Effective Field Theory for Cold Atoms IV

H.-W. Hammer

Institut für Kernphysik, TU Darmstadt and Extreme Matter Institute EMMI

School on Effective Field Theory across Length Scales, ICTP-SAIFR, Sao Paulo, Brazil, 2016

Agenda

1. EFT for Ultracold Atoms I: Effective Field Theories & Universality
2. EFT for Ultracold Atoms II: Cold Atoms & the Unitary Limit
3. EFT for Ultracold Atoms III: Weak Coupling at Finite Density
4. EFT for Ultracold Atoms IV: Few-Body Systems in the Unitary Limit
5. Beyond Ultracold Atoms: Halo Nuclei and Hadronic Molecules

Literature

G.P. Lepage, TASI Lectures 1989, arXiv:hep-ph/0506330

D.B. Kaplan, arXiv:nucl-th/9506035

E. Braaten, HWH, Phys. Rep. **428** (2006) 259 [arXiv:cond-mat/0410417]

Physics Near the Unitary Limit

TECHNISCHE UNIVERSITÄT DARMSTADT

- Unitary limit: $a \to \infty$, $R \to 0 \quad \Longrightarrow \quad \mathcal{T}_2(k,k) \propto i/k$
- Use as starting point for effective field theory description
  - Large scattering length: $|a| \gg R \sim r_e, R_{vdW}, \ldots$
  - Natural expansion parameter: $R/|a|$, kR,...
  - Universal dimer with energy $B_2 = \hbar^2/(ma^2)$ $\qquad (a > 0)$

    size $\langle r^2 \rangle^{1/2} \sim a \quad \Rightarrow \quad$ halo state
  - Reproduce tail of the wave function: $\psi(r) = \dfrac{e^{-r/a}}{r}$

- Nonperturb. resummation in EFT (van Kolck, Kaplan, Savage, Wise, 1998)

$$\mathcal{T}_2(k,k) \propto \frac{-a}{1+ika}\left[1 + \frac{r_e a k^2/2}{1+ika} + \ldots\right], \qquad 1/a \sim k \ll 1/r_e$$

$\Longrightarrow$ universal properties (and perturbative corrections)

Physical Systems with Large a

TECHNISCHE UNIVERSITÄT DARMSTADT

- Natural expansion parameter: $R/|a|$, kR,... $(R \sim r_e, R_{vdW}, ...)$
- Nuclear Physics: S-wave NN scattering, halo nuclei,...
  - ${}^1S_0$, ${}^3S_1$: $|a| \gg r_e \sim 1/m_\pi \longrightarrow B_d \approx 2.2$ MeV
  - neutron matter
  - halo nuclei
- Atomic Physics:
  - ^{4}He atoms: $a \approx 104$ Å $\gg r_e \approx 7$ Å $\sim R_{vdW}$, $\longrightarrow B_d \approx 100$ neV
  - ultracold atoms near Feshbach resonance $\Rightarrow$ variable a
- Particle Physics
  - Is the $X(3872)$ a $|D^0\bar{D}^{0*} + \bar{D}^0 D^{0*}\rangle$ molecule? $(J^{PC} = 1^{++})$

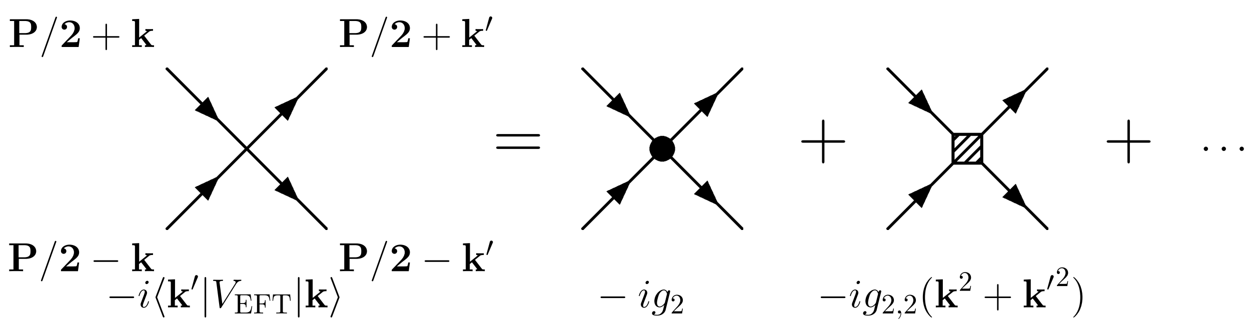
$$E_X = m_D + m_{D^*} - m_X = (0.1 \pm 0.2) \text{ MeV}$$

Two-Body System

- Large scattering length: $Q \sim k \sim 1/a \ll 1/R$
- Effective Lagrangian

$$\mathcal{L} = \psi^\dagger \left(i\partial_t + \frac{1}{2m}\vec{\nabla}^2 \right) \psi - \frac{g_2}{4}(\psi^\dagger\psi)^2 - \frac{g_{2,2}}{4}\left[\vec{\nabla}(\psi^\dagger\psi)\right]^2 + \ldots$$

- Vertices:

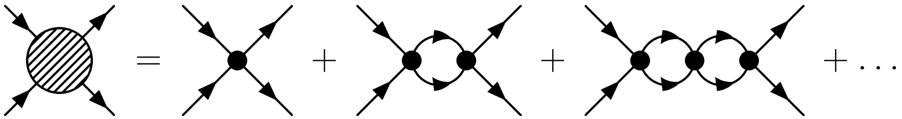


- Scaling: $g_2 \sim 1/(mQ), \quad g_{2,2} \sim R/(mQ^2), \quad g_{2,2j} \sim R^j/(mQ^{j+1})$
- Scaling of diagrams with Q: Q^ν

$$\nu = 5L - \underbrace{2I}_{2(L+V-1)} + \sum_j (2j-(j+1))V_{2j} = 3L+2+\sum_j (j-3)V_{2j} \geq -1$$

Two-Body System

- All diagrams with only g_2 vertices scale as $1/Q$
- Scattering Amplitude:



$$\mathcal{T}_2(E) = -ig_2 - \frac{ig_2^2}{2}\int_0^{\Lambda}\frac{d^3q}{(2\pi)^3}\frac{1}{m\tilde{E}-q^2} + .. = -ig_2 + \frac{ig_2^2}{4\pi^2}\left(\Lambda - \frac{\pi}{2}\sqrt{-m\tilde{E}}\right) + ..$$

$$\equiv \frac{8\pi}{m}\frac{1}{-1/a+\sqrt{-m\tilde{E}}} \qquad \text{matching!}$$

- Definition: $m\tilde{E} \equiv mE + i\epsilon$
- Geometric series $\Rightarrow$ can be summed
- Loop divergent $\Rightarrow$ regulate with momentum cutoff Λ
- Running coupling constant: $g_2(\Lambda) = \dfrac{8\pi}{m}\left(\dfrac{1}{a} - \dfrac{2}{\pi}\Lambda\right)^{-1}$

Two-Body System

TECHNISCHE UNIVERSITÄT DARMSTADT

- How does g_2 change with Λ (resolution scale)
- Renormalization group equation

$$\tilde{g}_2 \equiv \frac{m\Lambda}{4\pi^2} g_2 \implies \Lambda \frac{d}{d\Lambda} \tilde{g}_2 = \tilde{g}_2 (1 + \tilde{g}_2)$$

- Two fixed points:

  $- \ \tilde{g}_2 = 0 \ \leftrightarrow \ a = 0 \quad \implies$ no interaction

  $- \ \tilde{g}_2 = -1 \ \leftrightarrow \ 1/a = 0 \quad \implies$ unitary limit

  scale and conformal invariance

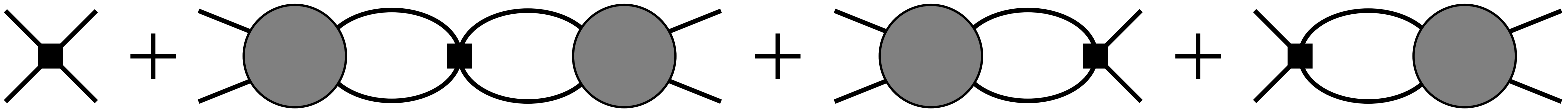
  (Mehen, Stewart, Wise, 2000; Nishida, Son, 2007; ...)

- Interesting many-body physics: BEC/BCS crossover, universal viscosity bound $\Rightarrow$ perfect liquid, ...

  (Kovtun, Son, Starinets, 2005; ...)

Two-Body System: Corrections

- Higher-order derivative terms are perturbative
- Diagrams with $g_{2,2}$ scale like Q^0: first correction



$$\mathcal{T}_2^0(E) = \frac{1}{\left(-1/a + \sqrt{-m\tilde{E}}\right)^2} \left(-2g_{2,2}mE\left(\frac{8\pi}{mg_2}\right)^2 + \text{const.}\right)$$

$$\equiv -\frac{8\pi}{m}\frac{mEr_e/2}{\left(-1/a + \sqrt{-m\tilde{E}}\right)^2} \qquad \text{matching!}$$

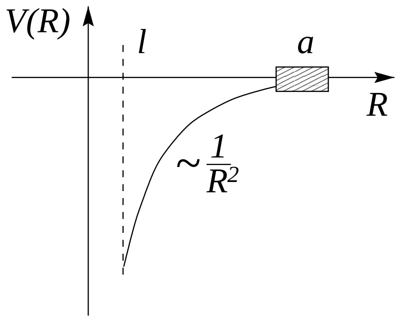
- Constant subtracted by counterterm $g_2^{(2)}$

Broken Scale Invariance

TECHNISCHE UNIVERSITÄT DARMSTADT

- Three-boson system near the unitary limit (Efimov, 1970)
- Hyperspherical coordinates: $R^2 = (r_{12}^2 + r_{13}^2 + r_{23}^2)/3$
- Schrödinger equation simplifies for $|a| \gg R \gg l$:

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial R^2} + \frac{s_0^2 + 1/4}{R^2}\right] f(R) = \underbrace{-\frac{\hbar^2\kappa^2}{m}}_{E} f(R)$$



- Singular Potential: renormalization required
- Boundary condition at small R: breaks scale invariance

  $\Longrightarrow$ scale invariance is anomalous
  $\Longrightarrow$ observables depend on boundary condition and a

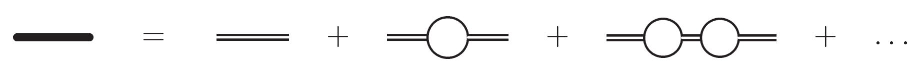
- Universality concept must be extended $\Rightarrow$ 3-body parameter
- Spin-1/2 fermions: repulsive interaction $\Rightarrow$ higher order

Three-Body System

- Use auxilliary field $d = \psi^2 + \ldots$ (cf. Hubbard-Stratonovich, Kaplan, ...)
- Effective Lagrangian

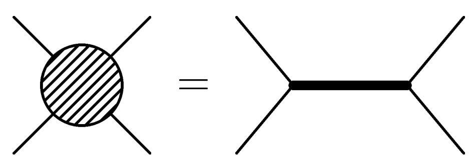
$$\mathcal{L}_d = \psi^\dagger \left(i\partial_t + \frac{1}{2m}\vec{\nabla}^2 \right) \psi + \frac{g_2}{4} d^\dagger d - \frac{g_2}{4}(d^\dagger \psi^2 + (\psi^\dagger)^2 d) - \frac{g_3}{36} d^\dagger d \psi^\dagger \psi + ..$$

- Equation of motion $\longrightarrow$ eliminate $d \longrightarrow \mathcal{L}$
- Full dimeron propagator:



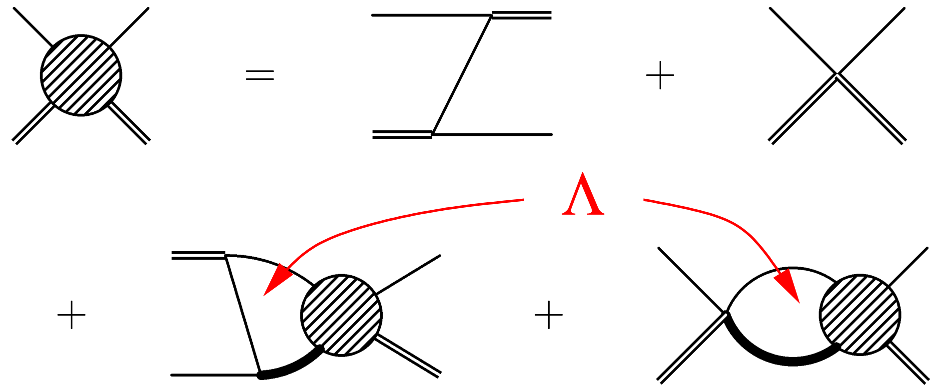
$$iD_d(P_0, P) = \frac{32\pi i}{g_2^2} \left[1/a - \sqrt{-mP_0 + P^2/4 - i\epsilon} \right]^{-1}$$

- Two-body amplitude:



Three-Body System in EFT

- Three-body equation:
  (S-waves)



$$\mathcal{T}_3(k,p) = M(k,p) + \frac{4}{\pi}\int_0^{\Lambda} dq\, \frac{q^2\, M(q,p)}{-\frac{1}{a} + \sqrt{\frac{3}{4}q^2 - mE - i\epsilon}}\, \mathcal{T}_3(k,q)$$

with $\quad M(k,p) \quad = \quad \underbrace{F(k,p)}_{\text{1-atom exchange}} \quad \underbrace{-\frac{g_3}{9g_2^2}}_{H(\Lambda)/\Lambda^2}$

($g_3 = 0$, $\Lambda \to \infty \longrightarrow$ Skorniakov, Ter-Martirosian ’57)

Renormalization

- Require invariance under $\Lambda \to \Lambda' = \Lambda + \delta\Lambda$

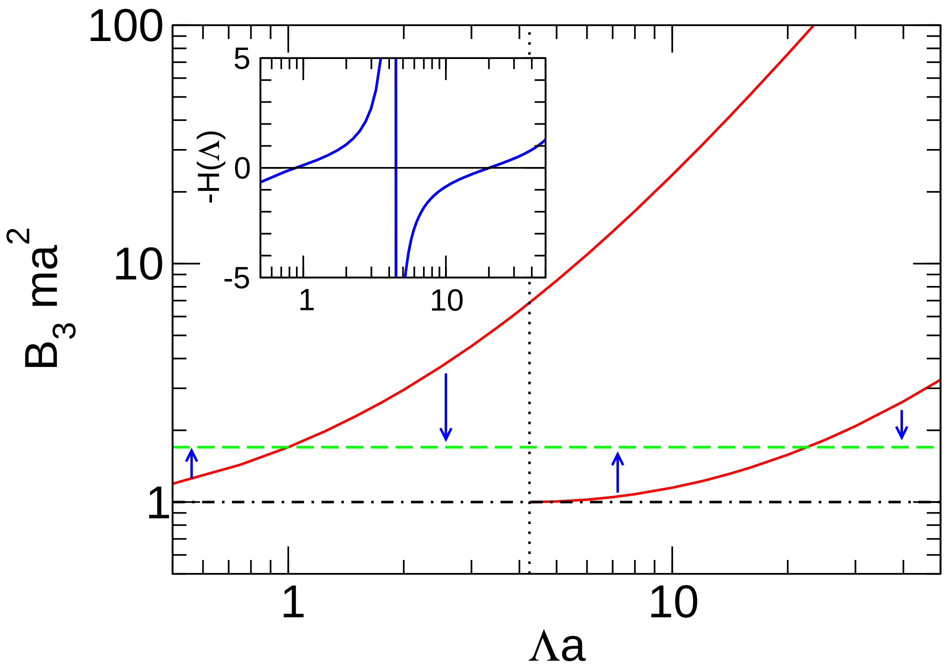
- $H(\Lambda)$ periodic: limit cycle

$$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda (22.7)^n$$

(Bedaque, HWH, van Kolck, 1999)

(Wilson, 1971)

- Anomaly: scale invariance broken to discrete subgroup



$$\boxed{H(\Lambda) \approx \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \qquad s_0 \approx 1.00624}$$

- Matching: $\Lambda_* \longleftarrow$ three-body observable
- Limit cycle $\Longleftrightarrow$ Discrete scale invariance $\Longleftrightarrow$ Efimov physics

Discrete Scale Invariance

TECHNISCHE UNIVERSITÄT DARMSTADT

- Similarity to Matrjoschka doll

  $\longrightarrow$ "Russian Doll Renormalization"

- Other examples
  - $1/r^2$ potential in QM
    (Beane et al., ...)
  - Field theory models
    (Wilson, Glazek, LeClair et al.,...)
  - Turbulence, earthquakes, finance,...
    (cf. Sornette, Phys. Rep. **297** (1998) 239)

- Observable consequences?

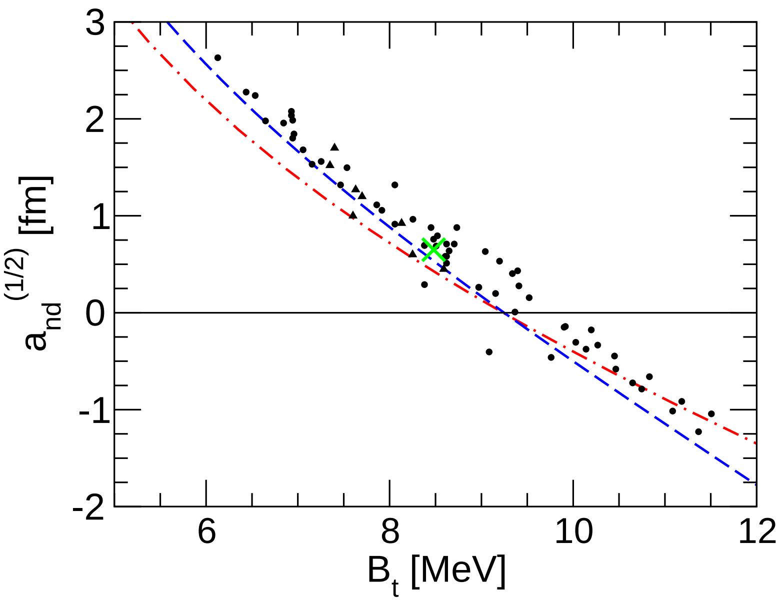
  $\longrightarrow$ Universal correlations (2 parameters at LO), Efimov effect, log-periodic dependence on scattering length,...

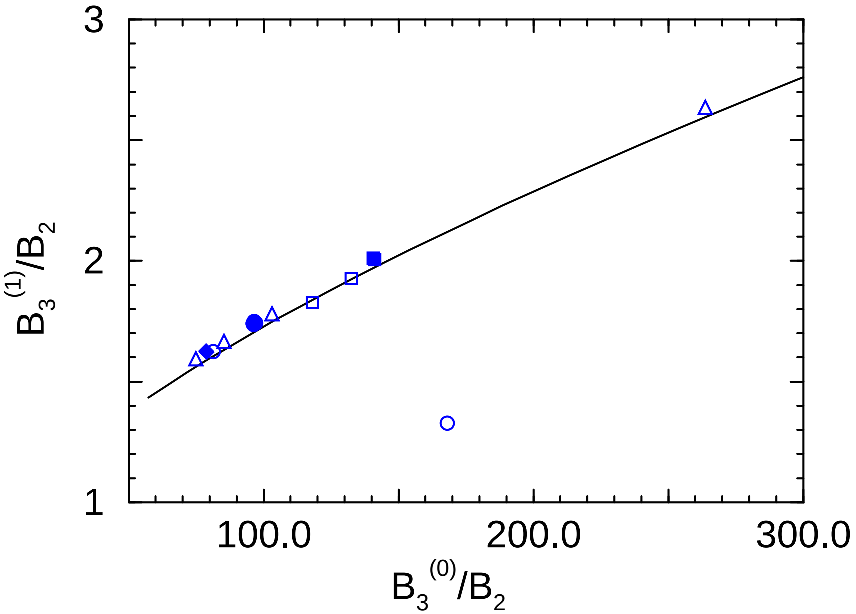
Universality in the 3-Body System

TECHNISCHE UNIVERSITÄT DARMSTADT

- Three-body observables for different potentials are correlated

  $\Longrightarrow$ Phillips line (Phillips, 1968)





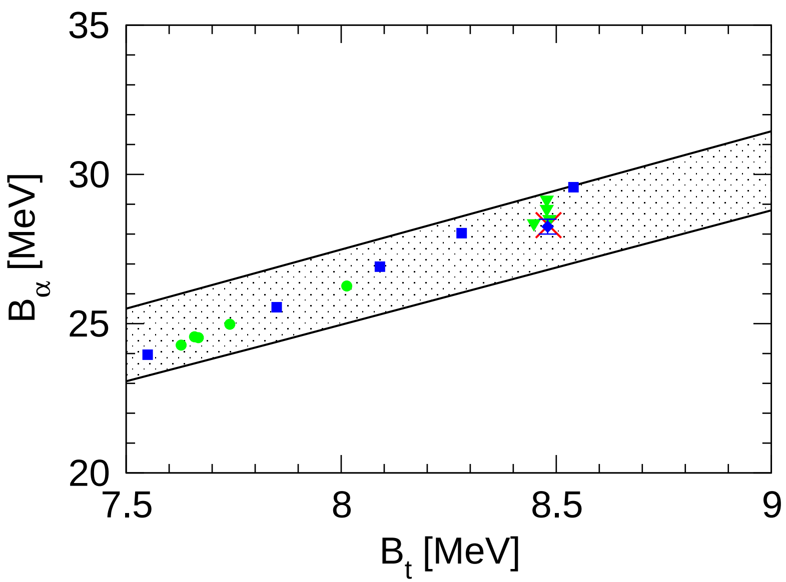
- Different values for Λ_* $\rightarrow$ physics at small distances
- Correlation universal: Nucleons, ^{4}He atoms,...

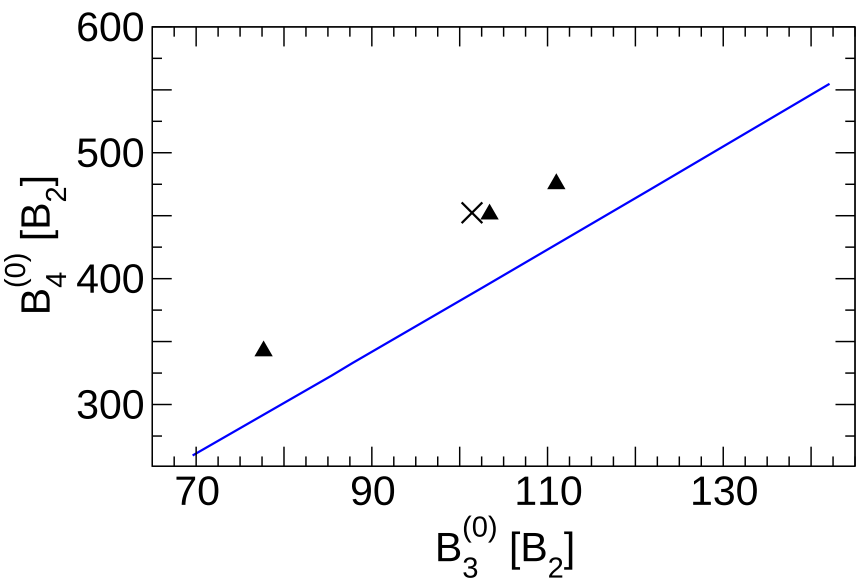
Universality in the 4-Body System

TECHNISCHE UNIVERSITÄT DARMSTADT

- No Four-body force required in LO (Platter, HWH, Meißner, 2004)
- Universal correlations persist in 4-body system

$\Longrightarrow$ Tjon line (Tjon, 1975)

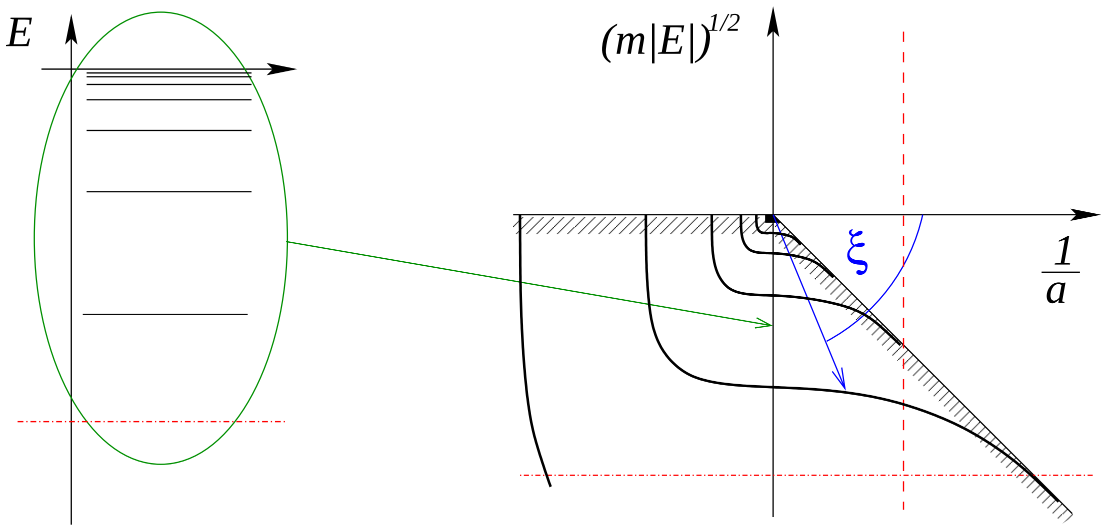




- Variation of Λ_* parametrizes Tjon line
- Correlation universal: Nucleons, ^{4}He atoms,...

Limit Cycle: Efimov Effect

- Universal spectrum of three-body states
  (Efimov, 1970)



  - Discrete scale invariance for fixed angle ξ
  - Geometrical spectrum for $1/a \to 0$

$$B_3^{(n)}/B_3^{(n+1)} \quad \xrightarrow{1/a \to 0} \quad \left(e^{\pi/s_0}\right)^2 = 515.035\ldots$$

- Ultracold atoms $\Longrightarrow$ variable scattering length

Limit Cycle: Scattering Observables

TECHNISCHE UNIVERSITÄT DARMSTADT

- Manifestation of limit cycle in scattering observables:

  $\Rightarrow$ log-periodic dependence on $a\Lambda_*$

$$Observable = f(s_0 \ln(a\Lambda_*)), \qquad f \text{ periodic}$$

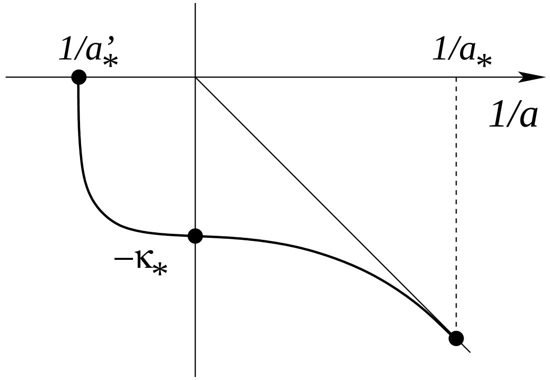
$$e.g. \quad a_{AD} = (1.46 - 2.15 \tan(s_0 \ln(a\Lambda_*) + 0.09))a$$

  $\Rightarrow$ indirect observation of Efimov effect

  $\Rightarrow$ Cold atoms/Feshbach resonances

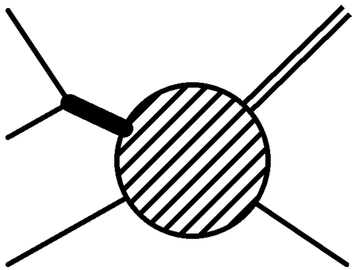
- Alternative definitions of 3-body parameter are possible: κ_*, a_*, a'_*, ...

  (Difference: constant factors...)



$$e.g. \quad s_0 \ln(a\Lambda_*) = s_0 \ln(a\kappa_*) + c \quad \Rightarrow \quad \Lambda_* = \kappa_* e^{c/s_0}$$

Three-Body Recombination



- Recombination into universal dimer:

  3 atoms $\rightarrow$ dimer + atom $\Rightarrow$ loss of atoms

- Recombination constant: $\dot{n}_A = -K_3\, n_A^3$
- Scattering length dependence for $a > 0$:

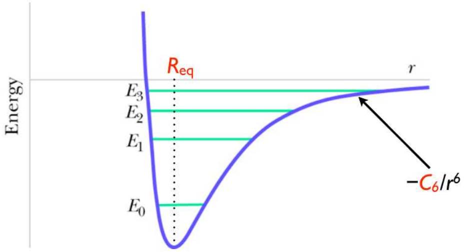
  (Nielsen, Macek, 1999; Esry, Greene, Burke, 1999; Bedaque, Braaten, HWH, 2000)

$$K_3 \approx 201.3 \sin^2\left[s_0 \ln(a\kappa_*) + 1.16\right] \frac{\hbar a^4}{m}\,, \qquad s_0 \approx 1.00624..$$

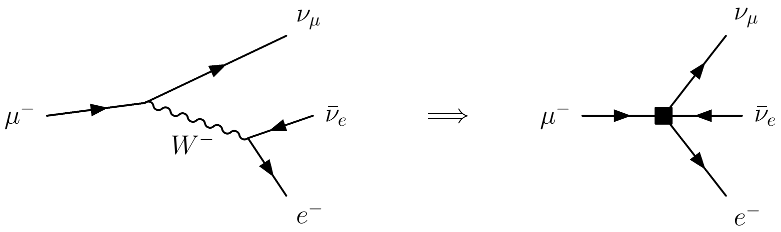
- Modification from deeply-bound dimer states?
- Recombination into deep dimers?
- How to include?

Recombination into Deep Dimers

- What about recombination into deep dimers?



- Large energy release $\Rightarrow$ final states have large momenta P_R $\Rightarrow$ outside of EFT...
- But effects of high-momentum particles on low-momentum particles can be incorporated in EFT
- Example: muon decay



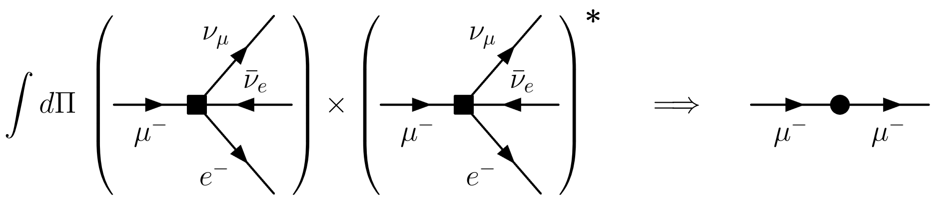
- Moreover: final state particles created in localized region $\sim \hbar/P_R$

$\Rightarrow$ effect on low-momentum particles can be accounted for by local anti-hermitian operators in $\mathcal{L}$

(cf. quarkonium annihilation in NRQCD: Bodwin, Braaten, Lepage, PRD **51** (1995) 1125)

Recombination into Deep Dimers

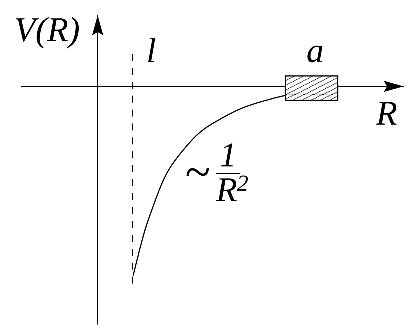
- Use optical theorem $\Rightarrow$ contribution to muon self energy



- Three-body recombination into deep dimers: anti-hermitian three-body interaction

$$\Longrightarrow \qquad s_0 \ln \kappa_* \to s_0 \ln \kappa_* + i\eta_*$$ (Braaten, HWH, 2004)

- Interpretation: $P_{loss} = 1 - \exp(-4\eta_*)$ is probability for atoms to get lost at short distances ($\eta_* \geq 0$)



- Nuclear physics analog: optical potential

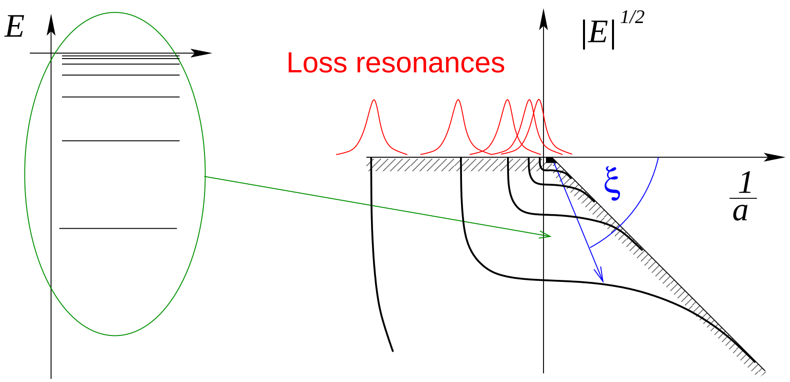
Three-body Recombination

TECHNISCHE UNIVERSITÄT DARMSTADT

- Consequences: modification of recombination into shallow dimer ($a > 0$) & recombination into deep dimers (all a)
- Efimov resonances in three-body recombination for $a < 0$

$$K_3^{\text{deep}} \approx \frac{13770 \sinh(2\eta_*)}{\sin^2[s_0 \ln(a/a'_*)] + \sinh^2 \eta_*} \frac{\hbar a^4}{m},$$

$\Rightarrow$ striking features in loss rates

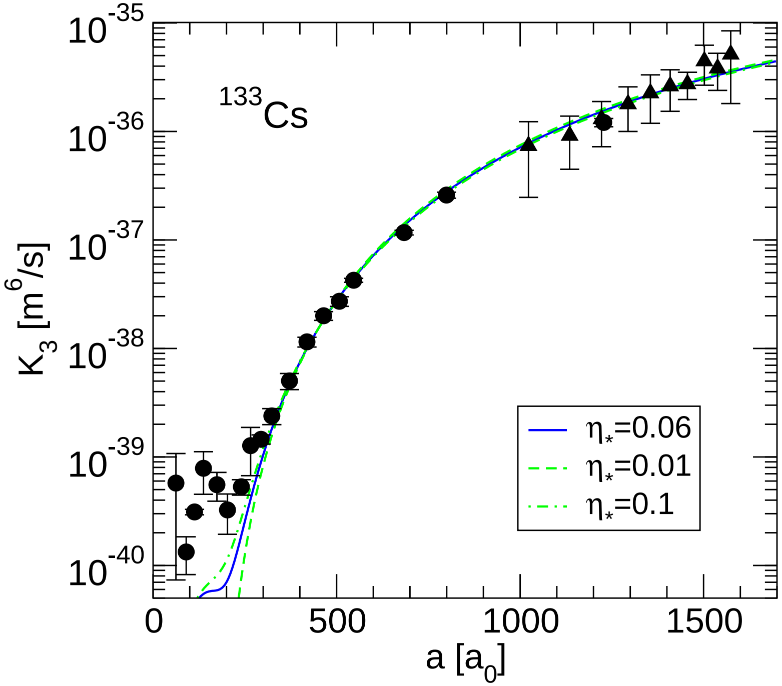


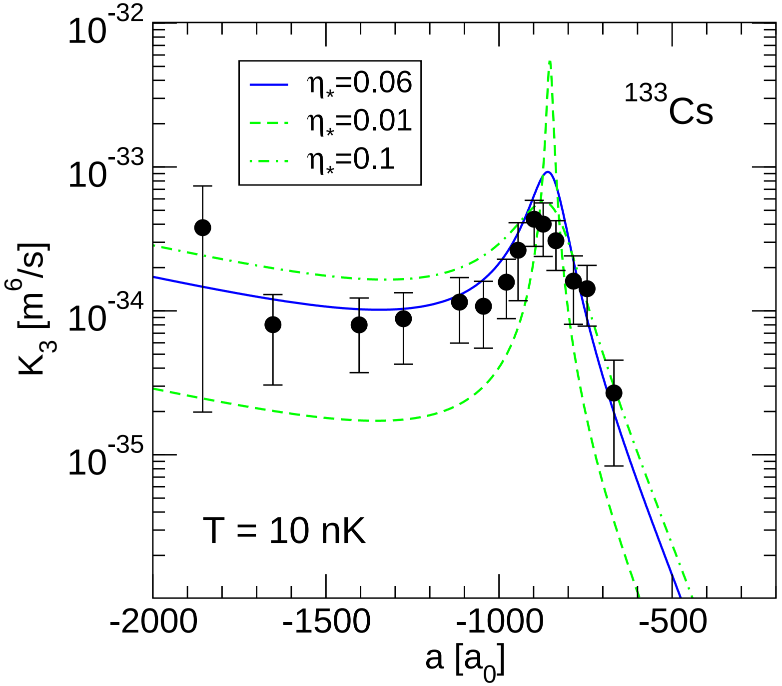
- Efimov resonances in atom-dimer recombination for $a > 0$
- Practical limitation at finite T: resonances will be washed out if

$$E_T \ll k_B T \quad \Leftrightarrow \quad |a| \gg \lambda_{th}$$

Observation of Efimov States

- Experimental evidence for Efimov states in ^{133}Cs
  (Kraemer et al. (Innsbruck), Nature **440** (2006) 315)
  - Identification via signature in recombination rate





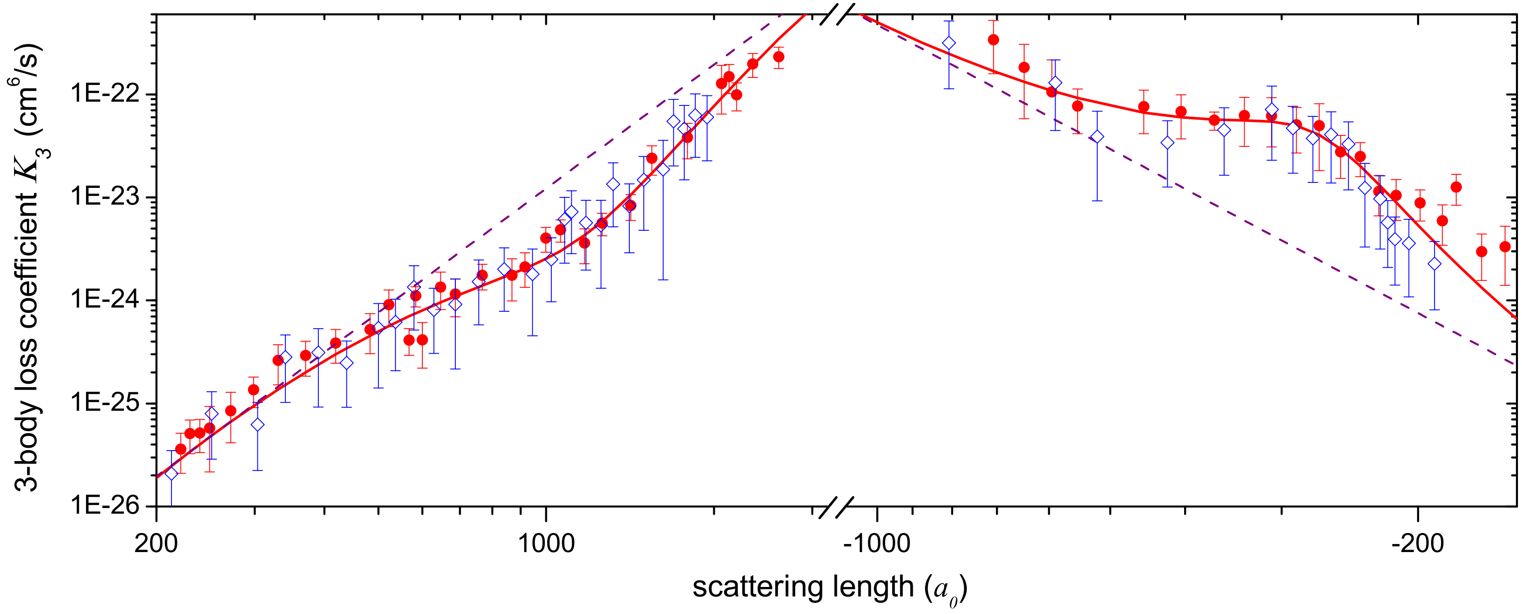
$a > 0$: $\kappa_* = 0.780/a_0$ $a < 0$: $\kappa_* = 0.945/a_0$

- Finite temperature effects for $a > 0$ ($T \geq 200$ nK)

Efimov States in Ultracold Atoms

TECHNISCHE UNIVERSITÄT DARMSTADT

- First experimental evidence in ^{133}Cs (Krämer et al. (Innsbruck), 2006)
  now also ^{6}Li, ^{7}Li, ^{39}K, ^{41}K/^{87}Rb, ^{6}Li/^{133}Cs
- Example: Efimov spectrum in ^{7}Li ($|m_F = 0\rangle$, $|m_F = 1\rangle$)
  (Gross et al. (Bar-Ilan Univ.), Phys. Rev. Lett. **105** (2010) 103203)

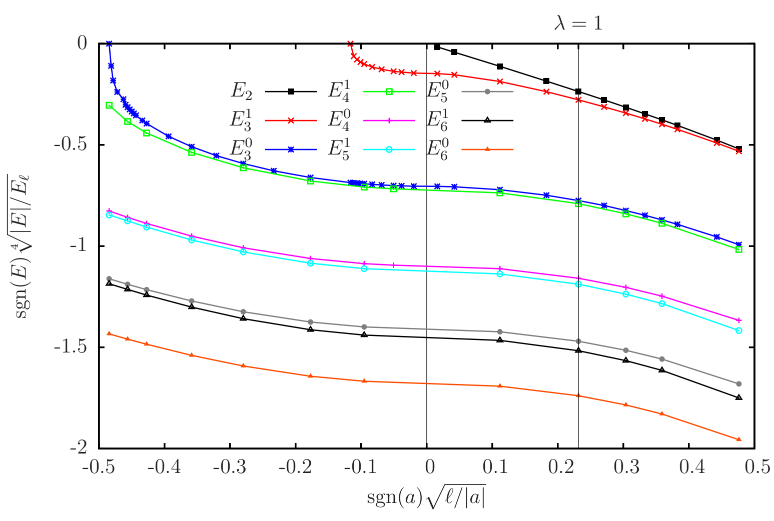


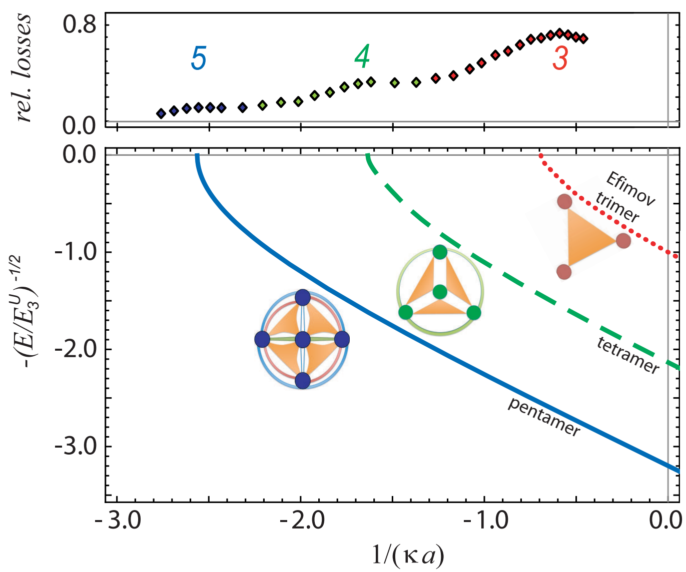
- Van der Waals tail determines position: $a_-/l_{vdW} \approx -10\ (\pm 15\%)$
  (Wang et al., 2012; Naidon et al. 2012, 2014; ...)
  ... but not width parameter η_* ...

Universal Tetramers and Beyond

TECHNISCHE UNIVERSITÄT DARMSTADT

- No four-body force required at LO (Platter, HWH, Meißner, 2004)
- Universal tetramers: $B_4^{(0)} = 4.610(1)\ B_3\,,\quad B_4^{(1)} = 1.00227(1)\ B_3$
  (Platter, HWH, 2004, 2007; von Stecher et al., 2009; Deltuva 2010-2013)
- Two tetramers attached to each trimer
- Universal states up to $N = 16$ calculated
  (von Stecher, 2010, 2011; Gattobigio, Kievsky, Viviani, 2011-2014)
- Observation up to $N = 5$ in Cs losses (Grimm et al. (Innsbruck), 2009, 2013)

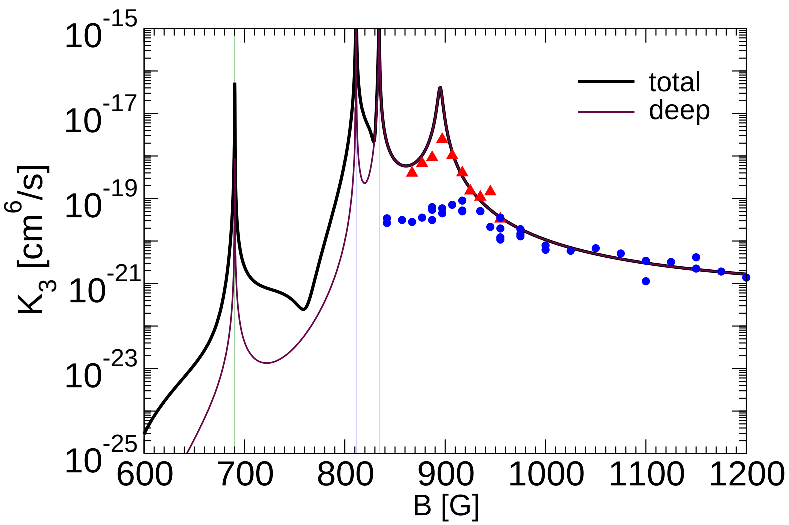


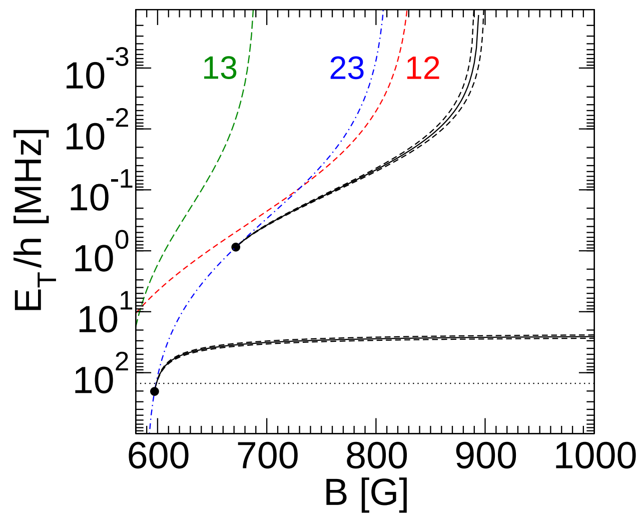


Efimov Physics with Fermions

TECHNISCHE UNIVERSITÄT DARMSTADT

- Recombination and bound state spectrum: ^{6}Li atoms





Williams et al. (Penn State), Phys. Rev. Lett. **103** (2009) 130404

Braaten, HWH, Kang, Platter, Phys. Rev. A **81** (2010) 013605

- Prediction: atom-dimer relaxation resonance around 680 G $(1-23)$
- Experiment: resonance at 685 G

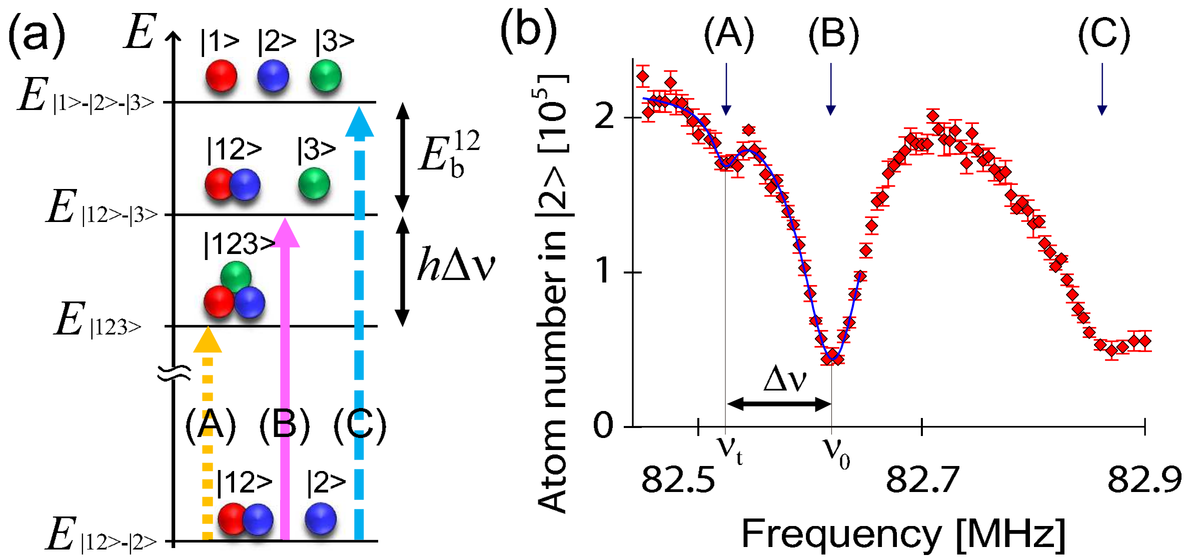
  (Lompe et al. (Heidelberg), 2010)
- Direct observation of Efimov trimers?

Efimov Physics with Fermions

- Direct observation of Efimov trimers through radio frequency association in ^{6}Li

  Lompe et al. (Heidelberg), Science **330** (2010) 940

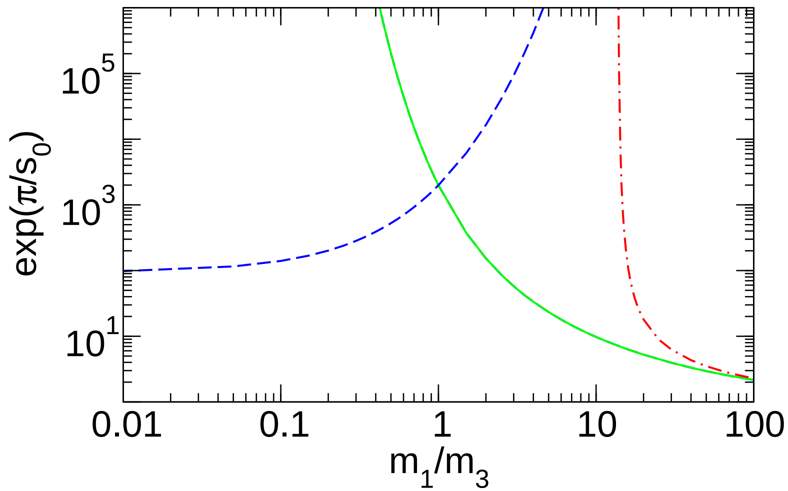
  Nakajima et al. (Tokyo), Phys. Rev. Lett. **106** (2011) 143201



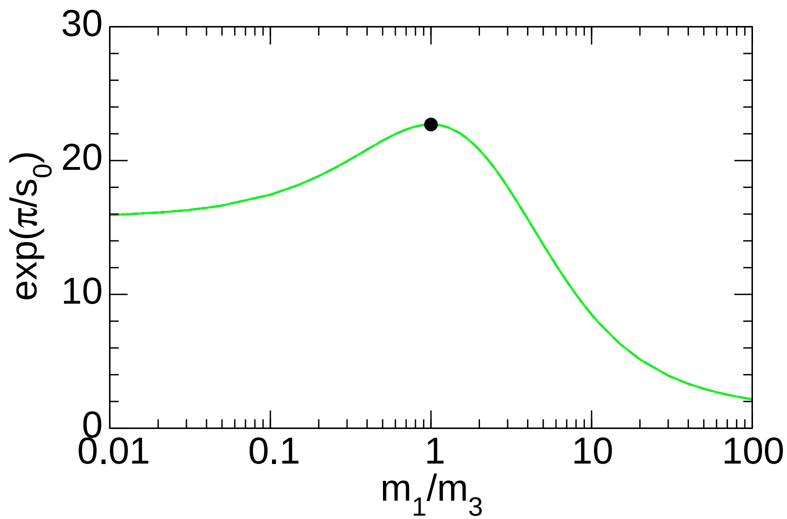
- Higher order corrections are required for quantitative description
- Phenomenological model with energy-dependent scattering length and three-body parameter can describe data

Efimov Physics in Mixtures

- Unequal masses: scaling factor changes
- Example: $m_1 = m_2, m_3$



2 interacting pairs



3 interacting pairs (bosons/distinguishable)

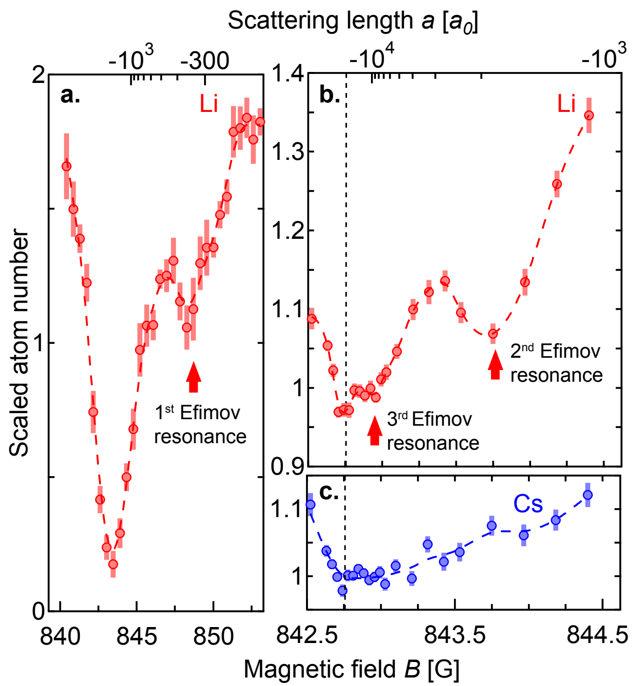
(Braaten, HWH, Phys. Rep. **428** (2006) 259)

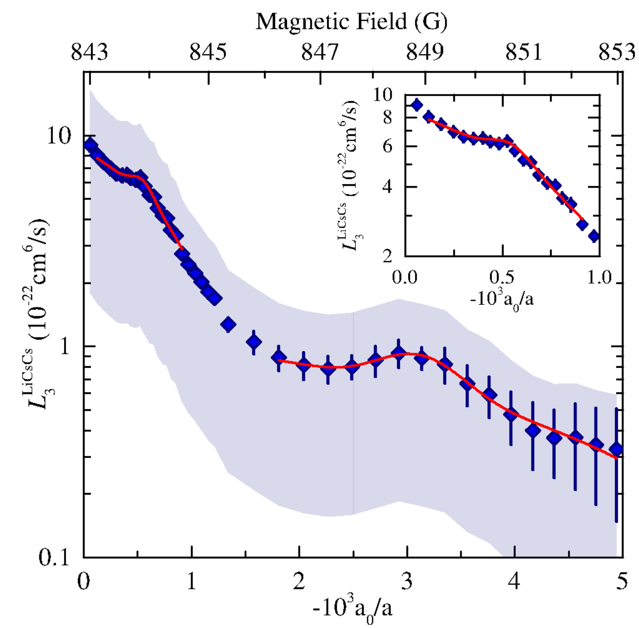
- More favorable scaling factor for two heavy & one light particle
- Experiments with mixtures

Efimov Physics in Mixtures

- Scaling factor can be significantly reduced in mixtures
- Example: Efimov spectrum in ^{6}Li/^{133}Cs mixture ($\lambda_0 \approx 4.9$)

Tung et al. (Chicago), PRL113, 240402 ('14) Pires et al. (Heidelberg), PRL112, 250404 ('14)





- Spectrum of universal states for $N > 3$

(Blume, Yang, 2014; Schmickler, HWH, Hiyama, in progress)

Summary

TECHNISCHE UNIVERSITÄT DARMSTADT

- Effective field theory for unitary limit
  - $\Rightarrow$ leading order interactions g_2, g_3 resummed to all orders
  - $\Rightarrow$ g_j terms for $j \geq 4$ are higher order
  - $\Rightarrow$ higher-derivative terms are perturbative
- Scale invariance anomalously broken for 3 and more particles
  - $\Rightarrow$ Efimov states and cousins
- Universal correlations between few-body observables
- Efimov physics in ultracold atoms

Additional Transparencies

N-Boson Droplets in 2D

- Study universal properties of N-boson droplets in 2 spatial dimensions

$$H = \int d^2x \left(\frac{\hbar^2}{2m} |\nabla\psi|^2 - \frac{g}{2}(\psi^\dagger\psi)^2 + \ldots \right)$$

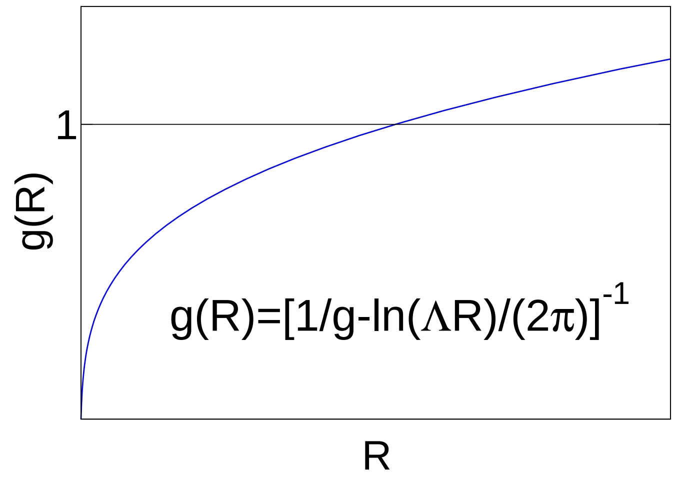
- Weakly attractive interaction: $g > 0$ (cf. Landau-Lifshitz)

  $\longrightarrow$ exponentially shallow dimer: $B_2 \sim \Lambda^2 \exp(-4\pi/g)$

- Define running coupling $g(R)$

  $\rightarrow$ asymptotic freedom

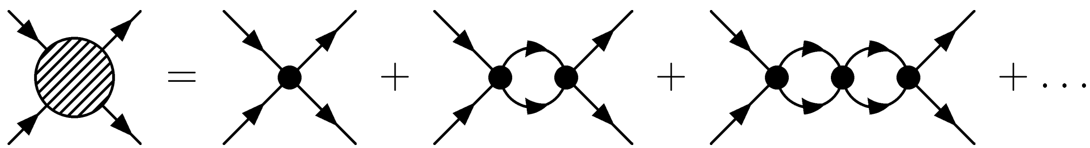
  $\rightarrow$ calculate shallow N-body states



N-Boson Droplets in 2D

TECHNISCHE UNIVERSITÄT DARMSTADT

- All N-body energies are proportional to B_2
- Use classical field theory with running coupling $g(R)$ to calculate $B_N \Rightarrow$ RG improved variational calculation



- Ansatz: $$\psi(\mathbf{r}) = \frac{\sqrt{N}}{R\sqrt{2\pi C}} f\left(\frac{r}{R}\right) \longrightarrow N = \int d^2x \, \psi^\dagger \psi$$

  $\rightarrow$ minimize H with respect to size R and shape $f(r/R)$
- Minimization with respect to size R: $\quad g(R) \sim 1/N + \mathcal{O}(1/N^2)$
- Both T and V are of $\mathcal{O}(N)$ $\rightarrow g(R)$ stabilizes droplet
- Minimization with respect to shape f $\rightarrow$ bell shape

N-Boson Droplets in 2D: Results

TECHNISCHE UNIVERSITÄT DARMSTADT

- N-body bound states show exponential behavior

$$\boxed{B_N = \left(c_0 + c_{-1}/N + c_{-2}/N^2 + \ldots\right)\, 8.567^N}$$

$$\Rightarrow \boxed{B_N/B_{N-1} \approx 8.567, \qquad R_N/R_{N-1} \approx 0.3417, \qquad N \gg 1}$$

(HWH, D.T. Son, Phys. Rev. Lett. **93** (2004) 250408)

- Finite range interactions: $1 \ll N \ll N_c \approx 0.931 \ln(R_2/r_0)$
- How large is large?
- Few-body calculations for $N = 3, 4$ are available

Bruch, Tjon, Phys. Rev. A **19** (1979) 425

Nielsen, Jensen, Fedorov, Few-Body Syst. **27** (1999) 15

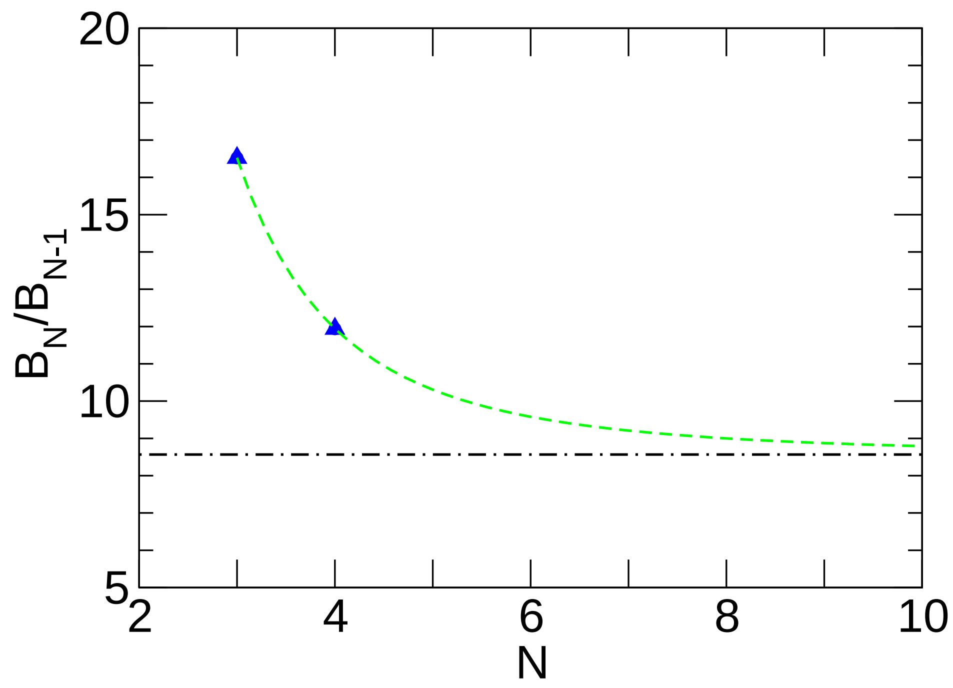
Platter, Hammer, Meißner, Few-Body Syst. **35** (2004) 169

N-Boson Droplets in 2D: Results

TECHNISCHE UNIVERSITÄT DARMSTADT

- How is the large-N limit approached?

$$N \gtrsim 6 \Leftrightarrow \frac{a}{r_e} \gg 600$$



- Calculate explicitly for larger N

(cf. Blume, Phys. Rev. B **72** (2005) 094510; Lee, Phys. Rev. A **73** (2006) 063204)

- Realization in experiment?