# Holographic duality, strange metals and entanglement.


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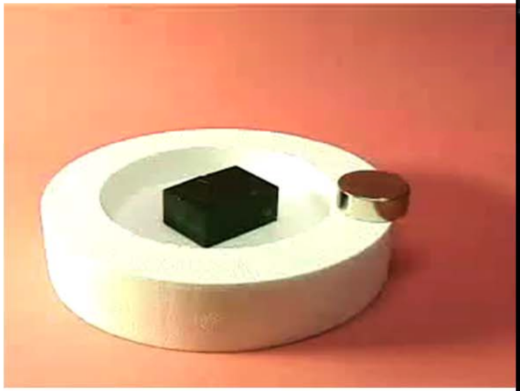
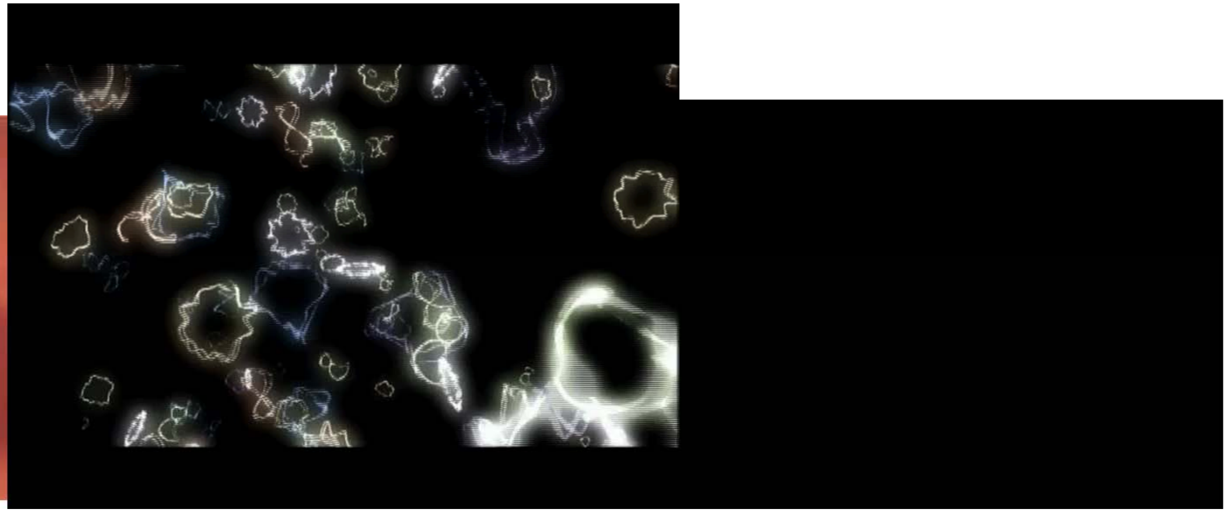
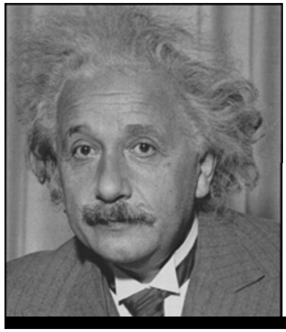
Universiteit Leiden

Instituut-Lorentz for theoretical physics

JOHN TEMPLETON FOUNDATION
SUPPORTING SCIENCE - INVESTING IN THE BIG QUESTIONS

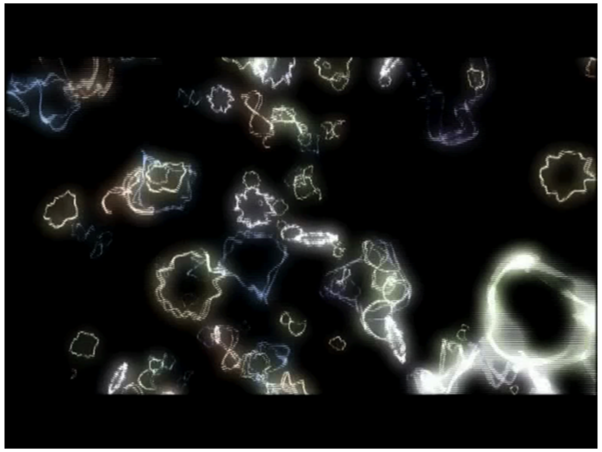





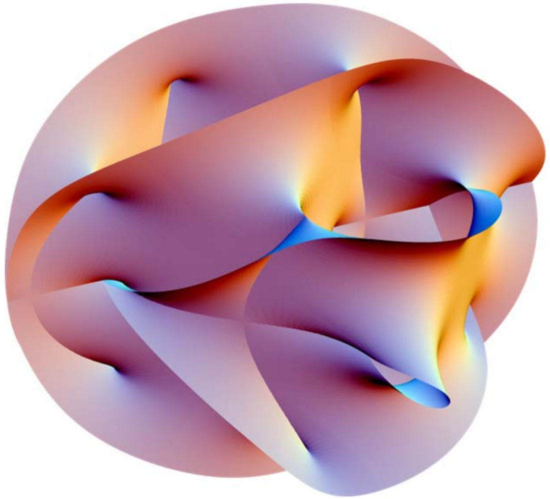


# String theory

$$\mathcal{S} = \frac{T}{2}\int \mathrm{d}^2\sigma \sqrt{-h}\, h^{ab} g_{\mu\nu}(X)\partial_a X^\mu(\sigma)\partial_b X^\nu(\sigma)$$



Quantum physics of strings



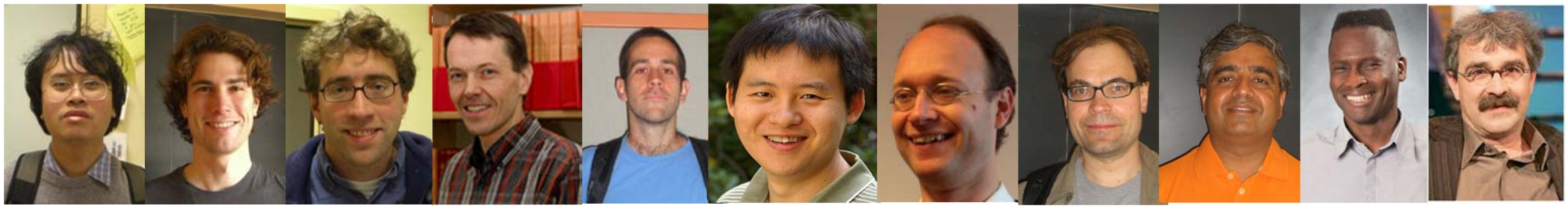
Beautiful mathematics (Calabi-Yau, …)

Quantum space-time (big bang, …)

# String theory: what is it really good for?

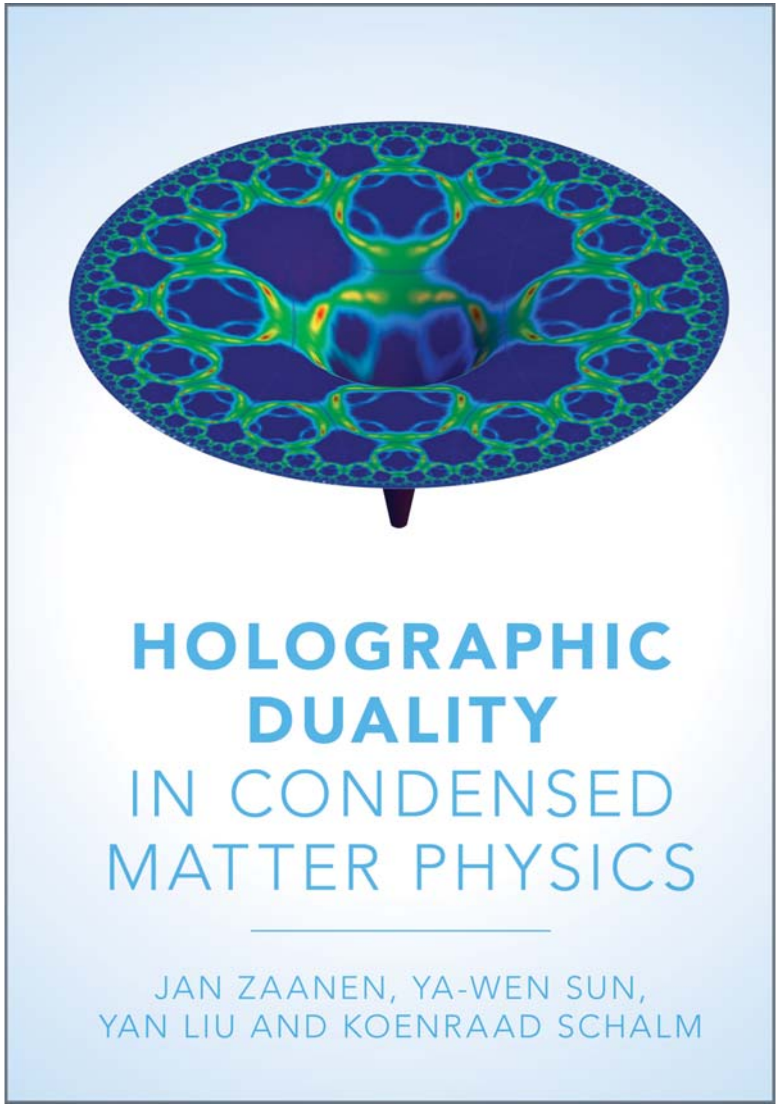
**- Quantum matter: heavy fermion systems, high Tc superconductors, CMR manganites…**


Polchinski Kachru Starinets Erdmenger Gubser Horowitz Kiritsis Gauntlett Policastro Tong



Son Hartnoll Herzog Thoracius McGreevy Liu Schalm Karch Sachdev Phillips Zaanen

# Book sales …



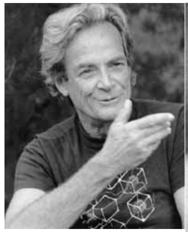

**Cambridge University Press**

**Release: October 28 2015.**

**It is 600 pages and only € 80!**

# Quantum field theory = Statistical physics.

$$Z = \sum_{configs.} e^{-\frac{E_{config}}{k_B T}}$$



**Path integral mapping**

**"Thermal QFT", Wick rotate:**

$$t \to i\tau$$

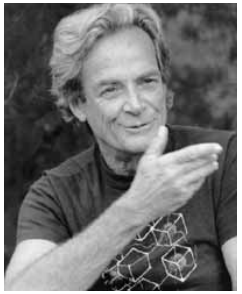
$$Z_\hbar = \sum_{worldhistories} e^{-\frac{S_{history}}{\hbar}}$$

**But generically: the quantum partition function is not probabilistic: "sign problem", no mathematical control!**

$$Z_\hbar = \sum_{worldhistories} (-1)^{history} e^{-\frac{S_{history}}{\hbar}}$$

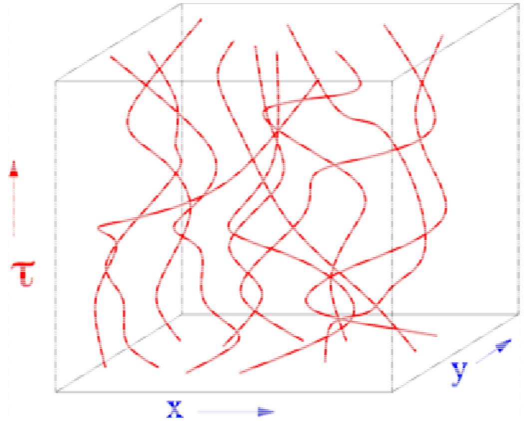
# Fermions at a finite density: the sign problem.

Imaginary time first quantized path-integral formulation



$$\begin{aligned}\mathcal{Z} &= \mathrm{Tr}\exp(-\beta\hat{\mathcal{H}}) \\ &= \int \mathrm{d}\mathbf{R}\rho(\mathbf{R},\mathbf{R};\beta)\end{aligned}$$

$$\mathbf{R} = (\mathbf{r}_1, \ldots, \mathbf{r}_N) \in \mathbb{R}^{Nd}$$




$$\begin{aligned}\rho_{B/F}(\mathbf{R},\mathbf{R};\beta) &= \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\rho_D(\mathbf{R},\mathcal{P}\mathbf{R};\beta) \\ &= \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\int_{\mathbf{R}\to\mathcal{P}\mathbf{R}}\mathcal{D}\mathbf{R}(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T}\mathrm{d}\tau\left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau)+V(\mathbf{R}(\tau))\right)\right\}\end{aligned}$$

Boltzmannons or Bosons:

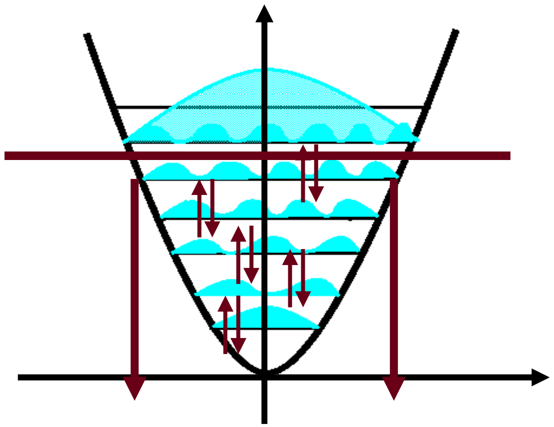
- integrand non-negative
- probability of equivalent classical system: (crosslinked) ringpolymers

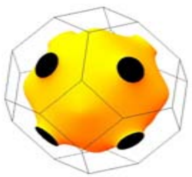
Fermions:

- negative Boltzmann weights
- non probablistic: NP-hard problem (Troyer, Wiese)!!!

# Fermions: the tiny repertoire …

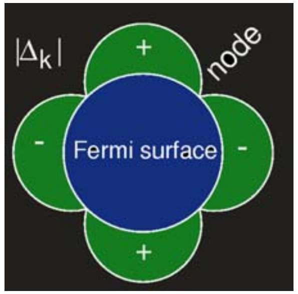
**Fermiology**

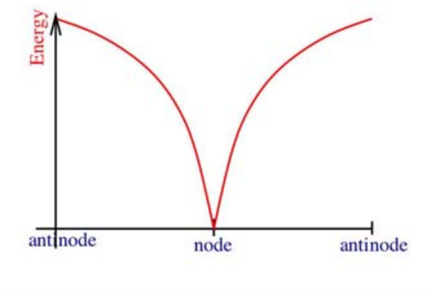




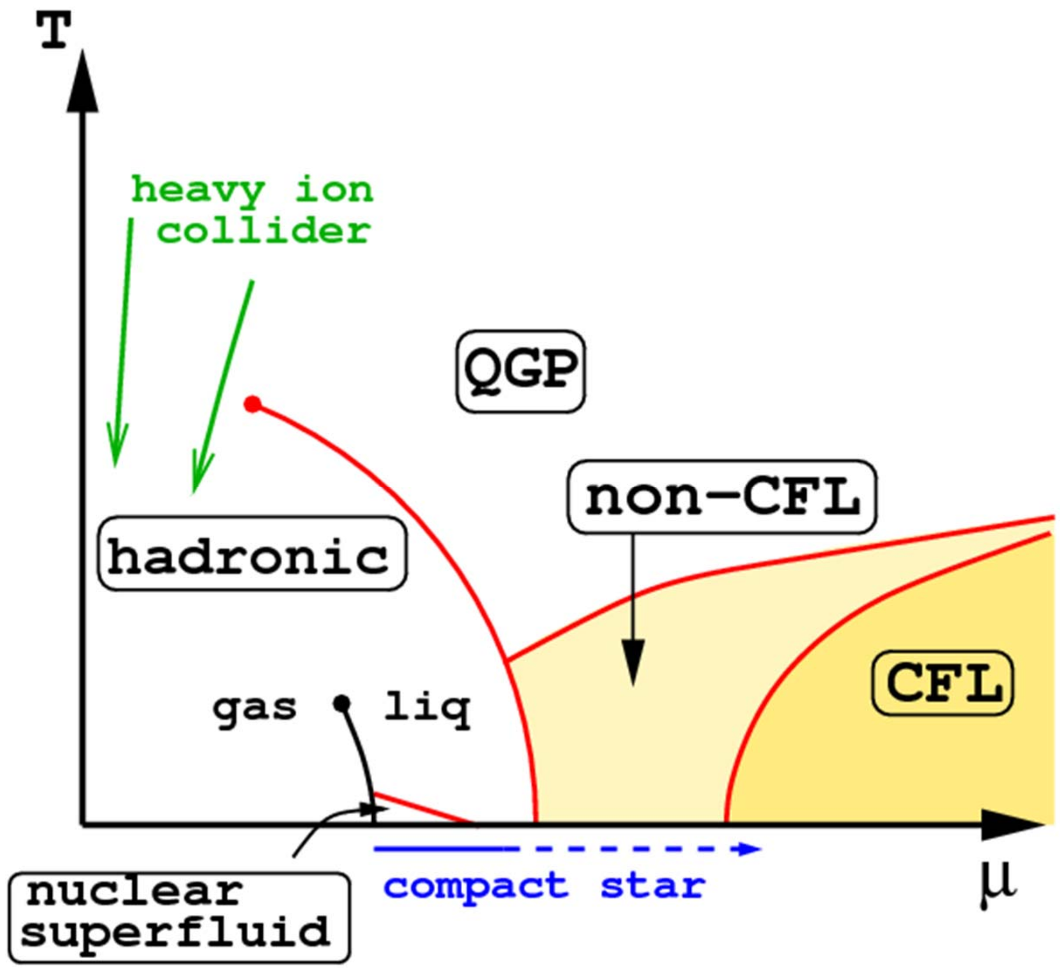
**BCS superconductivity**

$$\Psi_{BCS} = \Pi_k \left( u_k + v_k c^+_{k\uparrow} c^+_{-k\downarrow} \right) \left| vac. \right\rangle$$



# QCD at intermediate density ...



# 29 years later: the “consensus document”.


REVIEW

doi:10.1038/nature14165

## From quantum matter to high-temperature superconductivity in copper oxides

B. Keimer[1], S. A. Kivelson[2], M. R. Norman[3], S. Uchida[4] & J. Zaanen[5]

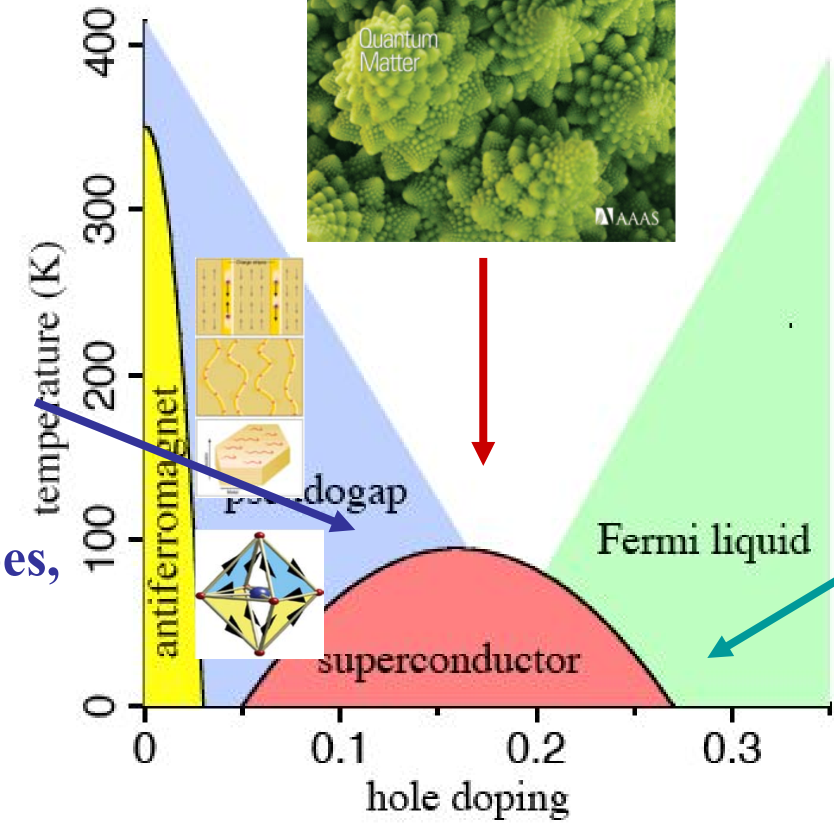
The discovery of high-temperature superconductivity in the copper oxides in 1986 triggered a huge amount of innovative scientific inquiry. In the almost three decades since, much has been learned about the novel forms of quantum matter that are exhibited in these strongly correlated electron systems. A qualitative understanding of the nature of the superconducting state itself has been achieved. However, unresolved issues include the astonishing complexity of the phase diagram, the unprecedented prominence of various forms of collective fluctuations, and the simplicity and insensitivity to material details of the ‘normal’ state at elevated temperatures.

# Phase diagram high Tc superconductors (Nature 518, 181, 2015)

**The clash: the quantum critical metal**

**… which is good for superconductivity!**

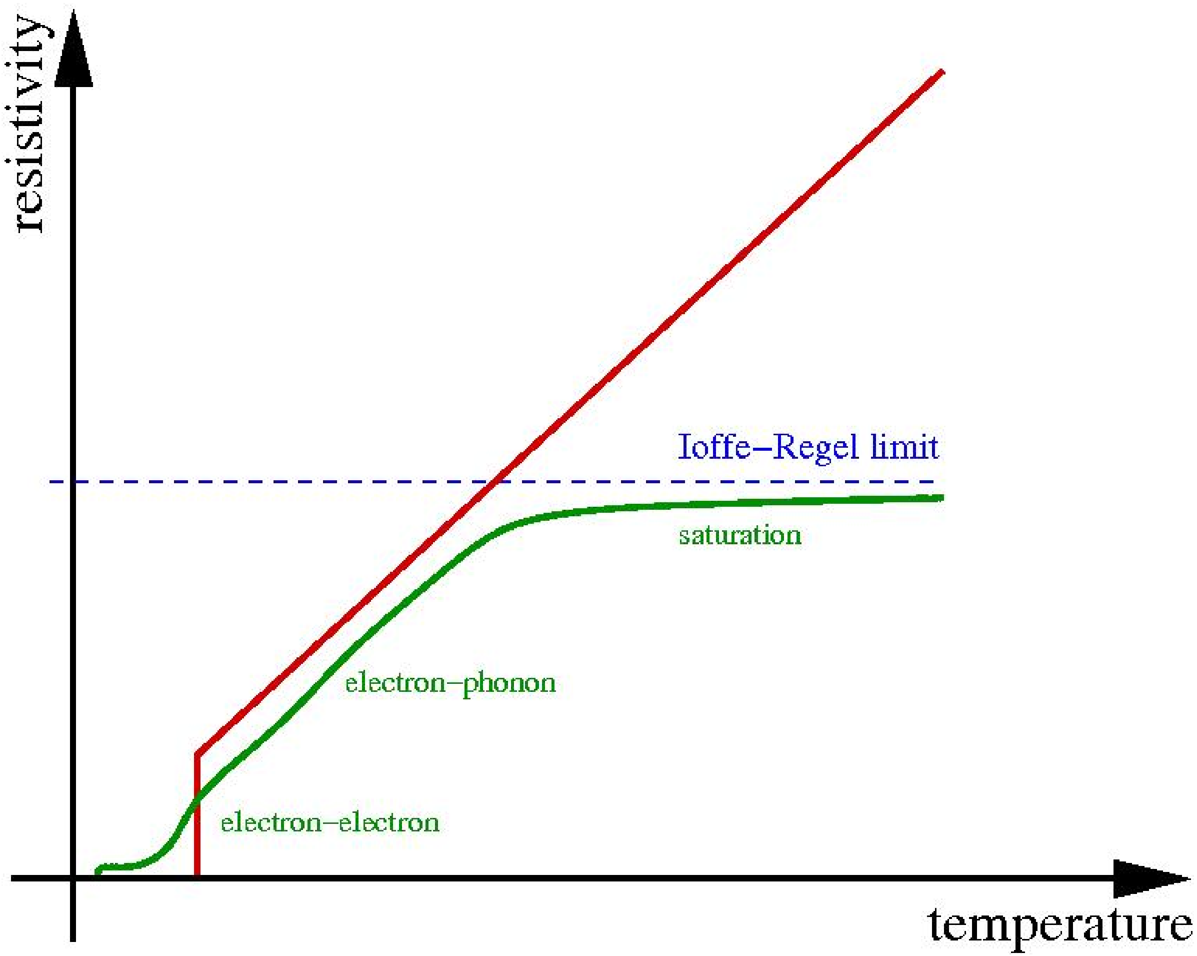

**The quantized traffic jam**




**The quantum fog (Fermi gas) returns**

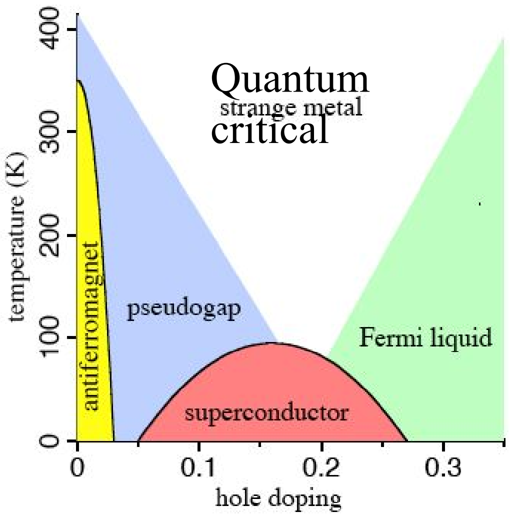
**Exotic orders: stripes, orbital currents, nematics …**

# Divine resistivity

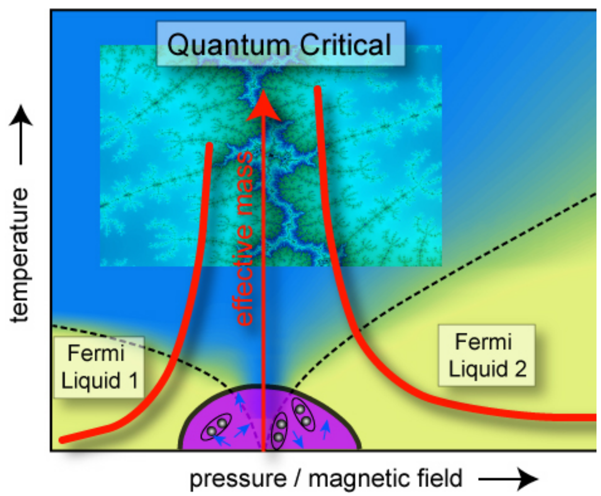


# A universal phase diagram

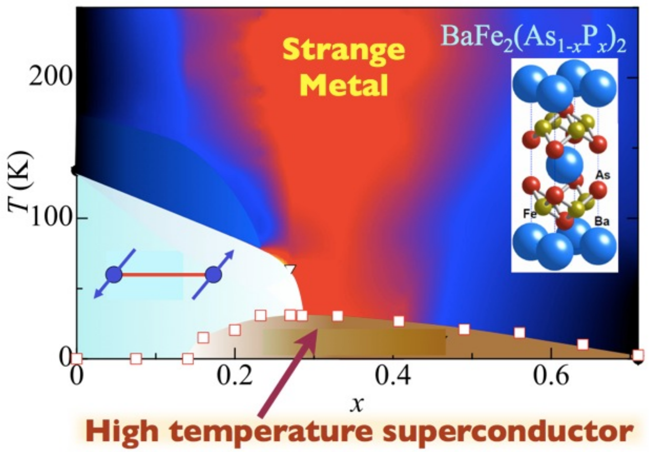
High Tc superconductors



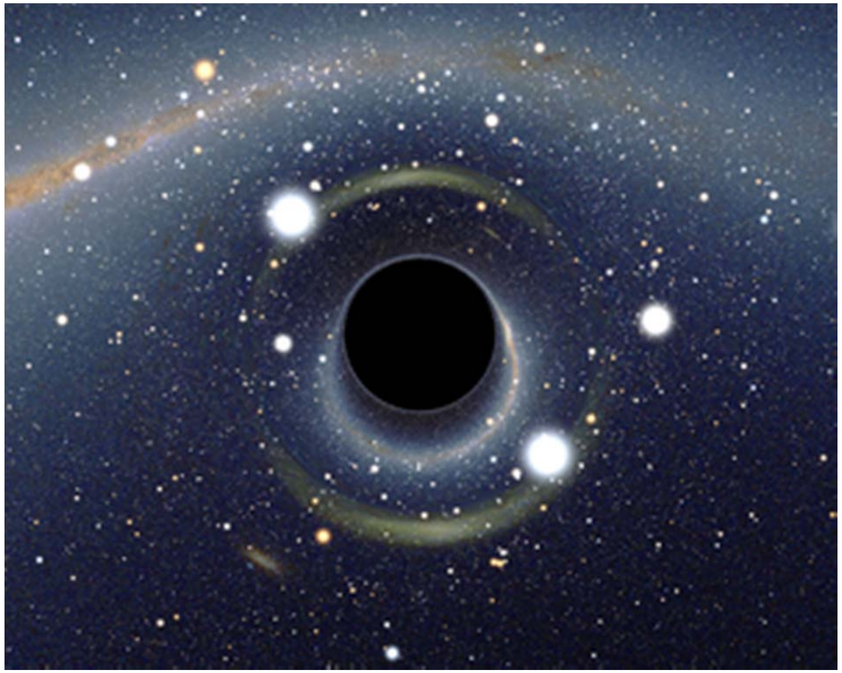

Heavy fermions




Iron superconductors (?)



# Black holes as "quantum matter computers" !?



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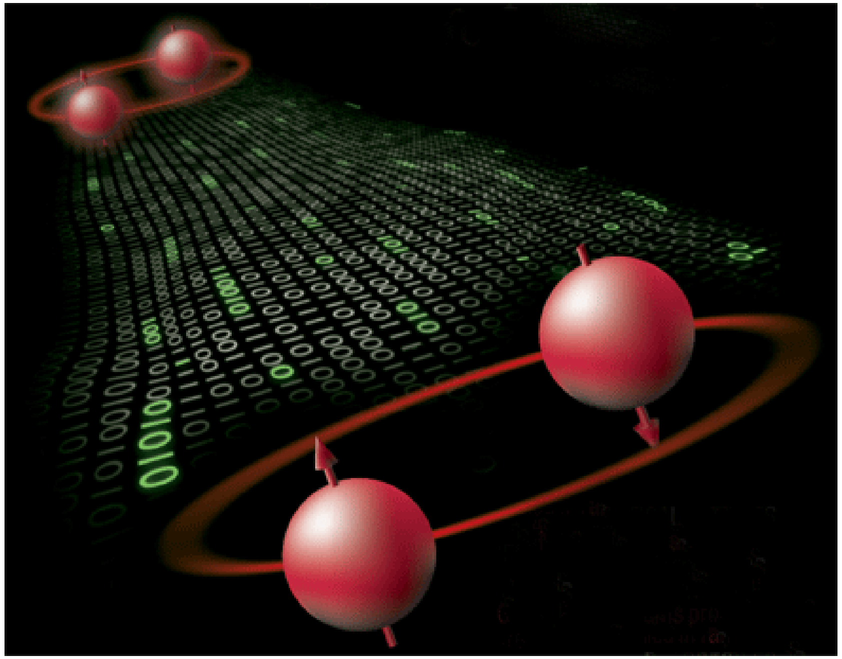
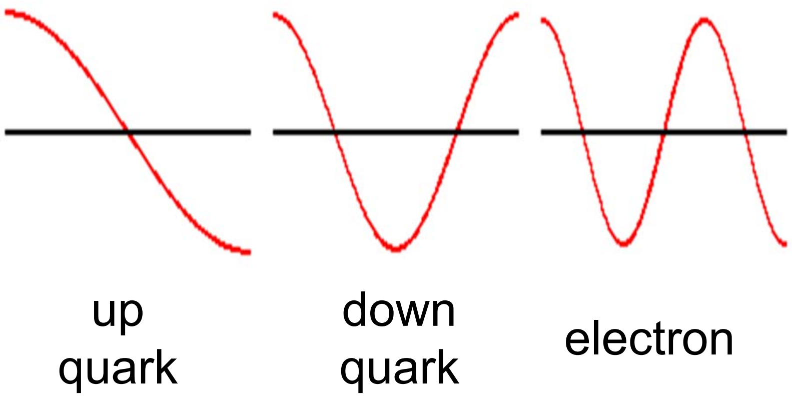
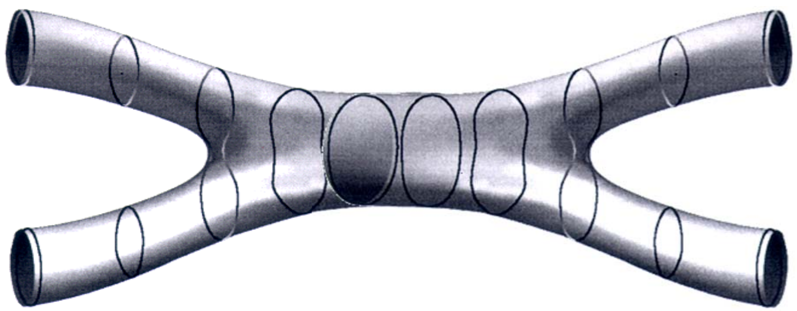


# Particles as string vibrations (1980's)

String vibration modes→
Different particles

Morphing strings →
Particle interactions

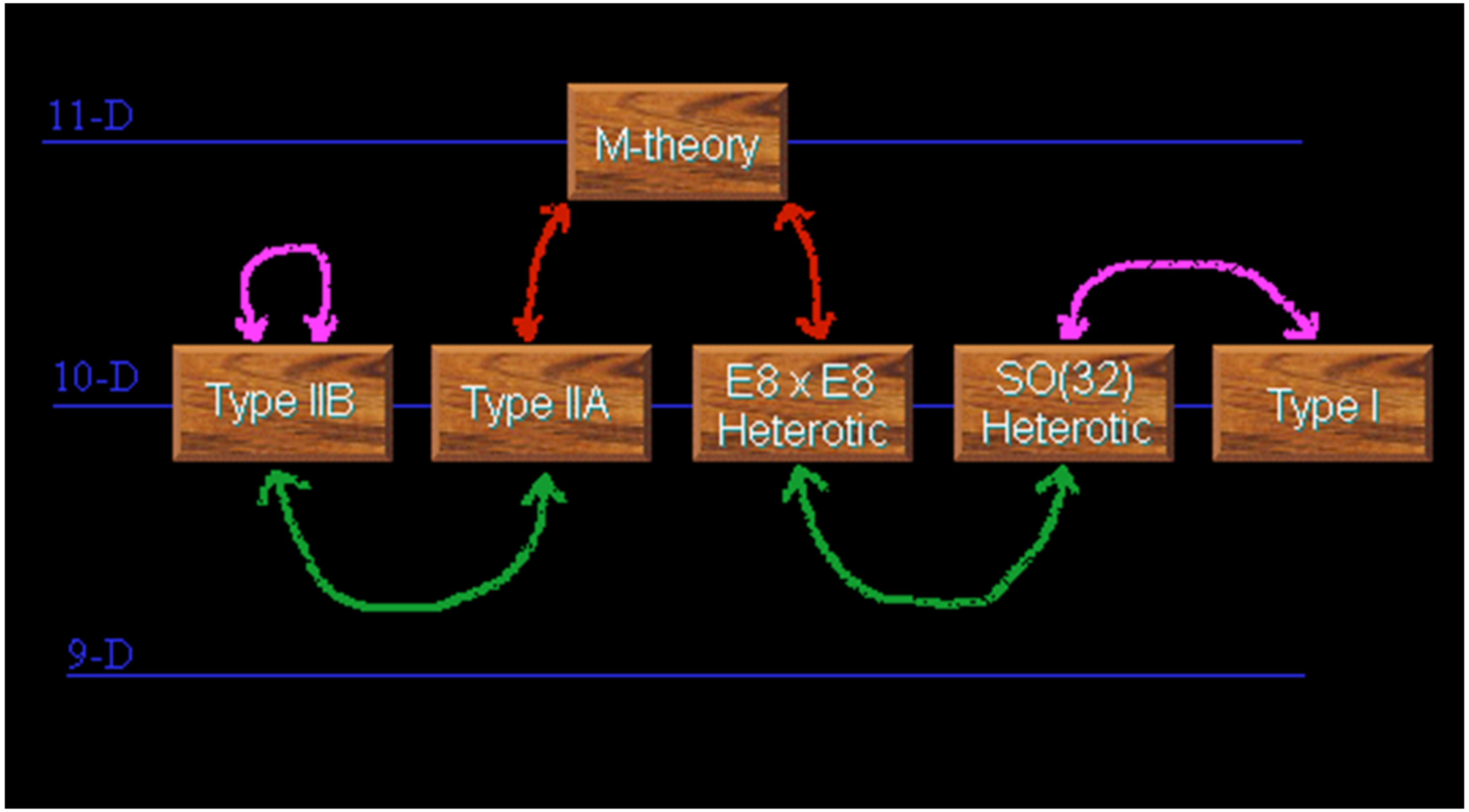


=>Unified theory: one string = all particles

=> Vibrations of "closed strings" describe gravitons
(quantum particles carrying gravitational force).

# The "second string revolution" (1995)




$\longleftrightarrow$

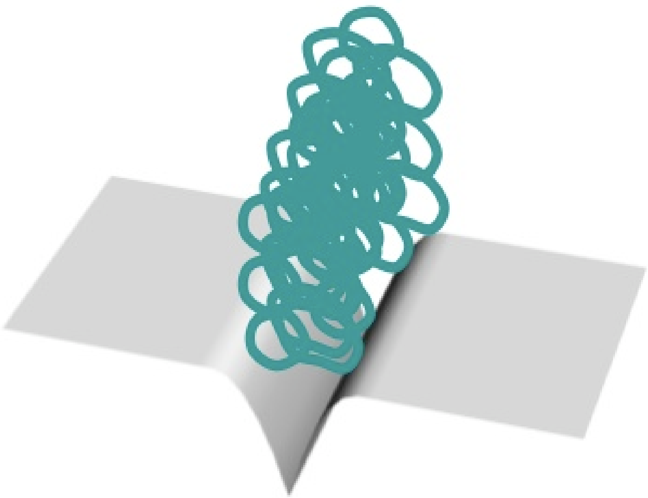
**Dualities**

# AdS/CFT correspondence

We have two different descriptions for same object!

Closed string description

Open string description



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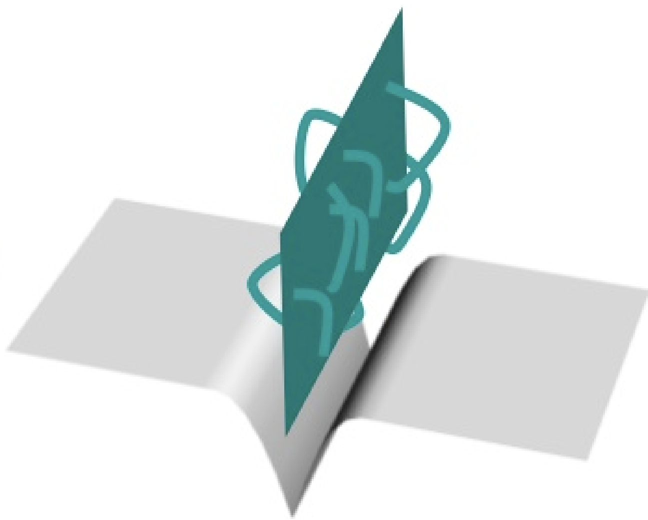
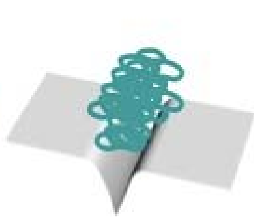
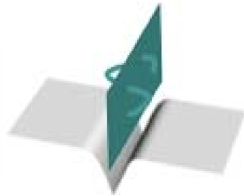


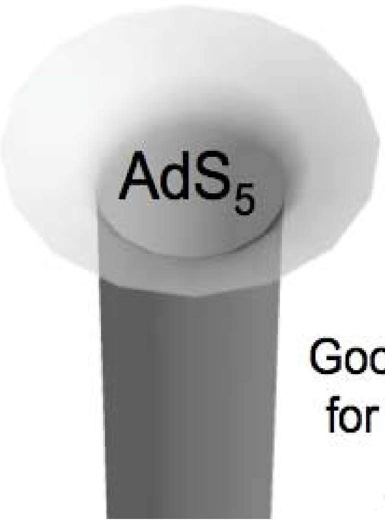
Especially, in the case of D3-brane, at low energy these two description will be approximated by ….

# AdS/CFT correspondence

Closed string description  = Open string description 

**Low energy limit with large N**

Supergravity on $AdS_5 \times S^5$ ⟺ dual ⟺ D=4 $\mathcal{N}$=4 U(N) Super Yang-Mills

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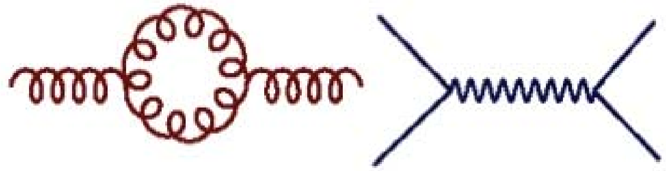


Good approximation for strong coupling

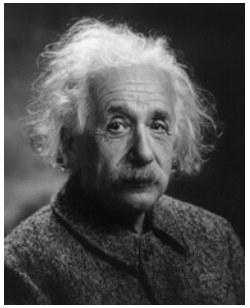
$$\lambda = g_{\rm YM}^2 N \gg 1$$

Good approximation for weak coupling

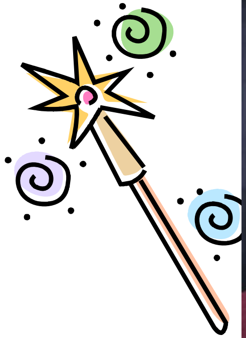
$$\lambda = g_{\rm YM}^2 N \ll 1$$

# General relativity "=" quantum field theory

**General relativity**

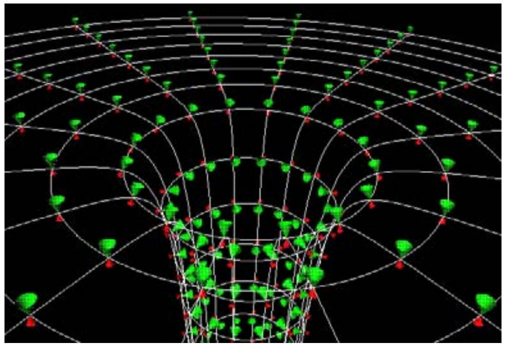


**'AdS/CFT'**



**Maldacena 1997**

**Quantum fields**



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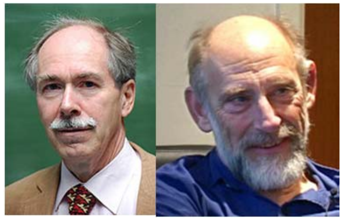
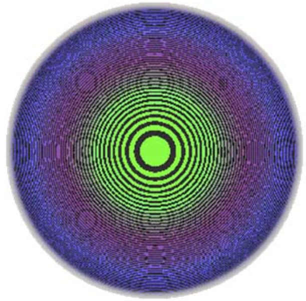
# Holographic gauge-gravity duality

**Einstein Universe "AdS"**

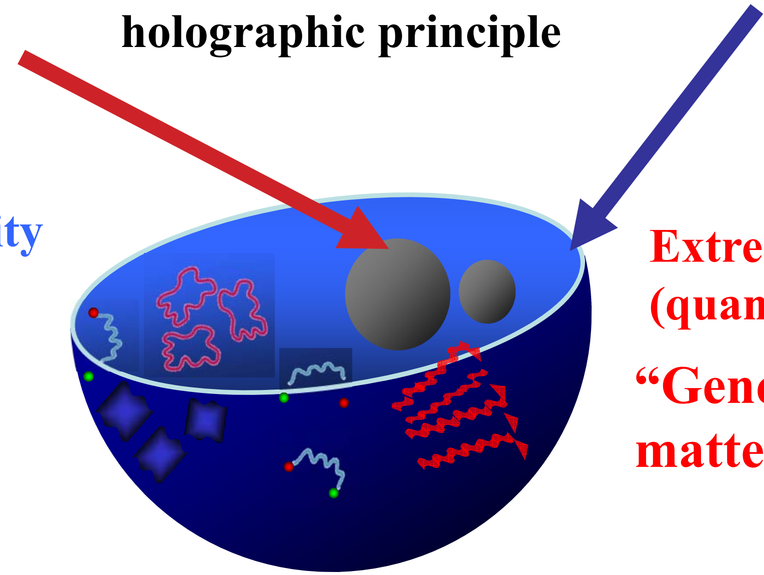


**'t Hooft-Susskind holographic principle**

**Quantum field world "CFT"**



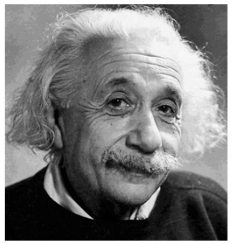
**Classical general relativity**



**Extremely strongly coupled (quantum) matter**

**Uniqueness of GR solutions**

**"Generating functional of matter emergence principle"**



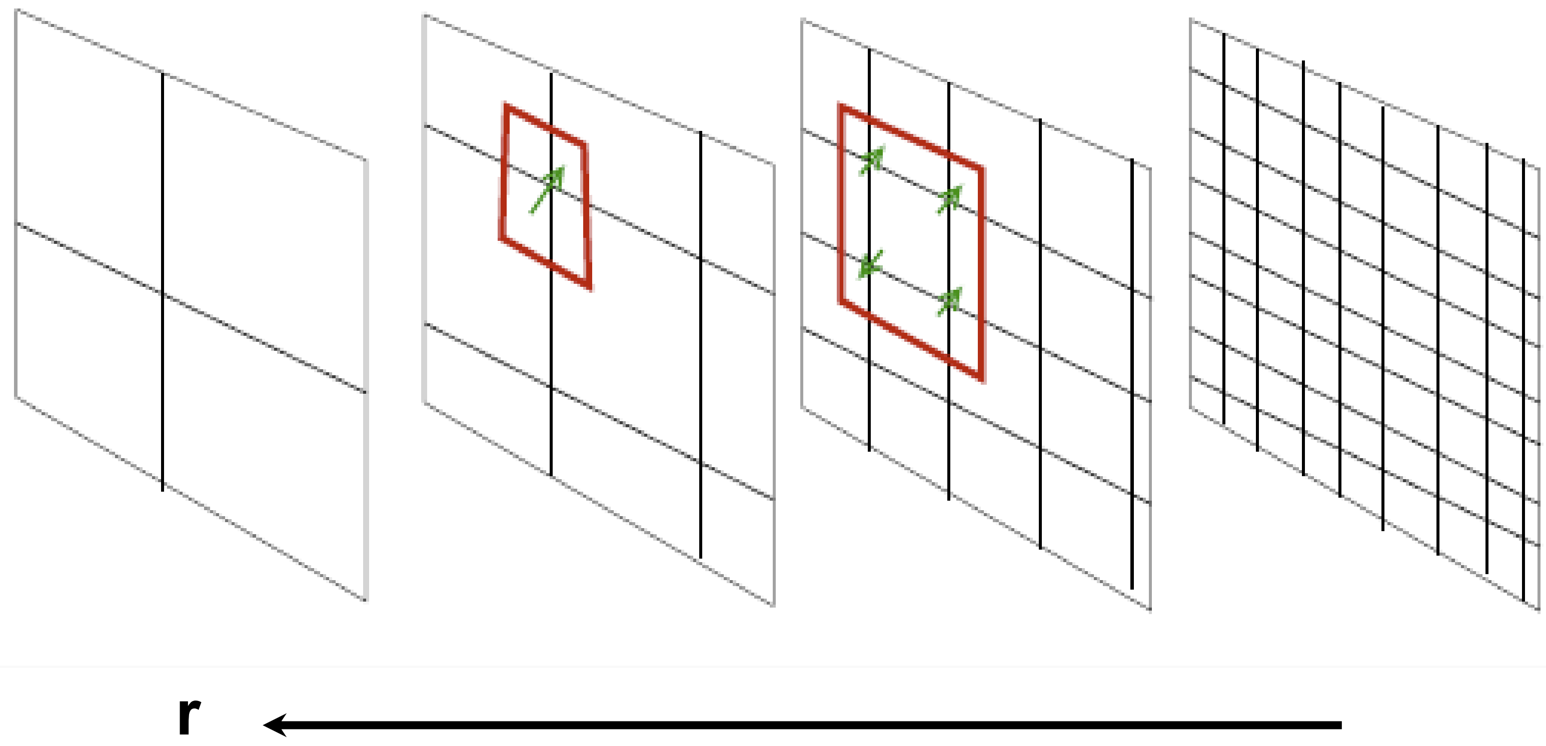
# General Relativity = Renormalization Group


Extra radial dimension of the bulk <=> scaling "dimension" in the field theory

Bulk AdS geometry = **scale invariance of the field theory**

$$dr^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$F(r) = -\Lambda r^2 + 1, \qquad \Lambda < 0$$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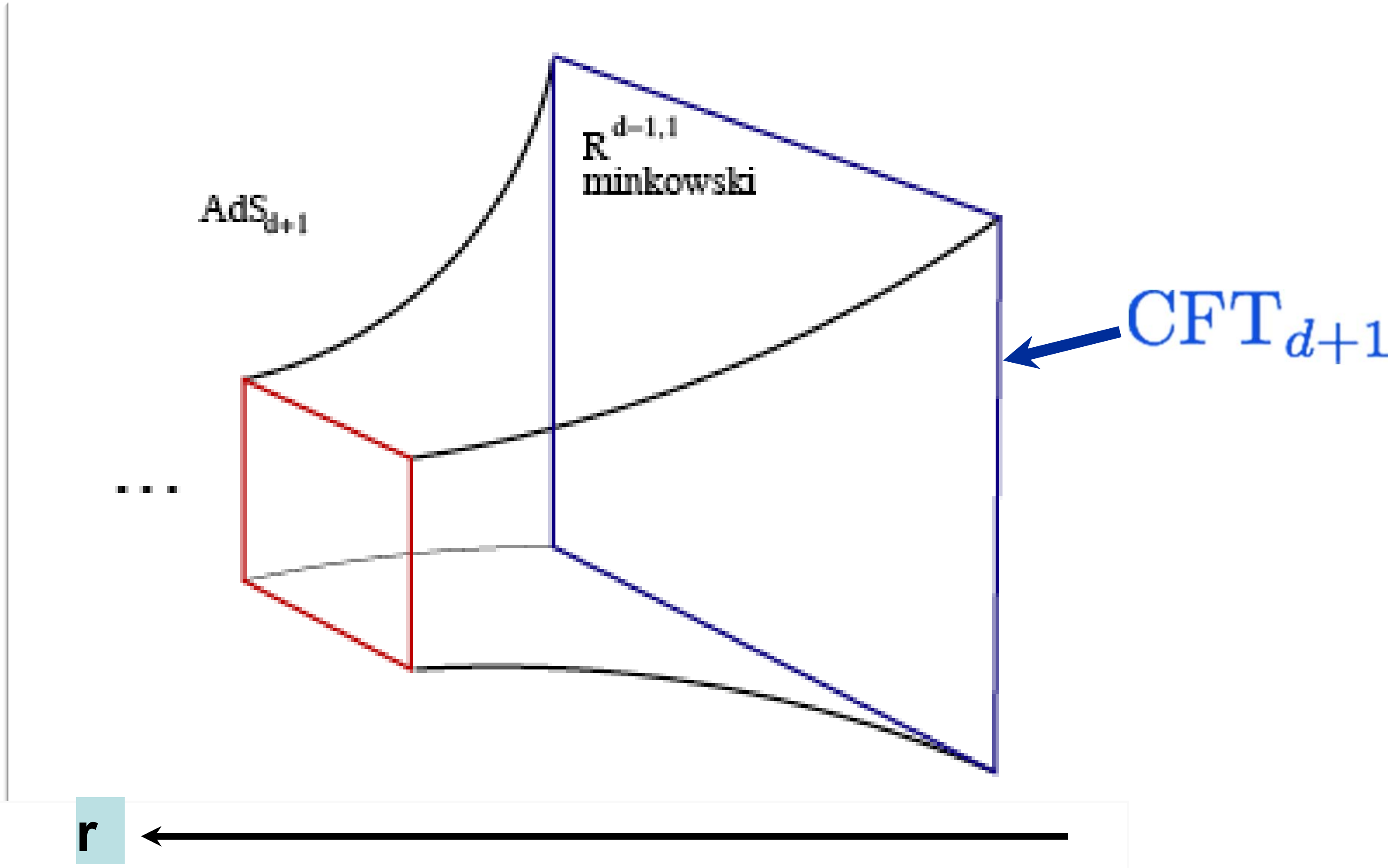
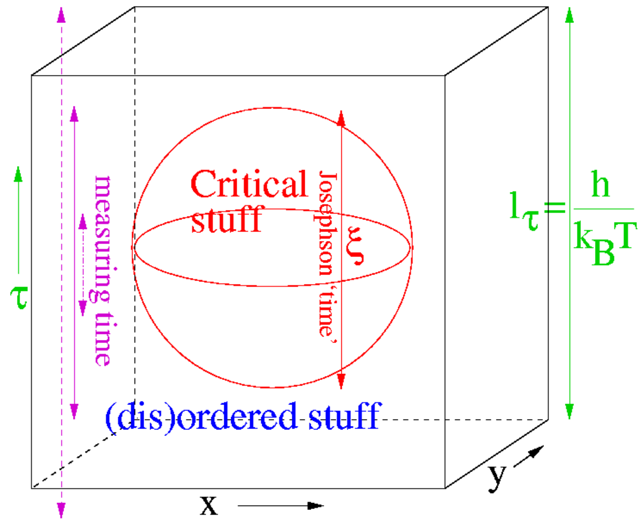


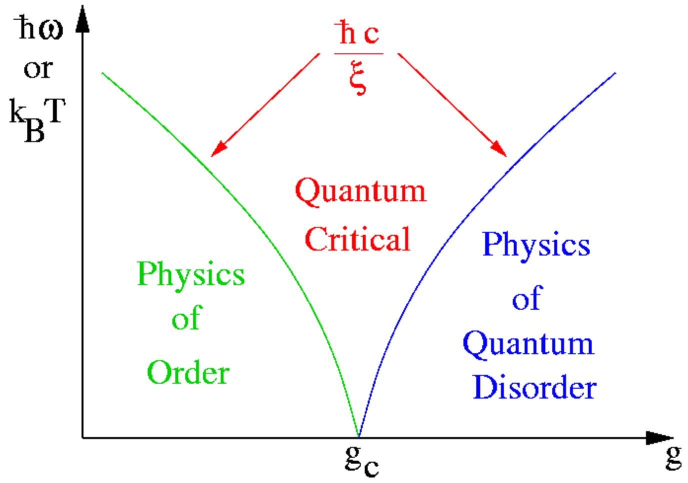

J. McGreevy, arXiv0909.0518

# Quantum criticality.

## Sachdev's book "quantum phase transitions"

**Scale invariance of the quantum dynamics (in space and time) is dynamically generated, as emergent phenomenon.**




**In the higher dimensional (bosonic) quantum field theories which are understood this only happens at isolated points in coupling constant space.**

# GKPW rule: propagators in QFT are classical waves in AdS

$$Z_{\mathrm{CFT}}(N) = \int \mathcal{D}\phi \, e^{iN^2 S_{\mathrm{AdS}}(\phi)}$$

$$\langle e^{\int d^{d+1}x J(x)\mathcal{O}(x)} \rangle_{\mathrm{QFT}} = \int \mathcal{D}\phi \; e^{iS_{\mathrm{bulk}}(\phi(x,r))|_{\phi(x,r=\infty)=J(x)}}$$

$$g_{YM}^2 N = \frac{R^4}{\alpha} \qquad g_{YM}^2 = g_s$$

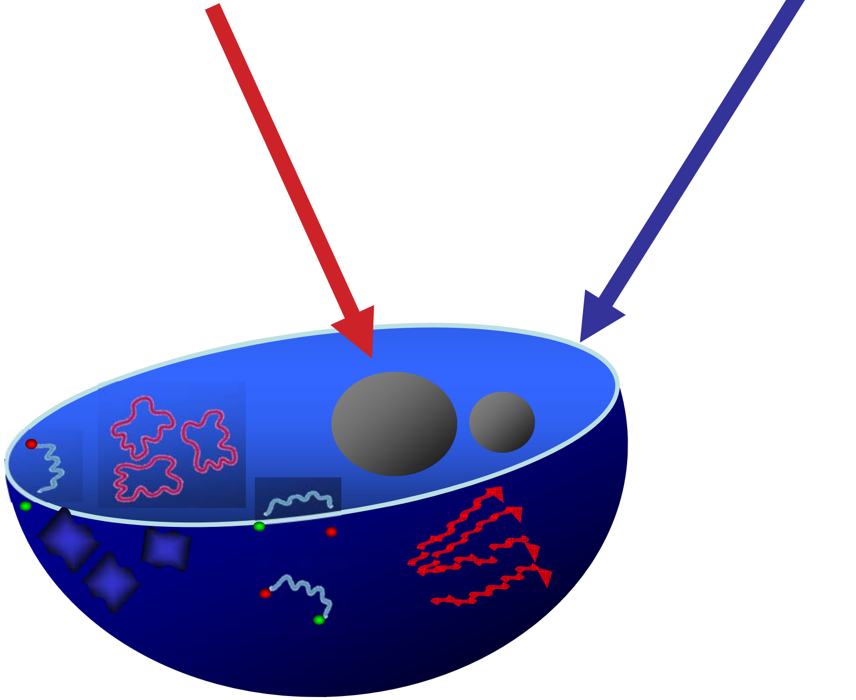
**Only in the *large N* limit the strongly coupled boundary field theory becomes dual to *classical* gravity!**


# MATRIX LARGE N


UV INDEPENDENCE

# The triumph: gravitational encoding of all thermal physics!

**Schwarzschild black hole**
**in the bulk**



**Boundary: the emergence theories of finite temperature matter.**

- **All of thermodynamics!** **Caveat: phase transitions are mean field (large N limit).**

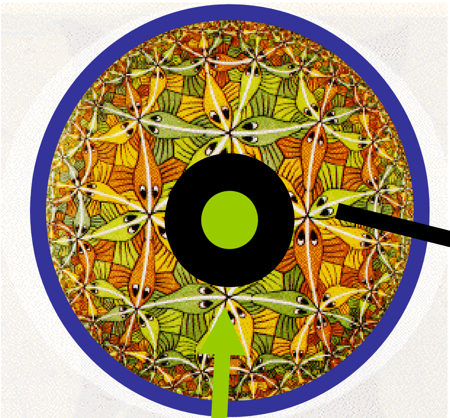
- **Precise encoding of Navier-Stokes hydrodynamics!** **Right now used to debug complicated hydrodynamics (e.g. superfluids).**

- **For special "Planckian dissipation" values of parameters** **(quantum criticality):**

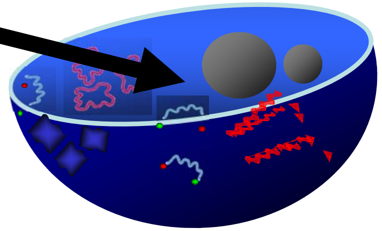
$$\tau_{\hbar} = const.\frac{\hbar}{k_B T}, \quad const. = O(1)$$

# The charged back hole encoding for finite density (2008 - ????)

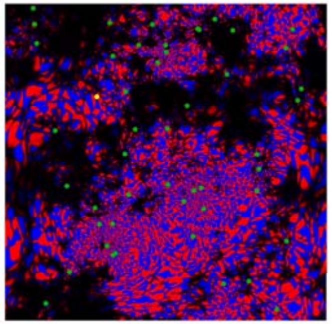
**Anti de Sitter universe.**



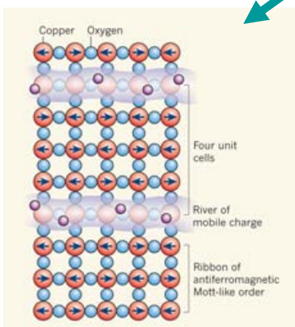
**Charged black hole in the middle**



**Finite density quantum matter:**



**Holographic strange metals**




**Stripy pseudogap orders**

**High Tc superconductors**

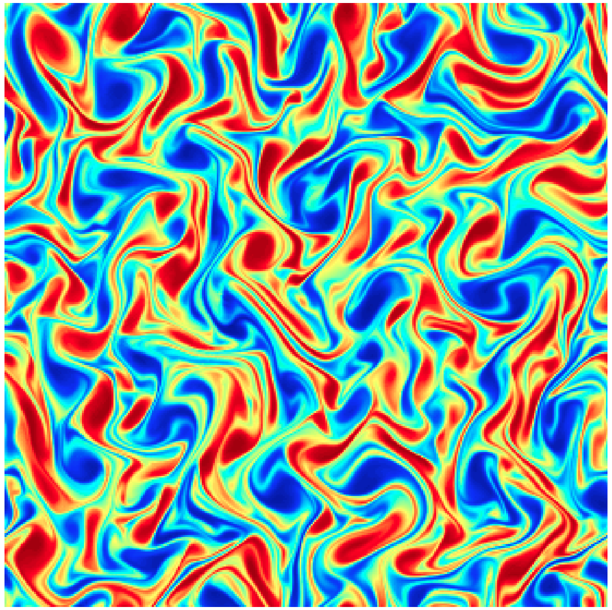
**Emergent Fermi liquids**

# Turbulence and fractal black hole horizons.

Chesler Yaffe

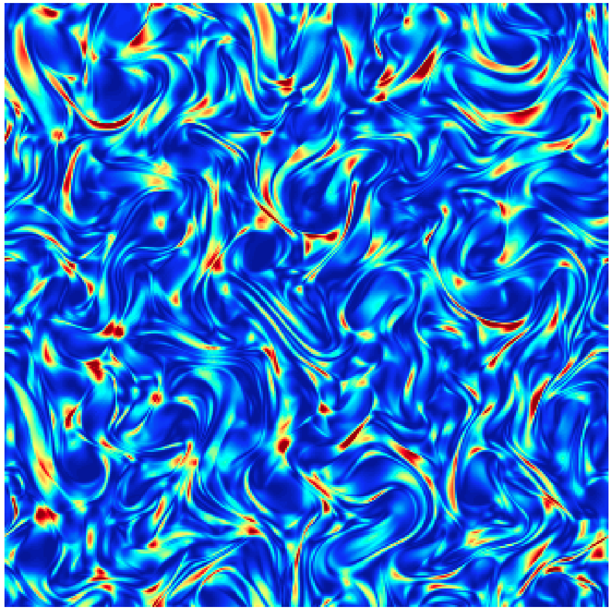
**Holography and numerical GR**

$\omega$



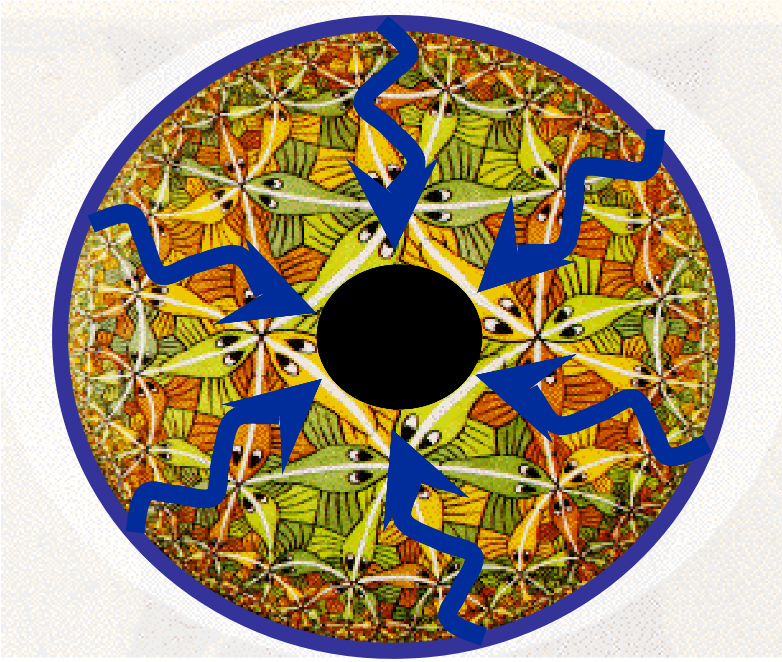
**Vorticity in the liquid (Kolmagorov scaling)**

$\partial A/\partial t$



**Near horizon geometry (fractal)**

# Dissipation = absorption of classical waves by Black hole!



**Policastro-Son-Starinets (2002):**

**Viscosity: absorption cross section of gravitons by black hole**

$$\eta = \frac{\sigma_{abs}(0)}{16\pi G}$$

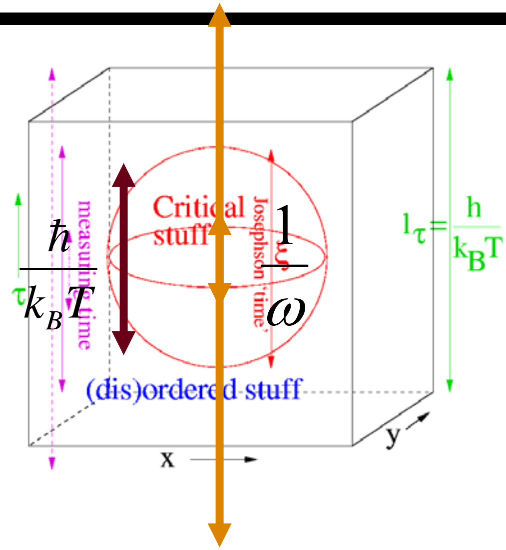
**= area of horizon (GR theorems)**

**Entropy density s: Bekenstein-Hawking BH entropy = area of horizon**

**Universal viscosity-entropy ratio for CFT's with gravitational dual limited in large N by:**

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

# Planckian dissipation …




Scaling form dynamical susceptibility:

$$\chi(\omega) \propto \frac{1}{T^{2-\eta}} \Psi\left(\frac{\hbar\omega}{k_B T}\right)$$

Quantum critical regime $\quad k_B T >> \frac{\hbar c}{\xi}$

$$\hbar\omega >> k_B T : \quad \chi(\omega) \propto \frac{e^{i\frac{\pi}{2}(2-\eta)}}{|\omega|^{2-\eta}}$$

$$\hbar\omega << k_B T : \quad \chi(\omega) \propto \frac{1}{T^{2-\eta}} \frac{1}{1 - i\omega\tau_\hbar}$$

Planckian dissipation: $\quad \tau_\hbar = const. \frac{\hbar}{k_B T}, \quad const. = O(1)$

# Quantum criticality and the dimension of viscosity ...

Viscosity: $\eta = f \tau_K$

Free energy density QC system: $f = sT$

**Planckian dissipation:** $\tau_K = A \frac{\hbar}{k_B T}$

$$\frac{\eta}{s} = AT \frac{\hbar}{k_B T} = A \frac{\hbar}{k_B}$$

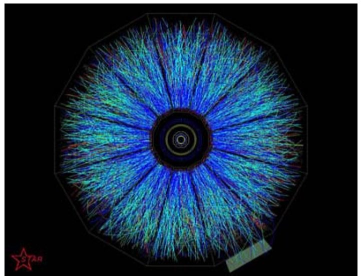
Large N SUSY Yang Mills: $A = \frac{1}{4\pi}$

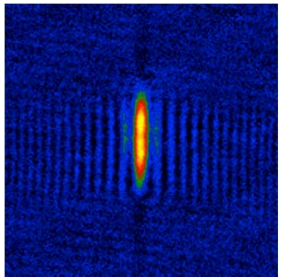
# Planckian dissipation

**Universal entropy production time in QC system:** $\tau = \tau_\hbar \approx \dfrac{\hbar}{k_B T}$

**Observed in Quark gluon plasma (heavy ion colliders RIHC, LHC) and cold atom "unitary fermi gas":**

$$\frac{\eta}{s} = T\tau_\hbar = \frac{1}{4\pi}\frac{\hbar}{k_B}$$





**Science March 4 2016:**

PHYSICS

***Electrons go with the flow in exotic material systems***

Electronic hydrodynamic flow—making electrons flow like a fluid—has been observed

*By* Jan Zaanen

the space they are moving in is made in-

REPORTS

ELECTRON TRANSPORT

**Negative local resistance caused by viscous electron backflow in graphene**

D. A. Bandurin,[1] I. Torre,[2] R. Krishna Kumar,[1,3] M. Ben Shalom,[1,4] A. Tomadin,[5] A. Principi,[6] G. H. Auton,[4] E. Khestanova,[1,4] K. S. Novoselov,[4] I. V. Grigorieva,[1] L. A. Ponomarenko,[1,3] A. K. Geim,[1*] M. Polini[7*]

ELECTRON TRANSPORT

**Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene**

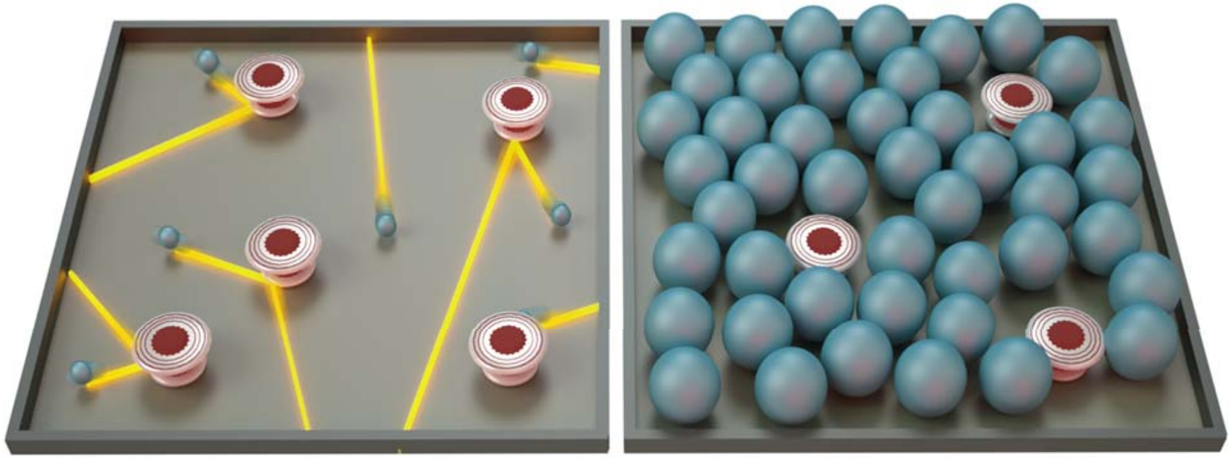
Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2*] Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] Kin Chung Fong[5*]

ELECTRON TRANSPORT

**Evidence for hydrodynamic electron flow in $PdCoO_2$**

Philip J. W. Moll,[1,2,3] Pallavi Kushwaha,[3] Nabhanila Nandi,[3] Burkhard Schmidt,[3] Andrew P. Mackenzie[3,4*]

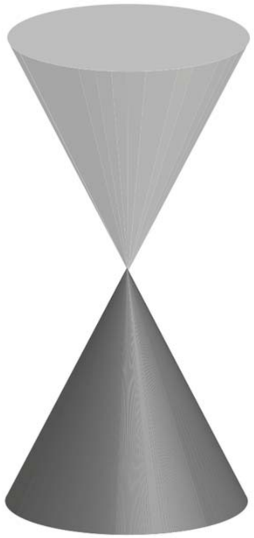
# Hydro-electricity exhibition in Science ..



**Science first march issue:**

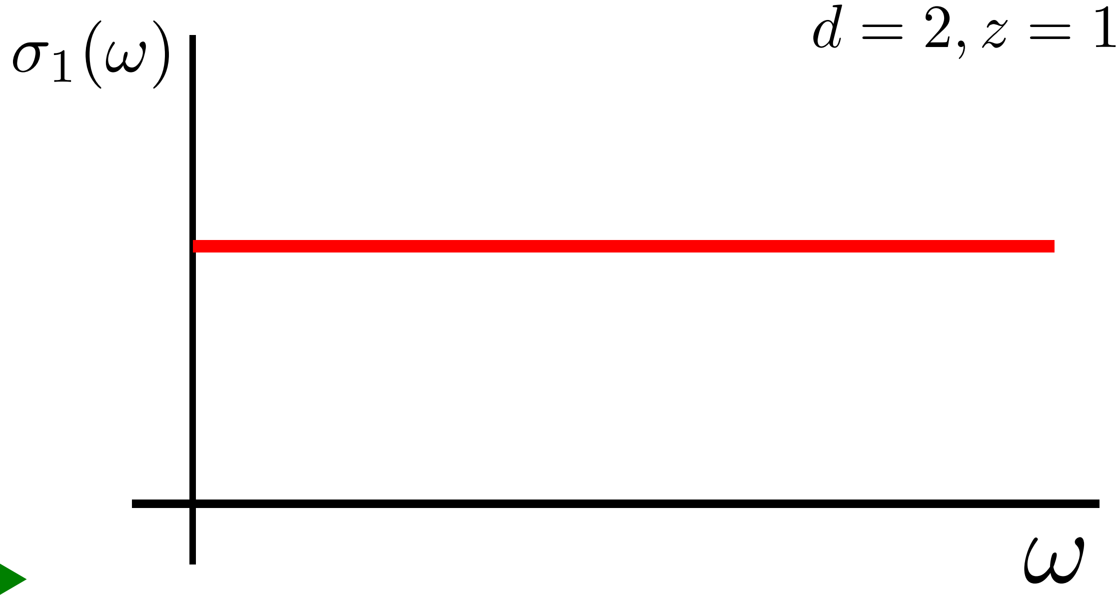
1. **JZ: Perspective**
2. **Crossno et al. (Harvard): WF violation in graphene.**
3. **Bandurin et al. (Manchester): “whirls” in graphene.**
4. **Moll et al. (Dresden): electron “pipes” in PdCoO2**

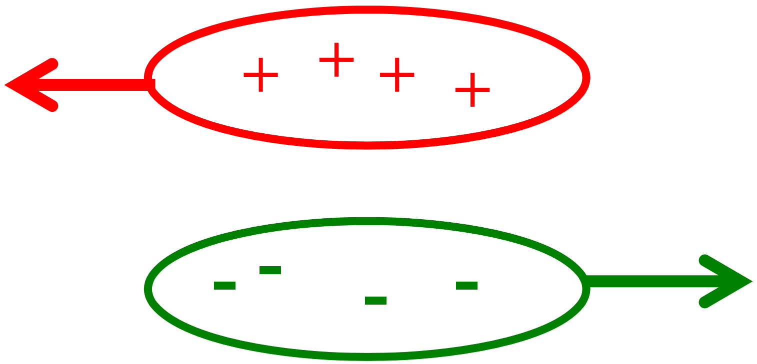
# Optical conductivity: zero density CFT.



**Current does not carry momentum**

$$\sigma_1(\omega) \sim \omega^{(d-2)/z}$$



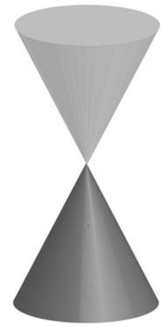



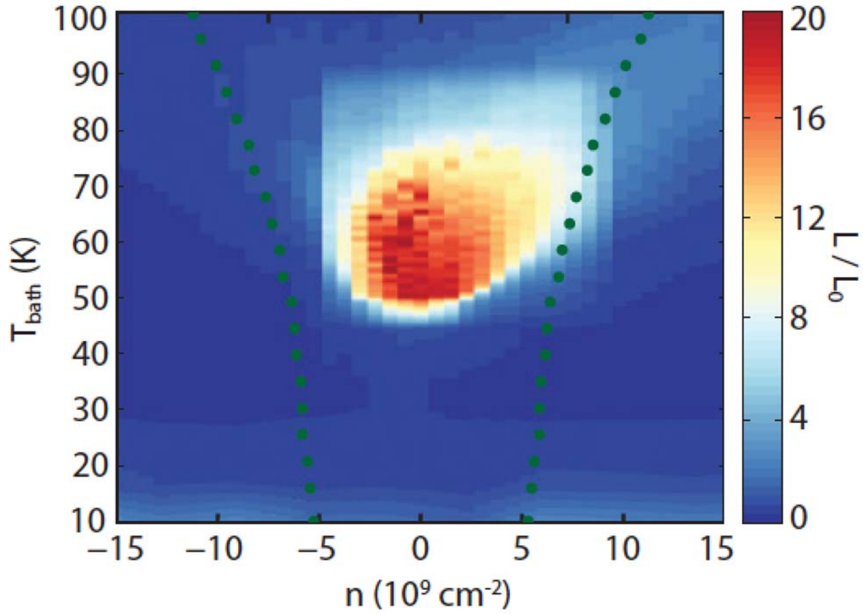
# Graphene as an interacting Dirac fluid.

Crossno et al., arXiv:1509.04713

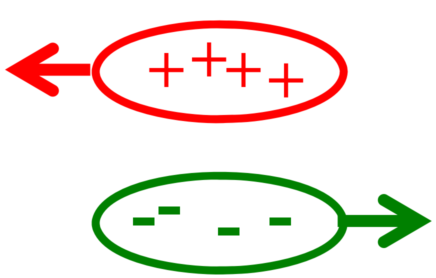
Wiedemann-Franz ratio: $\mathcal{L} = \dfrac{\kappa_e}{\sigma T}$

Lorentz-ratio (quasiparticles): $\mathcal{L}_0 = \dfrac{\pi^2}{3}\left(\dfrac{k_B}{e}\right)^2$

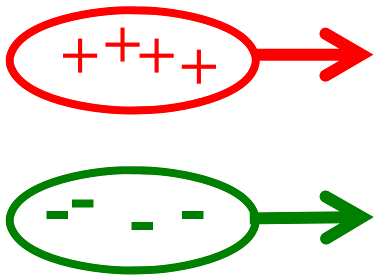





**Electric field**



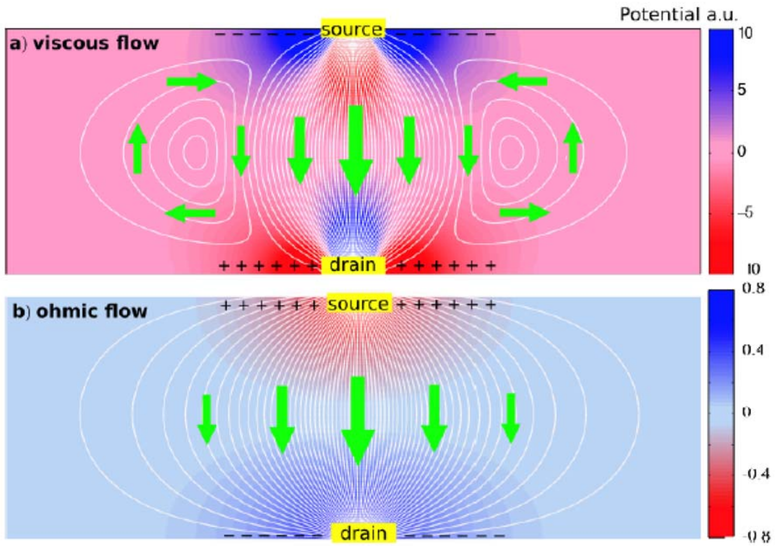
**Temp. gradient**



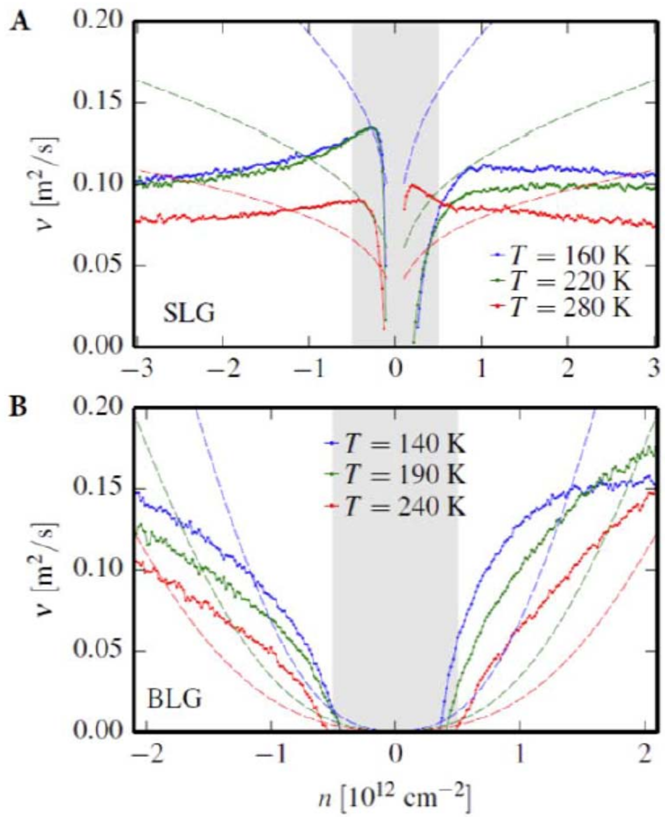
**Lucas et al. (arXiv:1510.01738): "minimalish" viscosity**

# Graphene's Hydrodynamical whirls and the negative local resistance .

**Prediction: Levitov, Falkovich, arXiv:1508.00836**

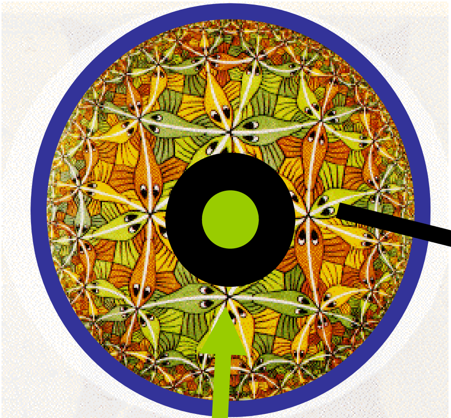



**Confirmation: Geim group, arXiv:1509.04165**

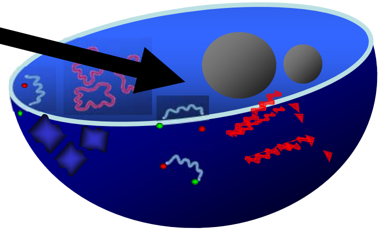


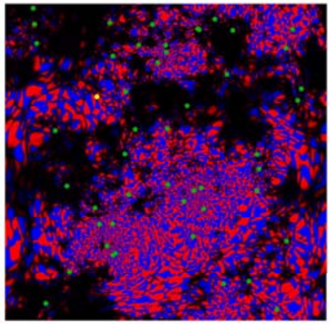
# The charged back hole encoding for finite density (2008 - ????)

**Anti de Sitter universe.**



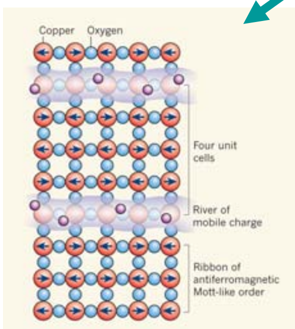
**Finite density quantum matter:**





**Holographic strange metals**

**Charged black hole in the middle**



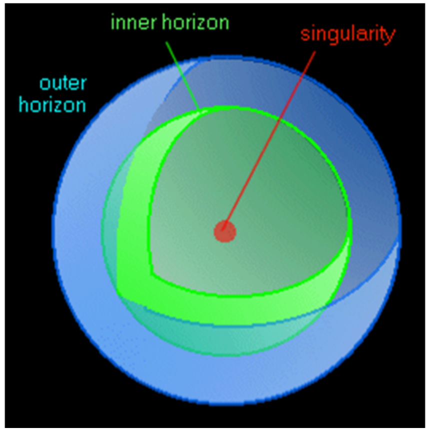

**Stripy pseudogap orders**

**High Tc superconductors**

**Emergent Fermi liquids**

# The holographic stable states: uncollapsing in an AdS "star".

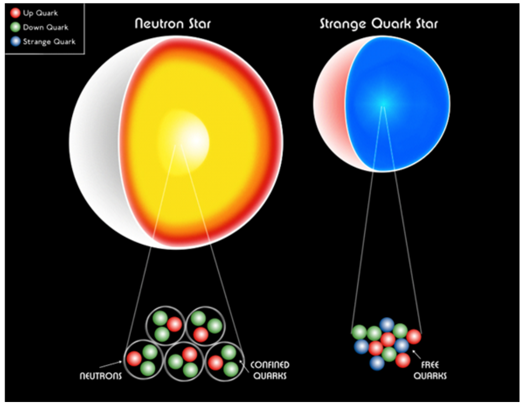
**(Reissner-Nordstrom)**
**"Black hole like object"**




**"uncollapse"**

**Phase transition**

**"star like object"**




**"fractionalized", "unstable":**
**strange metal**

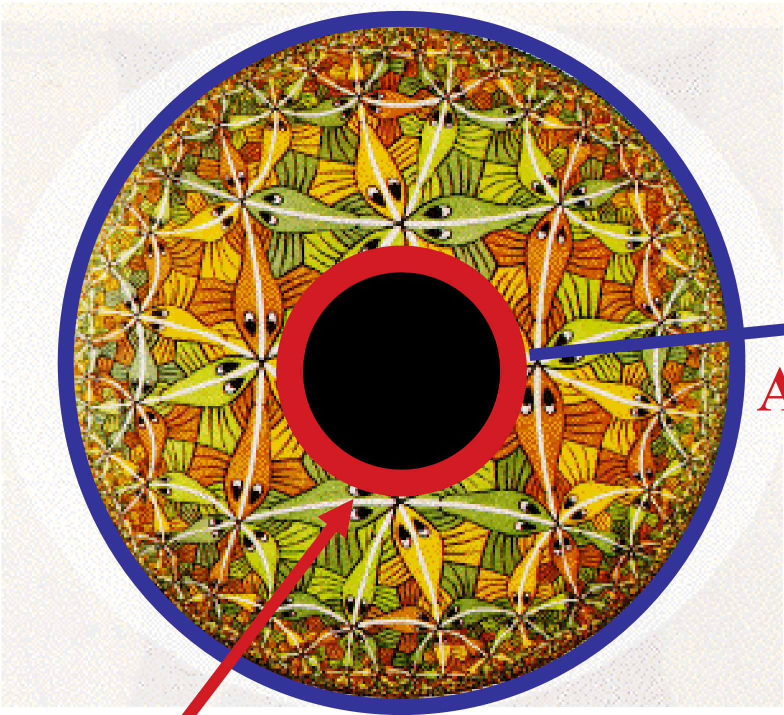
**"Cohesive state":**

**Symmetry breaking:**
**superconductor, crystal**
**("scalar hair")**

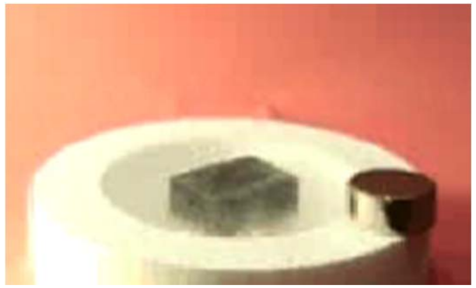
**Fermi-liquid ("electron star")**

# The holographic superconductor

Hartnoll, Herzog, Horowitz, arXiv:0803.3295



**Condensate (superconductor, … ) on the boundary!**



AdS-CFT

**'Super radiance' : in the presence of matter the extremal BH is unstable => zero T entropy always avoided by low T order!!!**

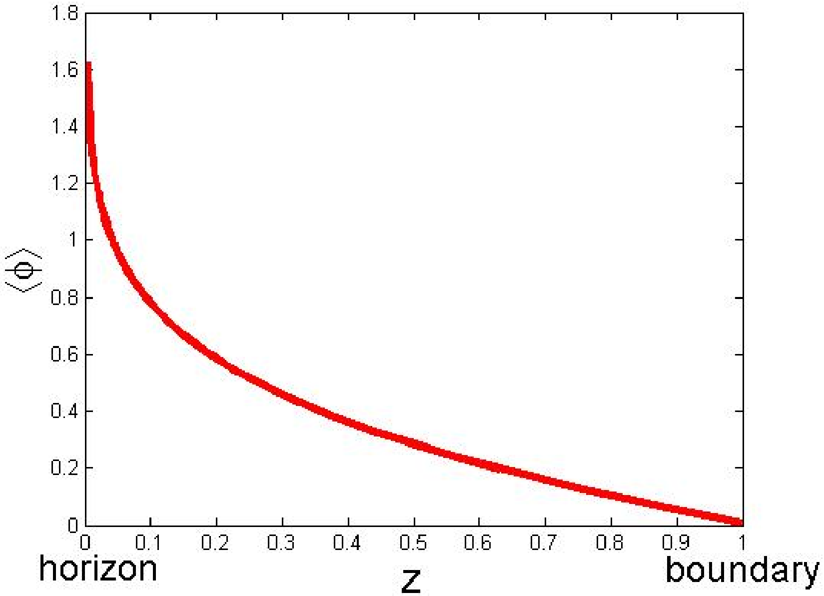
**(Scalar) matter 'atmosphere'**

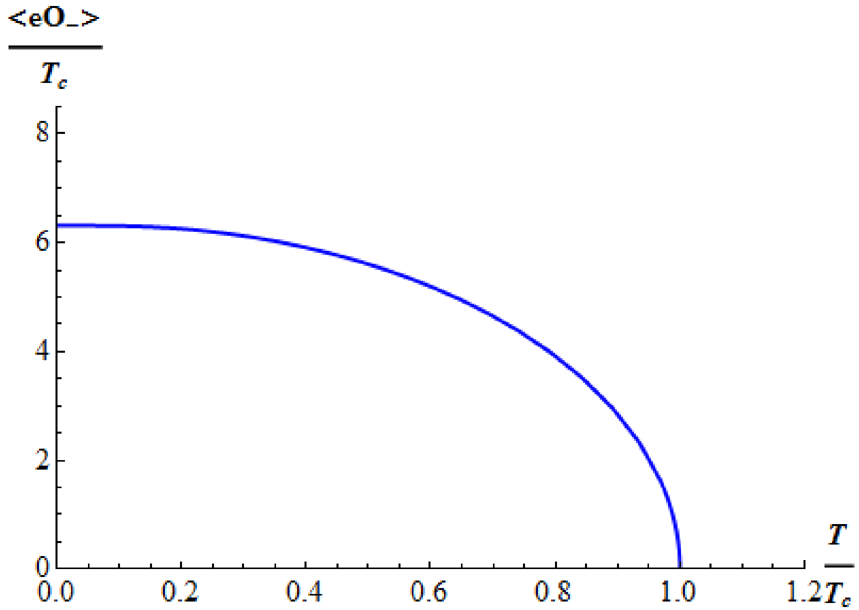
# The Bose-Einstein Black hole hair

Hartnoll Herzog Horowitz

Scalar hair accumulates at the horizon

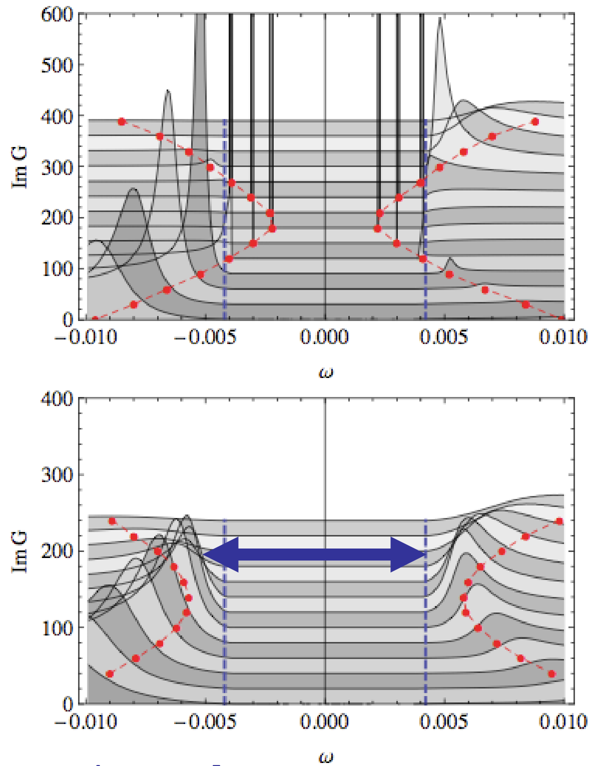
Mean field thermal transition.

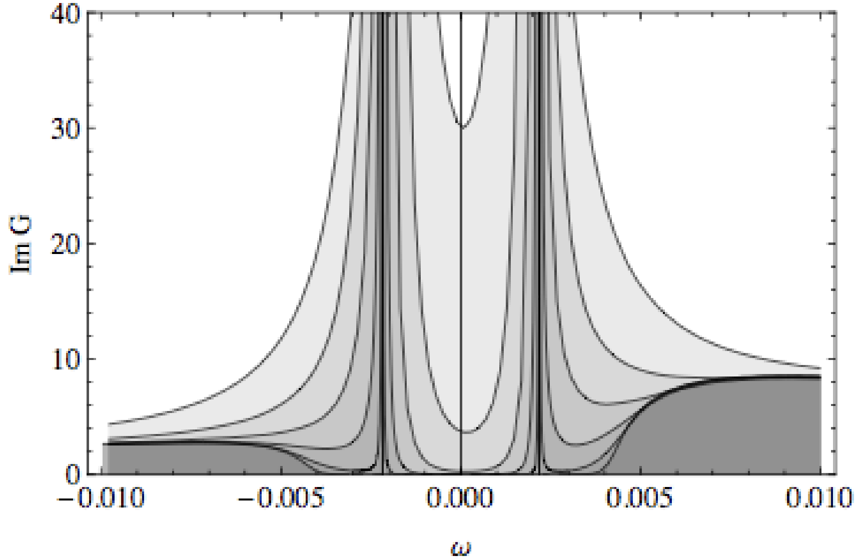


# Holographic superconductivity: stabilizing quasiparticles.

**Fermion spectrum for scalar-hair black hole (Faulkner et al., 911.340):**

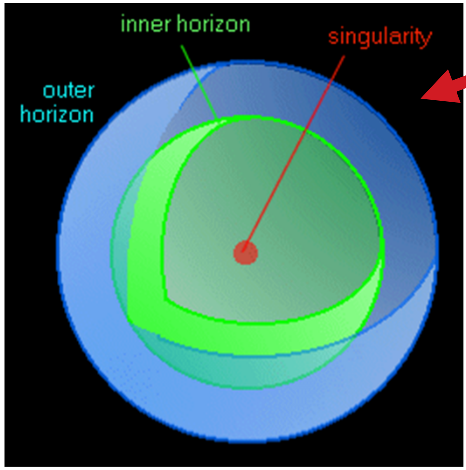



**'BCS' Gap in fermion spectrum !!**

**"Pseudogap" Temperature dependence**

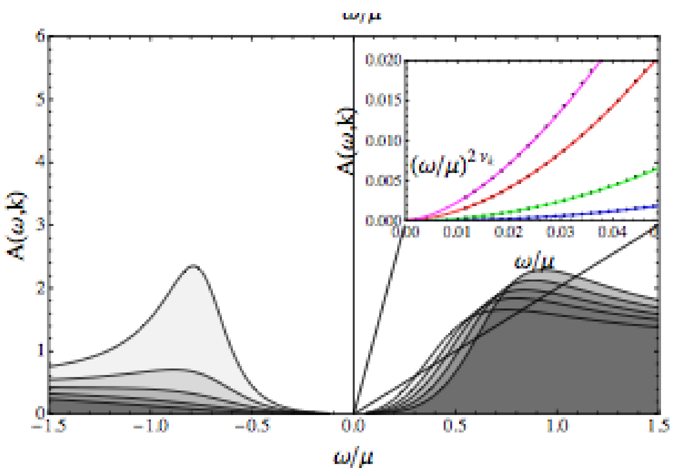
# Finite density: the Reissner-Nordstrom strange metals (Liu et al.).




**Near-horizon geometry of the extremal RN black hole:**

- **Space directions: flat, codes for simple Galilean invariance in the boundary.**

- **Time-radial(=scaling) direction: emergent $AdS_2$, codes for emergent temporal scale invariance!**




**Fermion spectral functions:**

$$A(k,\omega) \propto G''_{AdS_2}(k,\omega) \propto \omega^{2\nu_k}$$

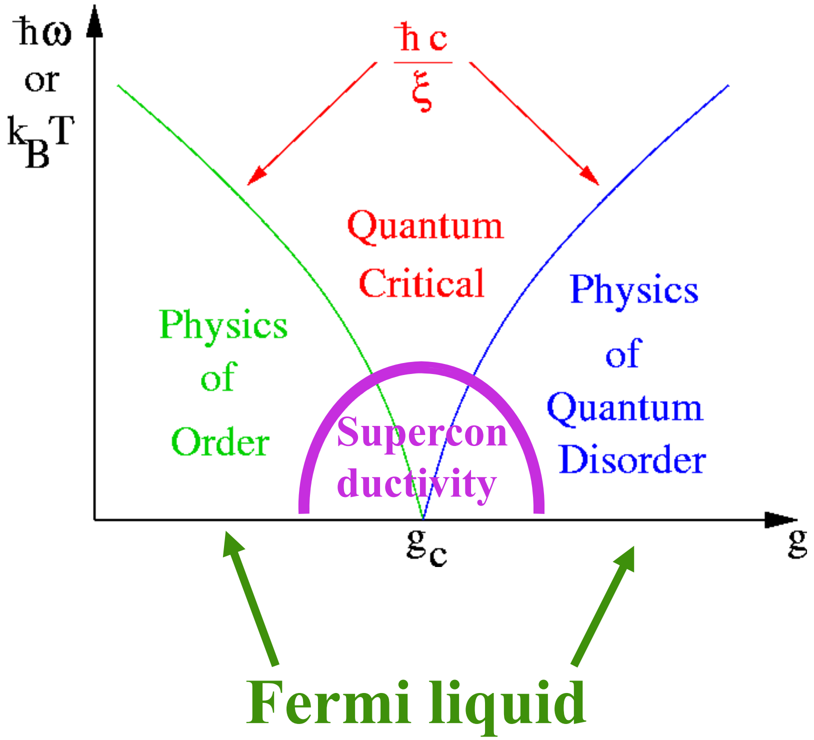
$$\nu_k = \frac{1}{\sqrt{6}} \sqrt{k^2 + \frac{1}{\xi^2}}$$

## "Un-particle physics!"

# Hertz-Millis metallic "Quantum Critical Point"

**Assertion: in the "UV" a *Fermi-liquid* is formed co-existing with an electronic *order parameter* (e.g. magnet) interacting via a Yukawa coupling.**

**The *order parameter* is subjected to *a bosonic quantum phase transition*: always isolated unstable fixed point (stat phys rule book).**




**Electrons: *Fermi-gas* = heat bath *damping bosonic critical fluctuations*.**

**Lingering singularities of QP *on the Fermi surface* due to critical bosons.**

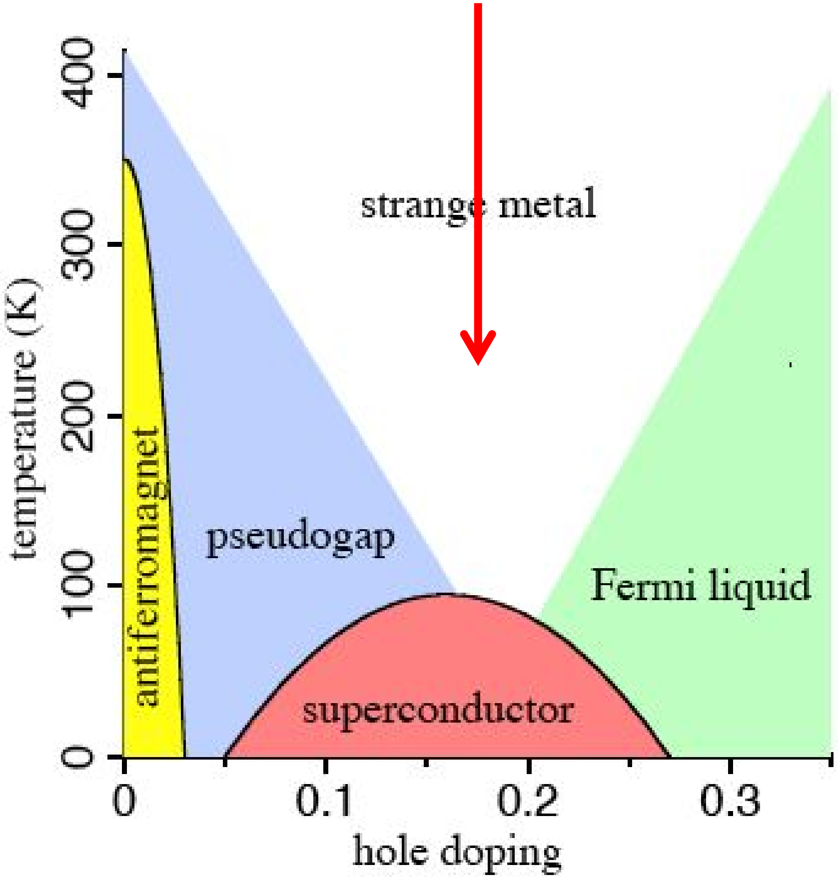
**"Critical Fermi-surface": Fermi surface survives but branch cut fermionic propagators.**

**Critical fluctuations acting as pairing glue: the "superconducting domes".**

# The unstable conformal metal of finite density holography.

Hong Liu

**Conformal metal: *quantum critical* fermionic *phase not requiring fine tuning!***




**Characterised by *non-bosonic scaling properties*: large (infinite) z, hyperscaling violation, …**

**Intrinsically *very unstable*: "mother" of Fermi-liquids, superconductors, stripes, CDW's, loop currents, electronic nematics, …**

**The finite temperature state: governed by *Planckian dissipation*.**

# "Scaling atlas" of holographic quantum critical phases.

Kiritsis

**Deep interior geometry sets the scaling behavior in the emergent deep infrared of the field theory. Uniqueness of GR solutions:**

**1. "Cap-off geometry" = confinement: conventional superconductors, Fermi liquids ….**

**2. Geometry survives: "hyperscaling violating geometries" (Einstein – Maxwell – Dilaton – Scalar fields – Fermions).**

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\vartheta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$x_i \to \zeta x_i, \quad t \to \zeta^z t, \quad ds \to \zeta^{\theta/d} ds$$

$$S \propto T^{(d-\theta)/z}$$

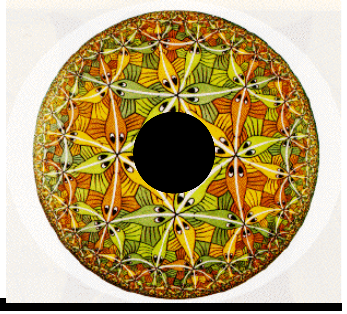
**Quantum critical phases with unusual values of:**

$z =$ **Dynamical critical exponent**

$\theta =$ **Hyperscaling violation exponent**

$\zeta =$ **Charge exponent**

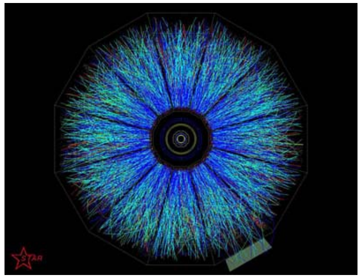
# Planckian dissipation

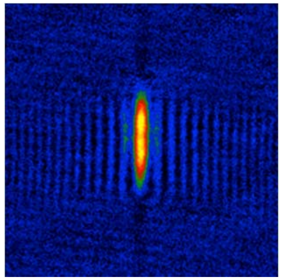


**Universal entropy production time in QC system:** $\tau = \tau_\hbar \approx \dfrac{\hbar}{k_B T}$

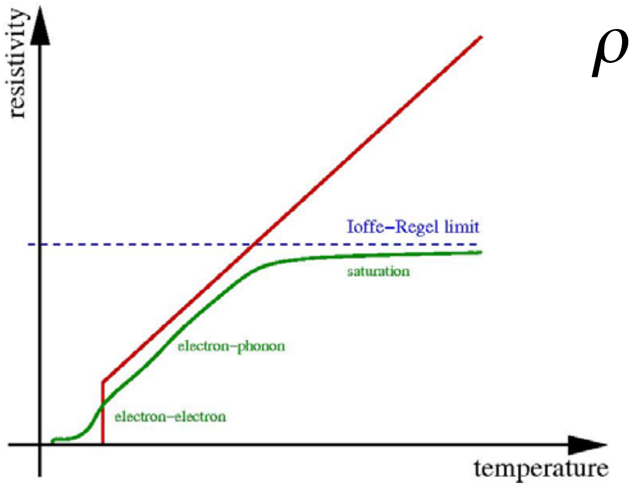
**Observed in Quark gluon plasma (heavy ion colliders RIHC, LHC) and cold atom "unitary fermi gas":**

$$\frac{\eta}{s} = T\tau_\hbar = \frac{1}{4\pi}\frac{\hbar}{k_B}$$



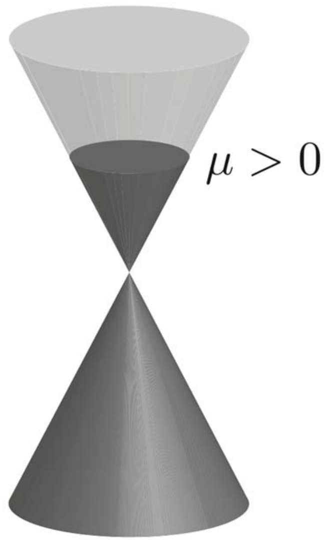


**Since early 1990's recognized as responsible for strange metal properties, e.g. linear resistivity high Tc metals:**

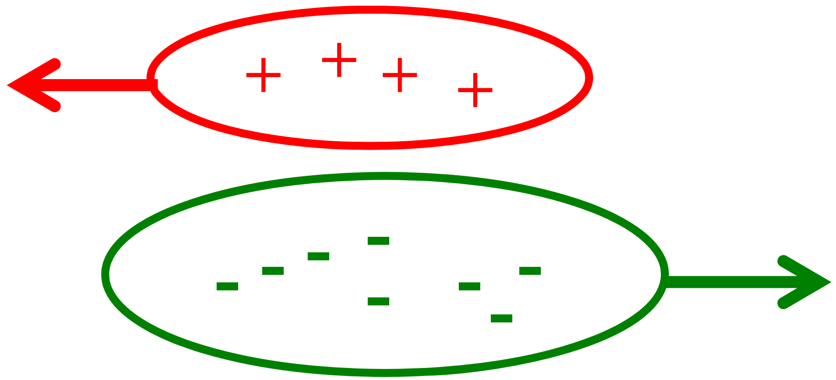


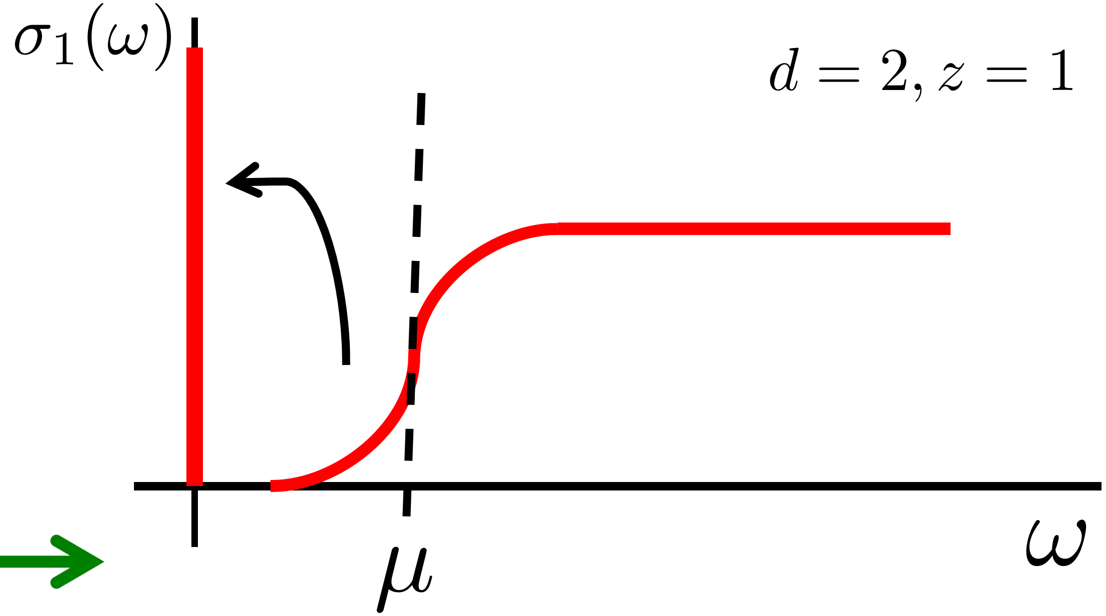

$$\rho \propto \frac{1}{\tau_\hbar} \propto k_B T$$

# Optical conductivity: finite density *particle* system.




**Current carries momentum: in Galilean continuum the weight below the chemical potential accumulates in the "perfect metal" Drude peak.**

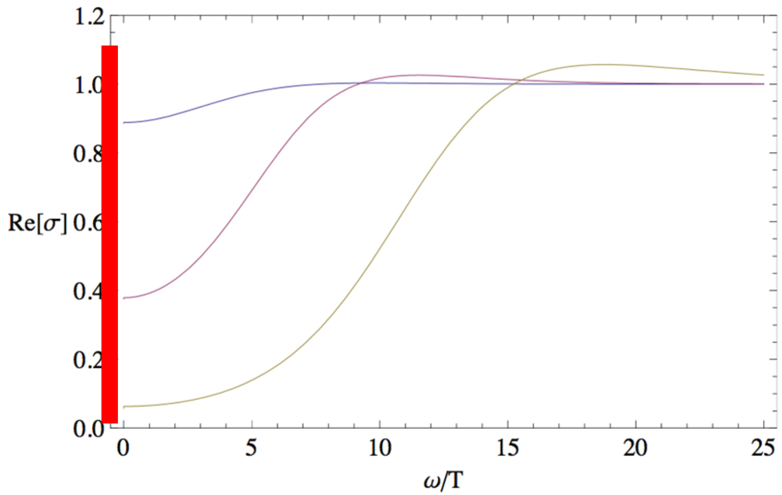


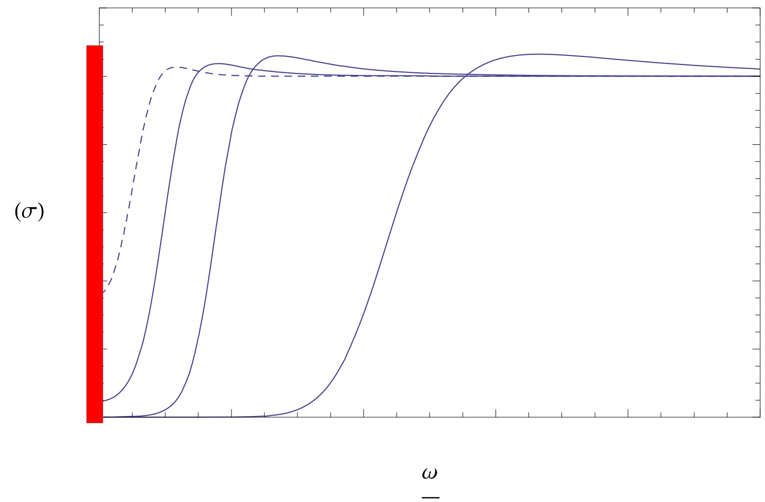
# Holographic optical conductivity (2+1D).

**Optical conductivity in finite density systems:**

**Strange metal.**

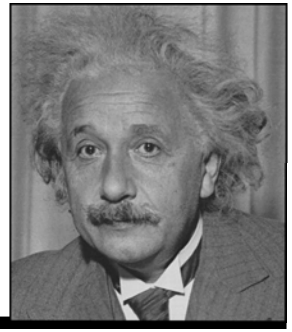



**Holographic superconductor.**



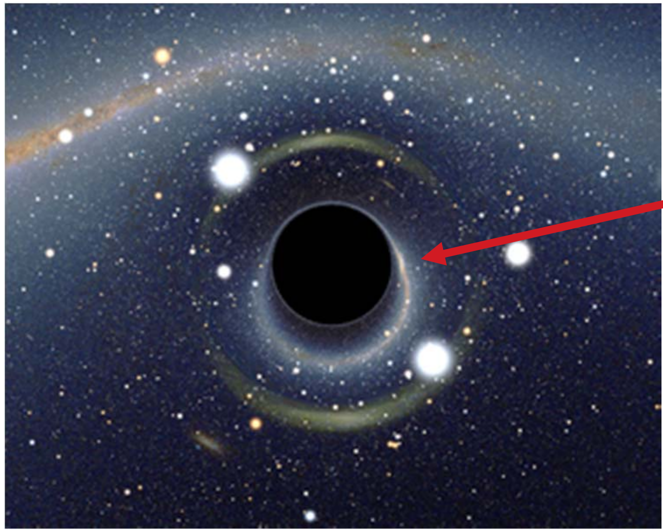

**"Gapping" as for a BCS superconductor!**

# Black holes with a corrugated horizon



**Charged Black Hole: describes finite density strange metal .**

**Breaking translational symmetry in the boundary:**



**Corrugate the black hole horizon ….**

**Not a favorite thing of general relativity -- hard work, still in progress!**

# Holographic quenched disorder.

David Vegh

**Dictionary entry "number one":**

**Global translational invariance in the boundary (energy-momentum conservation)**

**<==>**

**General covariance in the bulk (Einstein theory)**

**Breaking of Galilean invariance in the boundary = elastic scattering (?)**

**<==>**

**Fix the (spatial) frame in the bulk = "Massive gravity"**

$$S = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\left(\mathcal{R} + \frac{6}{L^2} - \frac{L^2}{4}F_{\mu\nu}F^{\mu\nu} + m^2\left(\alpha \mathrm{Tr}\left(\mathcal{K}\right) + \beta\left(\mathrm{Tr}\left(\mathcal{K}\right)^2 - \mathrm{Tr}\left(\mathcal{K}^2\right)\right)\right)\right)$$

$$\mathcal{K}^\mu_\alpha \mathcal{K}^\alpha_\nu = g^{\mu\alpha} f_{\alpha\nu}$$

**Couple the metric $g_{ab}$ to a fixed metric $f_{xx}=f_{yy}=1$**

# Holographic linear resistivity

(PRB 89, 2451161, 2014).

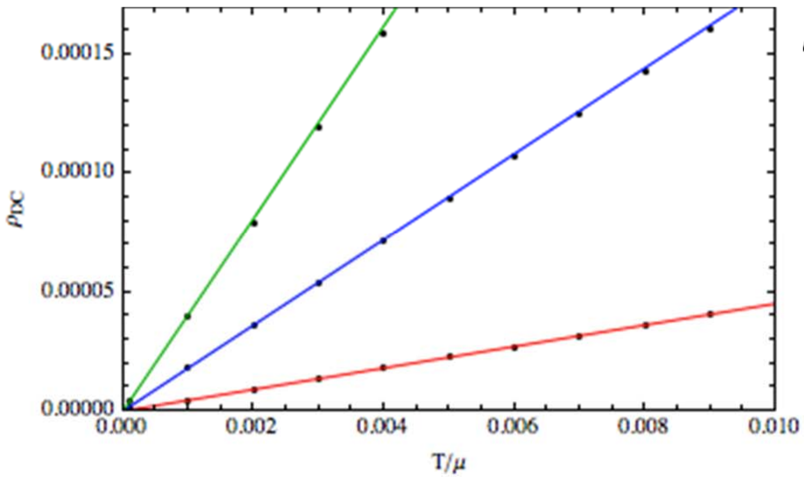
Richard Davison

Steve Gubser

**"Champion" strange metal: Einstein-Maxwell-dilaton (consistent truncation), local quantum critical, marginal Fermi-liquid (3+1D), susceptible to holo. superconductivity, healthy thermodynamics: unique ground state, Sommerfeld thermal entropy.**

David Vegh

**Breaking of Galilean invariance (finite conductivities) due to quenched disorder: "massive gravity" = "Higgsing" space-like diffeomorphisms in the bulk !?**




$$S = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\Big[\mathcal{R} - \frac{1}{4}e^{\phi}F_{\mu\nu}F^{\mu\nu} - \frac{3}{2}\partial_\mu\phi\partial^\mu\phi + \frac{6}{L^2}\cosh\phi - \frac{1}{2}m^2\left(\mathrm{Tr}\,(\mathcal{K})^2 - \mathrm{Tr}\,(\mathcal{K}^2)\right)\Big]$$

**Explicit holographic construction explaining linear resistivity!**

# The secret of the linear resistivity (PRB 89, 2451161, 2014).

Davison

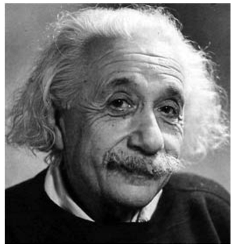
Hartnoll

**Planckian dissipation = very rapid local equilibration**: a hydrodynamical fluid is established before it realizes that momentum is non conserved due to the lattice potential (not true in Fermi-gas: Umklapp time is of order collision time).

Stokes

Resistivity in hydrodynamics

$$\rho(T) \propto \frac{1}{\tau_{rel}} = \frac{D}{l^2}$$



Einstein

Einstein relation:

$$D = \frac{\eta}{m_e n_e}$$

Sachdev

Son

**Planckian viscosity**

$$\eta = A\frac{\hbar}{k_B}s$$

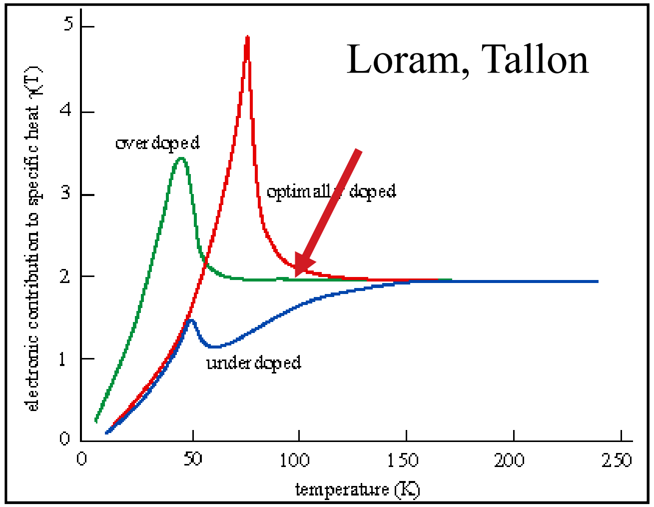
$$\rho(T) = \frac{1}{\omega_p^2 \tau_{rel}} = A\frac{\hbar}{\omega_p^2 l^2 m_e}\frac{S}{k_B}$$

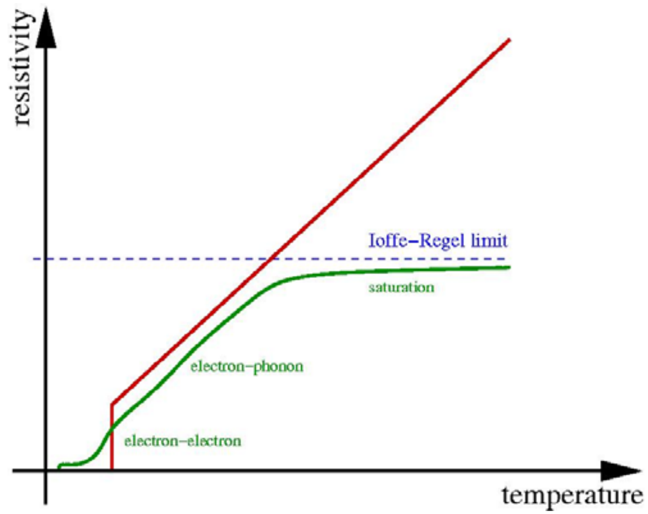
# Entropy versus transport: optimal doping

**Optimally doped**

$$C = \gamma T \Rightarrow S = T/\mu$$

$$\rho \propto \frac{1}{\tau_{rel}} \propto S \propto T$$



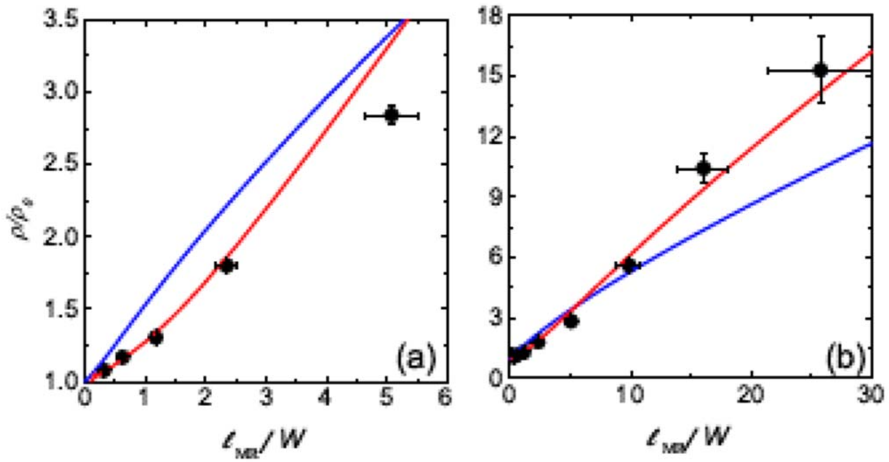

**Plugging in numbers: "mean-free path"** $l \approx 10^{-9}\, m$

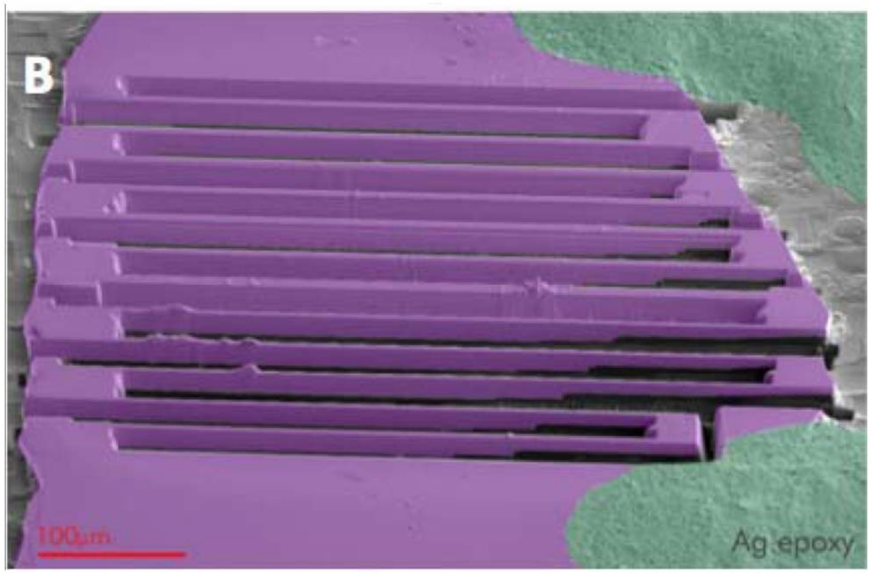
**Quite dirty but no residual resistivity since the fluid becomes perfect at T =0 !**

# Evidence for hydrodynamic electron flow in $PdCoO_2$

**P. J. W. Moll, P. Kushwaha, N. Nandi, B. Schmidt, and A. P. Mackenzie**

***arXiv:1509.05691***




**Hydrodynamic effect on transport.**

**The measured resistivity of $PdCoO_2$ channels normalised to that of the widest channel (ρ0), plotted against the inverse channel width 1/W multiplied by the bulk momentum- relaxing mean free path.**

# The tip of the empirical iceberg …

**- "Magneto-hydrodynamical" effects in microwave and optical response (vd Marel).**

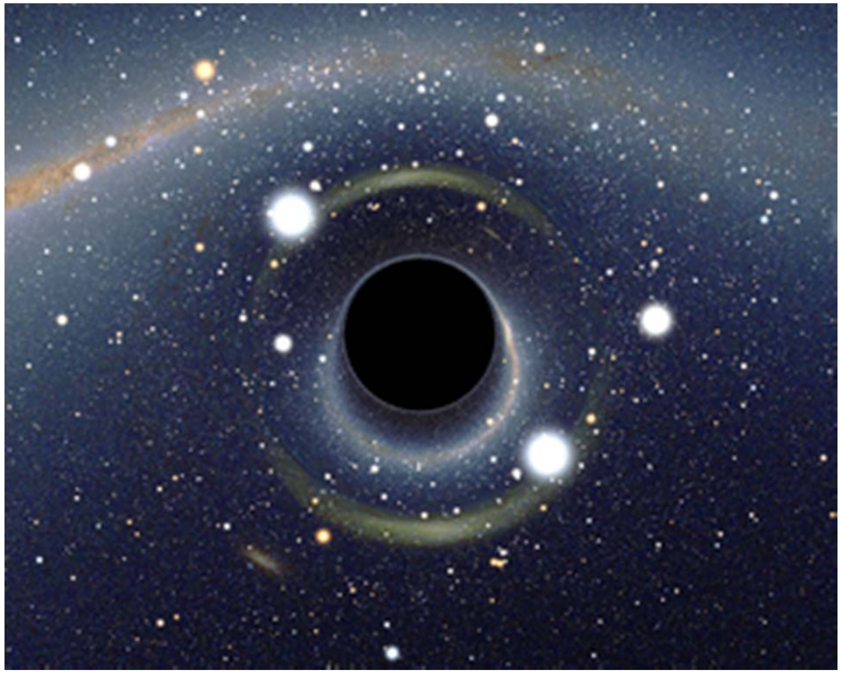
**- Direct evidence for a quantum critical phase in cuprates: doping dependent transport, the Hall angle, transversal WF (Hussey, …)**

**- Strange metal behaviour showing up in Photoemission and scanning tunneling spectroscopy (Dessau, …)**

**- (Dis)proving holographic superconductivity: measuring the conformal metal in the pair susceptibility**

**- "Charge renormalization": "fractional" Ahronov-Bohm effect (Phillips et al).**

# Black holes as “quantum matter computers” !?



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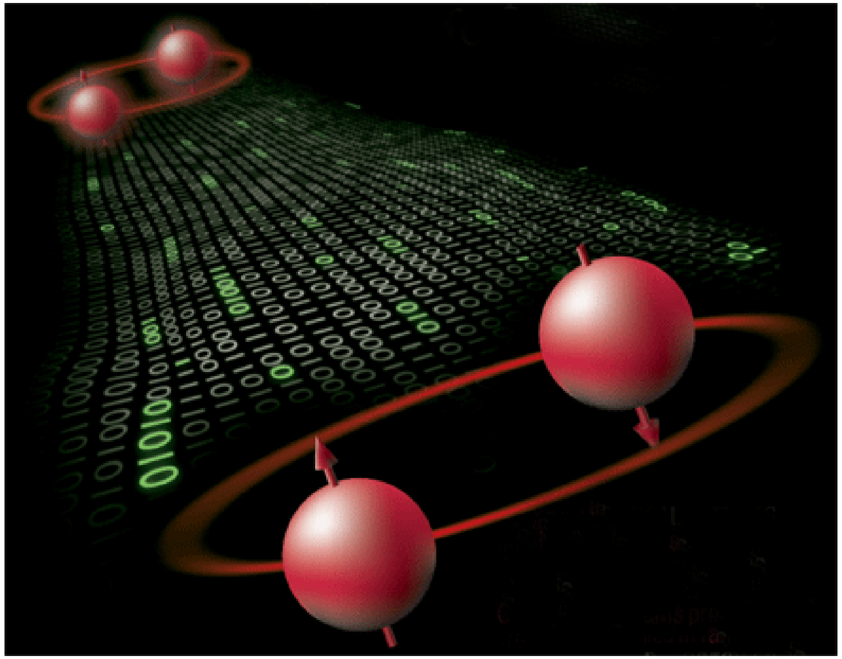


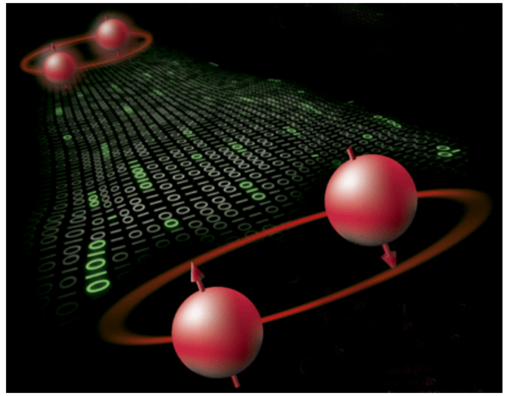
# Bell pairs and the "spooky action at a distance".

**Classical computers live in tensor product space:**

$$|\,product\rangle = |0\rangle_A \otimes |1\rangle_B \text{ or } |1\rangle_A \otimes |0\rangle_B$$

**Quantum computers exploit entangled states capable of "spooky action at a distance" (EPR paradox).**

$$|\,Bell\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B\right)$$



# "The classical condensates: from crystals to Fermi-liquids."

**States of matter that we understand are "short range entangled product"!**

$$\left|\Psi_0\{\Omega_i\}\right\rangle = \Pi_i \hat{X}_i^+(\Omega_i)\left|vac\right\rangle$$

**- Crystals: put atoms in real space wave packets** $\quad X_i^+(R_i^0) \propto e^{(R_i^0-r)^{\wedge}2/\sigma^2}\psi^+(r)$

**- Magnets: put spins in generalized coherent state**

$$X_i^+(\vec{n}_i) \propto e^{i\varphi_i/2}\cos(\theta_i/2)c_{i\uparrow}^+ + e^{-i\varphi_i/2}\sin(\theta_i/2)c_{i\downarrow}^+$$

**- Superconductors/superfluids: put bosons/Cooper pairs in coherent superposition** $\quad X_{k/i}{}^+ \propto u_k + v_k c_{k\uparrow}^+ c_{-k\downarrow}^+, \quad u_i + v_i e^{i\varphi_i} b_i^+$

**- Fermi gas/liquid special , but only "Fermi-Dirac entanglement"**

$$\left|\Psi_{FL}\right\rangle = \Pi_k^{k_F} c_k^+ \left|vac\right\rangle$$

# Quantum matter.

**"Macroscopic stuff that can quantum compute all by itself"**

$$|\Psi\rangle = \sum_{configs} A_{configs} |configs\rangle$$

**- Topological incompressible systems, no low energy excitations but the whole carries quantum information: fractional quantum Hall, top. Superconductors/insulators (Majorana' s, theta vacuum, ..)**

**- Compressible systems: strongly interacting bosonic quantum critical states have dense long range entanglements (Planckian dissipation)**

**- Compressible systems: are the strange metals of this kind??**

**Strongly interacting fermions at finite density: the fermion signs as entanglement resource!**

# Turning on the backflow

Kruger, JZ, PRB 78, 035104 (2008)

**Nodal surface has to become fractal !!!**

**Try backflow wave functions**

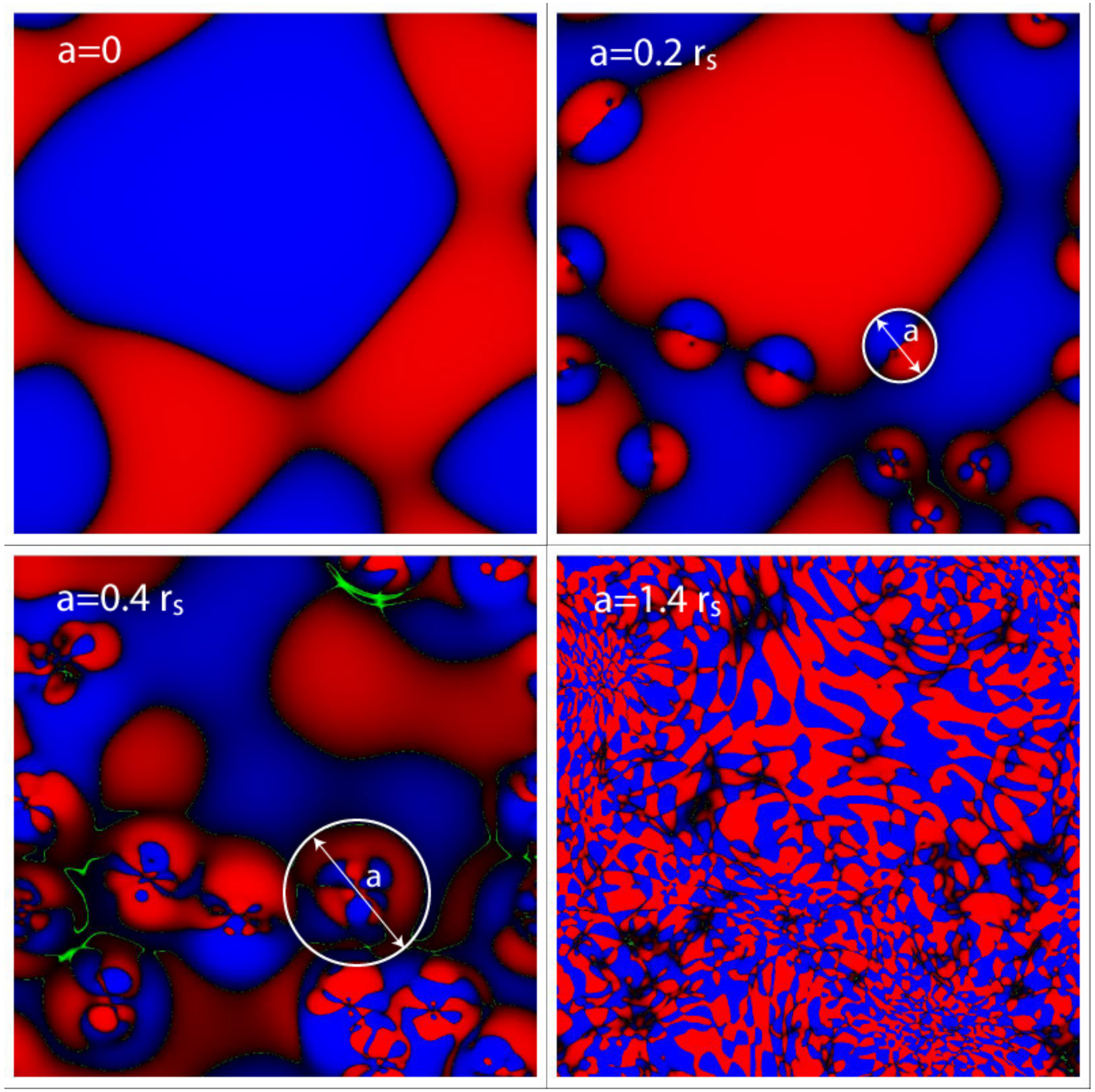
$$\psi_{bf}(\mathbf{R}) \sim \mathrm{Det}\left(e^{i\mathbf{k}_i\tilde{\mathbf{r}}_j}\right)_{ij}$$

$$\tilde{\mathbf{r}}_j = \mathbf{r}_j + \sum_{l(\neq j)} \eta(r_{jl})(\mathbf{r}_j - \mathbf{r}_l)$$

$$\eta(r) = \frac{a^3}{r^3 + r_0^3}$$

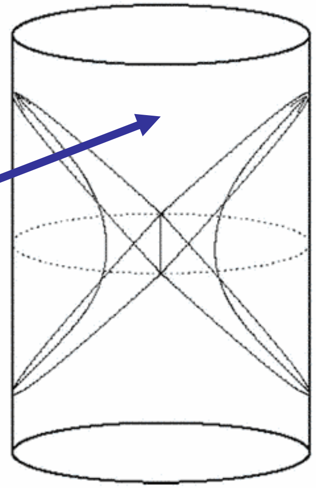
**Collective (hydrodynamic) regime:**

$$a \gg r_s$$



# Its from quantum bits ?

Van Raamsdonk



**Classical space time in bulk …**

Entangled state of pair of $H^d$ CFTs

$$|\Psi\rangle = \sum_i e^{-\pi R_H E_i} |E_i\rangle_{H^d} \otimes |E_i\rangle_{H^d}$$

**Encoded by quantum info (entanglement spectrum) in boundary**

“Rindler” description of pure global AdS

**also**

Hubeny Myers Swingle

**• • • • •**

# The tip of the iceberg.

- **Electron systems in solids: the glass ceiling of quantum field theory = *sign problem.***

- **Holographic duality = the generating functional of the mathematical theories describing strong emergence in quantum systems.**

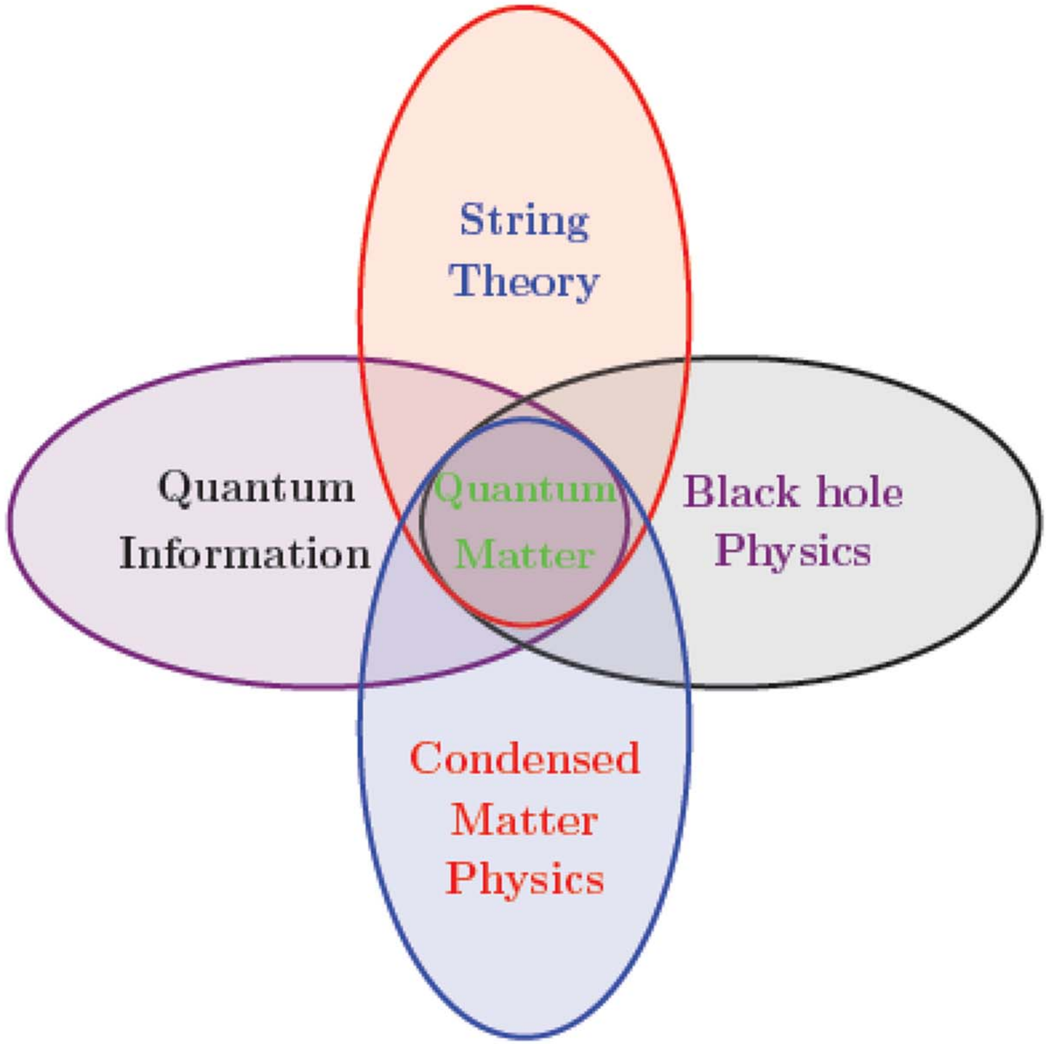
- **The extended conventional textbook of condensed matter physics is rewritten in terms of the equations of general relativity.**

- **Predicts "beauty behind the fermion brick wall": strange metals as self organized quantum critical phases with novel scaling properties, presently chased in the cond. mat. laboratories**

- **These are extremely densely entangled states of quantum matter.**

- **What does this all mean for the greater *quantum gravity* agenda?**

# Koenraad's cloverleaf ….



# Entanglement entropy

**Bipartite von Neumann entropy: measures entanglement = quantum information of Bell pairs.**

**Trace the full density matrix over B:** $$\rho_A = Tr_B \rho$$

**Compute the entropy associated with the reduced density matrix:**

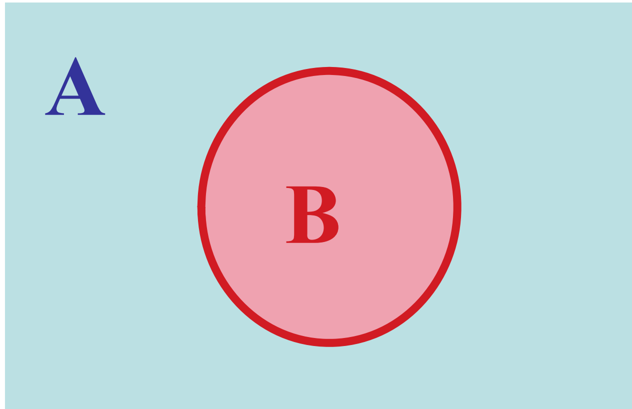
$$S_{vN,A} = Tr\left[\rho_A \ln \rho_A\right]$$

**Universal measure of two bit entanglement:**

$$|Bell\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B\right) \qquad S_{vN,A} = \sqrt{2}$$

$$|\mathrm{Pr}\,od\rangle = \frac{1}{2}\left(|0\rangle + |1\rangle\right)_A \otimes \left(|0\rangle + |1\rangle\right)_B \qquad S_{vN,A} = 0$$

# "Space bipartite" Von Neumann entropy: entanglements and fields




Divide **space in two**:

$$\rho_A = Tr_B \rho, \quad S_A = Tr\left[\rho_A \ln \rho_A\right]$$

**1+1 D CFT's** (Wilczek et al., Calabrese-Cardy):

$$S_A = \frac{c}{3} \log L$$ **c** = central charge

**Topological insulators** (Kitaev-Preskil, Levin-Wen):

$$S_A = \alpha \Sigma - \gamma + \cdots, \ \Sigma = L^{d-1}, \quad \gamma = \log D$$ ***D*** = total quantum dimension.

**Conformal fields in higher (e.g. odd) d** (Myers, Klebanov, …):

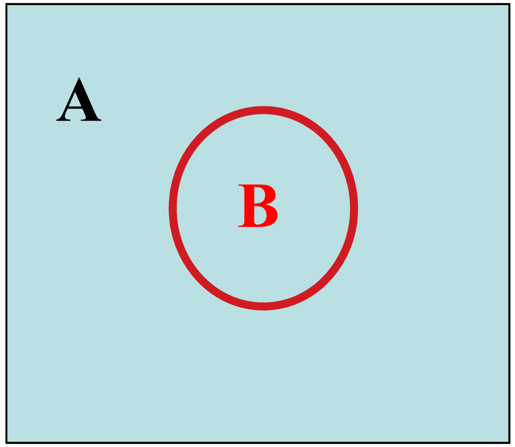
$$S_A = \alpha \Sigma + \cdots + (-1)^{(d-1)/2} s_d$$ $\mathbf{S_d}$ is universal

# Bipartite entanglement entropy and quantum field theory.


Wilczek

$$\rho_A = Tr_B[\,\rho\,]$$

$$S_{vN} = Tr[\rho_A \ln \rho_A]$$




**Measure of entanglement of degrees of freedom in spatial volume B with those in A.**

**Generic energy eigenstates: $S_{vN}$ scales with volume $L^d$ of B.**

**Ground states of bosonic systems: $S_{vN}$ scales with the area $L^{d-1}$ of B.**

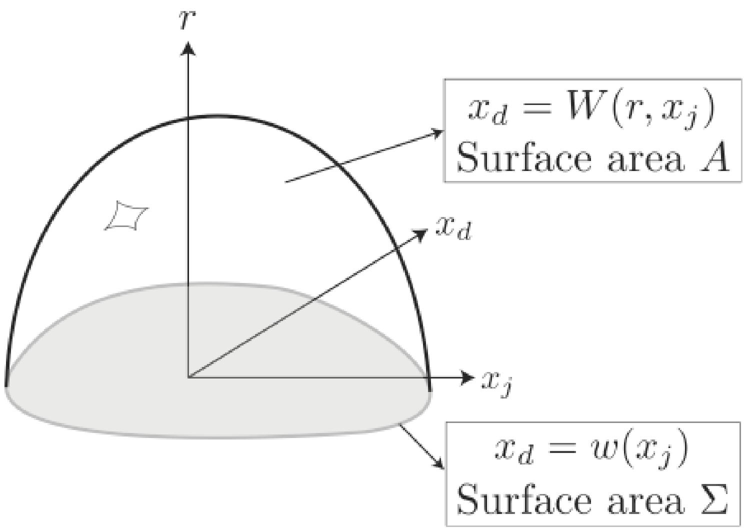
**Fermi gas: longer ranged "signful" entangled $S_{vN} \sim L^{d-1}\ln(L^{d-1})$**

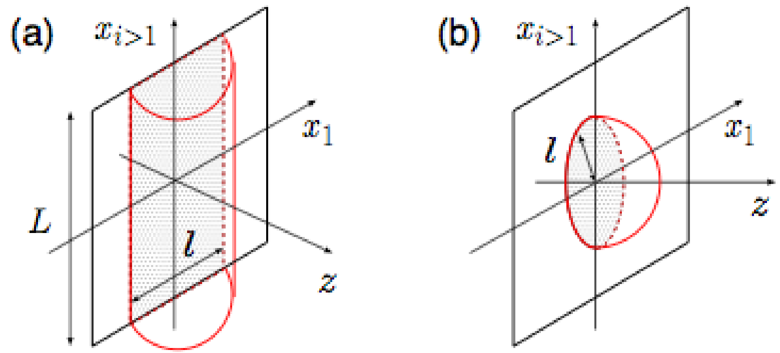
# Entanglement entropy versus AdS/CFT.

Takayanagi Ryu

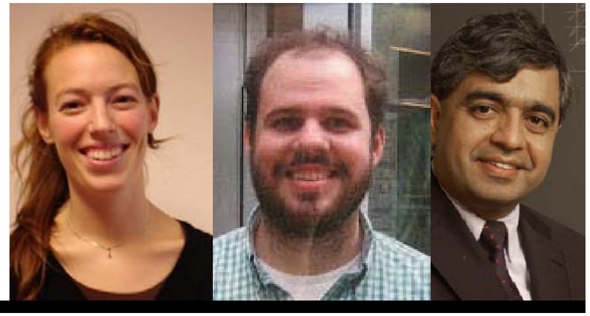
$$\rho_A = Tr_B[\rho] \qquad S_{vN} = Tr[\rho_A \ln \rho_A]$$

**The spatial bipartite entanglement entropy in the boundary is dual to the area of the minimal surface in the bulk, bounded by the cut in the space of the boundary**



# Holographic strange metal entanglement entropy.



Huijse Swingle Sachdev

PRB 85, 035121 (2012)

**Einstein-Maxwell-Dilaton bulk => "hyperscaling violating geometry" (Kiritsis et al.):**

$$ds^2 = \frac{1}{r^2}\left( -\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\vartheta/(d-\theta)} dr^2 + dx_i^2 \right)$$

**Boundary: interpolating between "normal" and RN strange metals.**

$$x_i \to \zeta x_i, \quad t \to \zeta^z t, \quad ds \to \zeta^{\theta/d} ds \qquad S \propto T^{(d-\theta)/z}$$

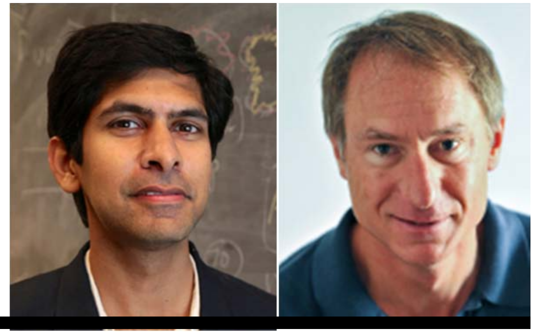
**Entanglement entropy:**

$$S_{vN} \propto L^{d-1} \qquad , \theta < d-1$$ **Bosonic fields**

$$\boxed{S_{vN} \propto L^{d-1} \ln L^{d-1}, \theta = d-1}$$ **Fermi liquid-like**

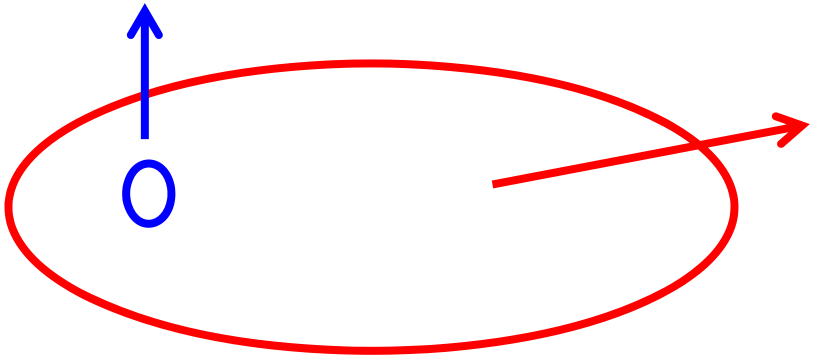
$$\boxed{S_{vN} \propto L^{\theta} \qquad , d-1 < \theta < d}$$ **But this is longer ranged !**

# Fermion signs and dense entanglement …



Grover Fisher
arXiv:1412.3534

$S_{vN}$ **area law: ground states of "sign-free" systems (bosons, tensor product states ..)**



**Energy eigenstates:**

$$|\Psi_i\rangle = \sum_{conf} (-1)^{i,conf} |A^i_{conf}| |conf\rangle$$

$$(-1)^{i,conf} =$$

**- Antisymmetrization => area log area $S_{vN}$ (Fermi gas)**

**- Random => Volume $S_{vN}$ (typical excited states)**

**The *quantum critical metallic phases* of holography are characterized by dense "sign driven" entanglement as characterized by the hyperscaling violation exponent!**

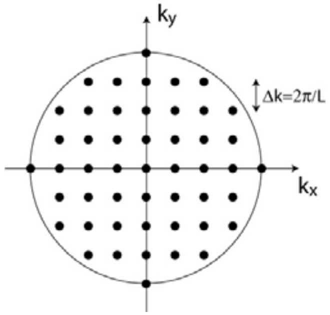
# The nodal hypersurface

**Antisymmetry of the wave function**

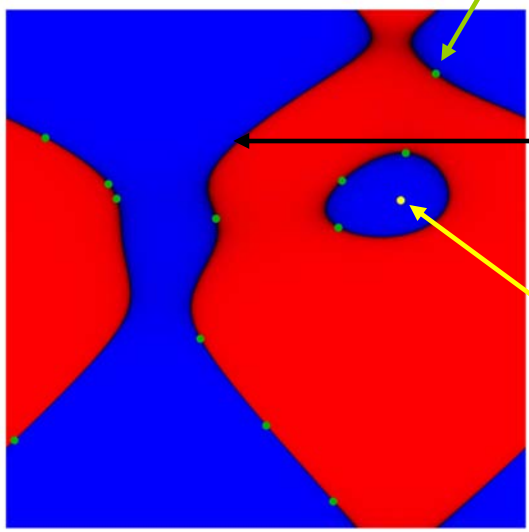
$$\Psi(\mathbf{r}_1,\ldots,\mathbf{r}_i,\ldots,\mathbf{r}_j,\ldots,\mathbf{r}_N) = -\Psi(\mathbf{r}_1,\ldots,\mathbf{r}_j,\ldots,\mathbf{r}_i,\ldots,\mathbf{r}_N)$$

**Free Fermions**

$$\Psi_0(\mathbf{R}) \sim \mathrm{Det}\left(e^{i\mathbf{k}_i\mathbf{r}_j}\right)_{ij}$$




**d=2**



**Pauli hypersurface**

$$P = \bigcup_{i\neq j} P_{ij}$$

$$P_{ij} = \{\mathbf{R} \in \mathbb{R}^{Nd} | \mathbf{r}_i = \mathbf{r}_j\}$$

$$\mathrm{dim}P = Nd - d$$

**Nodal hypersurface**

$$\Omega = \{\mathbf{R} \in \mathbb{R}^{Nd} | \Psi(\mathbf{R}) = 0\}$$

$$\mathrm{dim}\Omega = Nd - 1$$

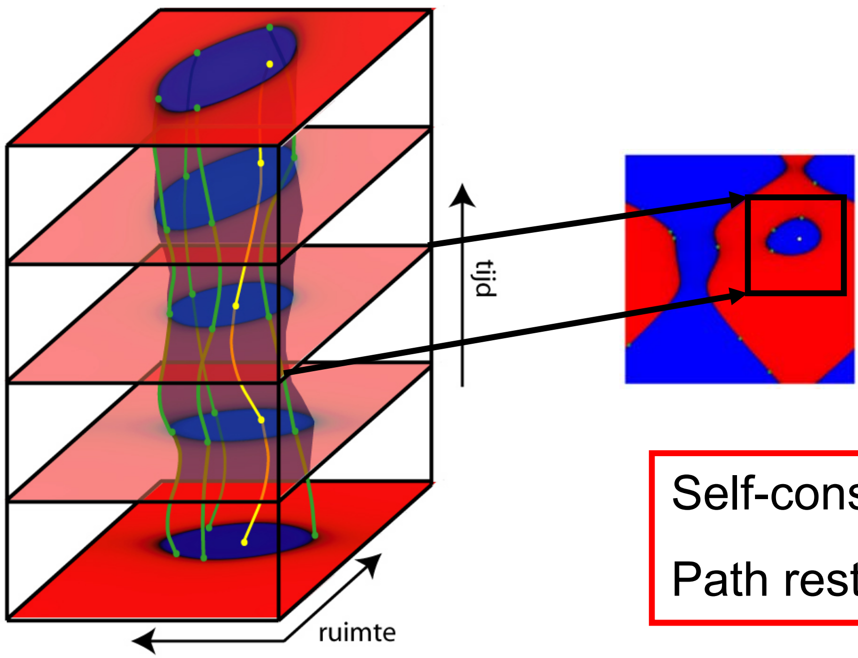
**Test particle**

# Constrained path integrals

Formally we can solve the sign problem!!

$$\rho_F(\mathbf{R},\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P},\text{even}} \int_{\gamma:\mathbf{R}\to\mathcal{P}\mathbf{R}}^{\gamma\in\Gamma(\mathbf{R},\mathcal{P}\mathbf{R})} \mathcal{D}\mathbf{R}(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T} \mathrm{d}\tau \left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau) + V(\mathbf{R}(\tau))\right)\right\}$$

$$\Gamma(\mathbf{R},\mathbf{R}') = \{\gamma : \mathbf{R} \to \mathbf{R}' | \rho_F(\mathbf{R},\mathbf{R}(\tau);\tau) \neq 0\}$$




*Ceperley, J. Stat. Phys. (1991)*

Self-consistency problem:

Path restrictions depend on $\rho_F$ !

# Reading the worldline picture

**Fermi-energy: confinement energy imposed by local geometry**

$$l^2(\tau) = \langle (\mathbf{r}_i(\tau) - \mathbf{r}_i(0))^2 \rangle = 2d\mathcal{D}\tau = 2d\frac{\hbar}{2m}\tau$$

$$l^2(\tau_c) \simeq r_s^2 \rightarrow \tau_c \simeq \frac{1}{2d}\frac{2m}{\hbar}n^{-2/d}$$

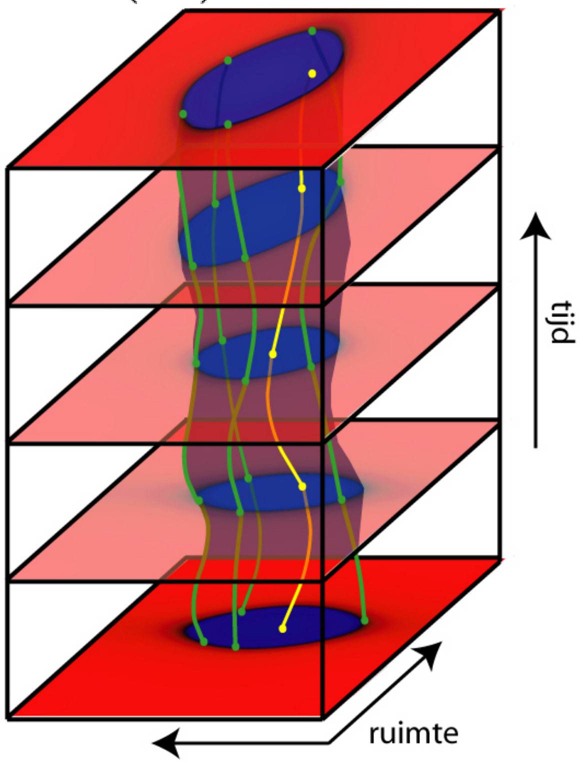
$$\hbar\omega_c = \frac{\hbar}{\tau_c} \simeq d\frac{\hbar^2}{2m}n^{2/d} \simeq E_F$$

**Average node to node spacing**

$$\sim r_s = \left(\frac{V}{N}\right)^{1/d} = n^{-1/d}$$

**Fermi surface encoded globally:** $\rho_F = Det\left(e^{ik_i r_j}\right) = 0$

**Change in coordinate of one particle changes the nodes everywhere**




**Finite T:** $\rho_F = (4\pi\lambda\beta)^{-dN/2} Det\left[\exp\left(-\frac{(r_i - r_{j0})^2}{4\lambda\tau}\right)\right]$

$\lambda = \hbar^2/(2M)$

**Non-locality length:** $\lambda_{nl} = v_F \tau_{inel} = v_F\left(\frac{E_F}{k_B T}\right)\left(\frac{\hbar}{k_B T}\right)$

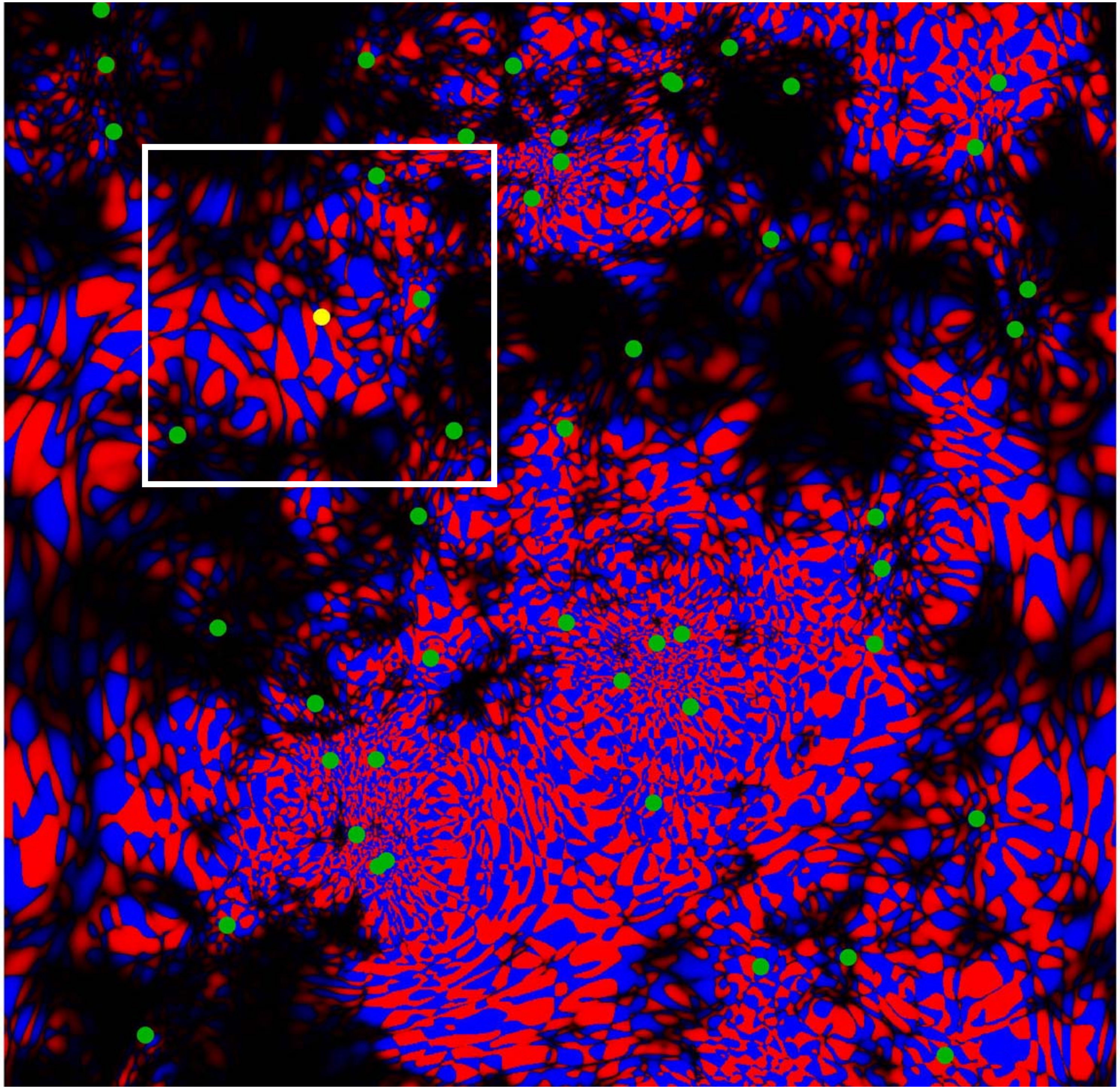
# Key to fermionic quantum criticality

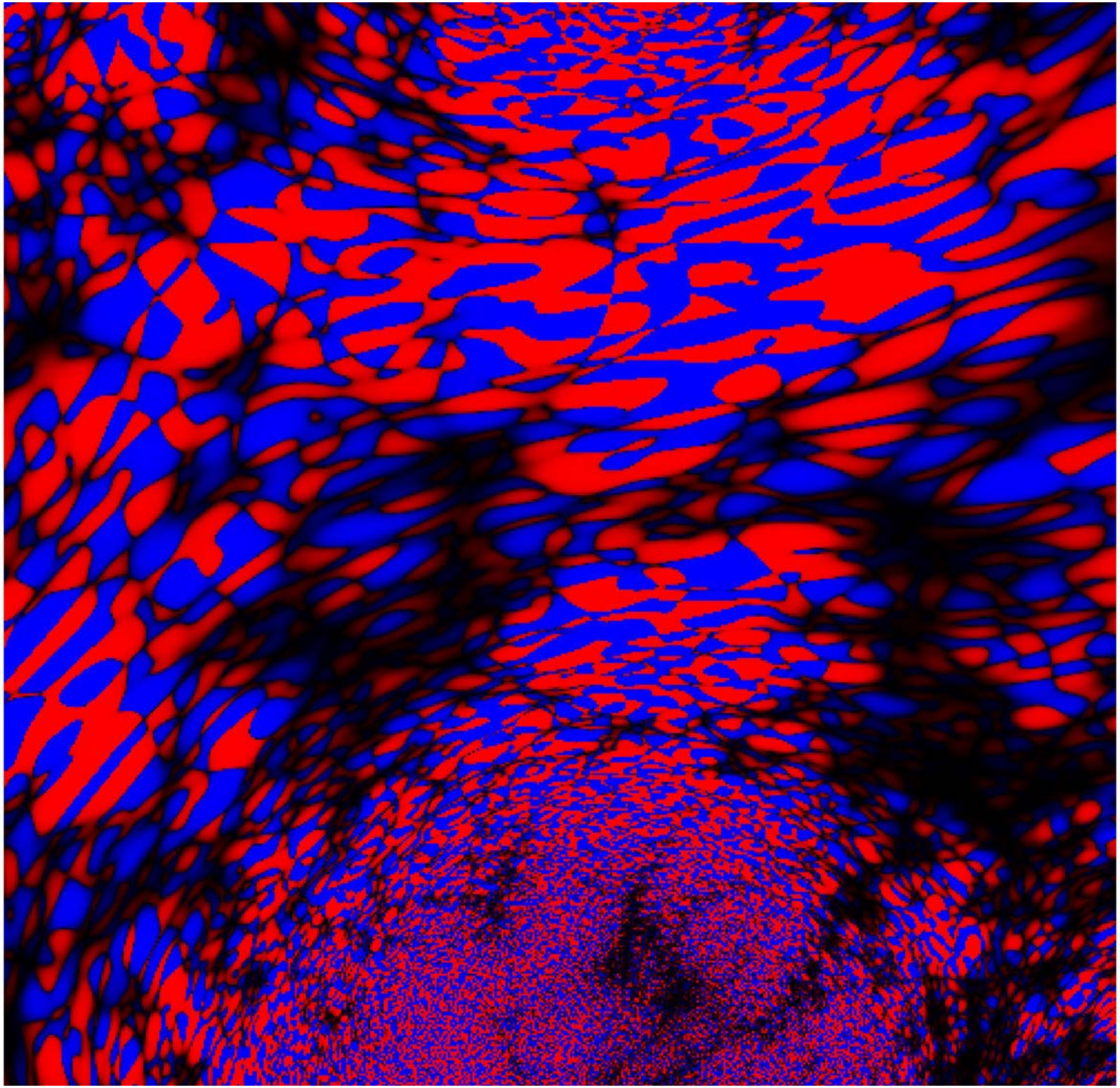

Kruger


JZ

Phys. Rev. B **78**, 035104 (2008)

**At the QCP scale invariance, no $E_F$** → **Nodal surface has to become fractal !!!**

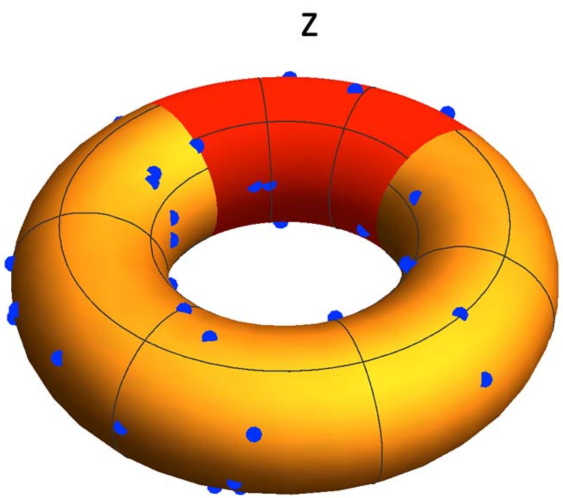




# Fractal nodes and entanglement entropy.


Kaplis Kruger

Second Renyi entropy: leading contribution scales like vN entropy.




$$S^q(z) = \frac{\ln(Tr\rho_A^q)}{1-q}, \quad q = 2$$

$$\rho(\mathbf{R}, \mathbf{R}') = \psi^*(\mathbf{R})\psi(\mathbf{R}')$$

$$\psi_{bf}(\mathbf{R}) \quad \sim \quad \mathrm{Det}\left(e^{i\mathbf{k}_i \tilde{\mathbf{r}}_j}\right)_{ij}$$

$$\tilde{\mathbf{r}}_j \quad = \quad \mathbf{r}_j + \sum_{l(\neq j)} \eta(r_{jl})(\mathbf{r}_j - \mathbf{r}_l)$$

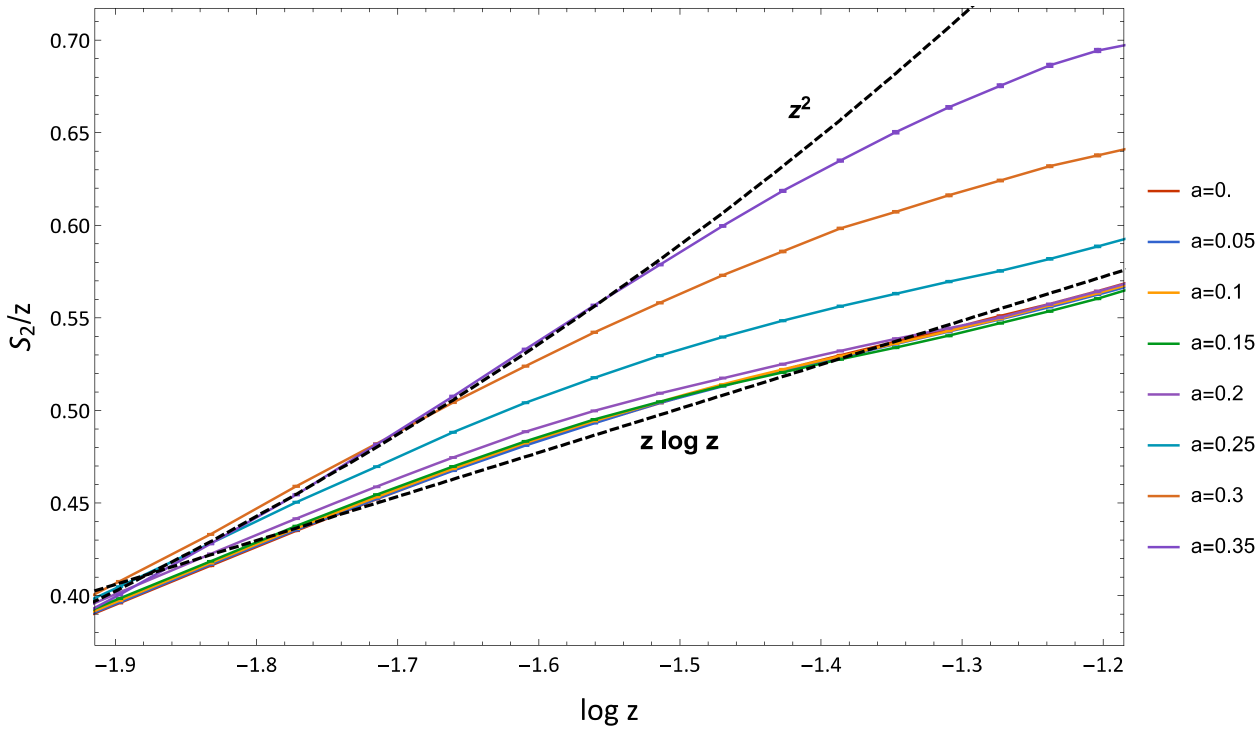
Backflow range exponent “eta” (=3 for hydro backflow) : $\eta(r) = \dfrac{a^\eta}{r^\eta + r_0^\eta}$

# Fractal nodes and entanglement entropy.

Kaplis Kruger

Second Renyi entropy: $S^q(z) = \dfrac{\ln(Tr\rho_A^q)}{1-q}, \quad q=2$

Hydrodynamical backflow, for increasing backflow length a ($a_c$ =0.5):



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