should be according to Eq. (1)). In this exercise the quantitiy to be varies is $\Delta\theta$, and the value of θ could be held fixed.

b) Using the approximation

$$f_n' = \frac{f_{n+1} - f_{n-1}}{2(\Delta\theta)} + \mathcal{O}((\Delta\theta)^2) \tag{2}$$

for the derivative $df(\theta)/d\theta = f\,'(\theta)$ of a function $f(\theta)$, where in this case $f_n = \sin(\theta_n) = s_n$, compare the accuracy of Eq.(2), i.e., compare $f_n{}' = \frac{f_{n+1}-f_{n-1}}{2(\Delta\theta)}$ with the exact result $d(\sin\theta)/d\theta = \cos(\theta)$, and check whether the error is proportional to the second power of $\Delta\theta$ (which it should be according to Eq. (2)).

c) By using Taylor series, show the validity of the truncation error estimate given in Eqs. (1) and (2).

Note: the write up of this project should follow the guidelines given for project 1a.