Numerical solution of the motion of the simple pendulum as a function of time.

Preliminary Math:
Consider a function of time $f(t)$ known at the equispaced times $t_{n-1} = t(n) = (n-1) * h$, where h is the time interval between two consecutive times, and n is an integer $1, 2, ...n_{\max}$. The time sequence thus is $0, h, 2h, ...(n_{\max} - 1)h$. The values of f at these times are denoted as $f_{n-1} = f(t_{n-1}) = f(n)$. For our case $f_{n-1} = theta(n)$, where $theta(n)$ is the angle in radians at the time $t_{n-1} = t(n)$.

According to Taylor's expansion we have

$$f_{n+1} = f_n + f\,'_n\, h + f\,''_n\, h^2/2 + f_n{}'''\, h^3/6 + \mathcal{O}(h^4) \tag{1}$$

where the primes denote the number of derivatives. For example $f_n{}''' = d^{\,3} f/dt^3$ evaluated at $t = t_n$.

Based on the equation above, it can be shown that

$$f\,'_n = \frac{f_{n+1} - f_{n-1}}{2h} + \mathcal{O}(h^2) \tag{2}$$

$$f\,''_n = \frac{f_{n+1} + f_{n-1} - 2\, f_n}{h^2} + \mathcal{O}(h^2). \tag{3}$$

Physics
Consider a mass m attached to the lower end of a string of length ℓ, and denote by θ (in radians) the angle which the string makes with the vertical direction. The equation of motion of the angle θ is

$$\frac{d^2}{dt^2}\theta = -\frac{g}{\ell}\sin(\theta). \tag{4}$$

Note that $\sqrt{\ell/g}$ has the dimension of time, (g is the acceleration of gravity in m/s^2) and $\sqrt{\ell/g}$ is the natural unit of time for this problem. By going to the dimensionless variable $\bar{t} = t/\sqrt{\ell/g}$, and by denoting $f\,' = df/d\bar{t}$, the equation of motion (4) becomes

$$\theta\,'' = -\sin(\theta) \tag{5}$$

where $\theta'' = d^2\theta/d\,\bar{t}^2$. Further, from conservation of energy one can show (can you?)

$$\theta\,' = \pm\, 2[\sin^2(\theta_0/2) - \sin^2(\theta/2)]^{1/2} \tag{6}$$

where θ_0 is the initial angle. This is also the maximum angle, since the pendulum is released from rest at this angle. Every half cycle the sign in Eq. (6) changes.