THE ASTROPHYSICS OF EMRIS

Capture of compact objects by SMBHs

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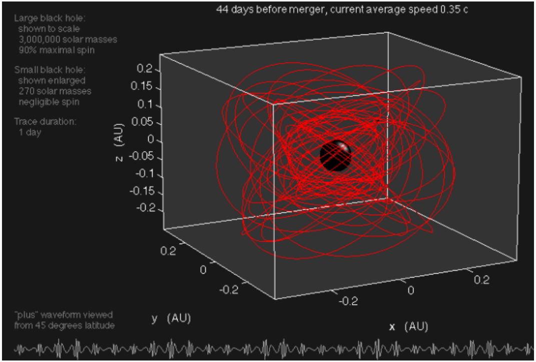
Max-Planck Institute for Gravitational Physics (Albert Einstein Institute)

CAPTURE OF COMPACT OBJECTS



[Figure Steve Drasco]

- Stellar mass object spiraling into $10^4 - 10^6\, M_\bullet$
- This range of masses corresponds to relaxed nuclei (!)
- Only compact objects (extended stars disrupted early)
- With eLISA stellar BH $z \gtrsim 0.7$

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- Bridge between astrophysics and GR

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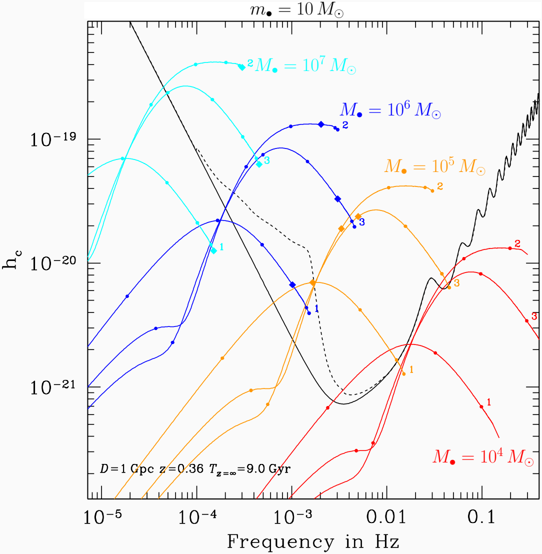
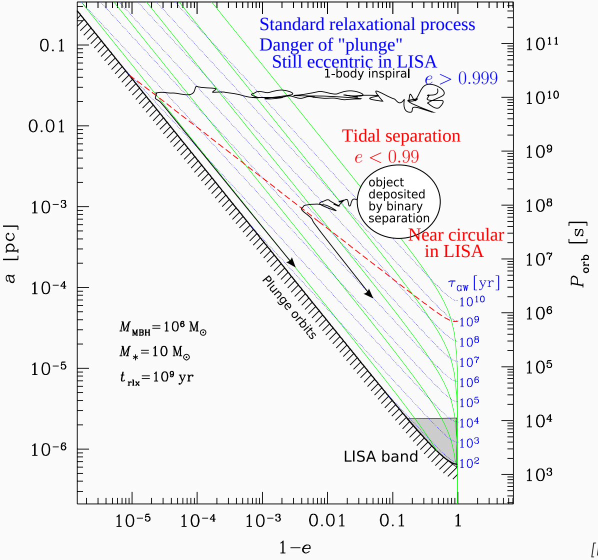
- In this talk we'll see some of these difficulties, and how we've made progress: Microphysics around SMBHs

RANGE OF MASSES



[PAS, LRR 2015]

DICHOTOMIZING AN EMRI



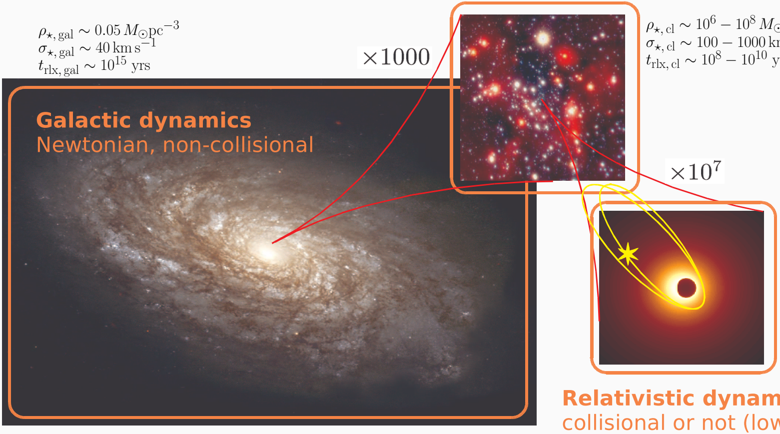
[PAS, LRR 2015]

DISTRIBUTION OF STARS AROUND SMBHS

Cluster dynamics
Newtonian, collisional

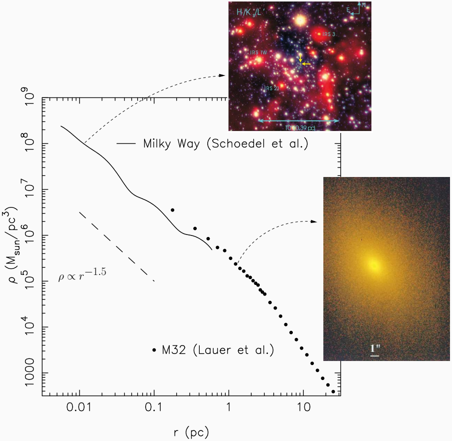
$\rho_{\star,\,\mathrm{gal}} \sim 0.05\,M_\odot \mathrm{pc}^{-3}$
$\sigma_{\star,\,\mathrm{gal}} \sim 40\,\mathrm{km\,s}^{-1}$
$t_{\mathrm{rlx,\,gal}} \sim 10^{15}$ yrs

$\rho_{\star,\,\mathrm{cl}} \sim 10^6 - 10^8\,M_\odot\,\mathrm{pc}^{-3}$
$\sigma_{\star,\,\mathrm{cl}} \sim 100 - 1000\,\mathrm{km\,s}^{-1}$
$t_{\mathrm{rlx,\,cl}} \sim 10^8 - 10^{10}$ yrs



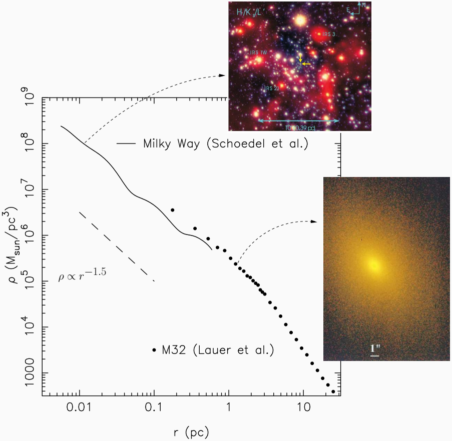
Relativistic dynamics
collisional or not (low *N*)
$\mathcal{M}_\bullet \sim 10^6 - 10^9\,M_\odot$
$R_{\mathrm{Schw}} = 10^{-7} - 10^{-4}$ pc

[PAS, LRR 2015]



[Adapted from Merritt 2006]

- ▷ 0th question: How many stars? How do they distribute?



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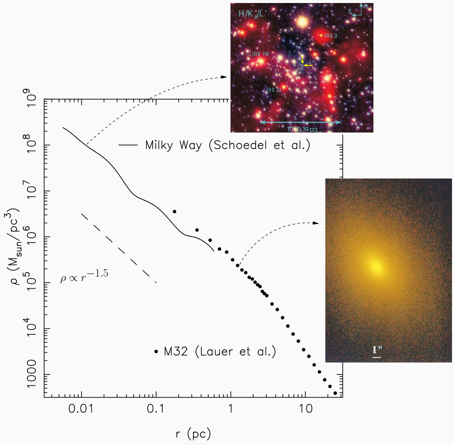
HOW MANY A YEAR?



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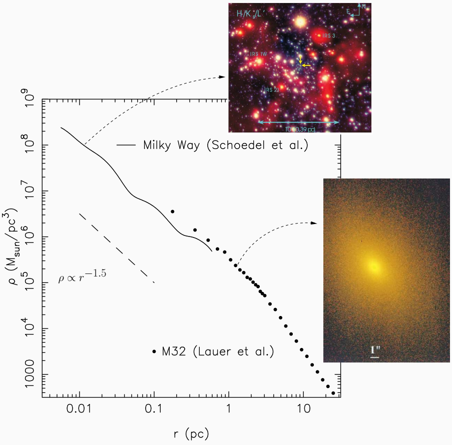

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  - Considerable amount of modelling: Are these profiles a coincidence?

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MASS SEGREGATION

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- There exists a Steady state with net inward flux of stars and energy *[Peebles 1972]*
- If single-mass: quasi-steady solution takes power-law form (isotropic DF) $f(E) \sim E^p$, $\rho(r) \sim r^{-\gamma}$, with $\gamma = 3/2 + p$
- Confirmed later with a detailed kinematic treatment for single-mass *[Bahcall & Wolf 1976]*: $\gamma = 7/4$ and $p = \gamma - 3/2 = 1/4$

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- Initial Mass Functions $\in [0.1, \sim 120] M_\odot$ to first order by two (well-separated) mass scales: $\mathcal{O}(1 M_\odot)$ (Main Sequence, White Dwarfs, Neutron Stars) and $\mathcal{O}(10 M_\odot)$ (Stellar Black Holes)

STRONG MASS SEGREGATION

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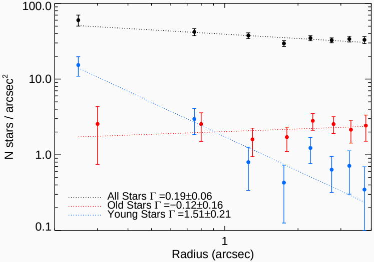
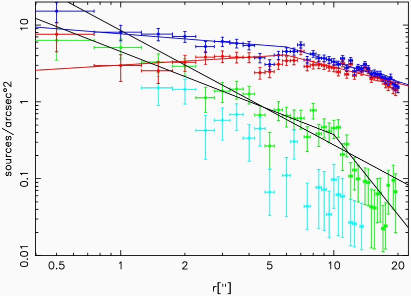
- Two branches for the solution: A "weak" (unrealistic) branch and a "strong" branch

  [Hopman & Alexander 2009, Preto & Amaro-Seoane 2010, Amaro-Seoane & Preto 2011]

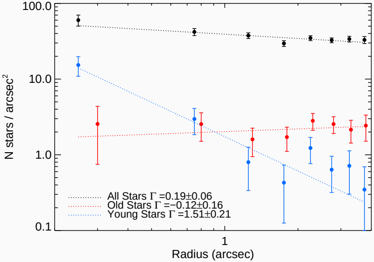
CUSPS IN DISTRESS

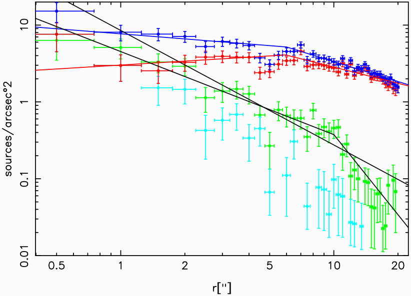




- ▷ Deficit of old stars based on number counts of spectroscopically identified, old stars in sub-parsec SgrA* (down to magnitude $K = 15.5$)

[Do et al. 2009, Buchholz et al 2009]





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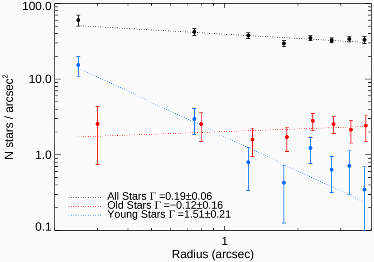

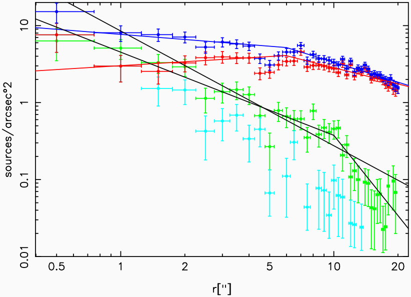

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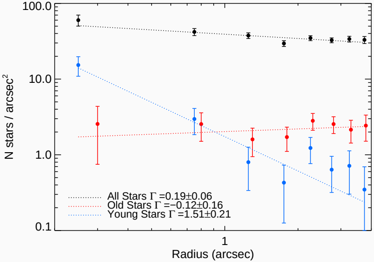

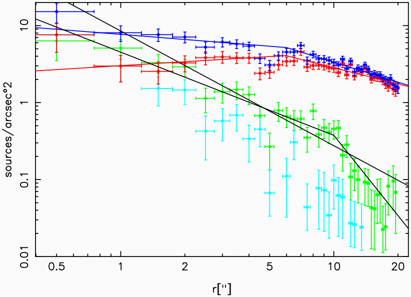

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- ▷ Possibility of a core with $\rho_\star$ decreasing, $\gamma < 0$
- ▷ Observers only see essentially late-type giants: Detectable stars are still a small fraction

1. Infalling clusters carve a hole – But need a steady inflow of one at roughly every 10^7 years

   [Baumgardt et al 2006, Portegies Zwart et al 2006]

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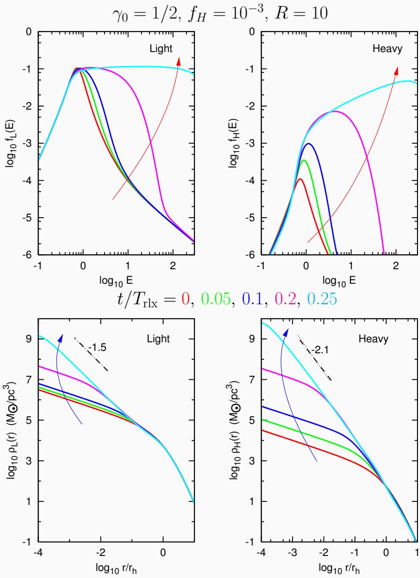
- ▷ We have now the correct, more efficient, solution of mass segregation



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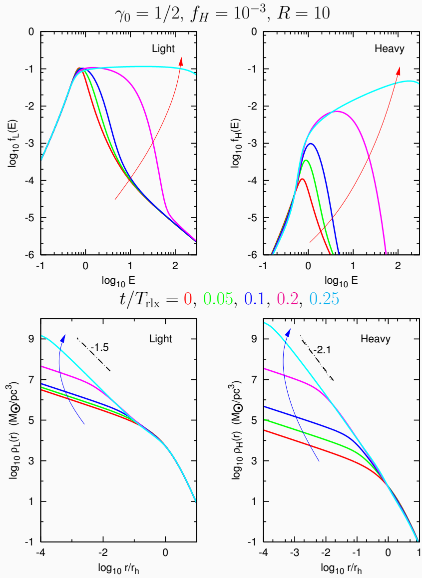

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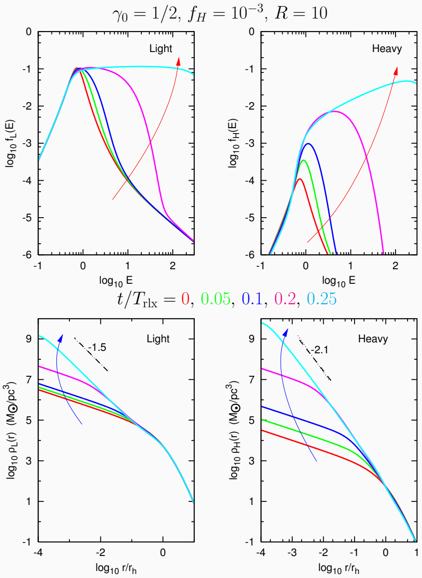

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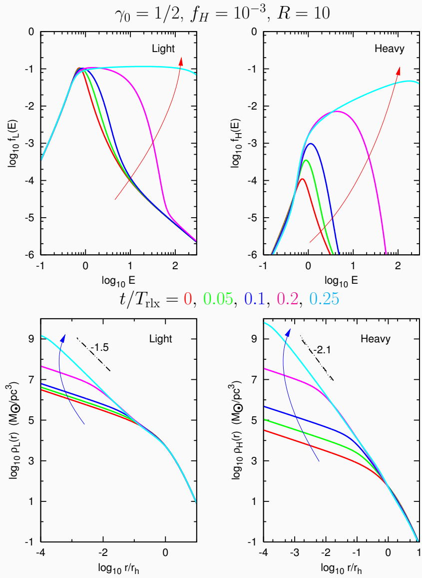
ISOCORE ... REGROWTH



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- Our results confirmed later

[Gualandris & Merritt 2011]

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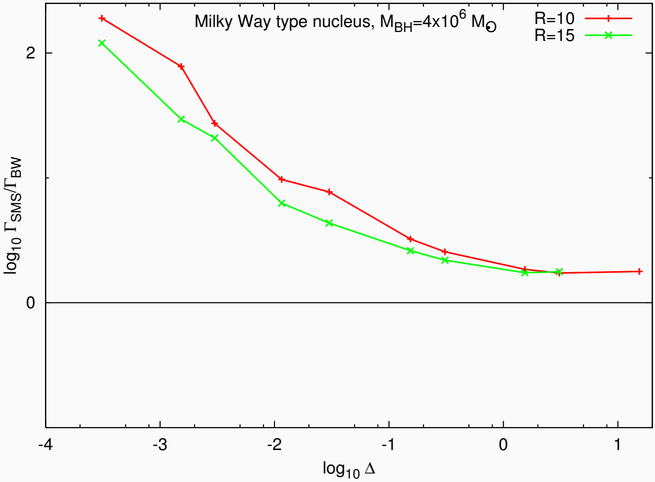
- The Milky Way nucleus is *not* necessarily the prototype of the nucleus from which e-LISA detections will be more frequent
- We still expect that a substantial fraction of EMRI events will originate from segregated stellar cusps, in particular with our new solution of mass segregation


Milky Way type nucleus, $M_{BH}=4x10^6$ $M_\odot$
R=10
R=15
2
0
$\log_{10} \Gamma_{SMS}/\Gamma_{BW}$
-4
-3
-2
-1
0
1
$\log_{10} \Delta$

DISGUISED CAPTURES

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THE ROLE OF THE SPIN

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- Plunges are more frequent than “adiabatic” EMRIs A common result to all event rate estimates
- What if these stars did not plunge? We’d have extremely eccentric sources, and event rates orders of magnitude larger

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NOT AN ACADEMIC EXERCISE

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- Take initial orbital parameters, evolve them and find the parameters corresponding to a plunge or unstable orbit
- (p, e, i), calculate constants of motion (E, L_z, C), then the average flux of these "constants", i.e. the average time evolution $(\dot{E}, \dot{L}_z, \dot{C})$

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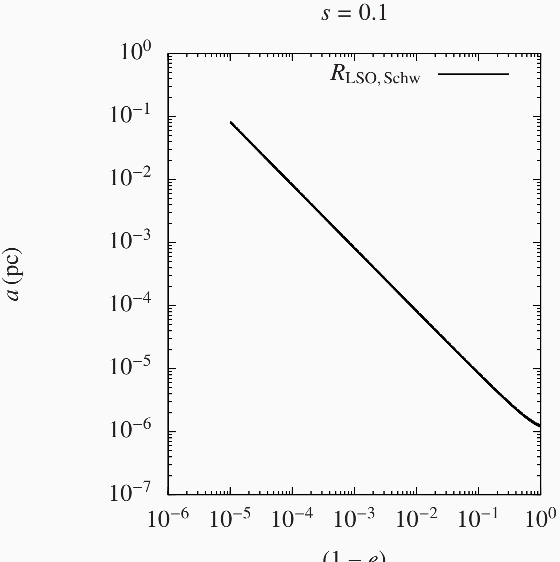
- Calculate time to go from apo to periapsis and back (radial periode) and thus the change in (E, L_z, C) and so the new constants of motion, therefore: $(p_{\rm new}, e_{\rm new}, i_{\rm new})$

SOME RESULTS DEPENDING ON THE SPIN

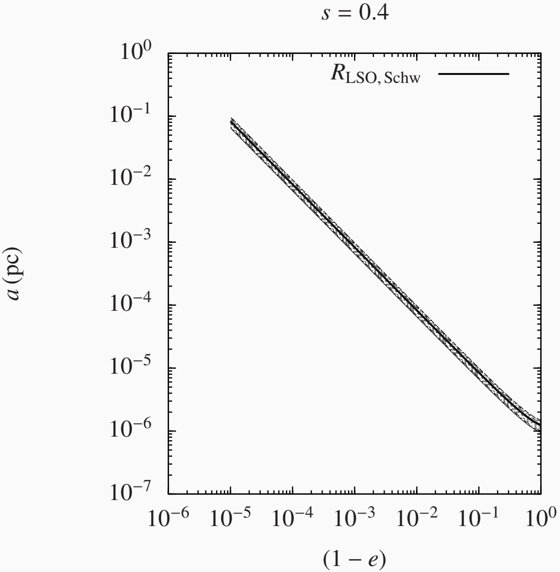
$\mathcal{M}_\bullet$	Spin (a/M)	a_0 (pc)	e_i	i (rad)	$\tau_{\rm mrg}$ (yrs)	$\tau_{\rm e-LISA}$	Peri (e-LISA)
3E6	0.990	8.6182E-4	0.9990	0.6	2.6755E3	6.8409E2	432503
1E6	0.990	2.8727E-4	0.9990	0.6	2.9743E2	1.1915E2	146074
1E6	0.500	2.8727E-4	0.9990	0.6	2.4714E2	9.8328E1	97715
3E6	0.500	8.6182E-4	0.9990	0.6	2.2229E3	5.6105E2	288372
1E6	0.900	2.3939E-4	0.9990	0.2	1.5328E2	6.8038E1	90555
3E6	0.900	7.1818E-4	0.9990	0.2	1.3785E3	3.9237E2	268423
3E6	0.900	7.1786E-3	0.9999	0.2	4.6101E3	3.9131E2	267802
3E6	0.900	5.7429E-3	0.9999	0.2	2.0757E3	1.9956E2	149747
3E6	0.900	5.0250E-3	0.9999	0.2	1.3164E3	1.3607E2	106563
1E6	0.900	1.6750E-3	0.9999	0.2	1.4843E2	2.3449E1	35889
1E6	0.900	1.4357E-3	0.9999	0.2	9.1260E1	1.5533E1	24593
1E6	0.900	1.4357E-3	0.9999	0.1	9.2711E1	1.5769E1	25038
3E6	0.900	4.3071E-3	0.9999	0.1	8.1857E2	9.1641E1	74371
5E6	0.900	7.1786E-3	0.9999	0.1	2.2652E3	2.0548E2	122993
1E6	0.900	1.4357E-3	0.9999	0.1	1.8272E2	3.1556E1	50075
4E6	0.700	6.7000E-3	0.9999	0	1.8937E3	1.7207E2	96284
4E6	0.998	6.7000E-3	0.9999	0	2.6993E3	2.4753E2	170494
4E6	0.998	9.5714E-3	0.9999	0	8.7952E3	6.6162E2	395248
4E6	0.998	7.6571E-3	0.9999	0	4.1097E3	3.5062E2	230973
4E6	0.998	6.7000E-3	0.9999	0	2.6993E3	2.4753E2	170494
4E6	0.998	5.7429E-3	0.9999	0	1.7598E3	1.7468E2	123868
4E6	0.998	5.7429E-3	0.9999	0.3	1.6574E3	1.6506E2	117974

Note: Prograde orbits, $m_\bullet = 10\,M_\odot$

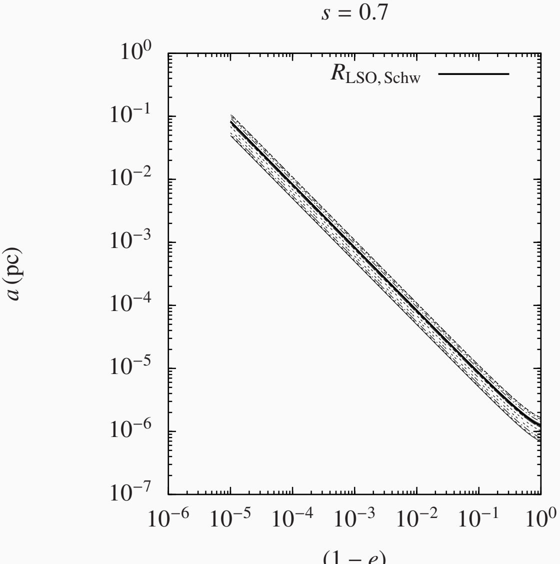
A FAMILY OF SEPARATRICES: $s = 0.1$



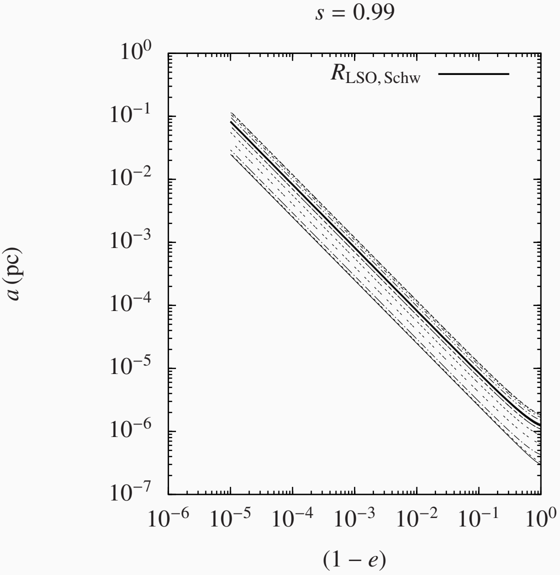
A FAMILY OF SEPARATRICES: $s = 0.4$

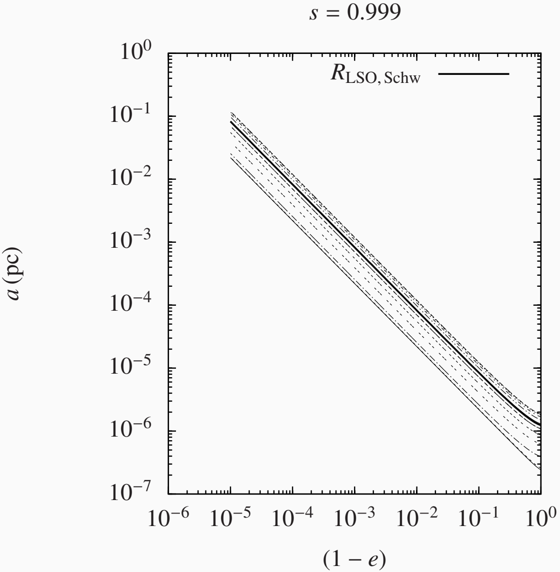


A FAMILY OF SEPARATRICES: $s = 0.7$



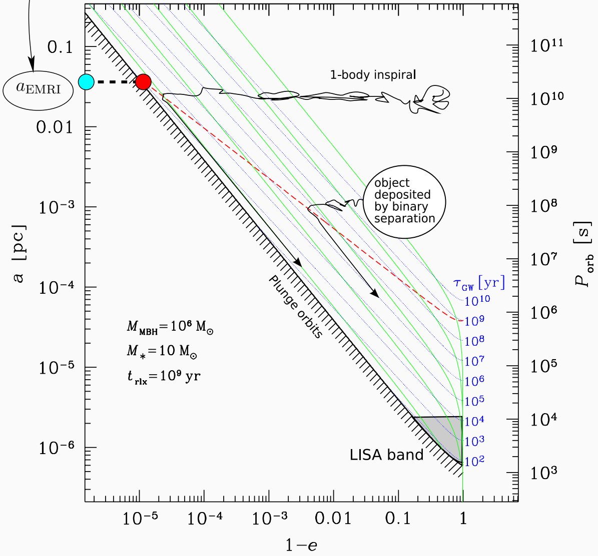
A FAMILY OF SEPARATRICES: $s = 0.99$




s = 0.999
R_LSO, Schw
a (pc)
(1 − e)
10^0
10^−1
10^−2
10^−3
10^−4
10^−5
10^−6
10^−7

IMPACT OF THE SPIN ON THE RATES?

$$\dot{N}_{\rm EMRI} \simeq \int_0^{a_{\rm EMRI}} \frac{dN_\bullet(a)}{\ln\left(\theta_{\rm LC}^{-2}\right) t_r(a)}$$



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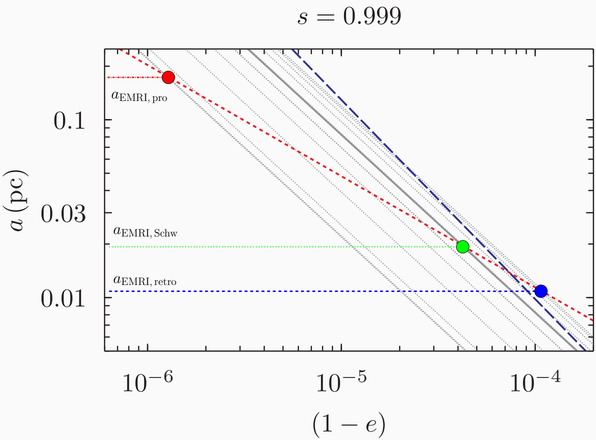

KERR VS. SCHWARZSCHILD

$$a_{\mathrm{EMRI}}^{\mathrm{Kerr}} = a_{\mathrm{EMRI}}^{\mathrm{Schw}} \times \mathcal{W}^{\frac{-5}{6-2\gamma}}(\iota, s)$$

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- When taking into account spinning MBHs EMRI rates are boosted

THE BUTTERFLY EFFECT

- ▷ What is the radius that contains *one* star?

WHERE'S THE ONE-STAR RADIUS?

- What is the radius that contains *one* star?
- Let us use the MW: half of the mass within the orbit of S-2 has $M_{\mathrm{encl}}/2 = \eta \times M_\bullet$ with $\eta \leq 0.040$ *[Gillessen et al 2009]*

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- ▷ Then

$$M(R) = \int_0^R 4\pi r^2 \rho(r)\, dr \propto \int_0^R r^{-\gamma+2} dr \propto R^{3-\gamma}$$

$$N(R) \simeq 8.6 \times 10^4 \left(\frac{R}{6 \times 10^{-4}\ \mathrm{pc}} \right)^{3-\gamma}$$

$$R_1 \simeq 6 \times 10^{-4}\ \mathrm{pc} \times \left(\frac{1}{8.6 \times 10^4} \right)^{\frac{1}{3-\gamma}}$$

▷ $R_1 \simeq 3 \times 10^{-7}$ pc for $\gamma = 1.5$

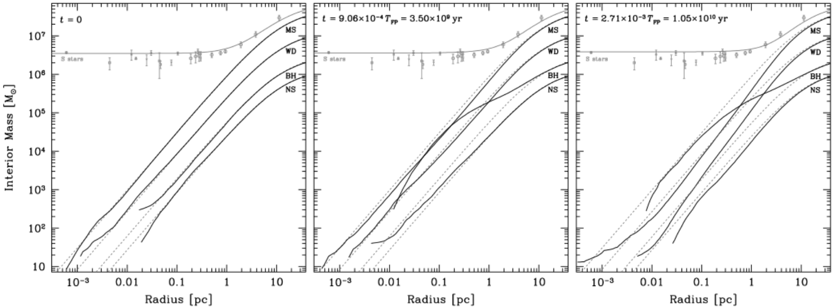

FIG. 11.—Evolution of the profiles of enclosed mass for GN25. The solid lines are the results of the MC simulation. For reference, the dashed lines show $\eta = 1.5$ profiles adjusted on the total mass and half-mass radius of each component. The top thin line is the total mass, including the central MBH; it is compared to the observational constrains for the MW center (see Fig. 3). [*See the electronic edition of the Journal for a color version of this figure.*]

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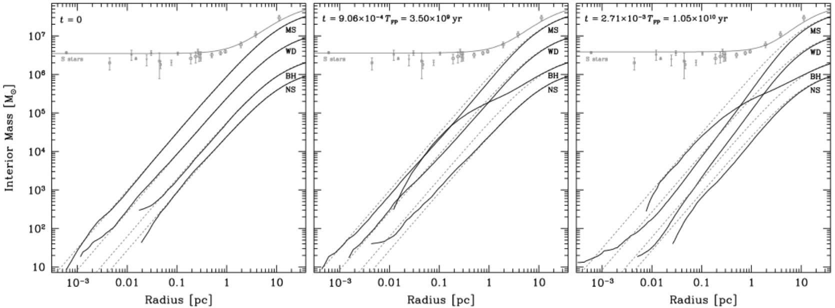

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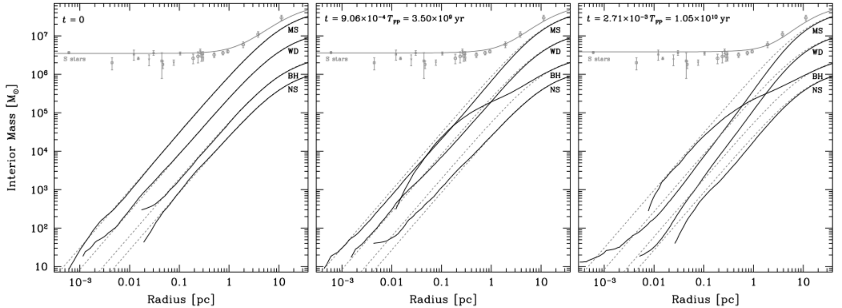
[Freitag, Amaro-Seoane & Kalogera 2006]



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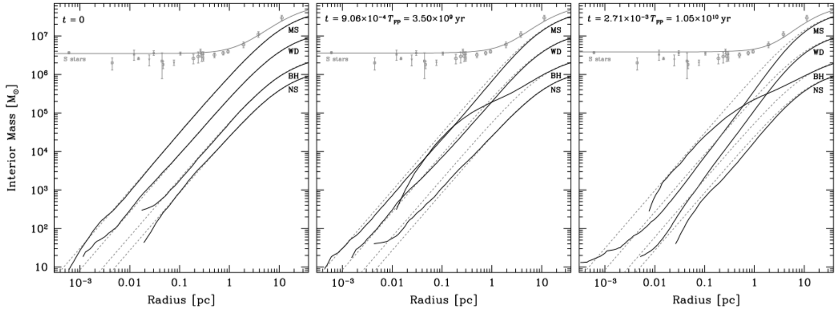
- Watch out: I am cheating



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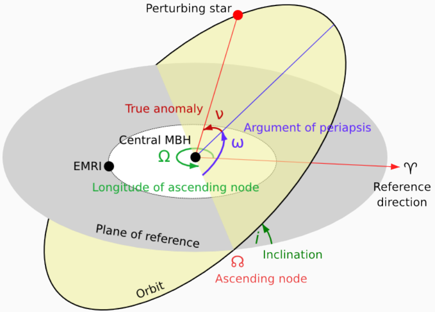

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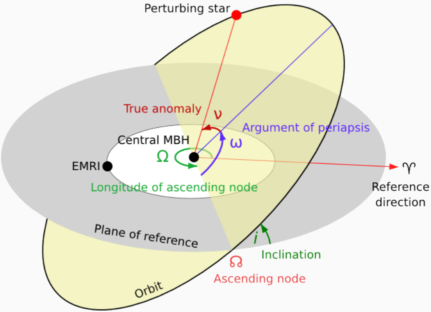

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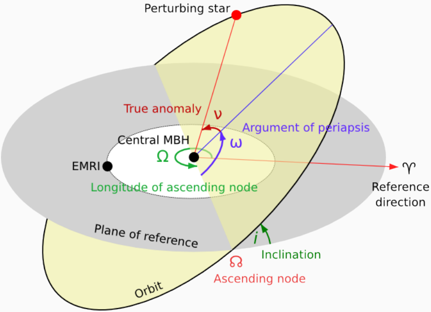

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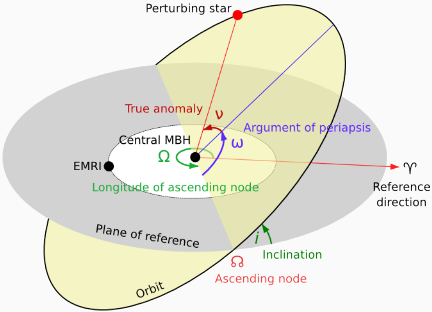

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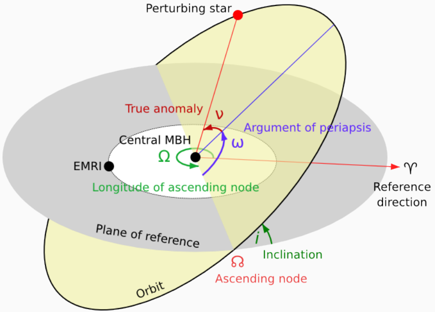
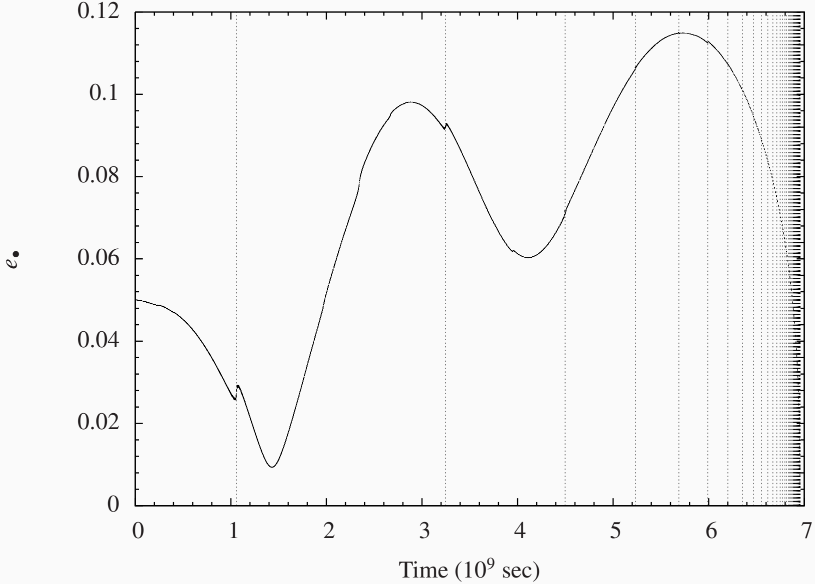
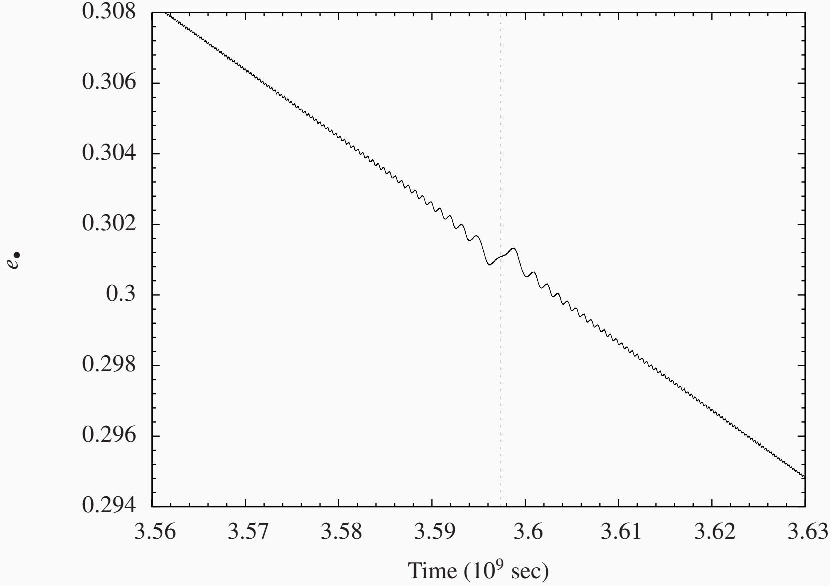
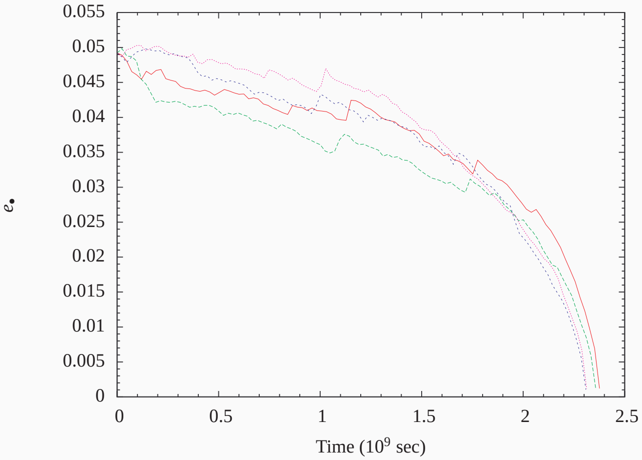
- Evolution of the eccentricity when taking **energy loss, i.e. 2.5 PN** into account?



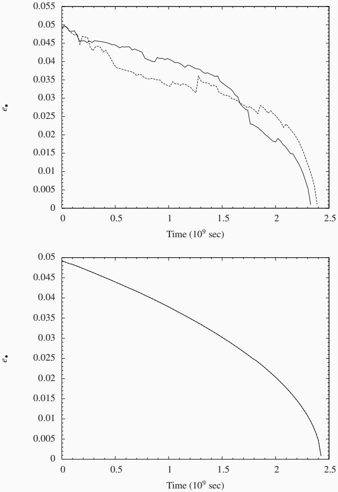

0.12
0.1
0.08
0.06
0.04
0.02
0
0
1
2
3
4
5
6
7
Time (10^9 sec)
$e_\bullet$


0.308
0.306
0.304
0.302
0.3
0.298
0.296
0.294
$e_\bullet$
3.56
3.57
3.58
3.59
3.6
3.61
3.62
3.63
Time (10^9 sec)



Red $i_\star = 30°$, green $i_\star = 30.001°$, blue fiducial plus a *ten billionth* of a degree, $i_\star = 30.0000000001°$ and magenta plus a *ten trillionth* of a degree, $i_\star = 30.0000000000001°$

NO, IT'S NOT A BUG



$a_\star = 4 \times 10^{-6}$ pc, 6×10^{-6} pc, 9×10^{-6} pc and 4.07243×10^{-5} pc

CHAOS?

- If at $T = 0$ two orbits differ by some small separation $\xi(0)$ in phase space...

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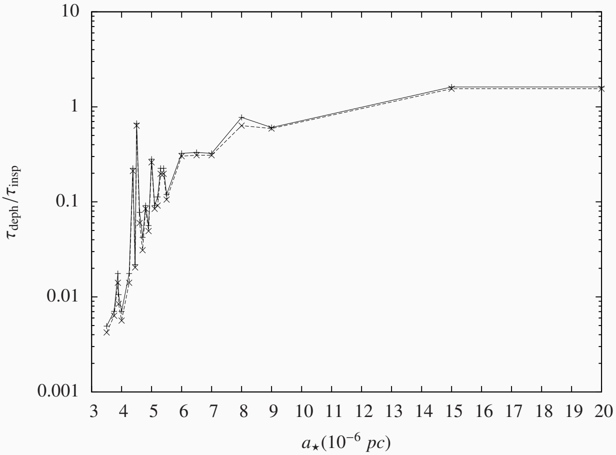
- How to characterise the chaos?



Start with a fiducial case and another one a *bit* different in phase space. Let them evolve. Calculate time for the "distance" to be $2 \times a_\bullet$ and divide it by the isolated inspiral time : Characteristic time

CONCLUSIONS

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- GR must not always be wrong: It could be an innocent star nearby

Questions?