Proj. 1 a

Truncation errors

Consider the Taylor series expansion for the sine function

$$\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \mathcal{O}(\theta^7) \tag{1}$$

Here θ is the angle in radians, $n!$ means $2 \times 3 \times 4 \times \cdots \times \ n$ (example $3! = 2 \times 3 = 6$), and $\mathcal{O}(\theta^7)$ means that the suppressed part of the expansion has a magnitude proportional to the seventh power of θ. This quantity $\mathcal{O}(\theta^n)$ is also called the "truncation" error of the algorithm. Obviously, the smaller θ, the smaller is $\mathcal{O}(\theta^7)/\sin(\theta)$, and hence, the smaller is the relative value of the truncation error. The purpose of this exercise is to have a numerical idea of the magnitude of the truncation error, and see if it conforms to the $\mathcal{O}(\theta^n)$ expectation.

Assignment:

In order to carry out this assignment, consider a sequence of equidistant angular points

$$\theta_{n-1} = theta(n) = (n-1) \times (\Delta\theta),\ n = 1, 2, 3, ...N, \tag{2}$$

where $(\Delta\theta)$ is a small angular increment of your choice, and N is a upper limit, also of your choice. You will have created $theta(n)$ which is a row vector with N entries. Define a related sequence of sine functions

$$s_{n-1} = s(n) = \sin(theta(n)),\ \ n = 1, 2, 3, ...N. \tag{3}$$

a) Compare $\sin(\theta)$ with θ, by numerically calculating the relative error for a range of values of θ. Make a nice graph of the absolute value of the truncation error as a function of θ. note: in MATLAB, θ has to be in radians.

b) The same as part a), but compare $\sin(\theta)$ with $\theta - \frac{\theta^3}{3!}$

c) The same as part a), but compare $\sin(\theta)$ with $\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!}$

Note: On the same graph you could show three lines, one for each of a, b, and c. Make use of the semilogy plot. The report, to be handed in at or before the deadline, should contain a short description of the purpose of the project, show your matlab program, show graphs with your results, and make a conclusion describing what you have learned by doing the project.