ICTP

SAIFR

March 18 to April 13, 2015
George Rawitscher
Univ. of Connecticut
george.rawitscher@uconn.edu

Numerical Spectral Methods for Solving Differential or Integral Equations

Internet Information

Heitor, 3393 7845 Terceiro andar
Jacqueline internal 7911

Files:
1 . Homework assignments
2. Lectures, written out
3. Solutions

-For your students get the files,
just **login as visitor and password: visitor**.

Open a browser and put on bar navigator: **http://172.16.108.186/georger**

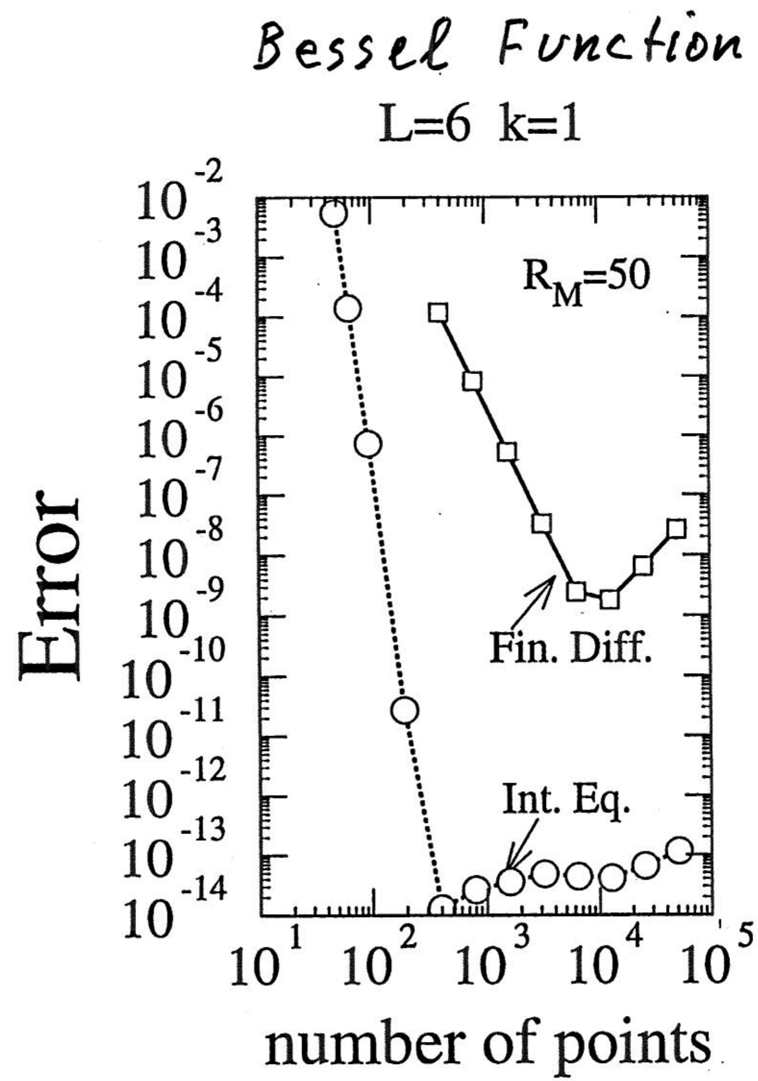
Obs.: It will open only for IFT. They cannot open the page outside of the IFT.

Lecture #1 : errors

- Computer numbers are discrete, and have round-off errors
- The functions are calculated at discrete mesh points
- The algorithms have their own approximations
  called truncation errors

Challenge: For each problem, find the best algorithm that leads to good accuracy, is fast, and has low complexity

Lecture 1



Gonzales
Eisert
Koltracht
Neumann
Rawitscher

J. Comp. Phys.
134 134 (1997)

IWOTA

The slide on the error of solving the Bessel equation
demonstrates the advantage of a spectral method
over a finite difference method (Numerov)

The horizontal axis shows the number of mesh-points "n"
in the radial interval [0,50]. The vertical axis shows the error.

For the Numerov method three points are taken at one time,
and the larger "n", the smaller the distance "h" between points

For the spectral method **all** n points are taken at one time.
The value of "n" is also the number of expansion polynomials

Please note that for the spectral method the error decreases very rapidly
(exponentially) once a certain smallest number of polynomials used is
exceeded. **The error behavior of the two methods is very different.**

Lecture #1 : errors, Homework

Homework #1a : Taylor series truncation

$$\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \mathcal{O}(\theta^7)$$

Homework 1 b: Taylor series truncation

$$\mathbf{f}_n^{\,\prime} = \frac{\mathbf{f}_{n+1} - \mathbf{f}_n}{(\Delta\theta)} + \mathbf{O}(\boldsymbol{\Delta\theta}) \qquad f(\theta) = \sin(\theta)$$

3:Homework 1 c: iteration

$$x_n = \frac{10}{3} x_{n-1} - x_{n-2};$$

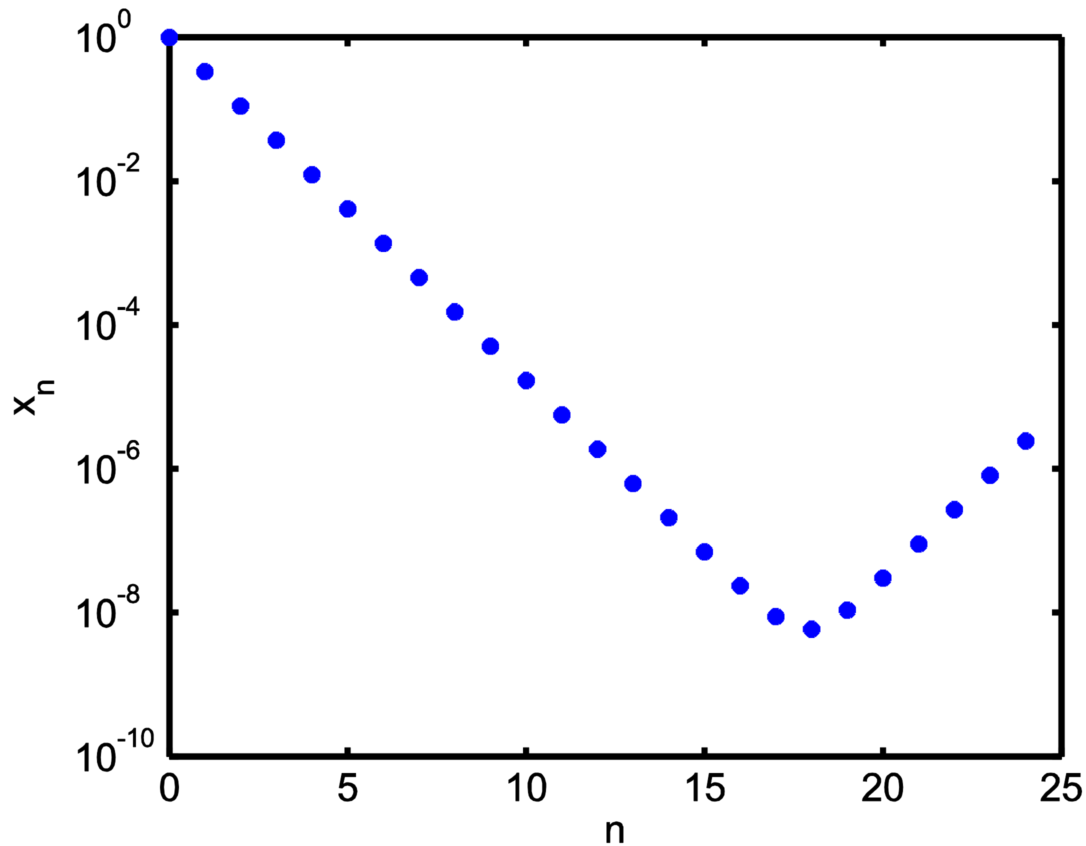
Lecture 1, Iteration example

$$x_n = \frac{10}{3} x_{n-1} - x_{n-2};$$ *with x_0=1, and x_1=1/3.*

expect

$$x_n = (1/3)^n$$

find



Lecture 1, Iteration example

$$x_n = \frac{10}{3} x_{n-1} - x_{n-2};$$ *with x_0=1, and x_1=1/3.*

expect $x_n = (1/3)^n$

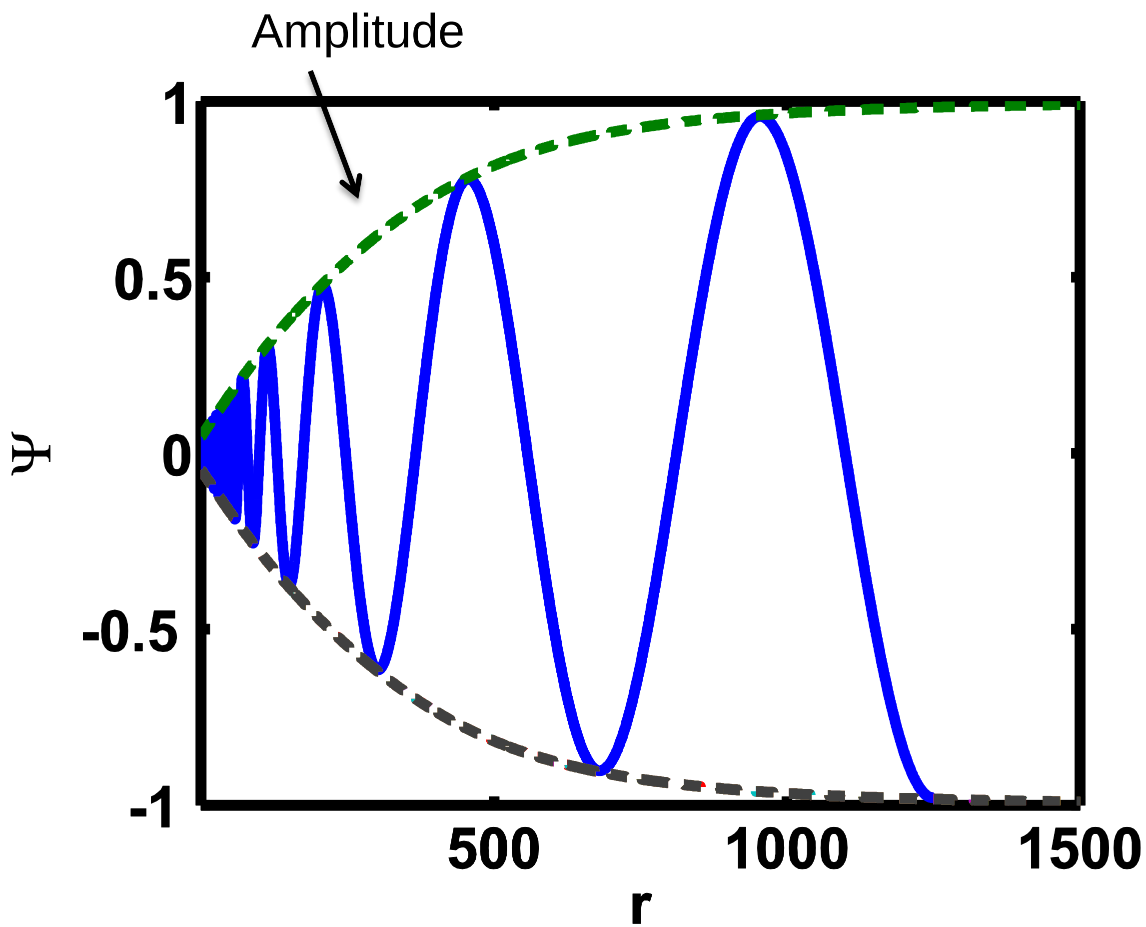
The plot on the previous slide shows that beyond n = 17 the value of x(n) increases again, i.e., (1/3)^n is no longer valid

This demonstrates that an error has crept in.
The nature of this error is a combination of round-off errors and the fact that the iteration equation has two solutions (1/3)^n and (3)^n as will be discussed during the homework session

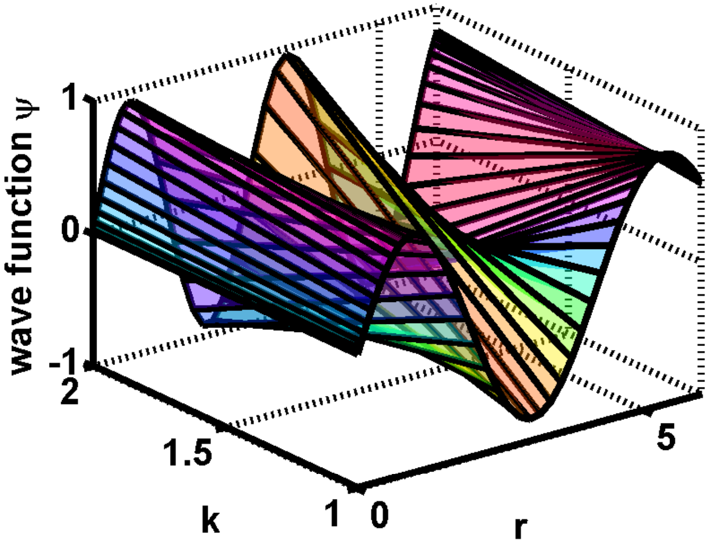
Lecture 10

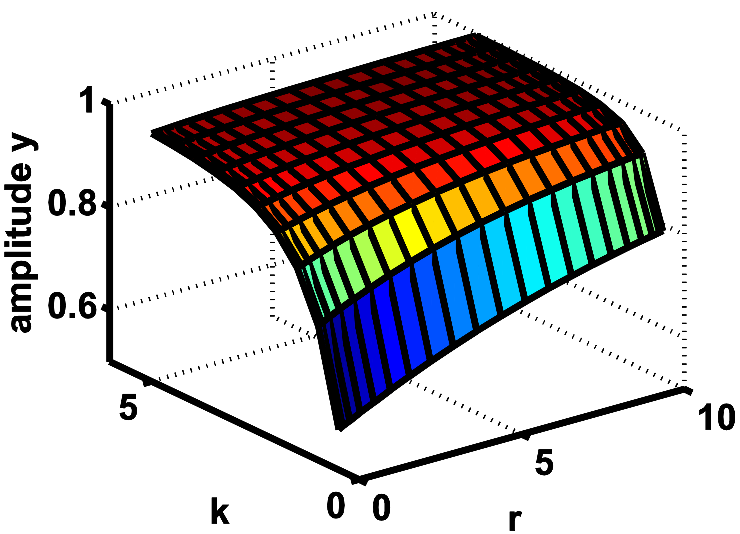
The Phase-Amplitude Method

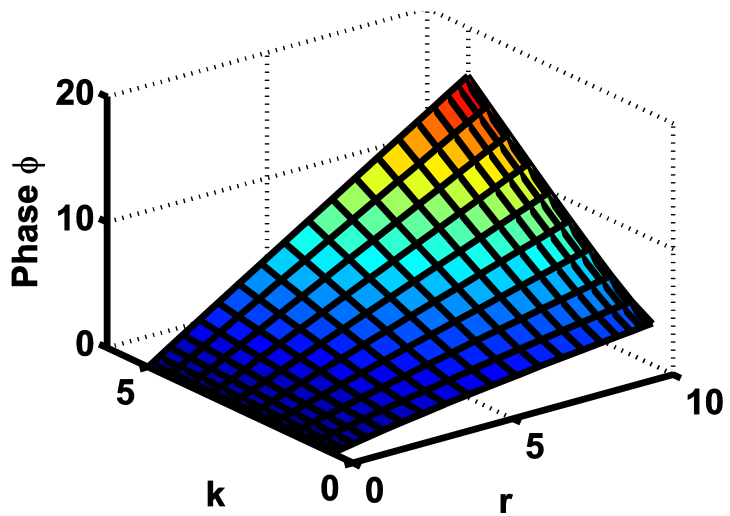
for extending a wave function to large distances



Lecture 10, Phase-Amplitude representation of a wave fctn







The amplitude function and phase function are continuous functions of distance.

The amplitude function obeys a non-linear differential eq.. In a future lecture we will show how this equation can be solved efficiently with a spectral method.