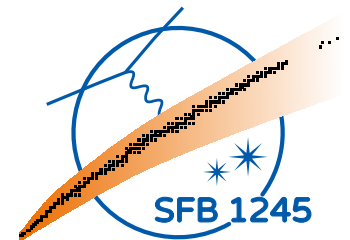


Beyond Ultracold Atoms: Halo Nuclei and Hadronic Molecules

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1. **EFT for Ultracold Atoms I: Effective Field Theories & Universality**
2. **EFT for Ultracold Atoms II: Cold Atoms & the Unitary Limit**
3. **EFT for Ultracold Atoms III: Weak Coupling at Finite Density**
4. **EFT for Ultracold Atoms IV: Few-Body Systems in the Unitary Limit**
5. **Beyond Ultracold Atoms: Halo Nuclei and Hadronic Molecules**

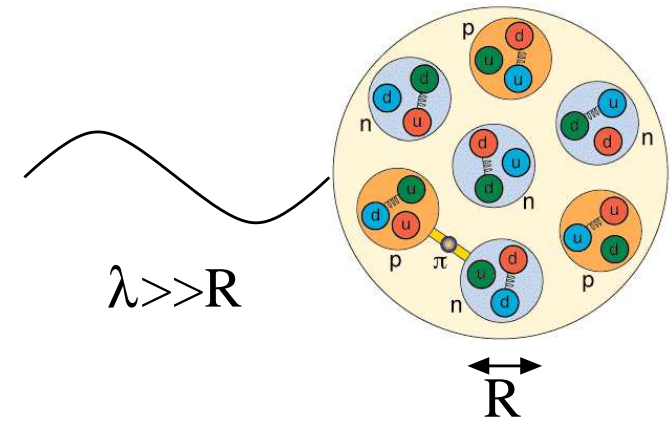
Literature

G.P. Lepage, TASI Lectures 1989, arXiv:hep-ph/0506330

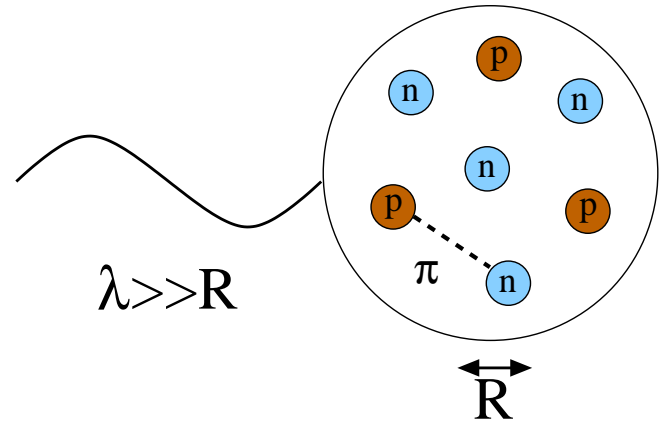
D.B. Kaplan, arXiv:nucl-th/9506035

E. Braaten, HWH, Phys. Rep. **428** (2006) 259 [arXiv:cond-mat/0410417]

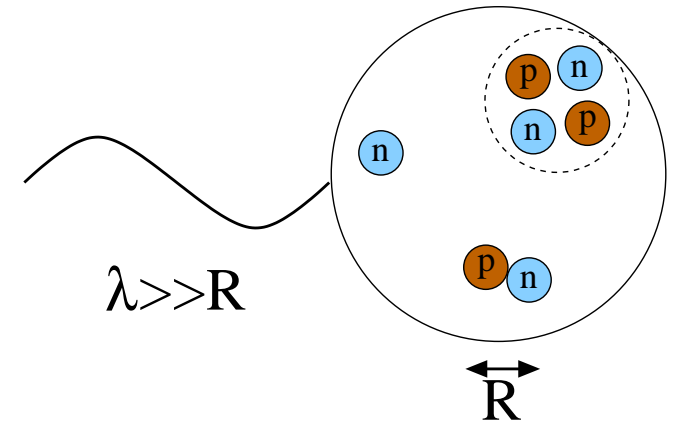
- Separation of scales:
 $1/k = \lambda \gg R$
- Limited resolution at low energy:
→ expand in powers of kR



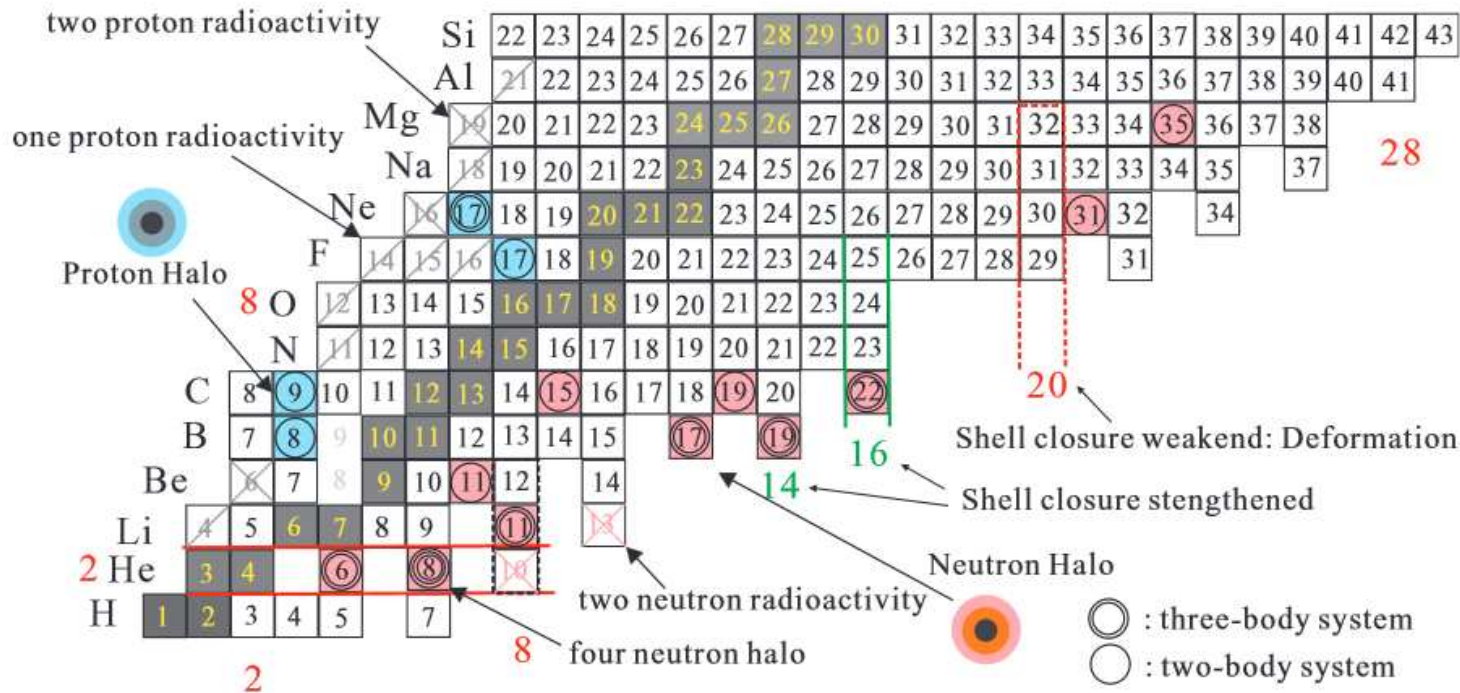
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- Short-distance physics not resolved
→ capture in low-energy constants using renormalization
→ include long-range physics explicitly
- Systematic, model independent



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→ include long-range physics explicitly
- Systematic, model independent
- Very low energies: only short range interactions
- Exploit cluster substructures \implies Halo EFT
- Universal properties



- Low separation energy of valence nucleons: $B_{valence} \ll B_{core}, E_{ex}$
 → close to “nucleon drip line” → **scale separation** → EFT

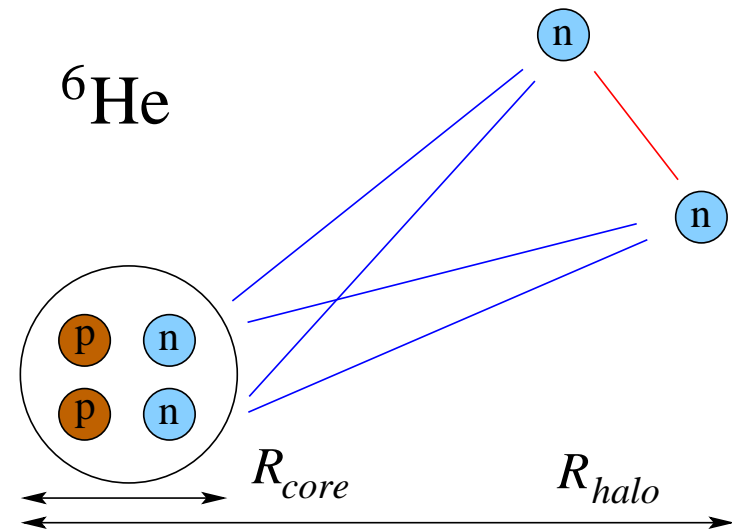


C.-B. Moon, Wikimedia Commons

- EFT for halo nuclei

(Bertulani, HWH, van Kolck, 2002; Bedaque, HWH, van Kolck, 2003; ...)

- Scales: $R_{halo} \gg R_{core} \sim \ell$
- Antisymmetrization with respect to neutrons in core?
- Core neutrons not active dof in halo EFT



- Physics: exchange of core nucleon and halo nucleon only contributes to observables if there is spatial overlap between wave functions of core and halo nucleon

\implies small for $R_{core} \ll R_{halo}$

- Effects subsumed in low-energy constants, included perturbatively in expansion in R_{core}/R_{halo}

- Effective Lagrangian (schematically)

$$\mathcal{L}_d = \psi^\dagger \left(i\partial_t + \frac{\vec{\nabla}^2}{2m} \right) \psi + \frac{g_2}{4} d^\dagger d - \frac{g_2}{4} (d^\dagger \psi^2 + (\psi^\dagger)^2 d) - \frac{g_3}{36} d^\dagger d \psi^\dagger \psi + ..$$

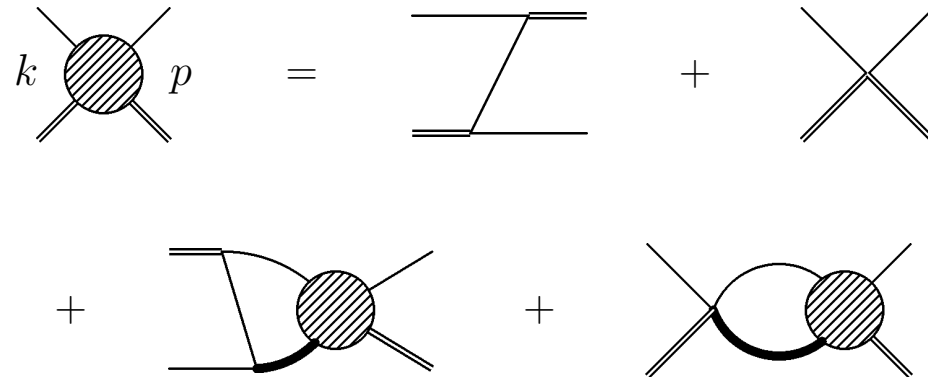
- 2- and 3-body interaction at leading order: g_2, g_3 enhanced!

- 2-body amplitude:



$$\text{---} = \text{==} + \text{---} \circ \text{---} + \text{---} \circ \circ \text{---} + \dots$$

- 3-body amplitude:



$$k \text{---} \text{---} p = \text{---} \text{---} \text{---} + \text{---} \times \text{---} + \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---}$$

- Structure of 2-neutron halo nuclei \rightarrow energies, matter form factors, radii

- Observables are independent of regulator/cutoff Λ

⇒ Running coupling $H(\Lambda) \propto \Lambda^2 g_3(\Lambda)$

- $H(\Lambda)$ periodic: **limit cycle**

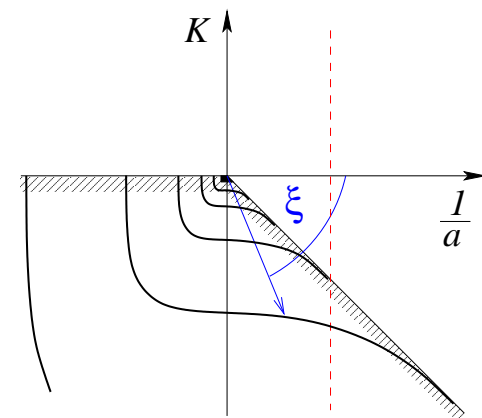
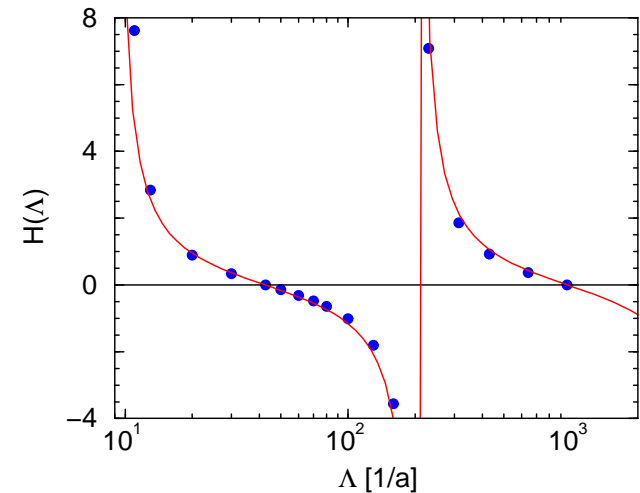
$$\Lambda \rightarrow \Lambda e^{n\pi/s_0} \approx \Lambda (22.7)^n$$

- Discrete scale invariance

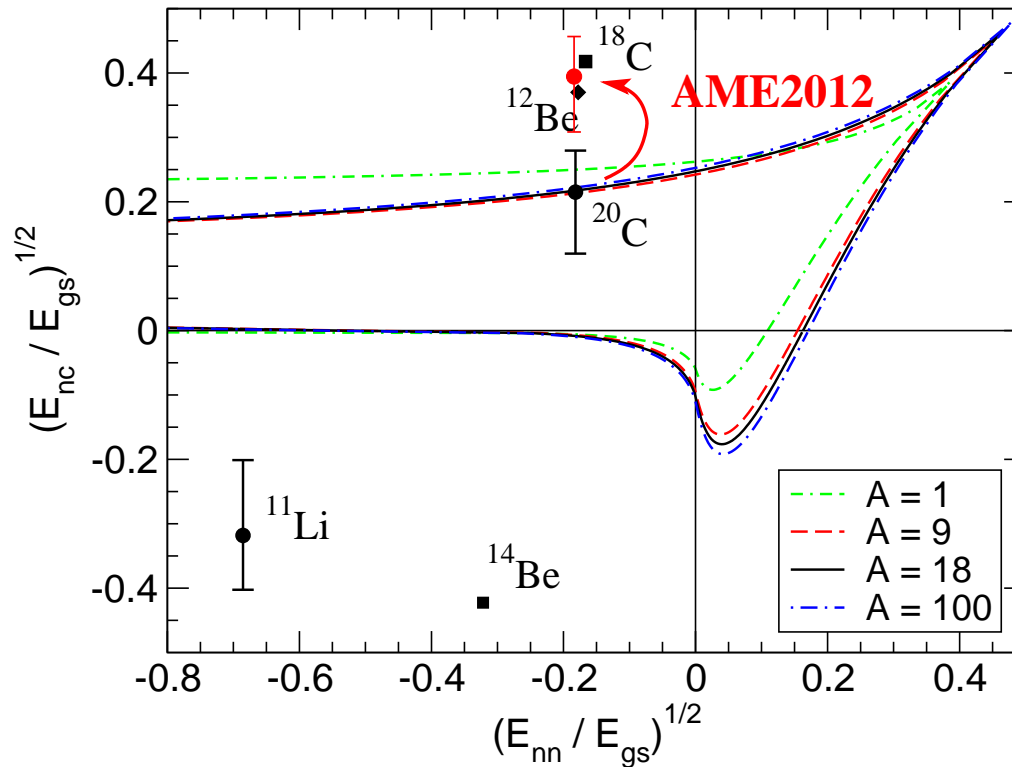
- Efimov effect (Efimov, 1970)

- Observed in ultracold atoms
(Krämer et al., 2006; ...)

- Relevant in halo nuclei?

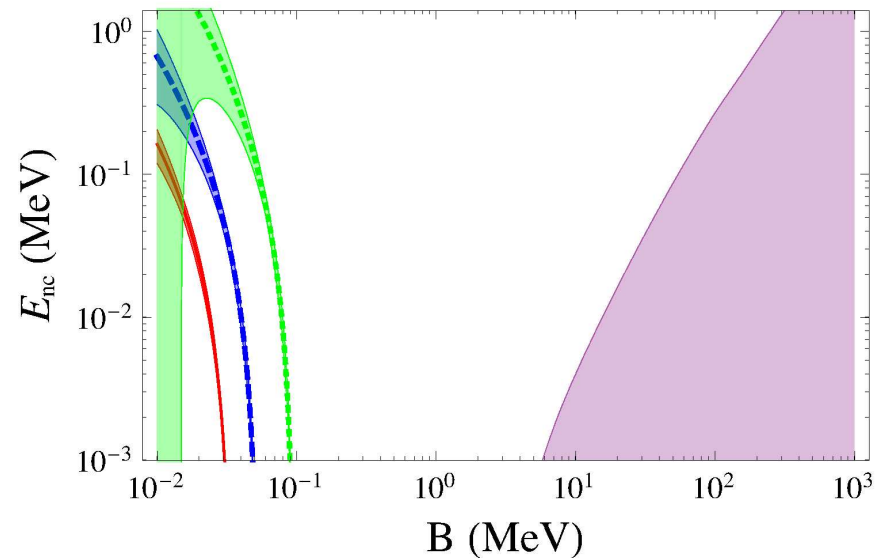
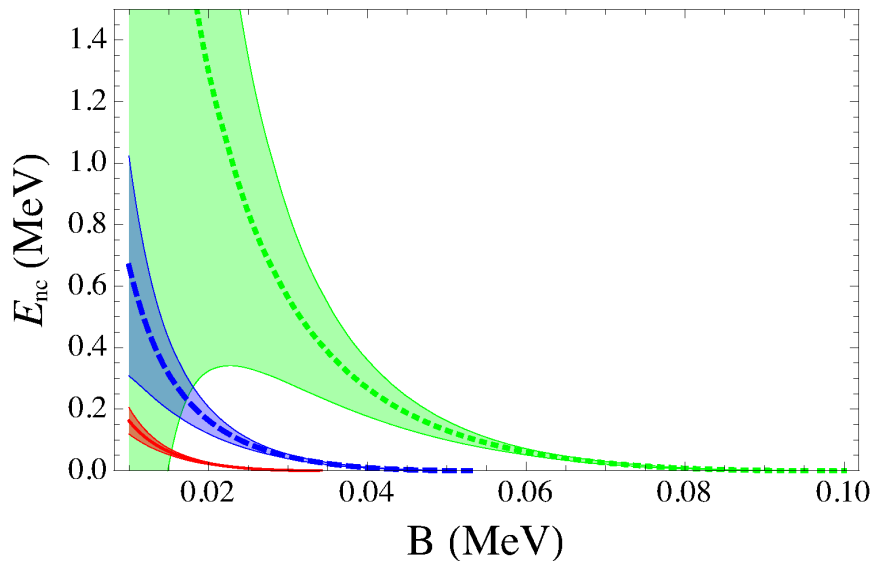


- Efimov effect in halo nuclei? (Fedorov, Jensen, Riisager, 1994)
 \implies excited states obeying scaling relations
- Correlation plot: $E_{nn} \leftrightarrow E_{nc}$ (Amorin, Frederico, Tomio, 1997)



Canham, HWH, Eur. Phys. J. A **37** (2008) 367

- Matter radius from $^{22}\text{C} + p$ & Glauber: $\langle r_0^2 \rangle^{1/2} = 5.4(9)$ fm
(Tanaka et al., Phys. Rev. Lett. **104** (2010) 062701)
- Halo EFT analysis of impact on other observables in ^{22}C
(Acharya, Ji, Phillips, Phys. Lett. B **723** (2013) 196)



Plots for $\langle r_0^2 \rangle^{1/2} = 4.5, 5.4, 6.3$ fm

- Excited Efimov states in ^{22}C appear to be ruled out

(G. Hagen, P. Hagen, HWH, Platter, Phys. Rev. Lett. **111** (2013) 132501)

- The Many and the Few: emergence of effective halo degrees of freedom
- Coupled cluster calculations of ^{60}Ca and ^{61}Ca using chiral N2LO two-body force and schematic three-body force:

^{61}Ca is a weakly bound S-wave state (or virtual state)

- Quantitative estimate: $S_n = B_{nc} = 5\dots 8$ keV

- Scattering Parameters:

$$a_{cn} = 54(1) \text{ fm}, r_{cn} = 9.0(2) \text{ fm} \quad \implies \quad r_{cn}/a_{cn} \approx 1/6$$

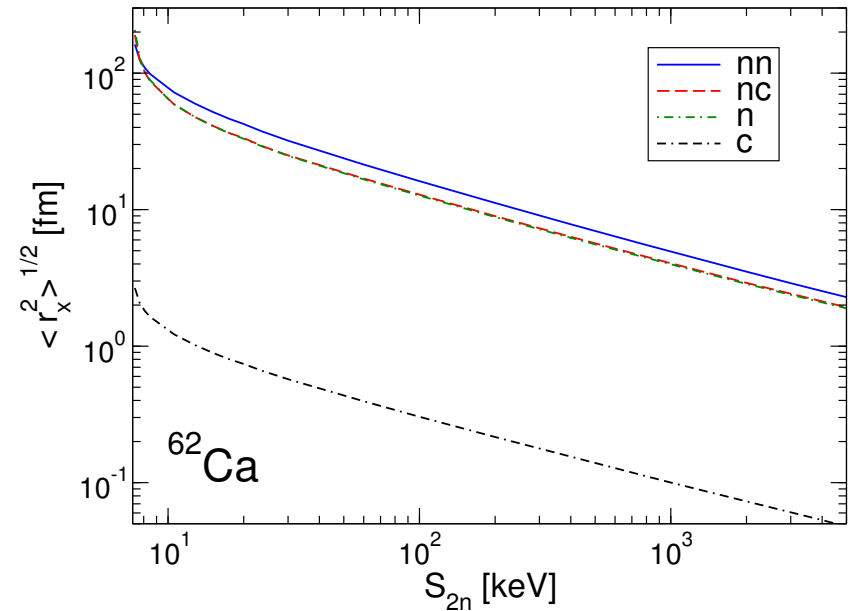
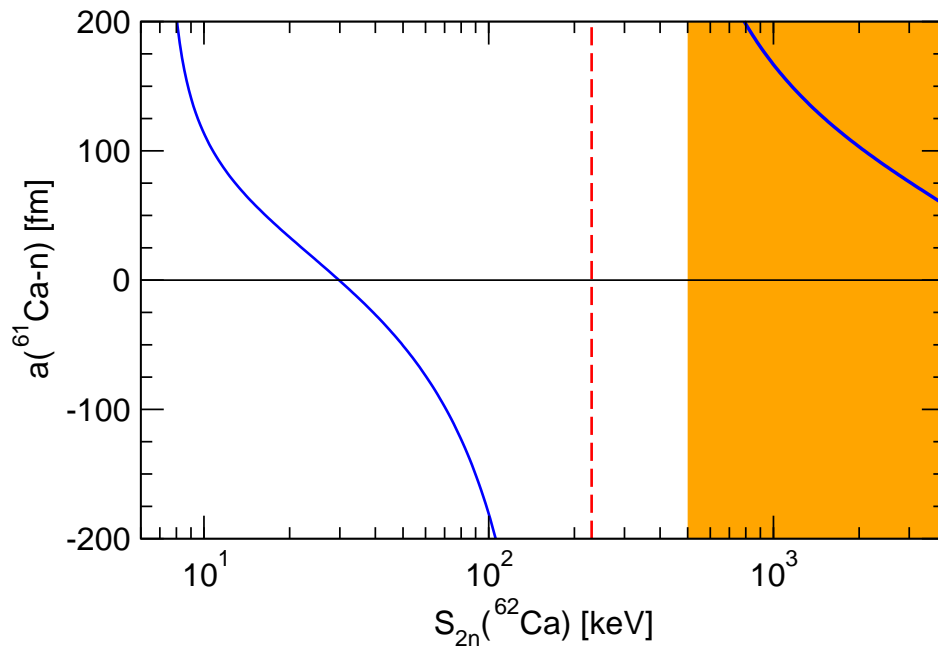
- Investigate consequences for ^{62}Ca using halo EFT

- Prospects for excited Efimov states in ^{62}Ca :

$$S_{\text{deep}} = 1/(\mu_{cn} r_{cn}^2) \approx 500 \text{ keV}, \text{ scaling factor } \lambda_0 \approx 16$$

$$\implies \text{ possible if } S_{2n} \gtrsim 230 \text{ keV}$$

- Universal correlations between S_{2n} , $^{61}\text{Ca}-n$ scattering length, ^{62}Ca matter, and charge radii

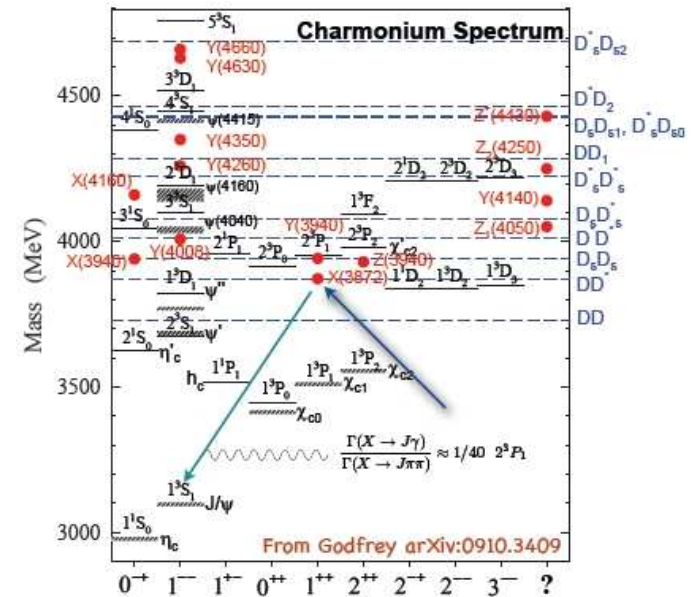


(G. Hagen, P. Hagen, HWH, Platter, Phys. Rev. Lett. **111** (2013) 132501)

- Excited Efimov state appears around $S_{2n} \approx 230$ keV
- Matter radii of order tens of Fermi possible

- How to study excited Efimov states experimentally?
(A. Macchiavelli, Few-Body Syst. **56**, 773 (2015))
- Consider transfer reactions for candidate nucleus ${}^A Z_N$
 - (a) One-neutron transfer: ${}^{(A-1)} Z_{(N-1)}(d, p){}^A Z_N$
 - (b) Two-neutron transfer: ${}^{(A-2)} Z_{(N-2)}(t, p){}^A Z_N$
- Back-of-the-envelope estimate \implies (a) is most promising
- Reaction calculation in Halo EFT would be useful

- New $c\bar{c}$ states at B factories: X , Y , Z
- Example: $X(3872)$ (Belle, CDF, BaBar, D0)
- No ordinary $c\bar{c}$ -state
 - Decays violate isospin
 - Measured mass depends on decay channel



$$m_X = (3871.69 \pm 0.17) \text{ MeV} \quad \Gamma < 1.2 \text{ MeV} \quad J^{PC} = 1^{++}$$

- Nature of $X(3872)$?
 $\bar{D}^0 D^{0*}$ -molecule, tetraquark, charmonium hybrid, ...
- Molecular nature \Rightarrow interaction of $X(3872)$ with D^0 , \bar{D}^0 , D^{0*} , \bar{D}^{0*} determined by large scattering length

- Nature of $X(3872)$ not finally resolved
- Assumption: $X(3872)$ is weakly-bound D^0 - \bar{D}^{0*} -molecule
 - $\implies |X\rangle = (|D^0\bar{D}^{0*}\rangle + |\bar{D}^0D^{0*}\rangle)/\sqrt{2}$, $B_X = (0.11 \pm 0.21) \text{ MeV}$
 - \implies **universal properties** (cf. Braaten et al., 2003-2008, ...)
 - Explains isospin violation in decays of $X(3872) \Rightarrow$ superposition of $I = 1$ and $I = 0$
 - Different masses due to different line shapes in decay channels
- Large scattering length to LO determines interaction of $X(3872)$ with D^0 and D^{0*}
- Higher orders: EFT with perturbative pions \Rightarrow **XEFT**
(Fleming, Kusunoki, Mehen, van Kolck, 2007; Fleming, Mehen, 2008)
(Braaten, HWH, Mehen, 2010; ...)

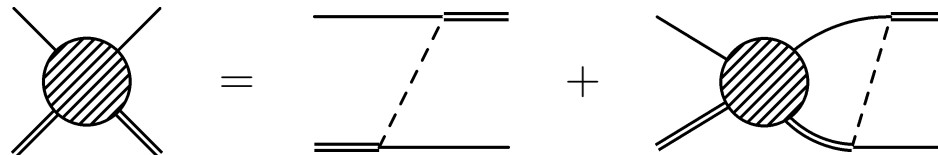
- Effective Lagrangian

$$\mathcal{L} = \sum_{j=D^0, D^{*0}, \bar{D}^0, \bar{D}^{*0}} \psi_j^\dagger \left(i\partial_t + \frac{\nabla^2}{2m_j} \right) \psi_j + \Delta X^\dagger X - \frac{g}{\sqrt{2}} \left(X^\dagger (\psi_{D^0} \psi_{\bar{D}^{*0}} + \psi_{D^{*0}} \psi_{\bar{D}^0}) + \text{H.c.} \right) + \dots,$$

- Propagator of the $X(3872)$

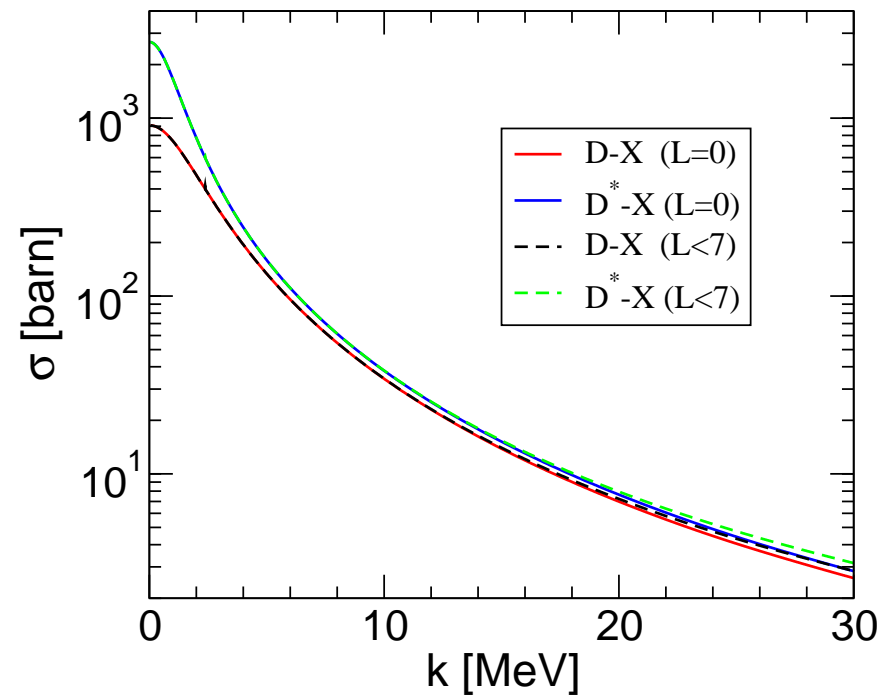
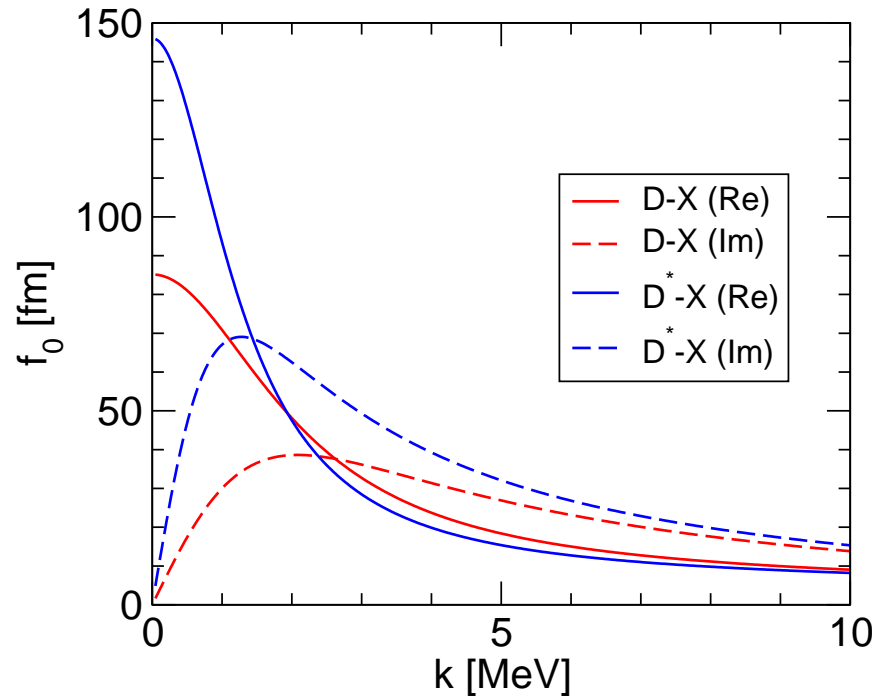
$$= = \text{====} + \text{====} \bigcirc \text{====} + \text{====} \bigcirc \text{====} \bigcirc \text{====} + \dots$$

- Three-body integral equation



$$\text{Shaded Circle} = \text{Diagram 1} + \text{Diagram 2}$$

- Predictions for scattering amplitude/cross section



Canham, HWH, Springer, Phys. Rev. D **80**, 014009 (2009)

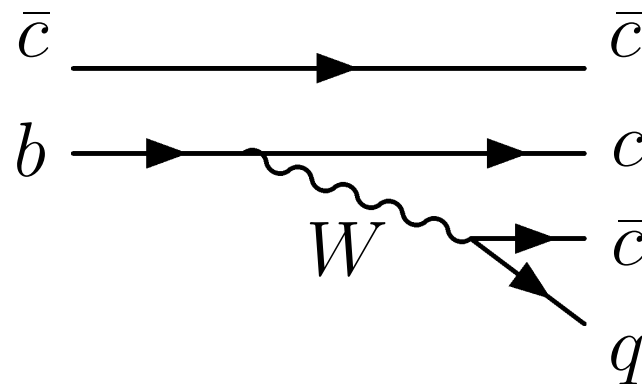
- Three-body scattering lengths:

$$a_{D^0 X} = -9.7a \approx -85 \text{ fm}$$

$$a_{D^{*0} X} = -16.6a \approx -146 \text{ fm}$$

Experimental Observation ?

- Behavior of $X(3872)$ produced in isolation should be distinguishable from its behavior when in the presence of $D^0, D^{*0}, \bar{D}^0, \bar{D}^{*0}$
- Final state interaction of D, D^* mesons in B_c -decays
- Example: quark-level B_c decay yielding three charmed/anticharmed quarks in final state



- Process in principle accessible at the LHC

- Cluster EFT for halo nuclei
 - ⇒ large scattering length/shallow states
 - Controlled, systematic approach ⇒ error estimates
 - Straightforward inclusion of external currents
- Universal theory has applications in atomic, nuclear, and particle physics
- Universality predicts correlations between observables
 - ⇒ input from theory or experiment
- Excited Efimov state possible in ^{62}Ca
- Calculations of EM structure and reactions