Emergent symmetries in disordered quantum spin chains

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Symmetry in nature

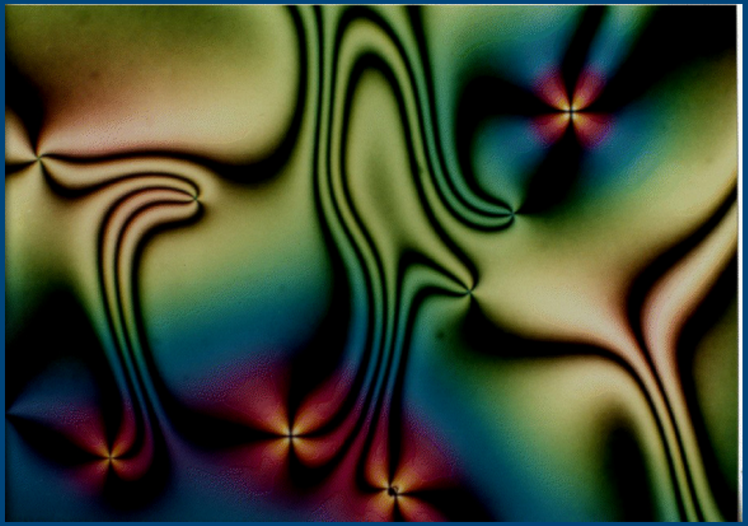
Usual scenario: physical systems are *less symmetric* at low temperatures/energies (via phase transitions):

- crystals vs gas/liquid
- (anti-)ferromagnets vs paramagnets
- liquid crystals vs normal liquids
- superconductors vs normal metals
- superfluids vs normal fluids
- SU(2) x U(1) vs $U_{em}(1)$….

1. Condensed matter physicists spend money to cool things down and find broken symmetries.
2. High energy physicists spend (a lot more) money to “heat things up” and access more symmetric states.

Water and ice

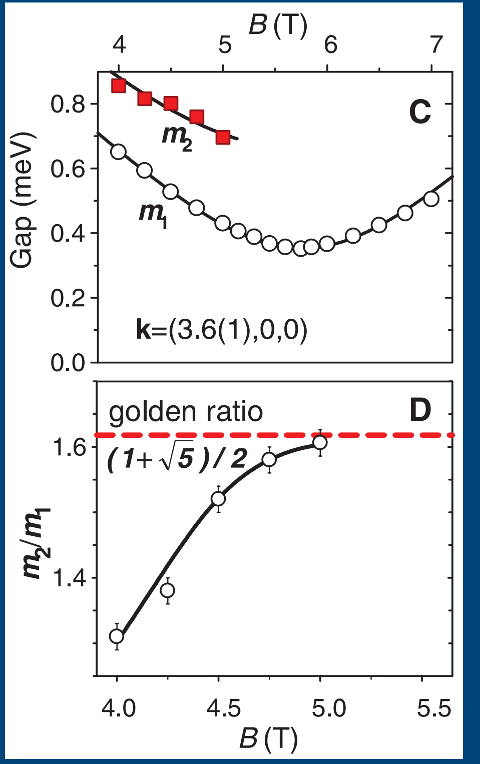
Liquid crystal



Less common: emergent symmetries

Emergent symmetries: the low-energy sector is *more symmetric* than the high-energy one (via crossovers)

- Critical Ising model in a small magnetic field: E_8 Lie group! (A. B. Zamolodchikov, Int. J. Mod. Phys. '89). Experimental realization: (R. Coldea *et al.*, Science **327**, 177 (2010))
- Tricritical Ising model: SUSY (D. Friedan, Z. Qiu, S. Shenker, PRL '85).
- Quantum spin-2 chains: SU(3) (P. Chen *et al.*, PRL '15)
- Certain quantum critical points: gauge symmetry (Senthil, Vishwanath, Balents, Sachdev, Fisher, Science **303**, 1490 (2004))….
- Symmetry-protected topological states: fermions and gauge fields (Xiao-Gang Wen)………..



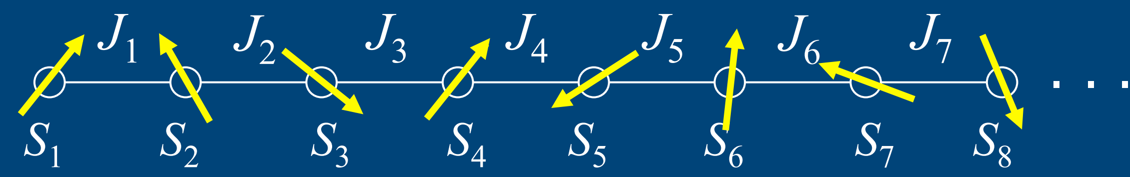
Less common: emergent symmetries

1. Known examples are few and far between.
2. Generic mechanism not known. General ideas:
   1. Stable low-T fixed point with group G
   2. Irrelevant operators break G into g ($g \subset G$)
3. No recipe for its construction: case by case…

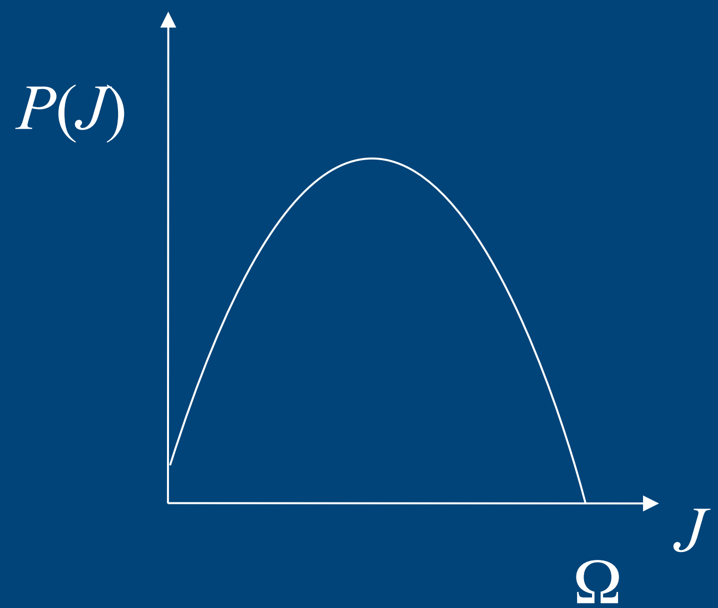
   (C. Itoi, S. Qin, and I. Affleck, PRB 61, 6747 (2000))

Disordered Heisenberg chain

$$H = \sum_i J_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$



$J_i > 0$: distributed according to $P(J;\Omega)$
Ω is the high energy cutoff



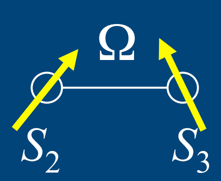
Strong disorder RG method

$$H = \sum_i J_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

$J_i > 0$ – distributed according to $P_0(J;\Omega)$;
Ω is the high energy cutoff

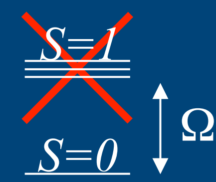
Decimation procedure:

1. Find the strongest coupling $\Omega=\max\{J_i\}$.

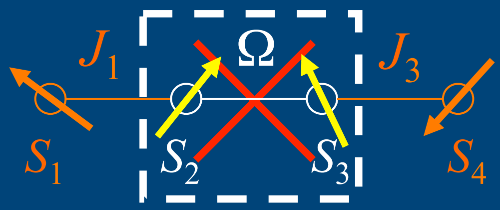


Diagonalize →

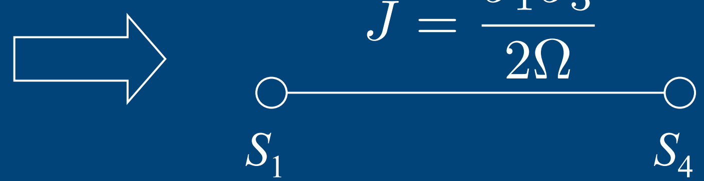
$$S_T = 1 \Rightarrow E_1 = \frac{1}{4}\Omega$$
$$S_T = 0 \Rightarrow E_0 = -\frac{3}{4}\Omega$$



2. Treat $H' = J_1 \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + J_3 \boldsymbol{S}_3 \cdot \boldsymbol{S}_4$ in 2nd order perturbation theory



$$\tilde{J} = \frac{J_1 J_3}{2\Omega}$$



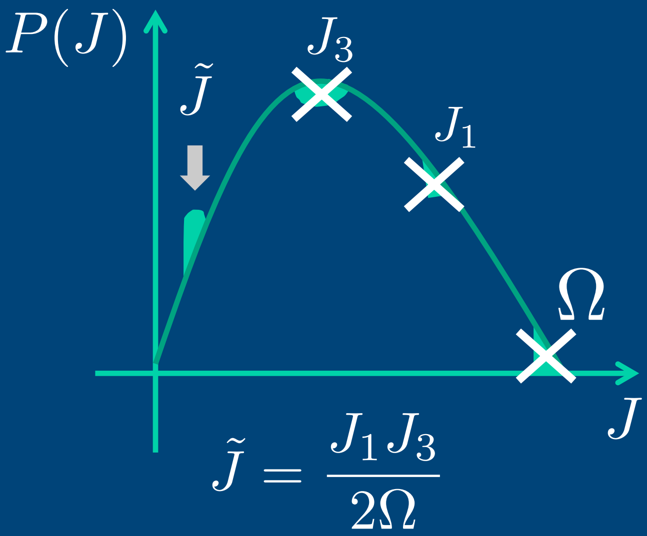
Note $\tilde{J} < J_1, J_2$

Net result: S_2 and S_3 disappear and new coupling between S_1 and S_4 appears
Topology of the (infinite) chain is preserved.

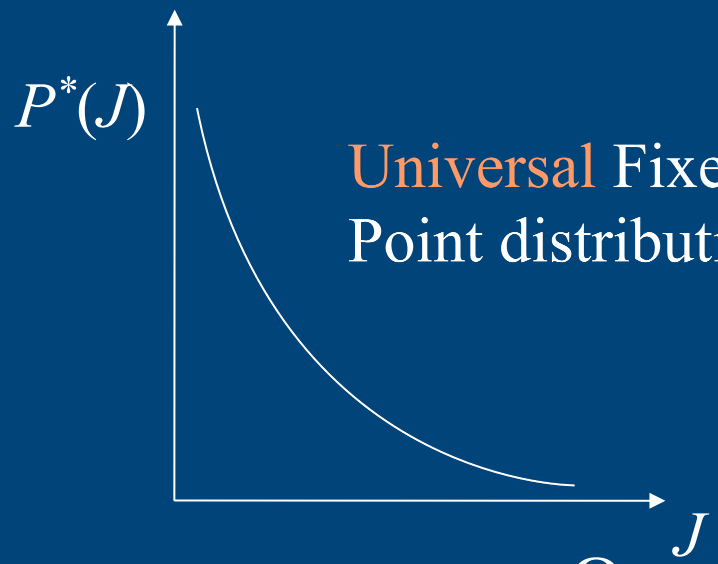
S. K. Ma, C. Dasgupta, and C.-K. Hu, Phys. Rev. Lett. **43**, 1434 (1979), C. Dasgupta, and S. K. Ma, Phys. Rev. B **22**, 1305 (1980).

Universality

All initial distributions have the same fate



Flow



$$P^*(J,\Omega) = \frac{\alpha}{\Omega}\left(\frac{\Omega}{J}\right)^{1-\alpha} \quad \alpha = \frac{1}{\ln(\Omega_0/\Omega)} \to 0$$

$$\frac{\Delta J}{\langle J \rangle} \sim \frac{1}{\sqrt{\alpha}} \to \infty$$

The effective disorder increases without limit!

The method is asymptotically exact: the wider the distribution, the more accurate the decimations.

Spatial distribution of strong bonds

Decimation procedure:

Spatial distribution of strong bonds

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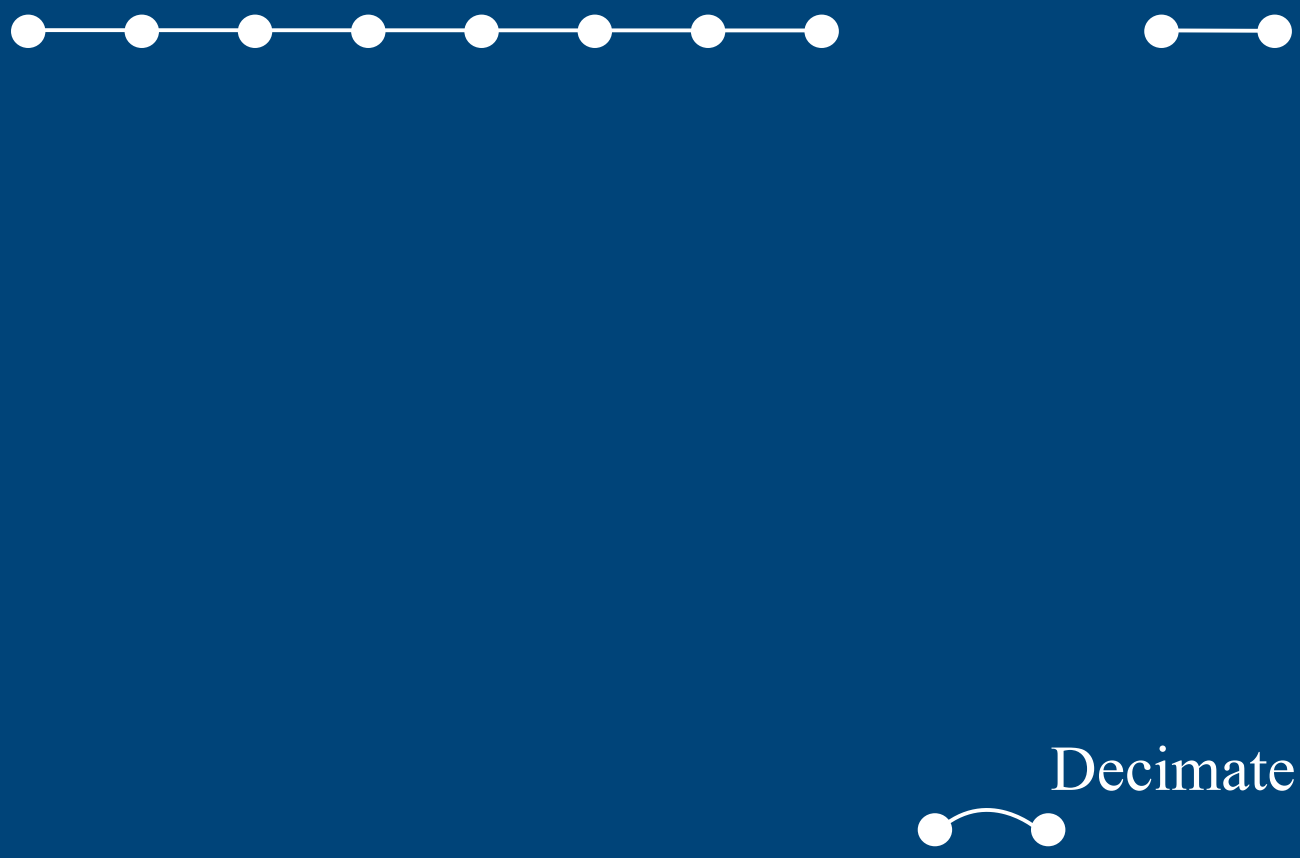

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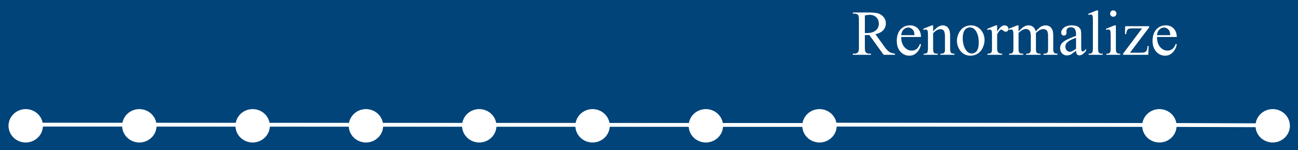

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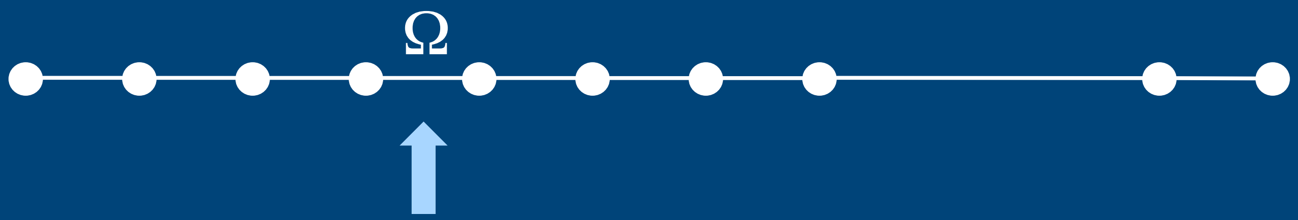

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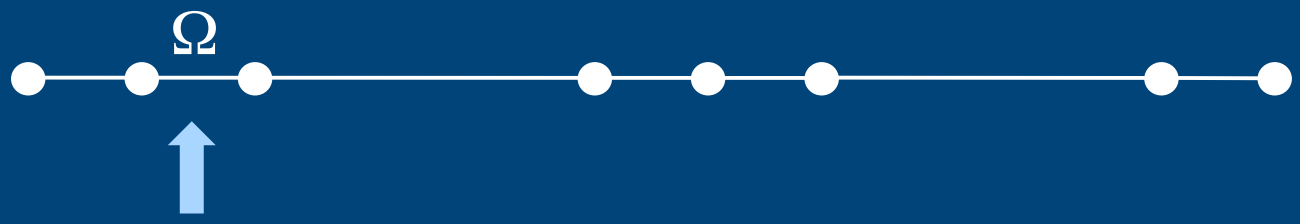

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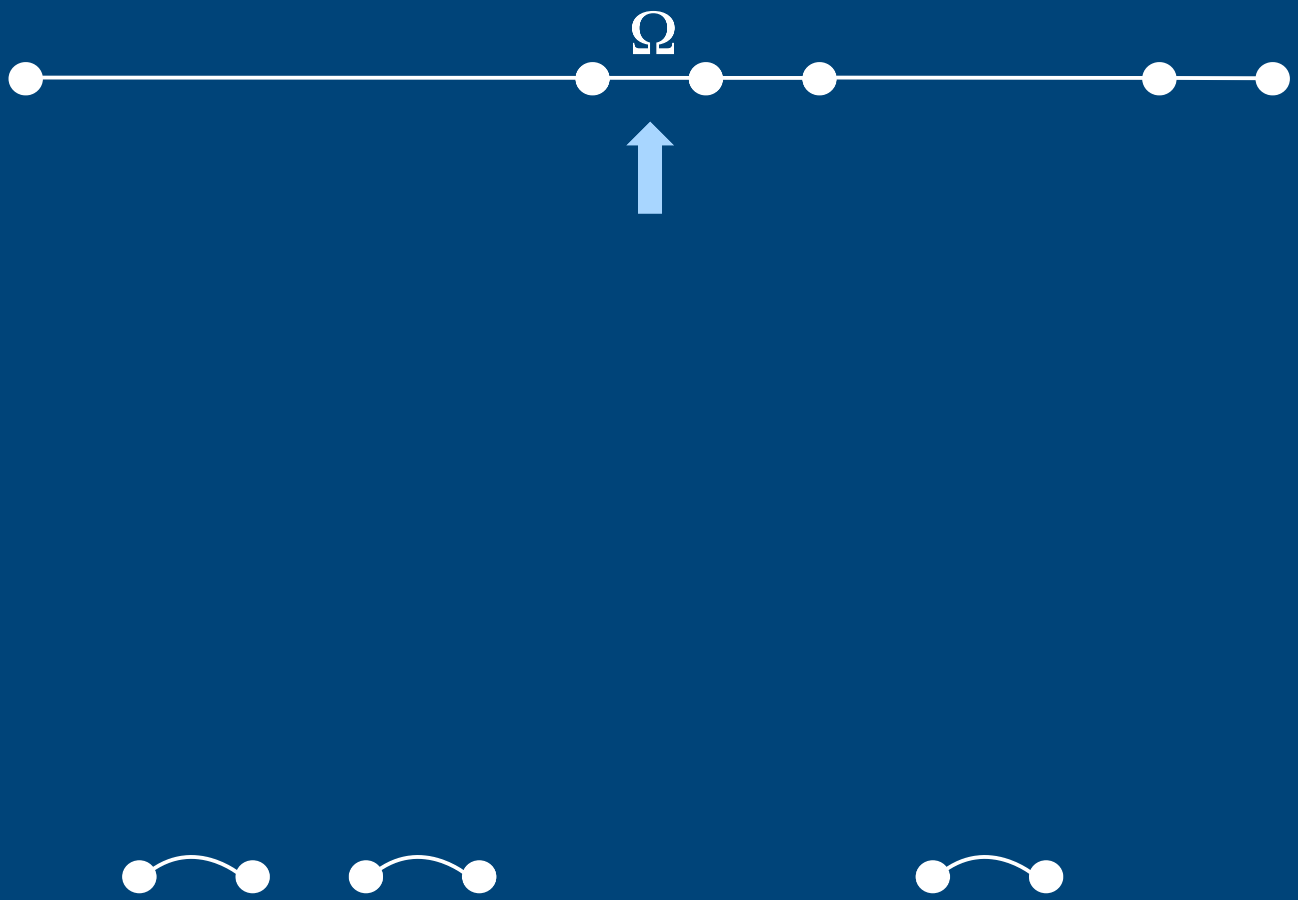

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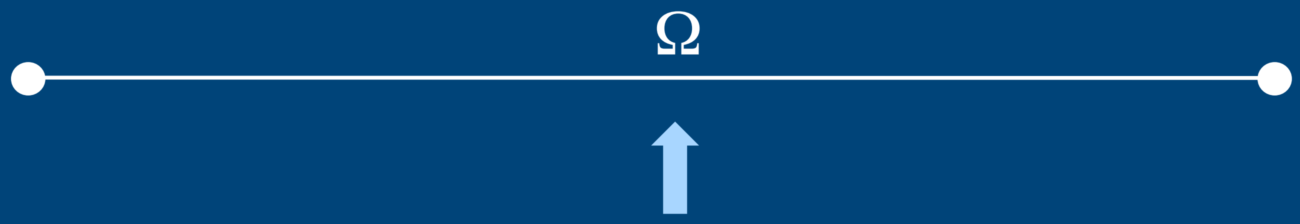
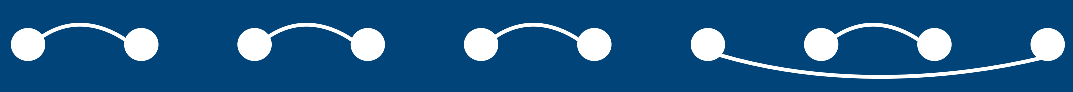




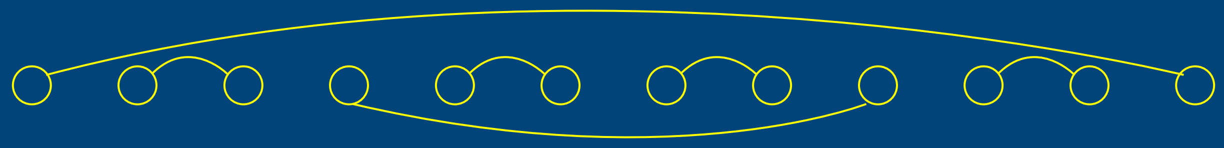
Random-Singlet ground state

Ground state

Random Singlet phase

Well-separated, strongly bound spin pairs

Excitations



Excitations are localized: breakup of long bonds.
Energy of an excitation of length L:

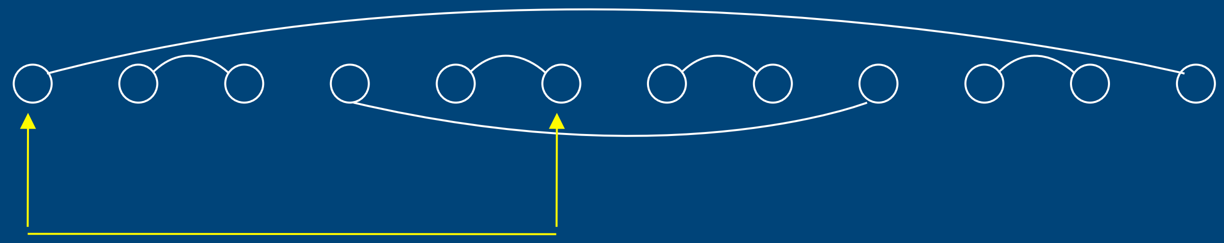
$$\Omega \sim e^{-L^{\psi}} \,, \boxed{\psi = \frac{1}{2}}$$

'Activated dynamical scaling'

With this scaling, we can get exact results for low-energy properties (susceptibility, specific heat), which I will not discuss.

D. S. Fisher, PRB **50**, 3799 (1994).

The correlation function at T=0



$$\langle \mathbf{S}_i \cdot \mathbf{S}_{i+r} \rangle \approx 0$$

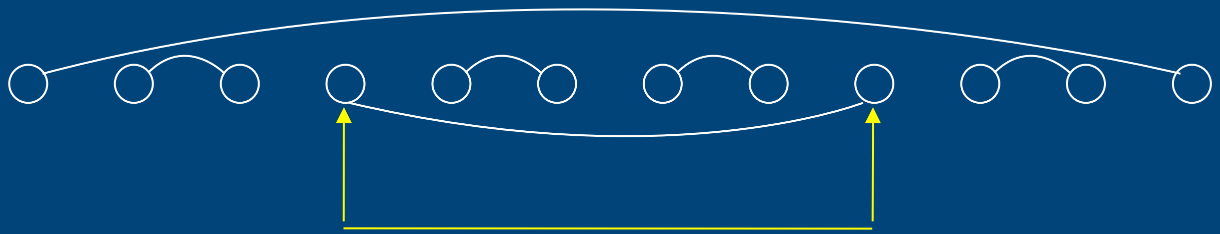
Typical pairs are weakly correlated

$$\langle \mathbf{S}_i \cdot \mathbf{S}_{i+r} \rangle_{typ} \sim \exp(-r^{\psi})$$

$$\boxed{\psi = \frac{1}{2}}$$

D. S. Fisher, PRB **50**, 3799 (1994).

The correlation function at T=0

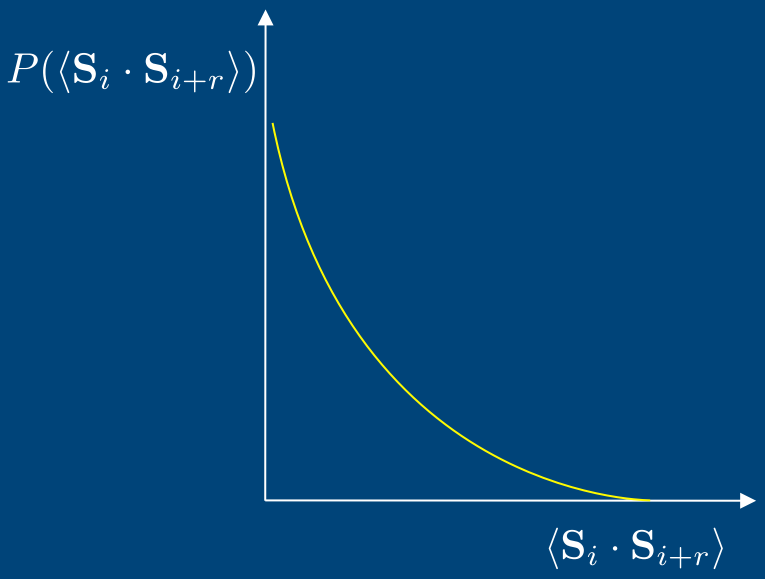


$$\langle \mathbf{S}_i \cdot \mathbf{S}_{i+r} \rangle \approx \mathcal{O}(1)$$

But average value is dominated by rare singlets

$$\langle \mathbf{S}_i \cdot \mathbf{S}_{i+r} \rangle_{av} \sim \frac{(-1)^r}{r^\phi}$$

$$\boxed{\phi = 2}$$



D. S. Fisher, PRB **50**, 3799 (1994).

Disordered spin-1 chains

The most general disordered spin-1 chain with global SU(2) invariance.

$$H_{JD} = \sum_i \left[J_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + D_i \left(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} \right)^2 \right]$$

Note: the two terms are linearly dependent for spin-1/2, but not for spin-1.

$$H_{JD} = \sum_i E_i \left[\cos\theta_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + \sin\theta_i \left(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} \right)^2 \right]$$

$$E_i = \sqrt{J_i^2 + D_i^2}; \quad \tan\theta_i = \frac{D_i}{J_i}$$

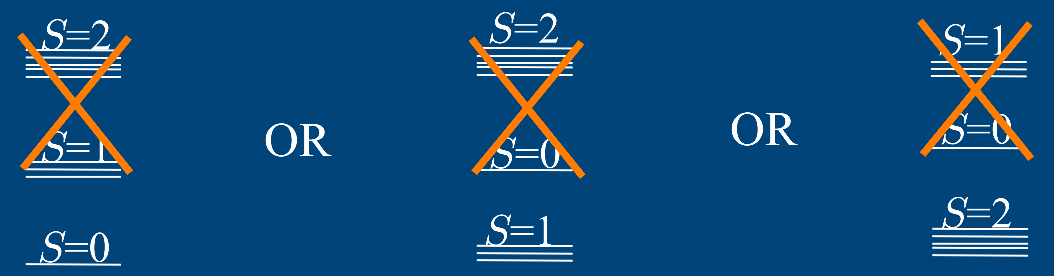
May be experimentally realized in optical lattices loaded with cold ^{23}Na

García-Ripoll, Martin-Delgado, & Cirac, PRL **93**, 250405 (2004)
Imambekov, Lukin, & Demler, PRA **68**, 063602 (2003)

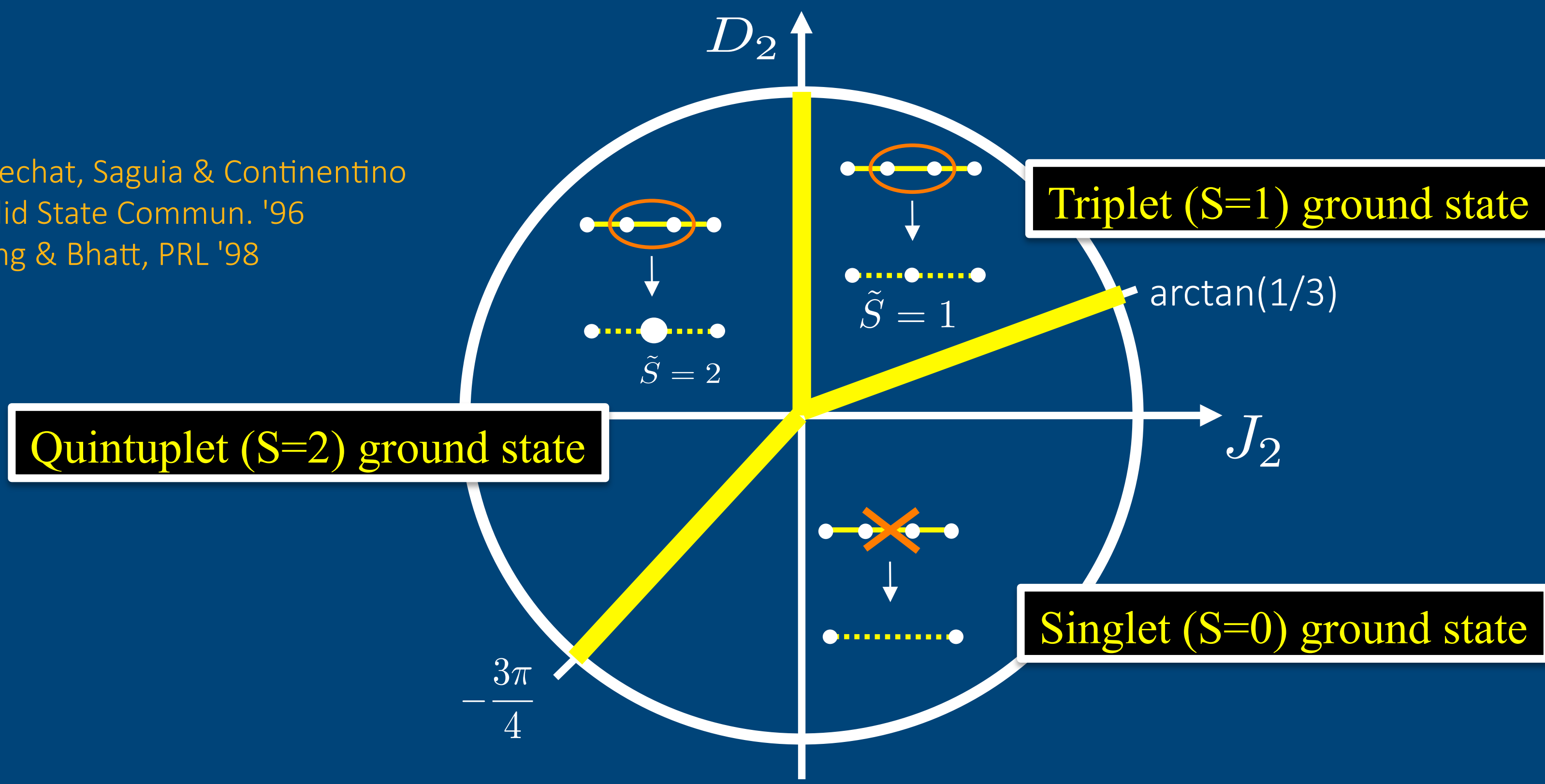
What is the behavior at strong disorder?

RG step for generic spin-1 chains

$$H_0 = J_2\,\mathbf{S}_2\cdot\mathbf{S}_3 + D_2\,(\mathbf{S}_2\cdot\mathbf{S}_3)^2$$



Boechat, Saguia & Continentino Solid State Commun. '96
Yang & Bhatt, PRL '98



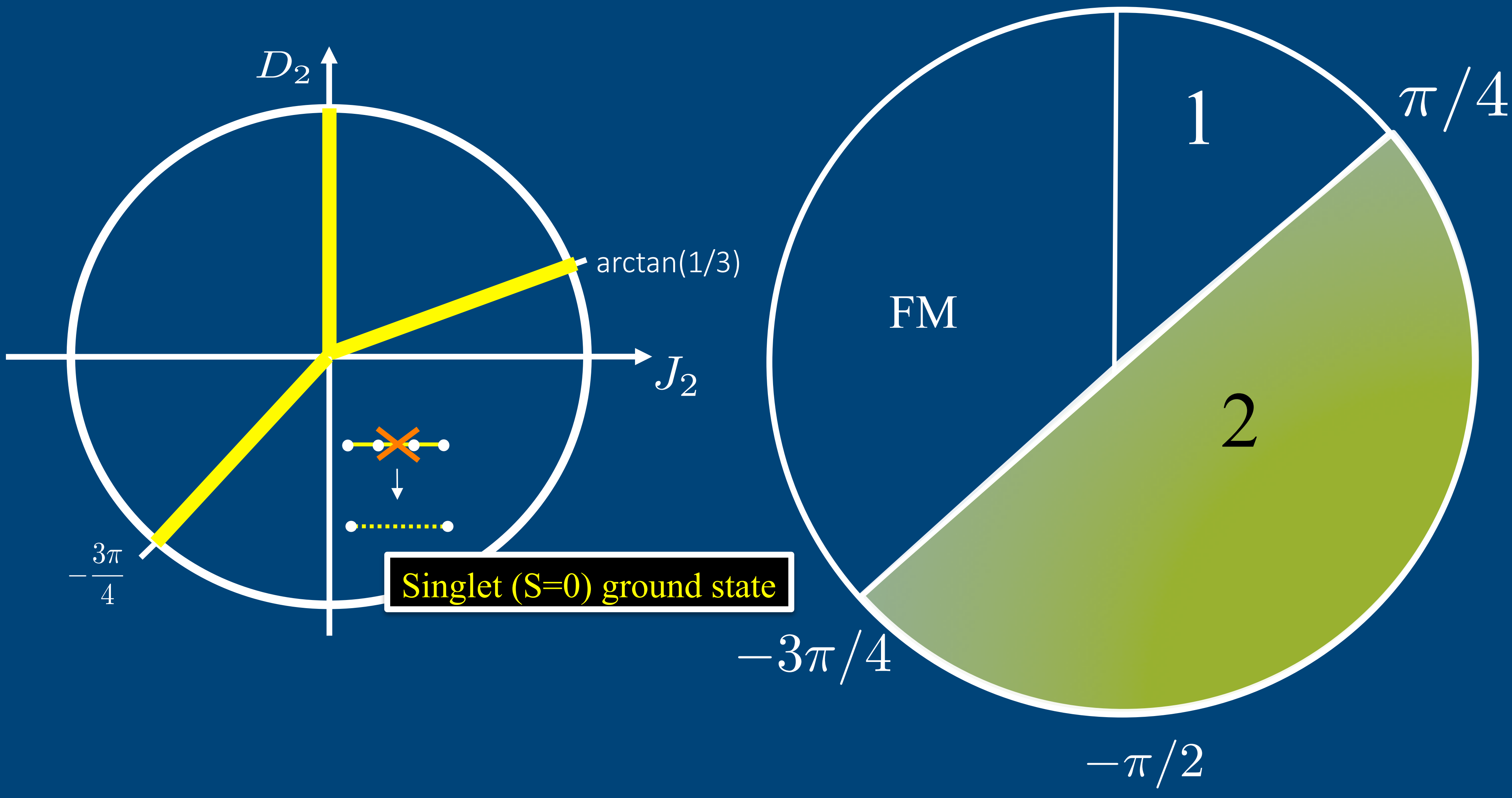
Disordered spin-1 chains: phase diagram

V. L. Quito, J. A. Hoyos, E.M., PRL **115**, 167201 (2015)

We first consider the case of random E_i but **fixed** θ

$$H_{JD} = \sum_i E_i \left[\cos\theta \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + \sin\theta \left(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}\right)^2\right]$$

Random singlet pairs



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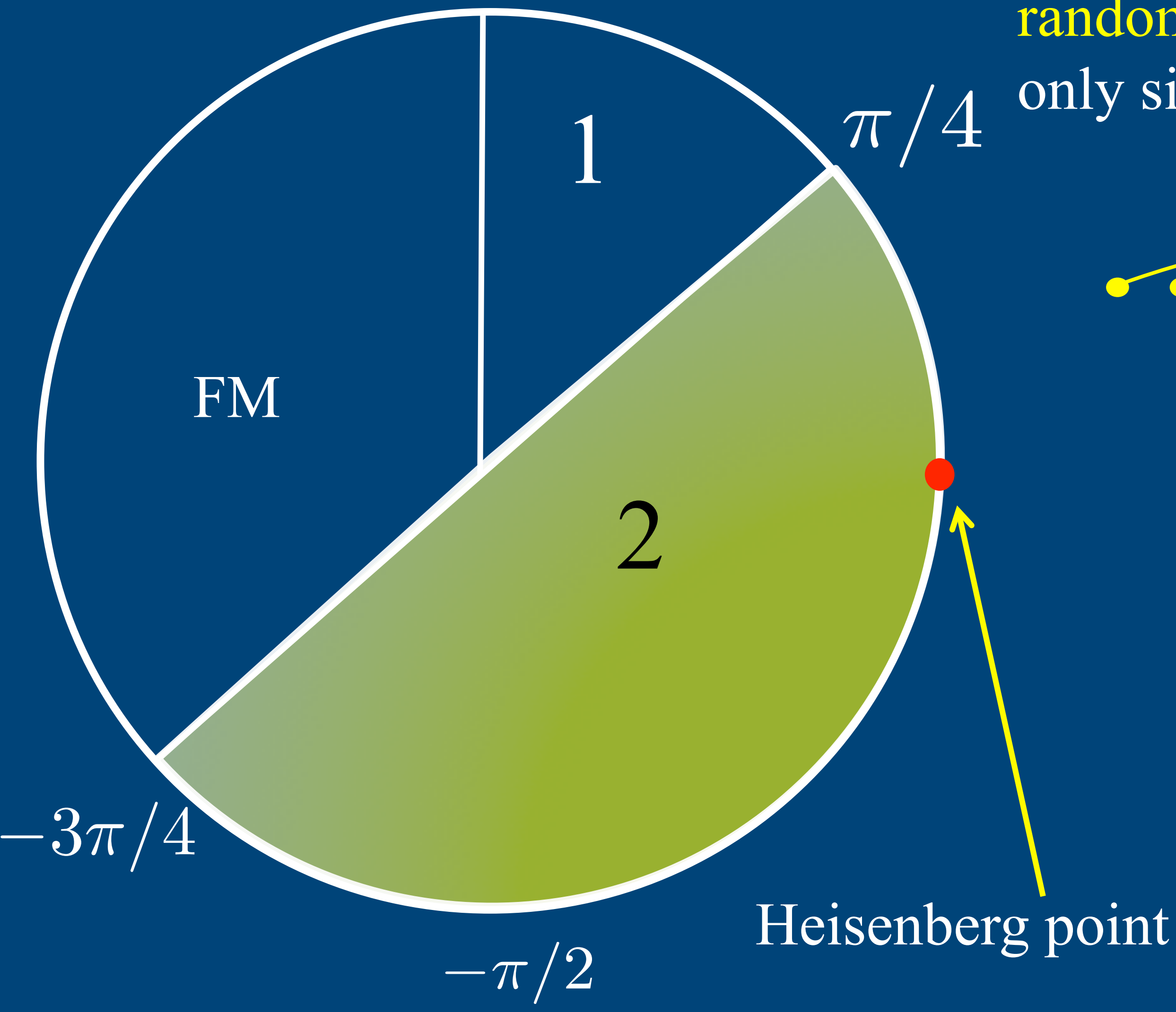
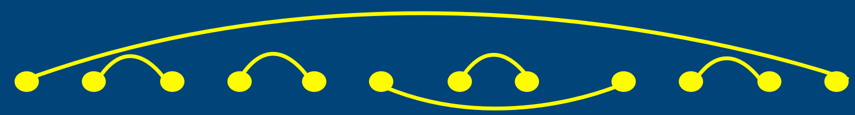


In phase 2, there is a conventional random singlet phase: asymptotically, only singlet-forming decimations occur

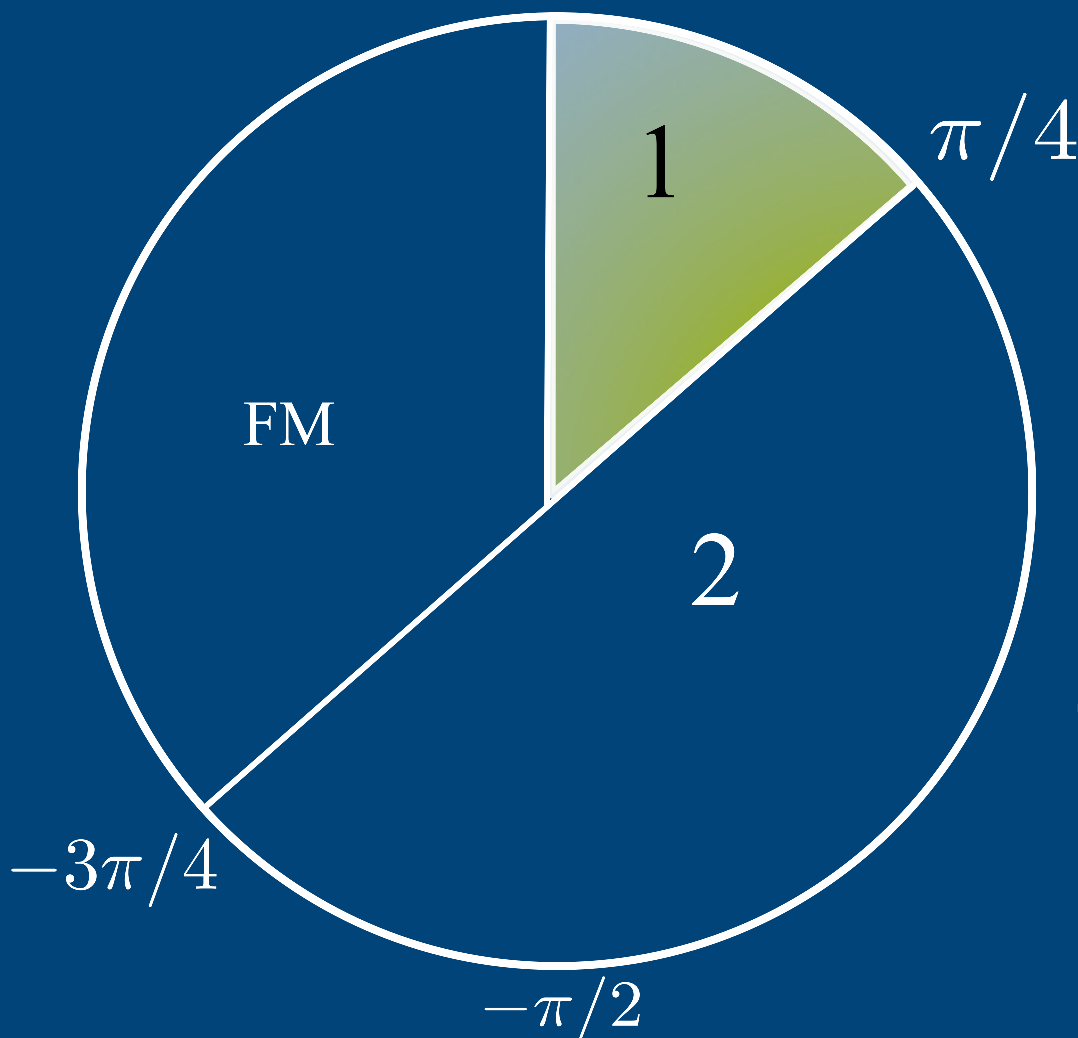


$$\Omega \sim e^{-L^{\psi}}$$

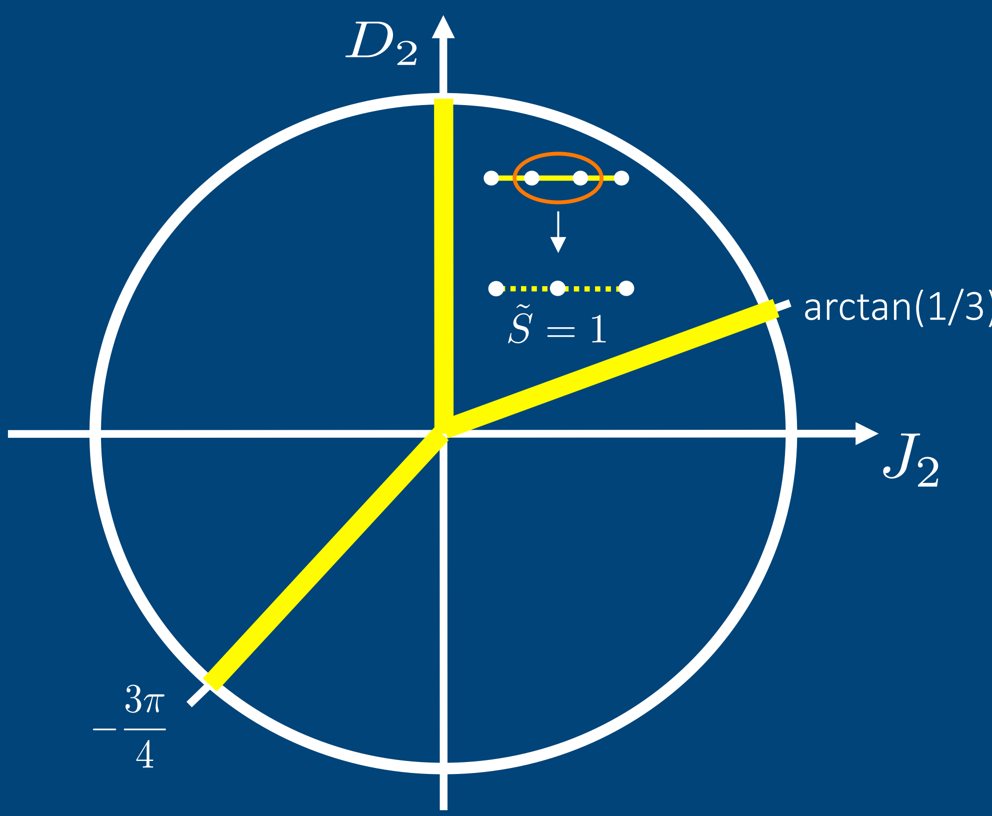
$$\langle \boldsymbol{S}_i \cdot \boldsymbol{S}_j \rangle_{av} \sim \frac{e^{iq(i-j)}}{|i-j|^{\phi}}$$

$$\psi_M = \frac{1}{2} \quad \phi_M = 2$$

Random singlet trios



In phase 1, the ground state is made of random spin trios (and less frequent sextets, etc.). At each step both singlets and spins-1 are formed.



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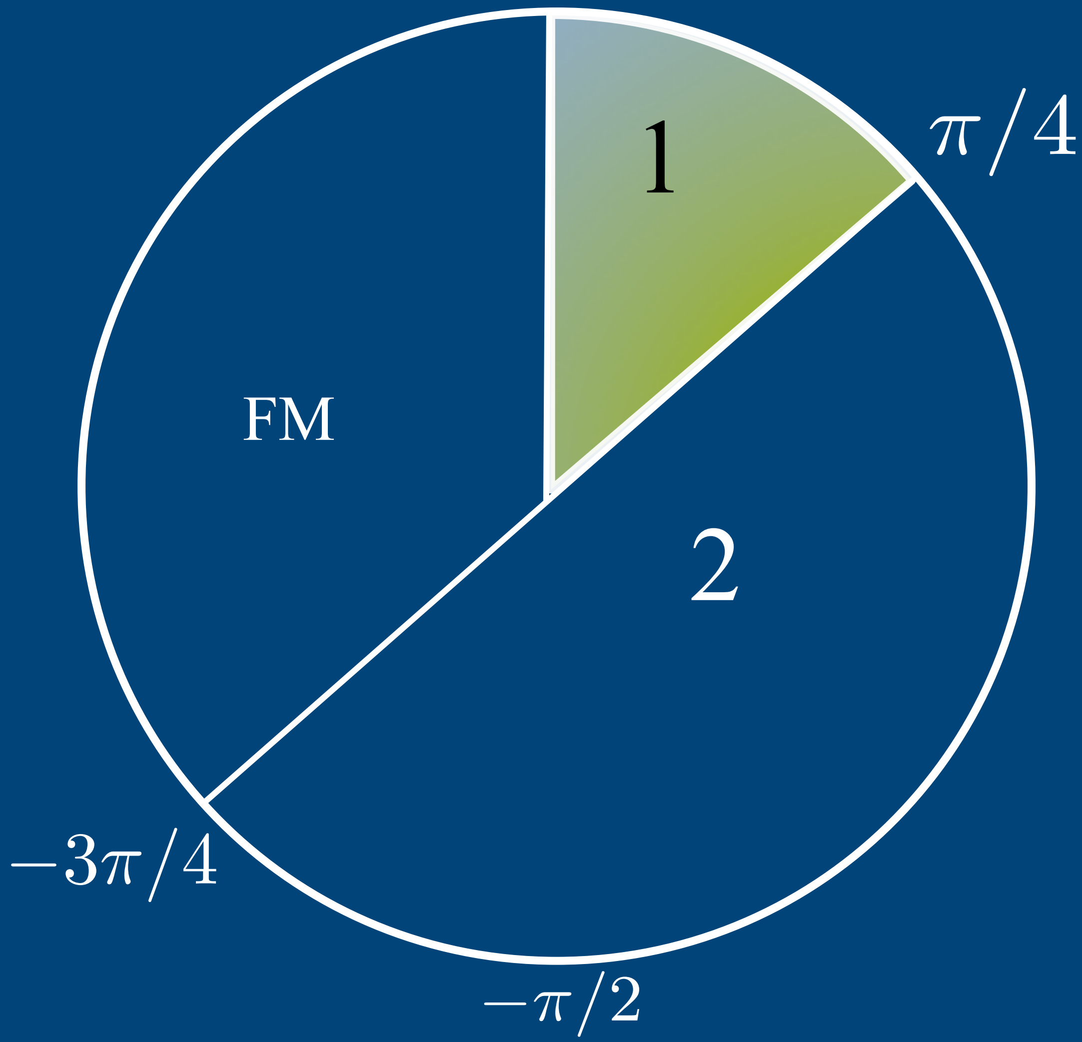

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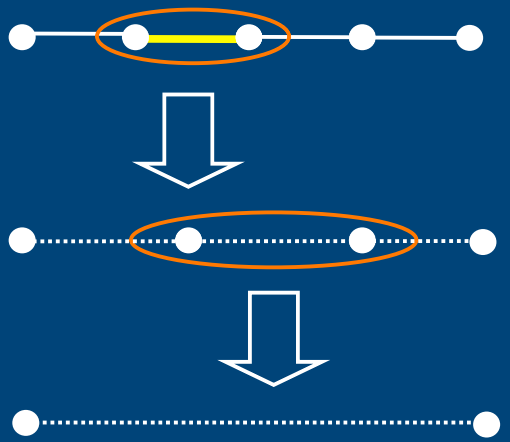

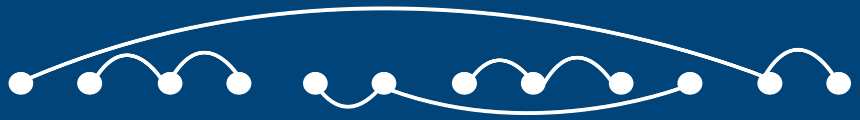

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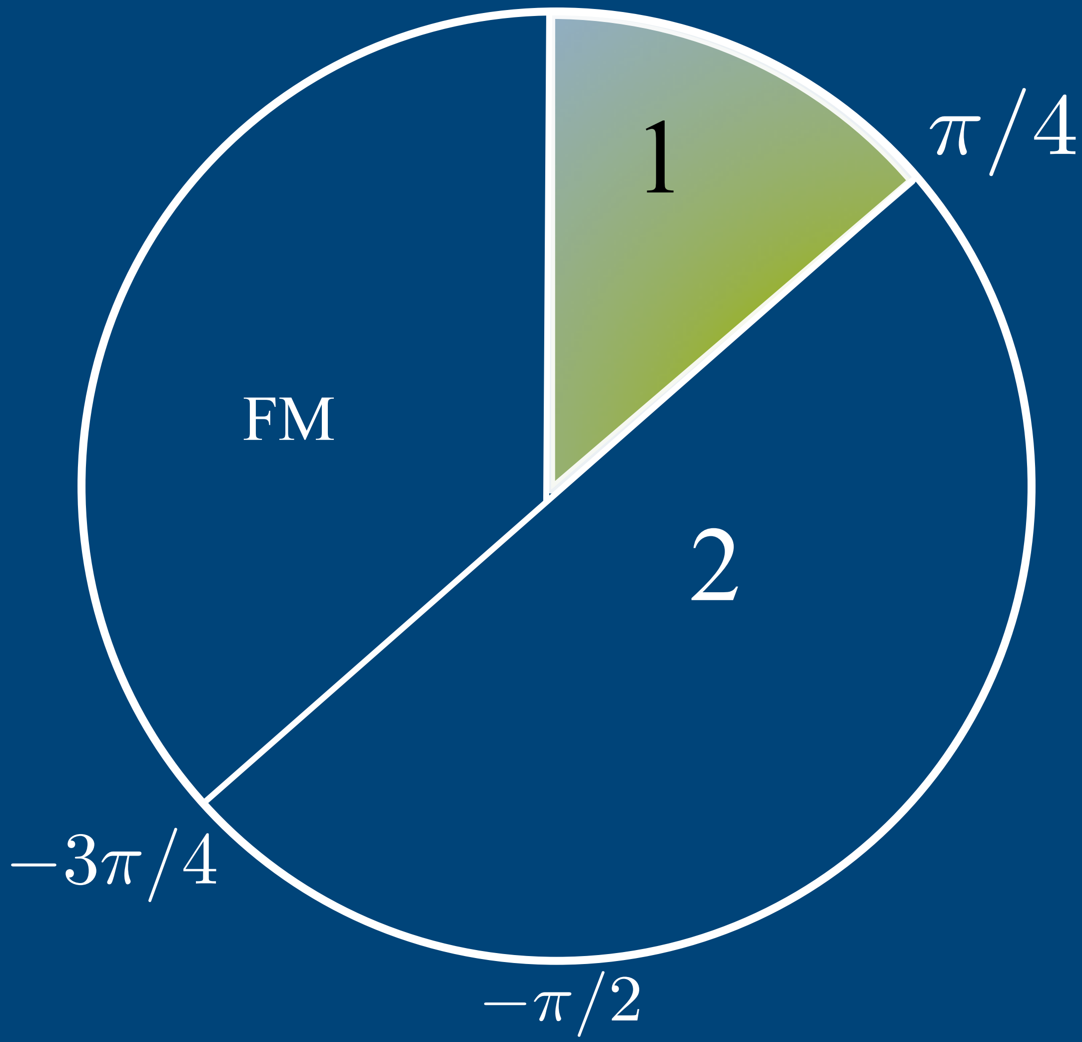

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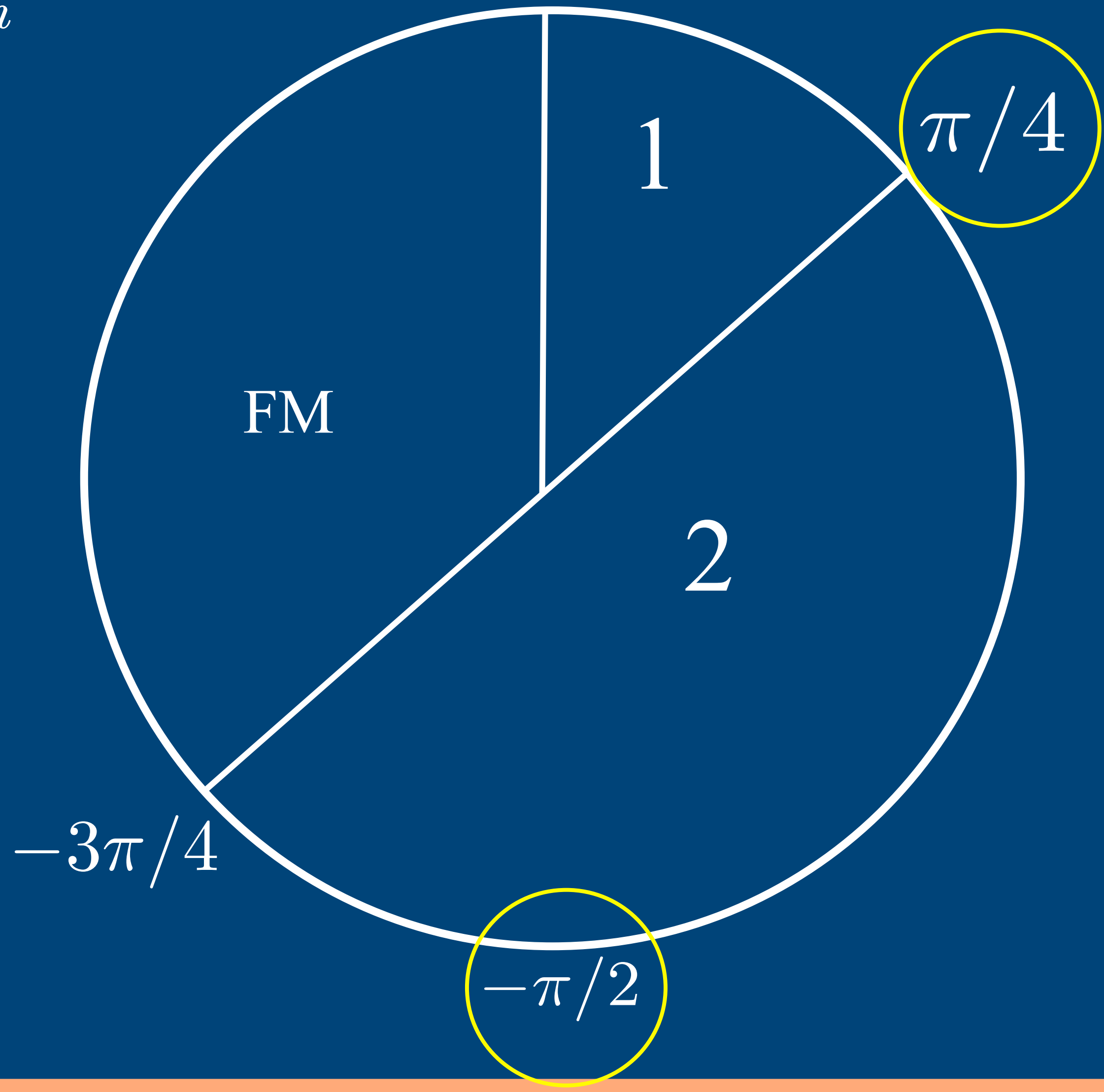
$$\psi_B = \frac{1}{3} \quad \phi_B = \frac{4}{3}$$

Emergent SU(3) symmetry

Special SU(3) points

$$H_{JD} = \sum_i E_i \left[\cos\theta \, \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + \sin\theta \left(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} \right)^2 \right]$$



Where did SU(3) come from?

SU(N): group of N x N unitary matrices with determinant equal to 1

$$\mathbf{U} = e^{i\mathbf{H}}$$ if H is Hermitian and traceless

For N=2, the Pauli matrices are a complete basis for traceless Hermitian matrices:

$$\mathbf{U} = e^{i\boldsymbol{\theta}\cdot\boldsymbol{\sigma}}$$

For N=3, the following 8 spin-1 operators form an analogous complete set:

$$\Lambda_1 = S_x, \quad \Lambda_2 = S_y, \quad \Lambda_3 = S_z,$$

$$\begin{aligned}
\Lambda_4 &= S_x S_y + S_y S_x, \\
\Lambda_5 &= S_x S_z + S_z S_x, \\
\Lambda_6 &= S_y S_z + S_z S_y, \\
\Lambda_7 &= S_x^2 - S_y^2, \\
\Lambda_8 &= \frac{1}{\sqrt{3}}\left(2S_z^2 - S_x^2 - S_y^2\right).
\end{aligned}$$

$$\mathbf{U} = e^{i\sum_{a=1}^{8}\zeta_a \Lambda_a}$$

These 8 operators are the generators of the fundamental ('quark') representation of SU(3).

Where did SU(3) come from?

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$$\Lambda_1 = S_x, \quad \Lambda_2 = S_y, \quad \Lambda_3 = S_z,$$

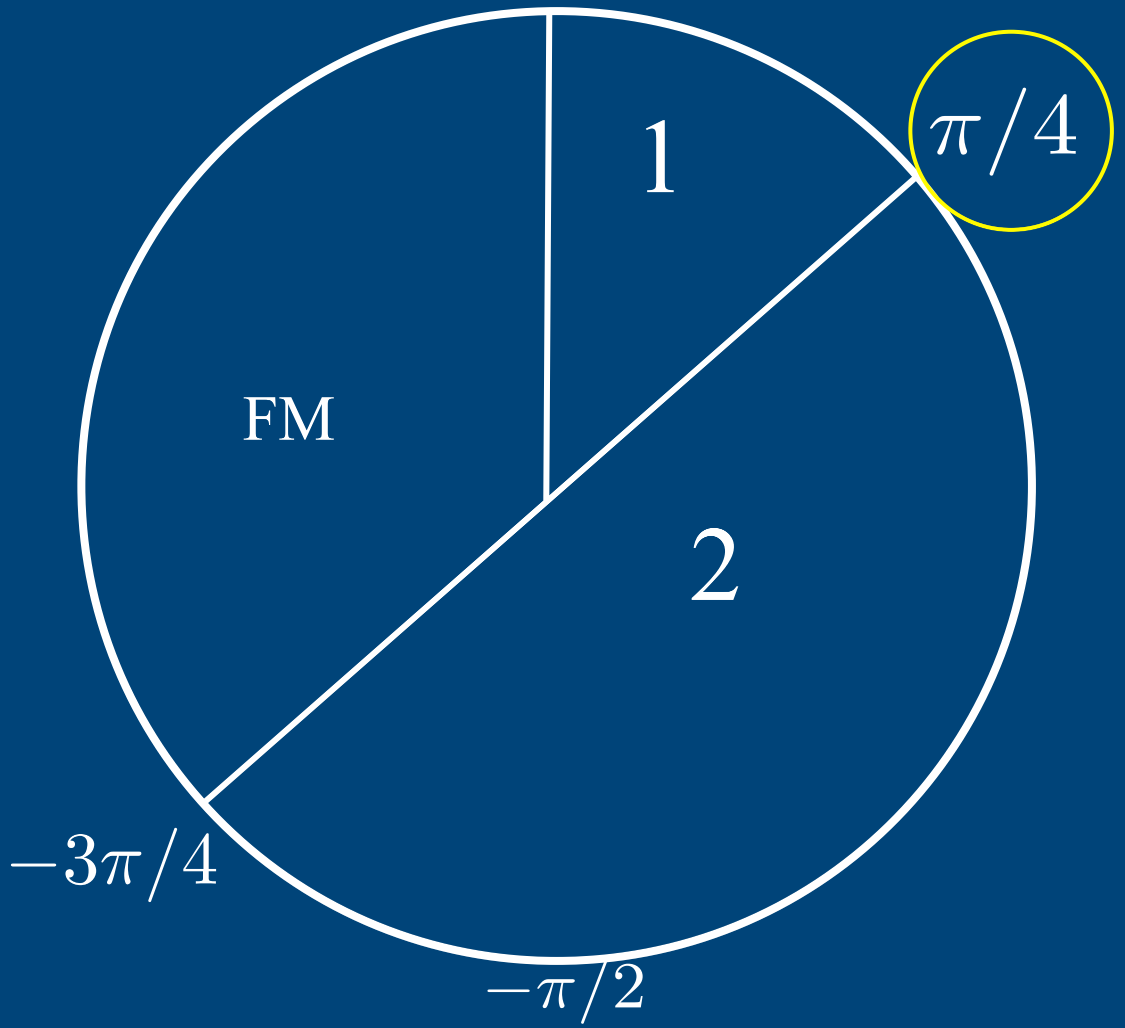
$$\Lambda_4 = -S_xS_y + S_yS_x,$$
$$\Lambda_5 = -S_xS_z + S_zS_x,$$
$$\Lambda_6 = -S_yS_z + S_zS_y,$$
$$\Lambda_7 = -S_x^2 - S_y^2,$$
$$\Lambda_8 = -\frac{1}{\sqrt{3}}\left(2S_z^2 - S_x^2 - S_y^2\right).$$

$$\mathbf{U} = e^{i\sum_{a=1}^{8}\zeta_a\Lambda_a}$$

If we change the sign of Λ_a (a=4,5,6,7,8) we have the 'antiquark' one.

Special SU(3) points

$$H\left(\theta=\frac{\pi}{4}\right)=\sum_{i,a}E_i\Lambda_{i,a}\Lambda_{(i+1),a}=\sum_i E_i\boldsymbol{\Lambda}_i\cdot\boldsymbol{\Lambda}_{i+1}$$



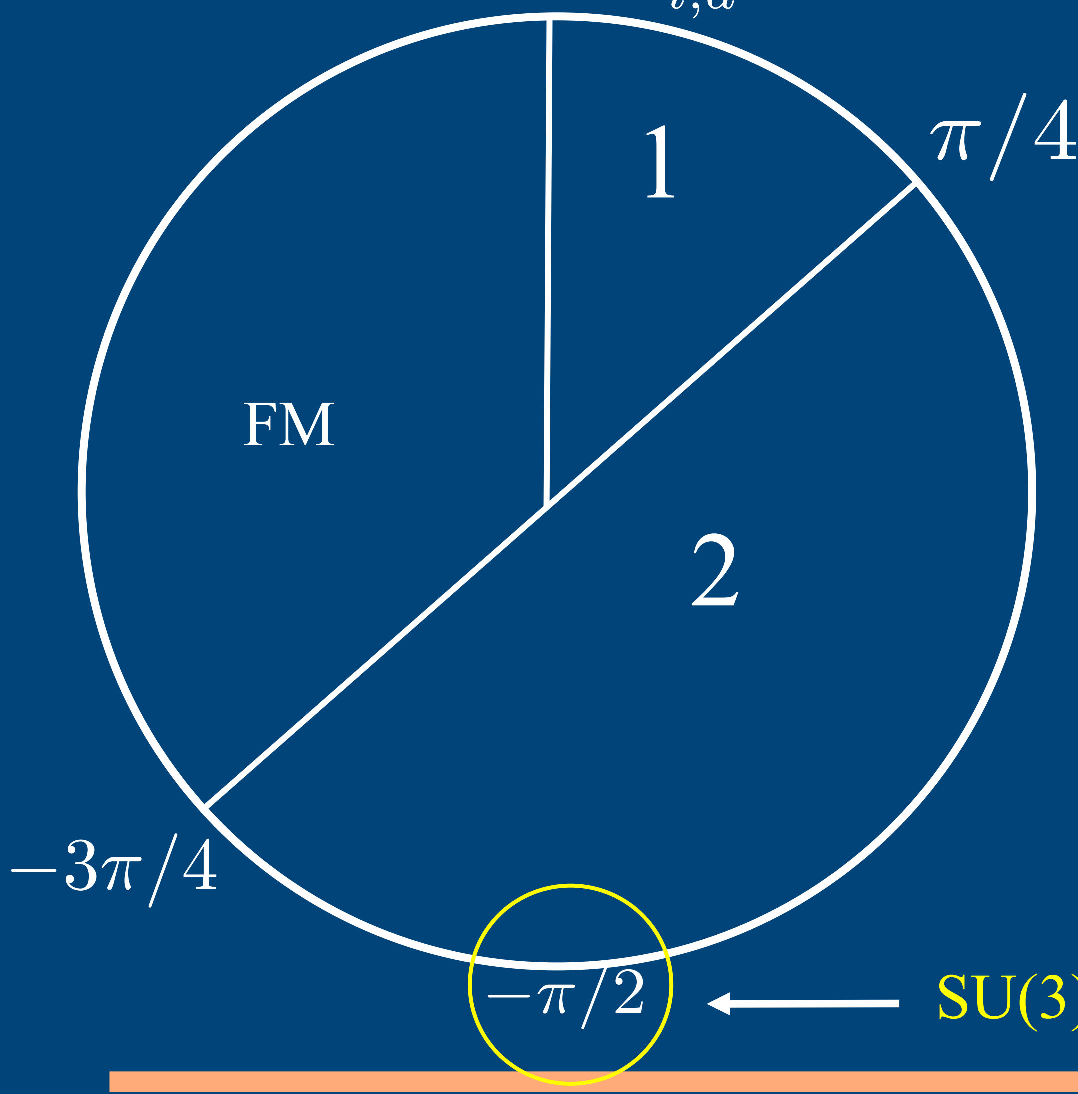
SU(3) - (quark-quark)

q — q — q — q — q — q — q

Quarks disguised as spins!

Special SU(3) points

$$H\left(\theta=-\frac{\pi}{2}\right)=\sum_{i,a}E_i\Lambda_{i,a}\Lambda_{(i+1),a}=\sum_i E_i\mathbf{\Lambda}_i\cdot\mathbf{\Lambda}_{i+1}$$



SU(3) - (quark-antiquark)

Quarks/antiquarks disguised as spins!

$\bar{q} — q — \bar{q} — q — \bar{q} — q — \bar{q}$

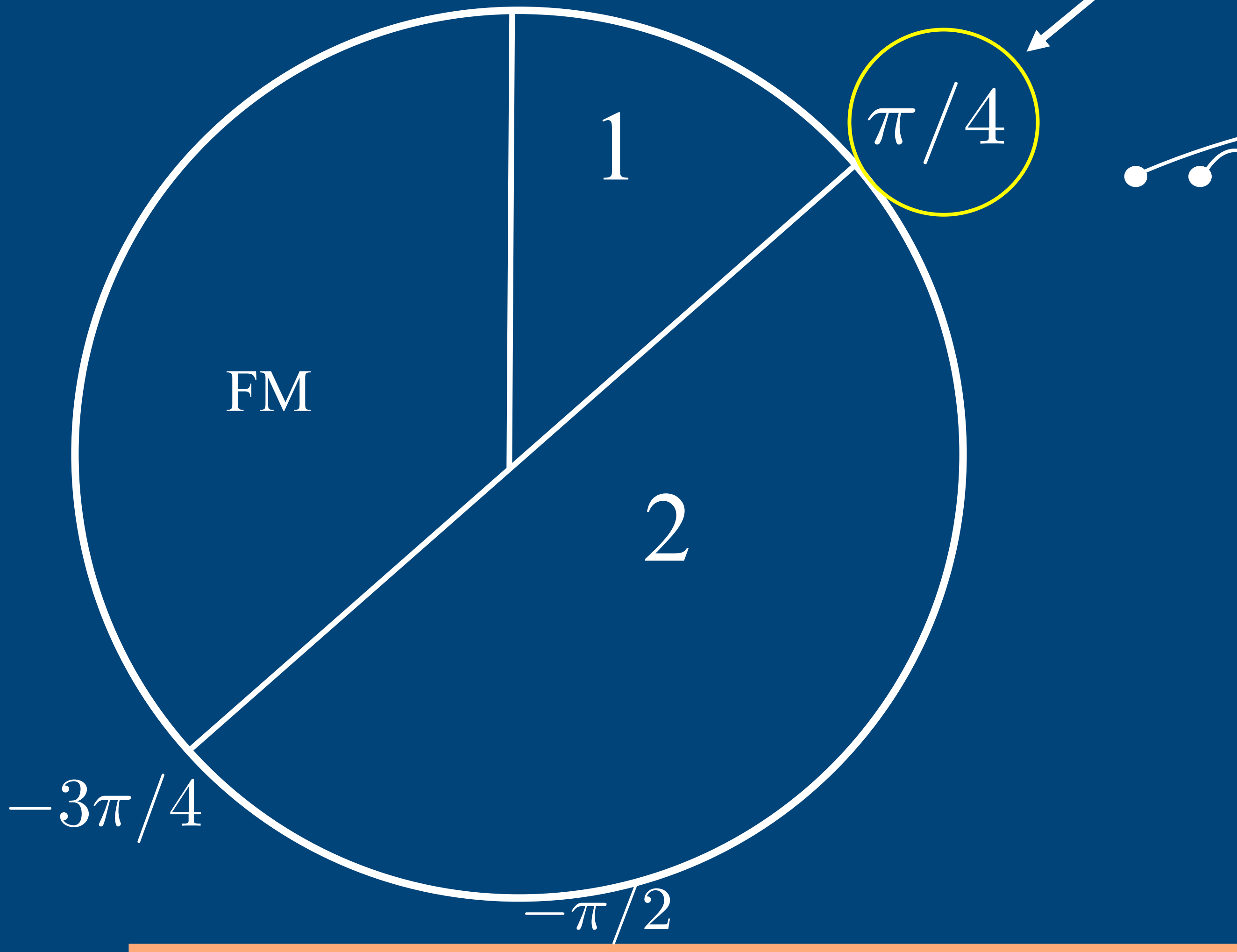
Special SU(3) points

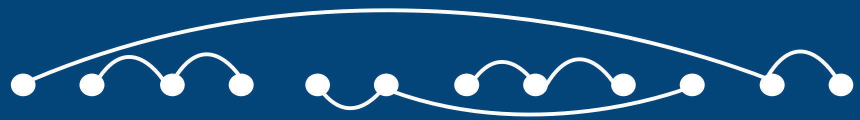
J. A. Hoyos and E. M., PRB **70**, 180401(R) (2004)

SU(3) - (quark-quark)

q — q — q — q — q — q — q

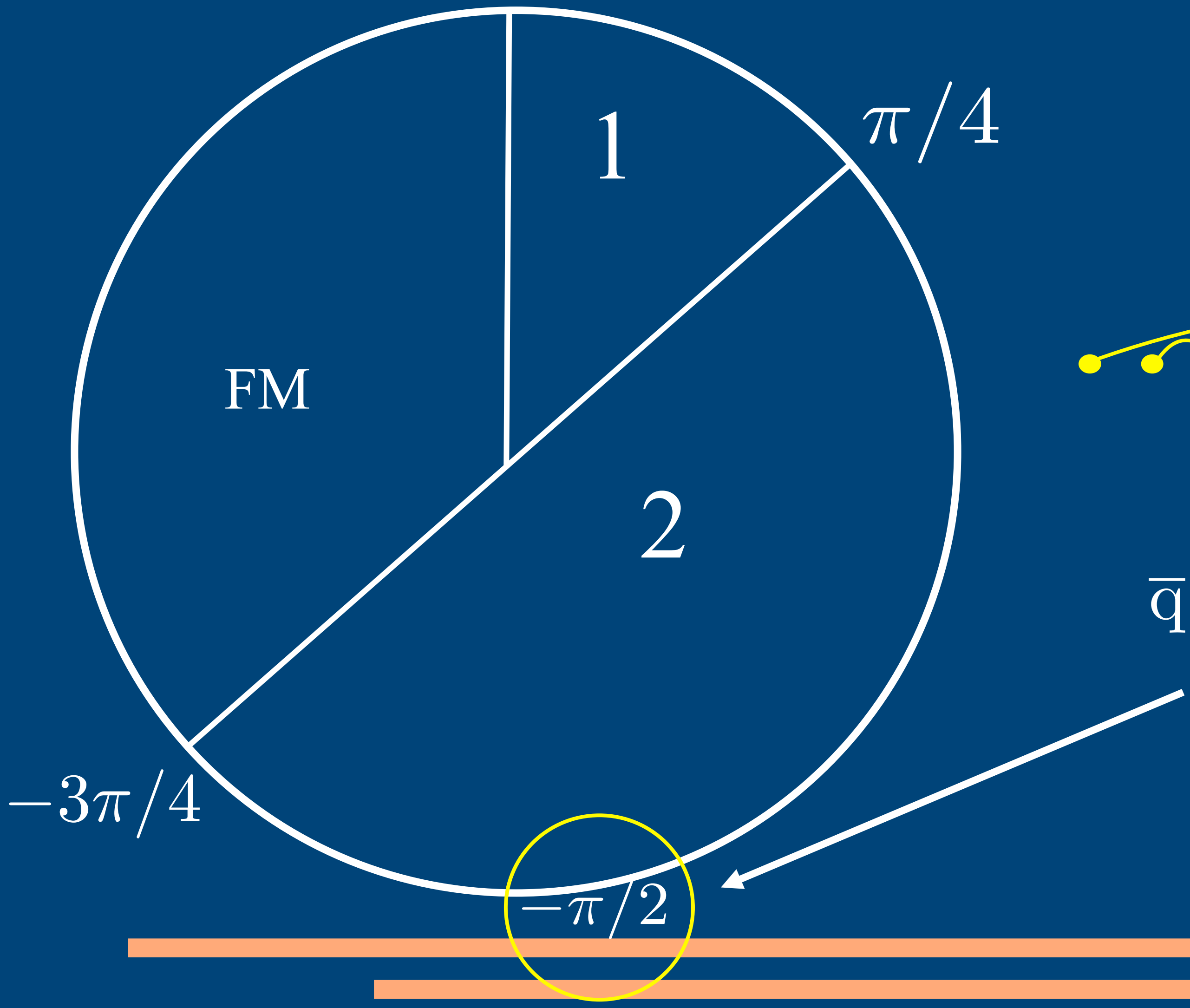
$$\psi_B = \frac{1}{3} \quad \phi_B = \frac{4}{3}$$

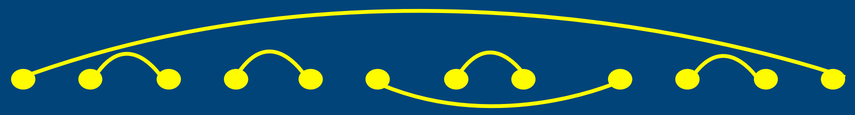




Special SU(3) points

J. A. Hoyos and E. M., PRB **70**, 180401(R) (2004)



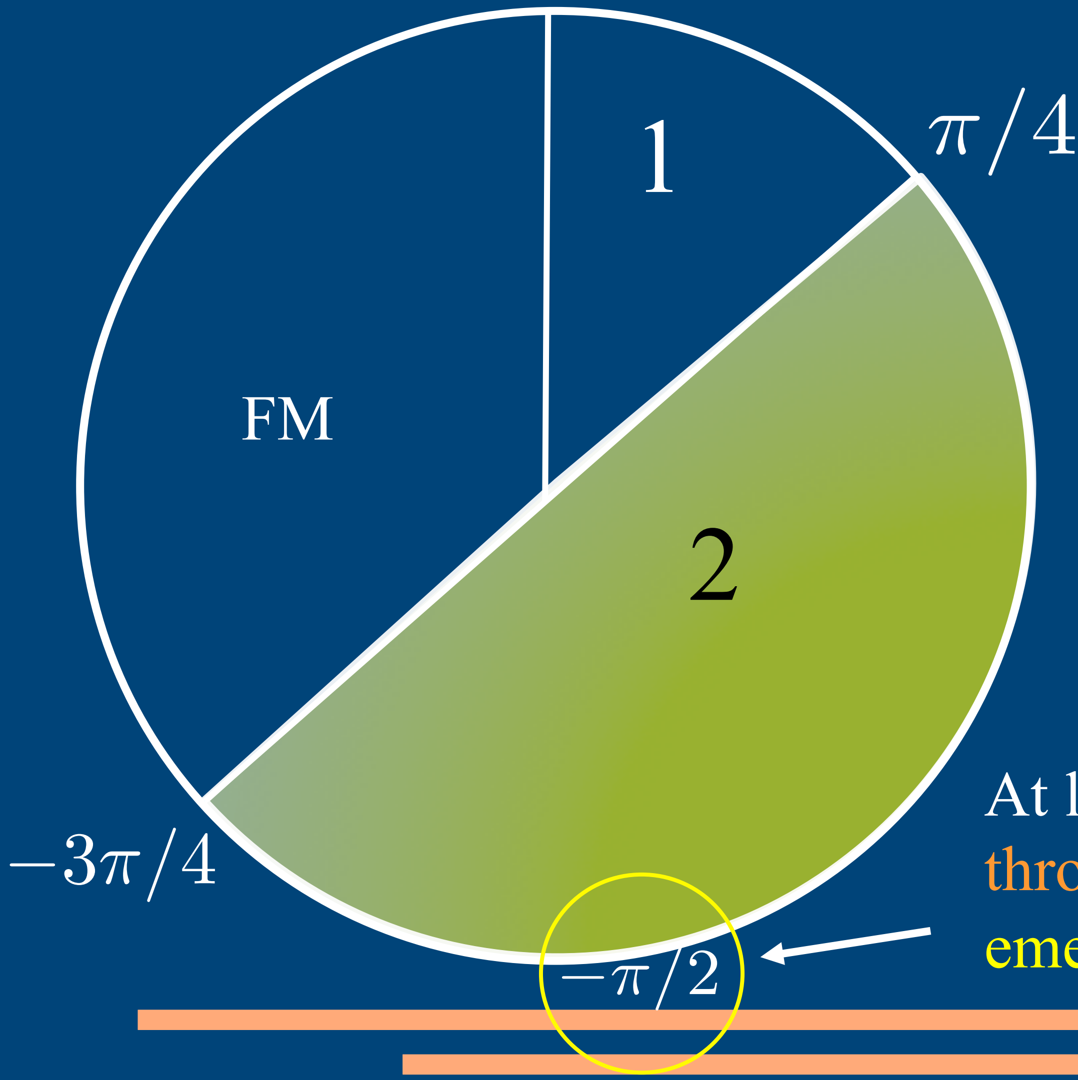


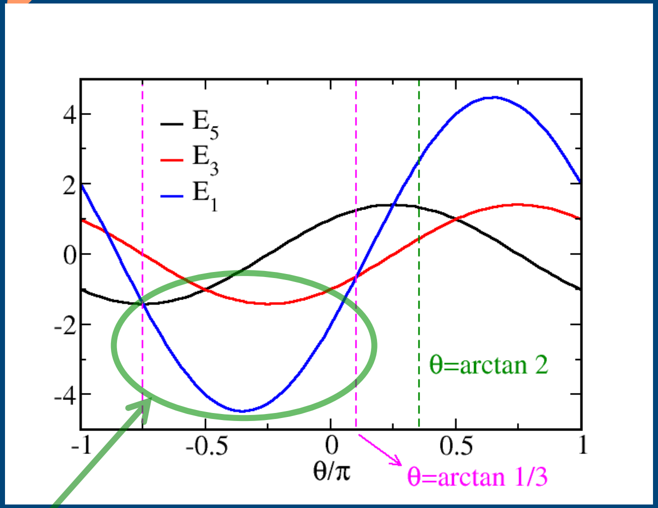
SU(3) - (quark-antiquark)

$$\overline{\mathrm{q}} - \mathrm{q} - \overline{\mathrm{q}} - \mathrm{q} - \overline{\mathrm{q}} - \mathrm{q} - \overline{\mathrm{q}}$$

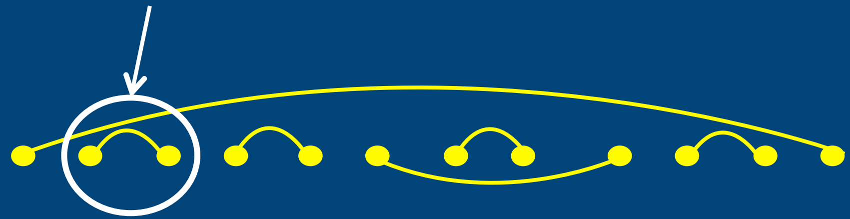
$$\psi_M = \frac{1}{2} \quad \phi_M = 2$$

Emergent SU(3) symmetry





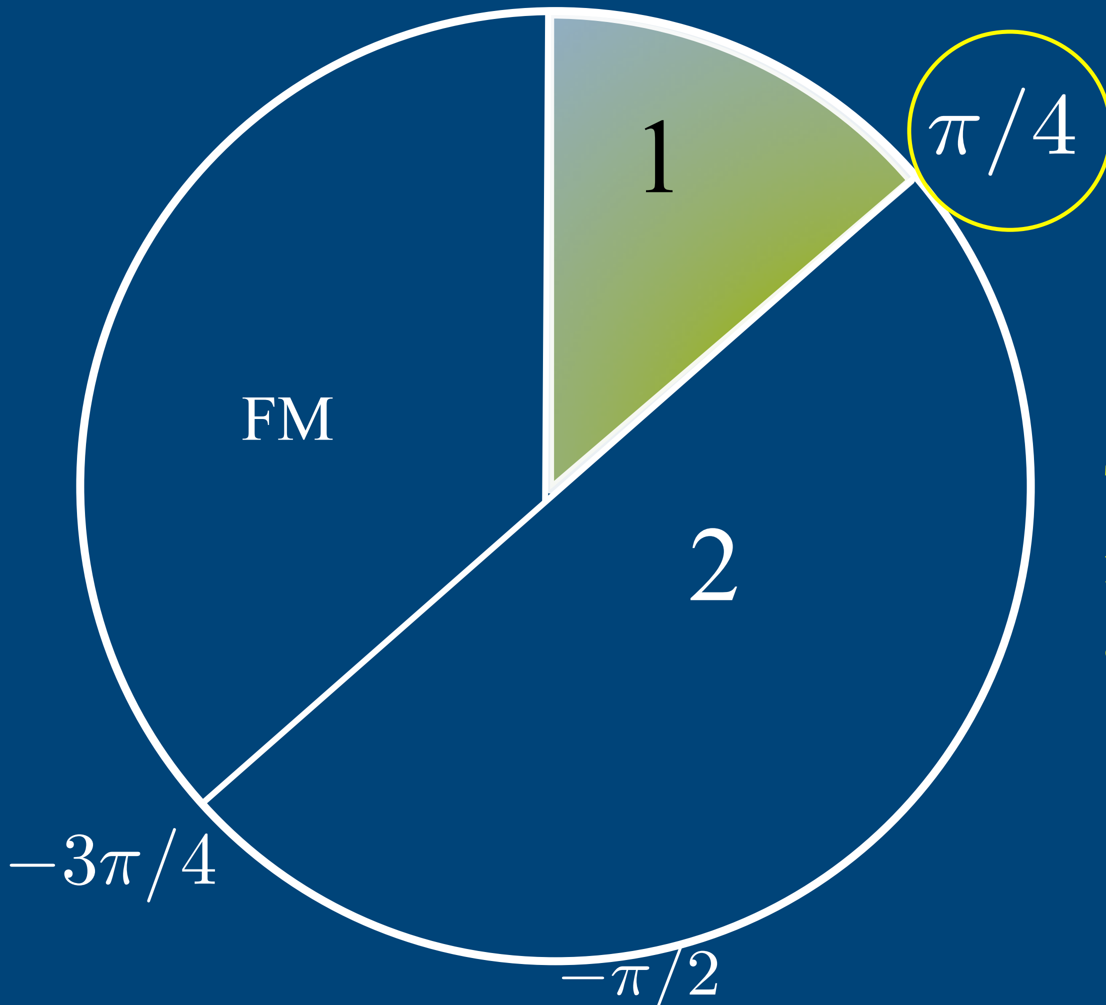
The pair singlet is angle-independent: it is both an SU(2) and an SU(3) singlet.

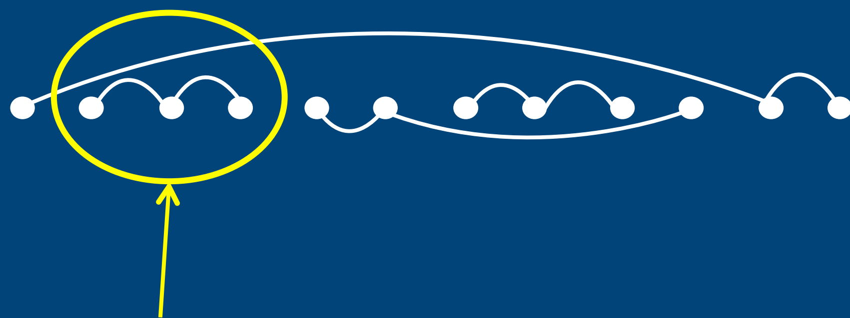


At long length scales, the **state** is the same throughout region 2, including –π/2: emergent SU(3)

Emergent SU(3) symmetry

At long length scales, the **state** is the same throughout region 1, including π/4: emergent SU(3)

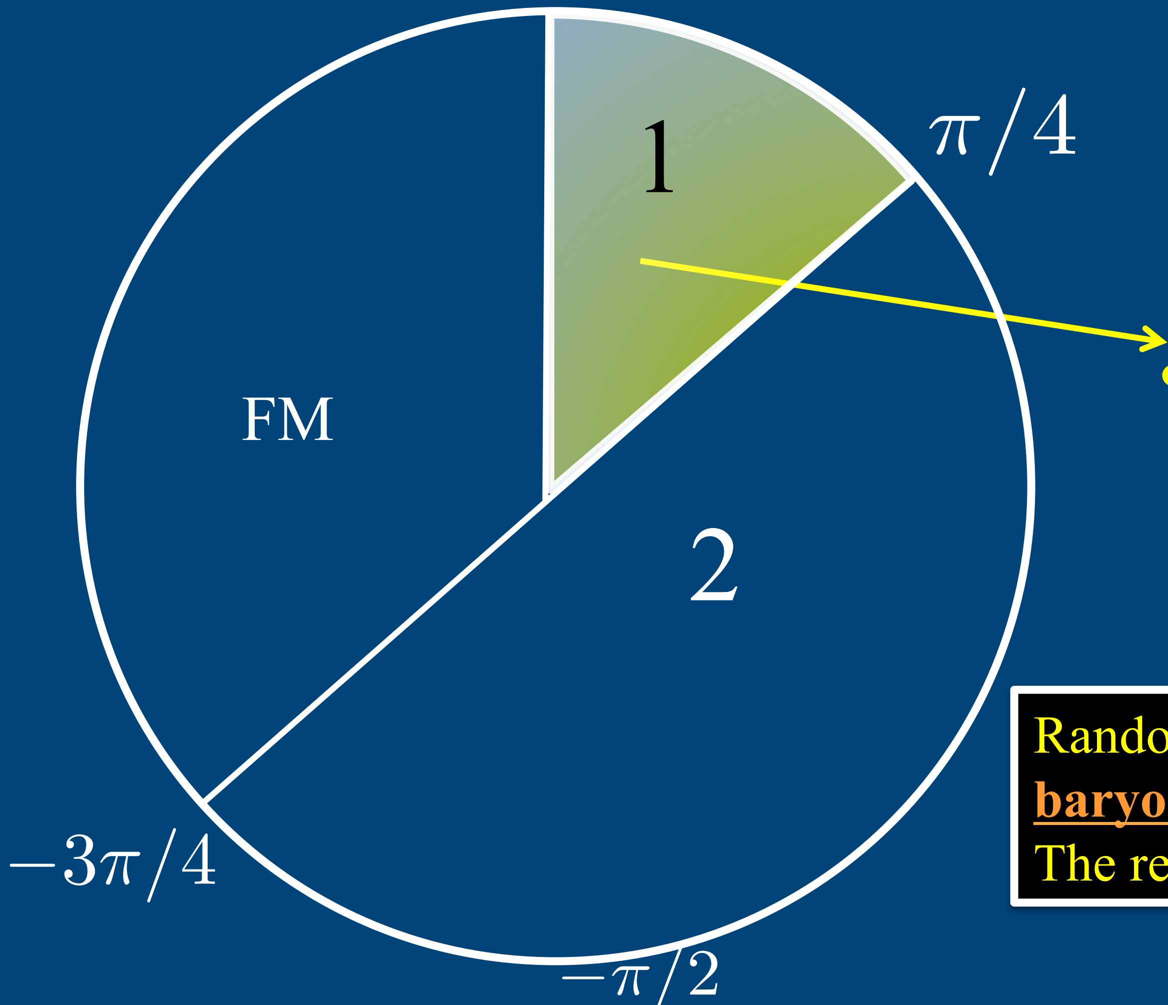


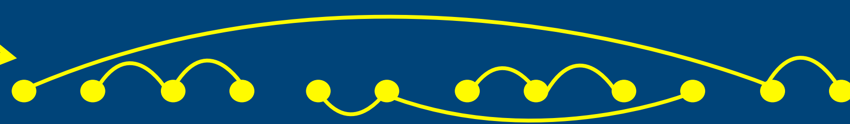


The trio singlet is also angle-independent: it is both an SU(2) and an SU(3) singlet.

Baryonic random singlet phase

$$H = \sum_i E_i \left[\cos\theta \, \mathbf{S}_i \cdot \mathbf{S}_{i+1} + \sin\theta \, (\mathbf{S}_i \cdot \mathbf{S}_{i+1})^2\right]$$

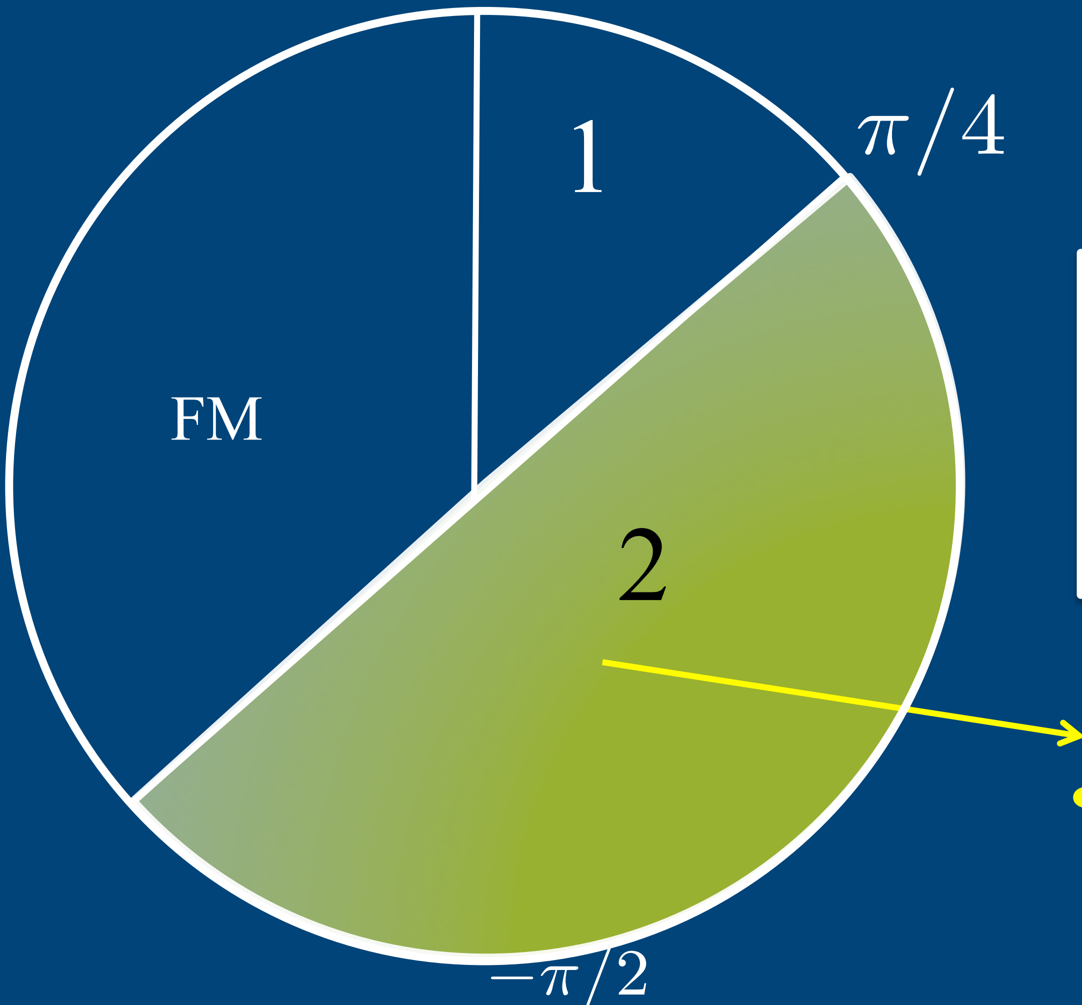




Random three-quark SU(3) singlets (baryons):
baryonic random singlet phase
The response of the state is SU(3) symmetric

Mesonic random singlet phase

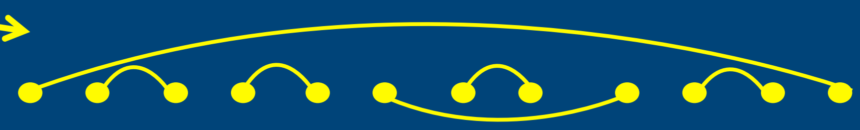
$$H = \sum_i E_i \left[\cos\theta\, \mathbf{S}_i \cdot \mathbf{S}_{i+1} + \sin\theta\, (\mathbf{S}_i \cdot \mathbf{S}_{i+1})^2\right]$$



Random quark-antiquark SU(3) singlets (mesons):

mesonic random singlet phase

Again, the response of the state is SU(3) symmetric



Emergent SU(3) symmetry

$$\langle \boldsymbol{S}_i \cdot \boldsymbol{S}_j \rangle_{av} \sim \frac{e^{iq(i-j)}}{|i-j|^{\phi}} \Rightarrow \langle \Lambda_{ai} \Lambda_{aj} \rangle_{av} \sim \frac{e^{iq(i-j)}}{|i-j|^{\phi}} \quad (a = 1, \ldots, 8)$$

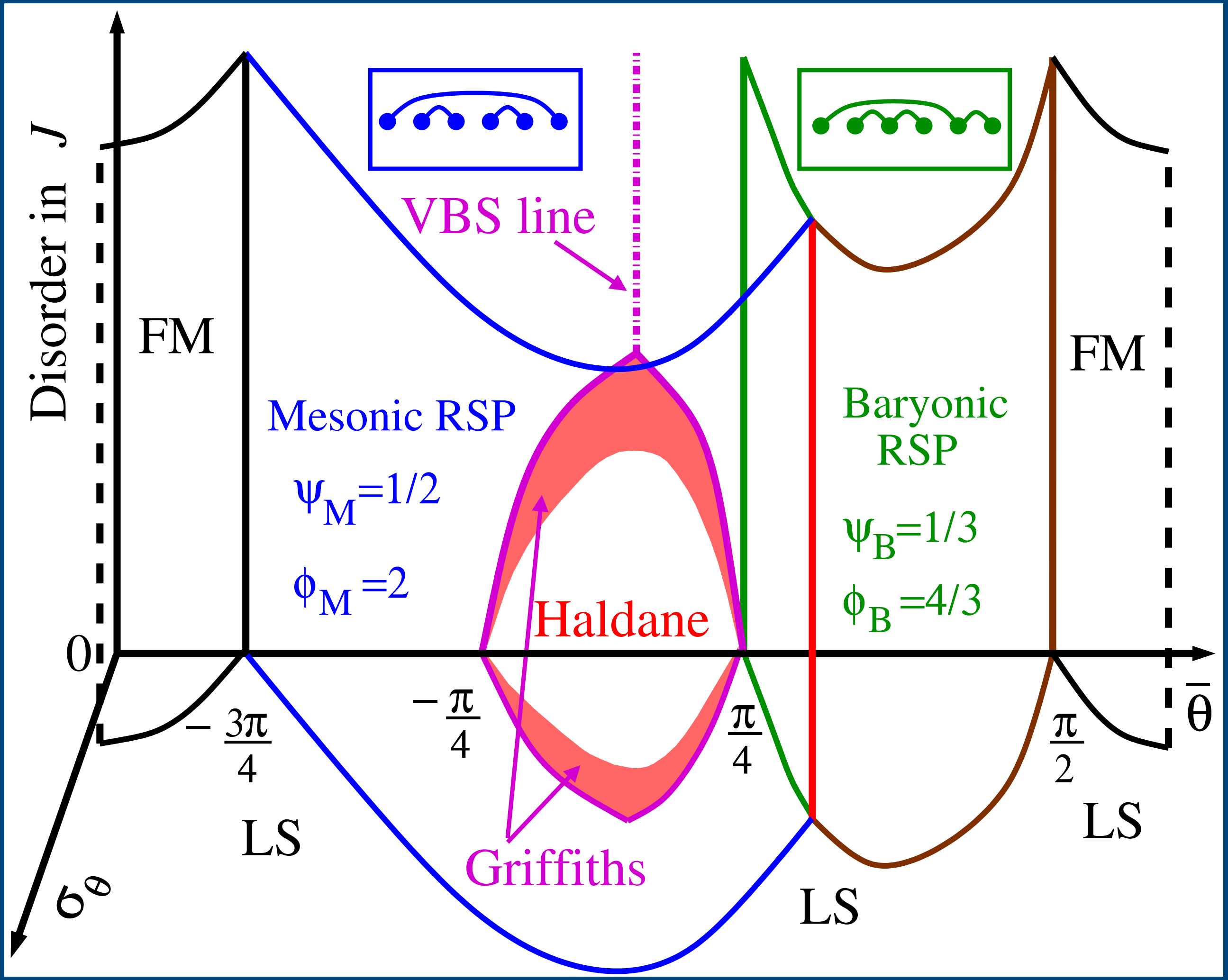
$$\begin{aligned}
\Lambda_1 &= S_x, \\
\Lambda_2 &= S_y, \\
\Lambda_3 &= S_z, \\
\Lambda_4 &= S_x S_y + S_y S_x, \\
\Lambda_5 &= S_x S_z + S_z S_x, \\
\Lambda_6 &= S_y S_z + S_z S_y, \\
\Lambda_7 &= S_x^2 - S_y^2, \\
\Lambda_8 &= \frac{1}{\sqrt{3}} \left(2S_z^2 - S_x^2 - S_y^2 \right).
\end{aligned}$$

But note:

- The exponents are all the same.
- The numerical pre-factors are the same only at the SU(3) points or at very strong initial disorder. The emergent SU(3) only appears asymptotically, the procedure is inaccurate at the beginning of the flow.

Full phase diagram

Allowing for spatial fluctuations of θ in the initial distribution



Can this be a more general phenomenon?

Chains with spins $S > 1$: (V. L. Quito, J. A. Hoyos, E.M., arXiv:1512.04542)

$$H = \sum_i \alpha_i^1 (\mathbf{S}_i \cdot \mathbf{S}_{i+1}) + \alpha_i^2 (\mathbf{S}_i \cdot \mathbf{S}_{i+1})^2 + \ldots + \alpha_i^{2S} (\mathbf{S}_i \cdot \mathbf{S}_{i+1})^{2S}$$

- Conventional random singlet phases with SU(2S+1) symmetry: only spin pairs (mesons), $\psi = ½$,…
- Phases with $\psi = 1/3$, but **no** emergent higher symmetry.

Can we find baryonic phases (more than two spins per singlet) with emergent symmetries?

Can this be a more general phenomenon?

But $S = 1$ is also the fundamental representation of SO(3).

- So instead of

$$\mathrm{SU}(2) \text{ with } S=1 \rightarrow \mathrm{SU}(2) \text{ with } S > 1$$

- we tried

$$\mathrm{SO}(3) \rightarrow \mathrm{SO}(N) \text{ (with the fundamental repres.)}$$

(V. L. Quito, P. L. S. Lopes, J. A. Hoyos, E.M., in progress)

Can this be a more general phenomenon?

$$H_{SO(N)} = \sum_i \left[J_i \sum_{a<b} L_i^{ab} L_{i+1}^{ab} + D_i \left(\sum_{a<b} L_i^{ab} L_{i+1}^{ab} \right)^2 \right]$$

- Most general SO(N) invariant Hamiltonian.
- L^{ab} generates rotations is the *ab* plane

$$L^{ab} \left| b \right\rangle = i \left| a \right\rangle$$

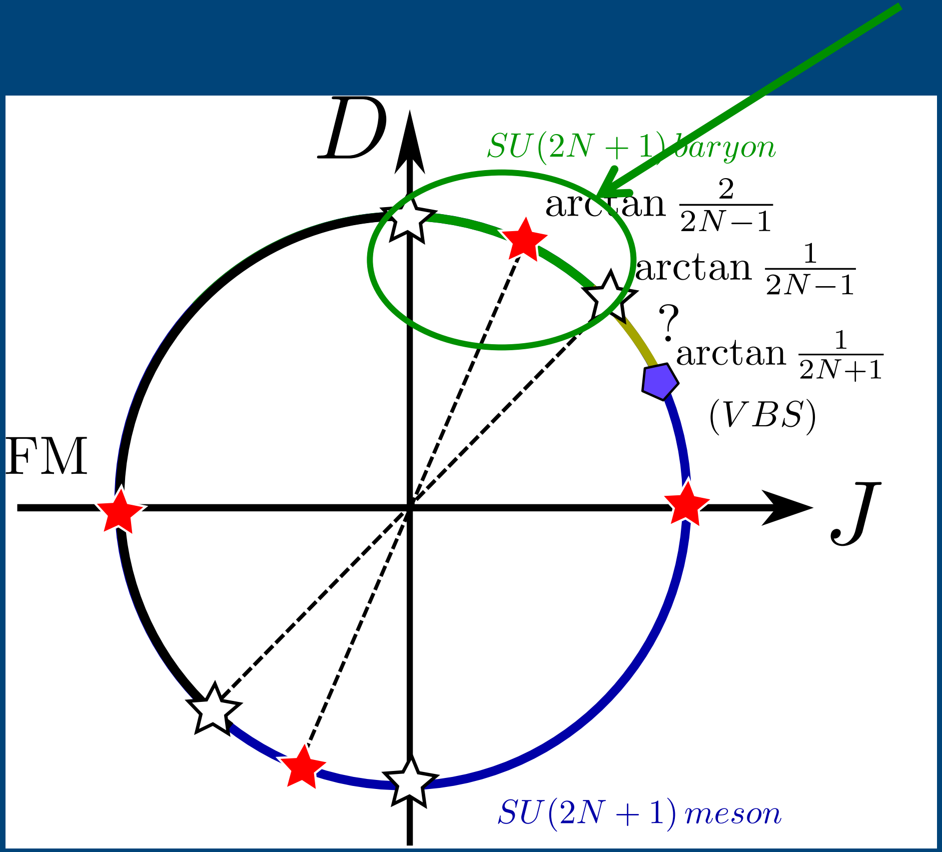
$$L^{ab} \left| a \right\rangle = -i \left| b \right\rangle$$

$$L^{ab} \left| c \right\rangle = 0 \quad c \neq a, b$$

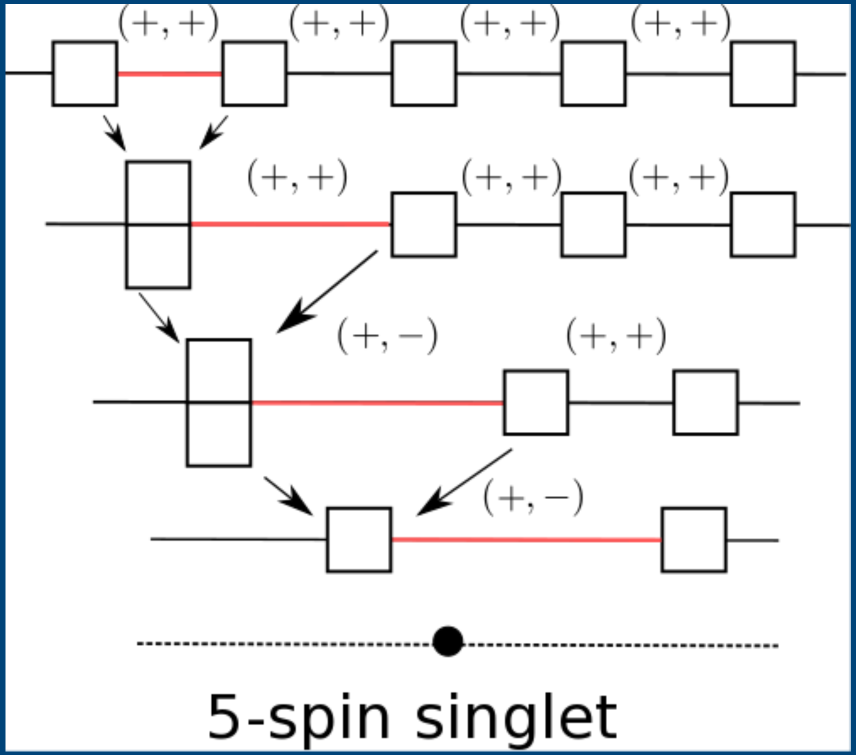
Note that SO(5) symmetry can be realized in S=3/2 fermionic cold atoms (C. Wu PRL **95**,155115 (2005))

Emergent SU(2N+1) in SO(2N+1) chains

Baryonic phase with $\psi = 1/(2N+1)$ and emergent SU(2N+1) symmetry



SO(5) case



The case of SO(2N) is more involved and is still in progress…

Conclusions

- Infinite effective disorder in spin-1/2 and spin-1 chains.
- Emergence of SU(3) symmetry in an SU(2)-invariant system:
  Hadrons in condensed matter physics.
- Emergence of SU(2N+1) symmetry in SO(2N+1) chains:
  Composite singlets with 2N+1 constituents.