Floquet topological insulators: quantum engineering using driving fields

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Next generation quantum materials, São Paulo, April 4, 2016

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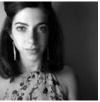
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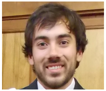

F. Mangussi (PhD)

Outline

- Topological Insulators.

  QHE (broken TRS, B field),TIs (SO, TRS)⇒protected edge states

- Floquet Topological Insulators.

  New tuning tool. Floquet theory. What's similar? what's different?

- Irradiated graphene as a paradigmatic example.

  Edge states. Two terminal conductance. Multi-terminal case. Hall conductance ($G = \frac{e^2}{h} N_{\mathrm{edges}}?$). States Hierarchy. Defects and adatoms. Anomalous Goos-Hänchen effect.

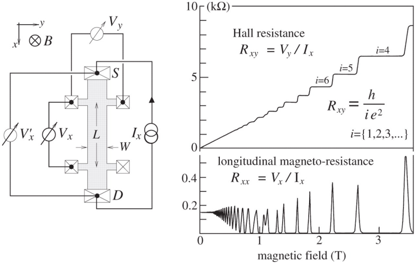
Take home message

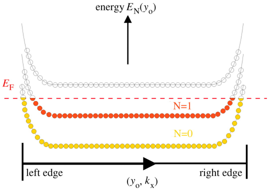
Periodic time dependent potentials can induce topological properties (edge modes) on 'normal' materials with clear transport signatures. Not all of them contribute the same to spectral and transport properties, there is a hierarchy.

Topological insulators

Integer QHE (an anomalous bulk insulator)

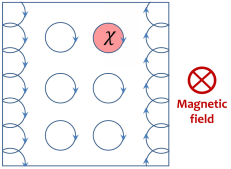
A 2DEG with a $\perp B$ field





$$G_{xy} = N\frac{e^2}{h}$$

- TRS is broken $\Rightarrow$ chiral edge states.
- N is the number of edge states (filled Landau levels)
- There is no backscattering (chirality) $\Rightarrow$ channels are ballistic.
- Effect is robust agains imperfections



D. Carpentier lectures

Integer QHE

A topological point of view

Thouless (TKNN), PRL (1982)

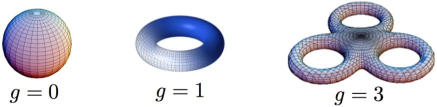
- N is a topological invariant ($\sum_n C_n$).
- Describes a bulk insulator (gap required) $\Rightarrow$ not all insulators are equal.
- Property of the Bloch wavefunctions $|u_{n\boldsymbol{k}}\rangle$.
- Insensitive to small perturbations (geometry, disorder, weak interactions)

$$C_n = \frac{1}{2\pi} \int_{BZ} \nabla \times \boldsymbol{A}_n \cdot d\boldsymbol{S}$$

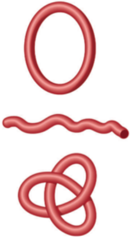
Chern number

$$\boldsymbol{A}_n = i \langle u_{n\boldsymbol{k}} | \nabla_{\boldsymbol{k}} | u_{n\boldsymbol{k}} \rangle$$

Berry curvature



Gauss-Bonnet theorem

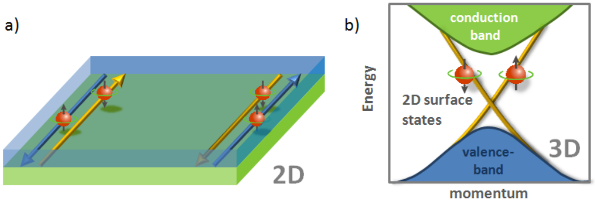


Moore, Nat. (2010)

New class of topological insulators

⇒2D Quantum Spin Hall (QSHE): Proposed by Kane & Mele PRL (2005); Bernevig, Hugues & Zhang, Science (2006); observed by Köning *et al* Science (2007)

⇒3D Topological Insulators: Proposed by Fu, Kane & Mele PRL (2007); Moore & Balents PRB (2007); experiments by Hsieh *et al* Nature (2008)

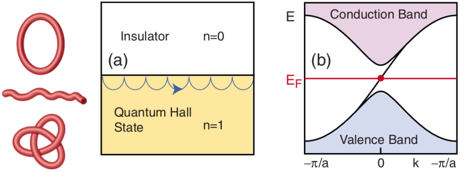


Helical states

New class of topological insulators

Key role of spin-orbit coupling $\Rightarrow$ no TRS breaking

- Bulk gap.
- Conducting boundary states at the interface with a non-topological material (e.g. vacuum)
- Non-trivial Hall response.



Hasan and Kane, RMP (2010)

Bulk-boundary correspondence

Bulk determines what happens at the boundary (microscopic details are irrelevant)

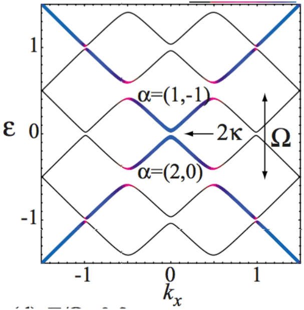
Requires material engineering (complex materials)

Is there another approach?

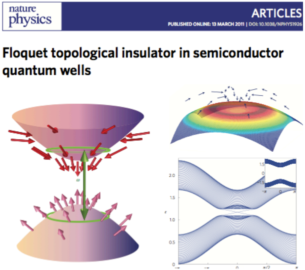
Floquet topological insulators

Time dependent periodic potentials

⇒ Induce a new (out of equilibrium) 'band structure'



Oka & Aoki, (2009)

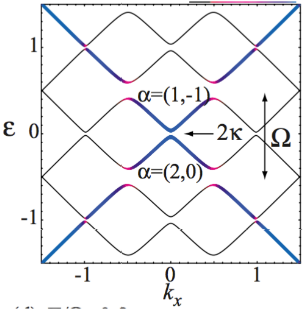


Lindner, Refael & Galitski (2010)

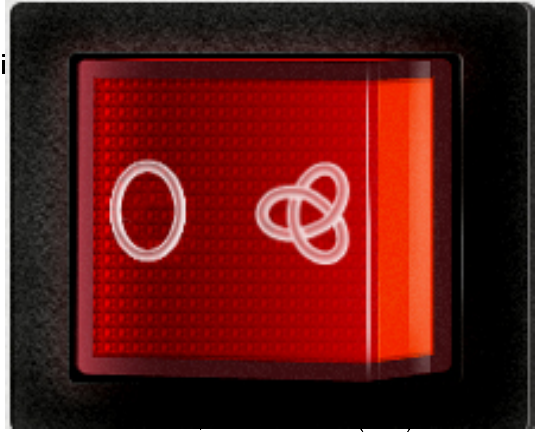
Floquet topological insulators

Time dependent periodic potentials

⇒ Induce a new (out of equi



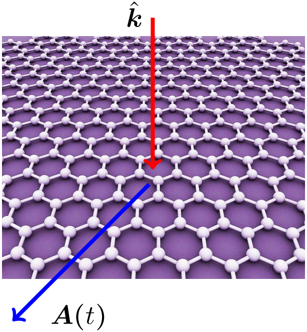
Oka & Aoki, (2009)



A topological switch!

Floquet Dirac fermions

Graphene in the presence of an EM field



- Normal incidence $\Rightarrow \boldsymbol{E}$ in graphene's plane.
- Choose gauge such that $\boldsymbol{E}(t) = -\frac{1}{c}\partial_t \boldsymbol{A}$.
- Monocromatic field, frequency Ω.
- Uniform field, $\boldsymbol{A}(\boldsymbol{r}, t) = \boldsymbol{A}(t)$.

The time dependent hamiltonian is

$$\hat{\mathcal{H}}(t) = v_F\, \boldsymbol{\sigma} \cdot \left[\boldsymbol{p} + \frac{e}{c}\boldsymbol{A}(t)\right]$$

$\boldsymbol{A}(t+T) = A(t) \qquad T = \frac{2\pi}{\Omega} \Leftarrow$ periodic perturbation.

Circularly polarized case: $\boldsymbol{A} = A_0(\cos\Omega t\,\hat{\boldsymbol{x}} + \sin\Omega t\,\hat{\boldsymbol{y}})$.

Floquet theory

Equivalent to Bloch theorem for periodic time dependent systems

Bloch (space)

$$\hat{\mathcal{H}}(\boldsymbol{r} + \boldsymbol{R}) = \hat{\mathcal{H}}(\boldsymbol{r})$$

$$\Rightarrow \psi_{\boldsymbol{k}}(\boldsymbol{r}) = e^{\mathrm{i}\,\boldsymbol{k}\cdot\boldsymbol{r}}\ \phi_{\boldsymbol{k}}(\boldsymbol{r})$$

quasi-momentum

Bloch function

$$\phi_{\boldsymbol{k}}(\boldsymbol{r} + \boldsymbol{R}) = \phi_{\boldsymbol{k}}(\boldsymbol{r})$$

$k \in \left[-\frac{\pi}{a}, \frac{\pi}{a}\right]$ (Brillouin zone)

or extended zone picture

Floquet (time)

$$\hat{\mathcal{H}}(t + T) = \hat{\mathcal{H}}(T)$$

$$\Rightarrow |\psi_\alpha(t)\rangle = e^{-\mathrm{i}\,\varepsilon_\alpha\, t/\hbar}\ |\phi_\alpha(t)\rangle$$

quasi-energy

Floquet eigenvector

$$|\phi_\alpha(t + T)\rangle = |\phi_\alpha(t)\rangle$$

$\varepsilon_\alpha \in \left[-\frac{\hbar\Omega}{2}, \frac{\hbar\Omega}{2}\right]$ (Floquet BZ)

or extended zone picture

Shirley, Phys. Rev (1965); Sambe, PRA (1973).

Floquet theory

The extended Floquet (or Sambe) space: $\mathcal{R} \otimes \mathcal{T}$

- space of periodic functions $\mathcal{T} \Rightarrow |\phi_\alpha(t)\rangle = \sum_{m=-\infty}^{\infty} |u_m^\alpha\rangle e^{\mathrm{i}m\Omega t}$

$$\boxed{H_F|\phi_\alpha\rangle = \varepsilon_\alpha|\phi_\alpha\rangle}$$

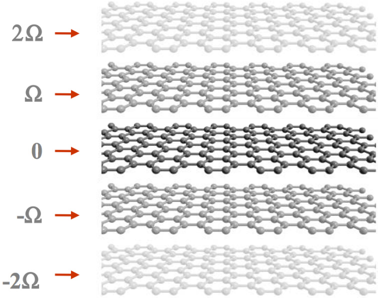
- H_F time independent infinite matrix.

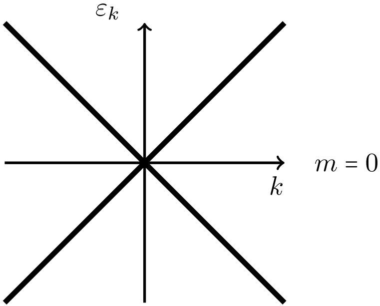
$$\hat{\mathcal{H}}(t) = \hat{\mathcal{H}}_0 + \hat{V} e^{\mathrm{i}\Omega t} + \hat{V}^\dagger e^{-\mathrm{i}\Omega t}$$

$$H_F = \begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \iddots \\ \cdots & H_0 + 2\hbar\Omega \boldsymbol{I} & V & \mathbf{0} & \mathbf{0} & \cdots \\ \cdots & V^\dagger & \boxed{H_0 + \hbar\Omega \boldsymbol{I}} & V & \mathbf{0} & \cdots \\ \cdots & \mathbf{0} & V^\dagger & \boxed{H_0} & V & \cdots \\ \cdots & \mathbf{0} & \mathbf{0} & V^\dagger & H_0 - \hbar\Omega \boldsymbol{I} & \cdots \\ \iddots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Floquet theory

Physical interpretation: ‘electron ⊗ photons’

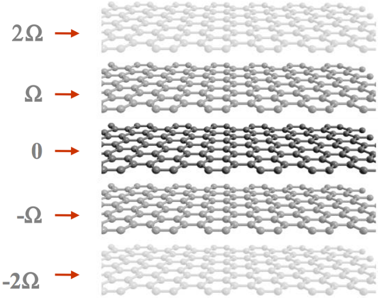


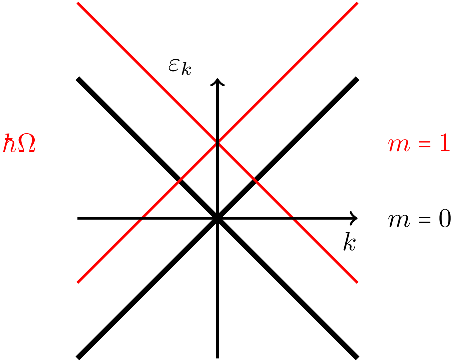


- Copies of the original system coupled by EM field
- Different number of photons

Floquet theory

Physical interpretation: ‘electron ⊗ photons’





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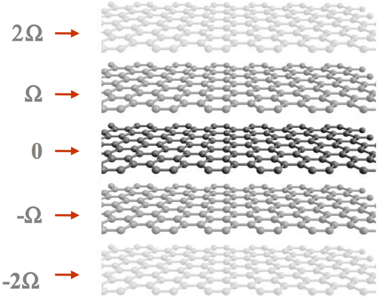

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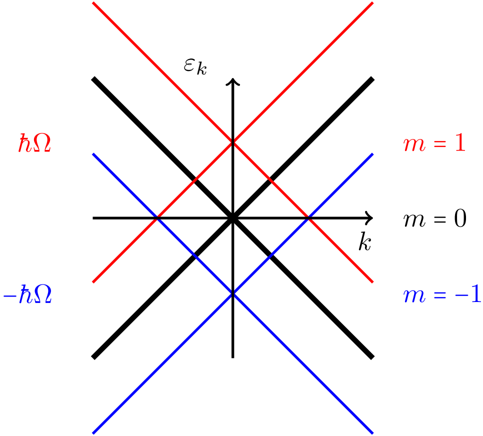

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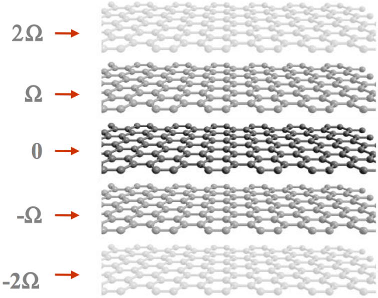

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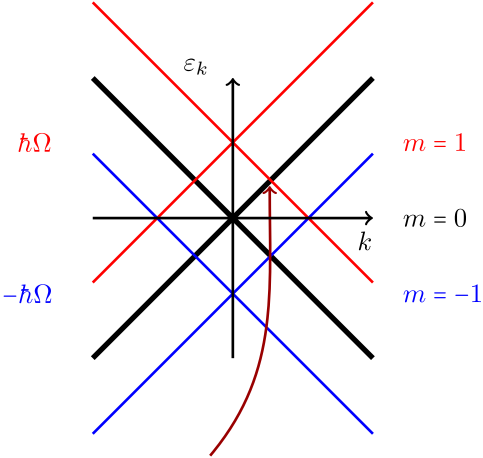
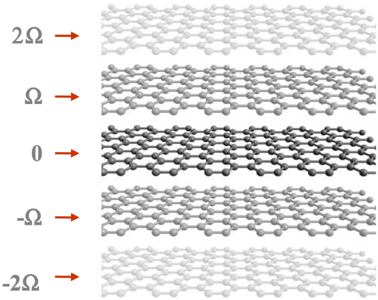


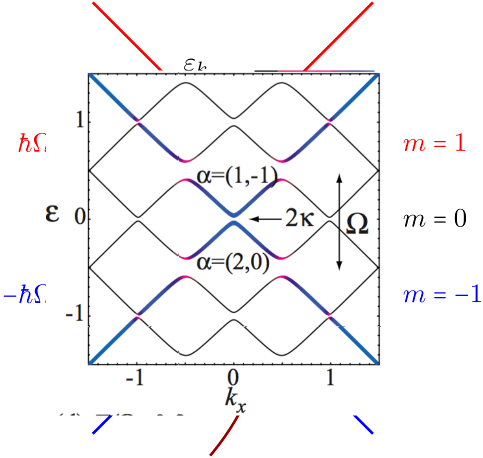
Degeneracy point at $\hbar\Omega/2$
$\Rightarrow$ dynamical gap

Floquet theory

Physical interpretation: ‘electron ⊗ photons’



- Copies of the original system coupled by EM field
- Different number of photons



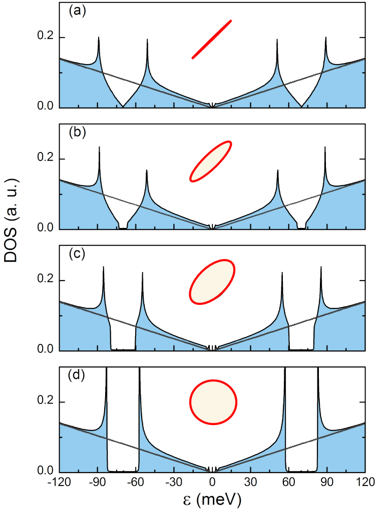
Degeneracy point at $\hbar\Omega/2$
$\Rightarrow$ dynamical gap

Dynamical gap

Projected density of states

$$\bar{\rho}(\varepsilon) = -\frac{1}{\pi}\mathrm{Im}\left(\mathrm{Tr}\left[\boldsymbol{G}_F^{00}(\varepsilon)\right]\right)$$

- Floquet-Green's function:
  $\boldsymbol{G}_F(\varepsilon) = (\varepsilon + \mathrm{i}0^+ - H_F)^{-1}$
- $\boldsymbol{G}_F^{00}(\varepsilon)$ projection onto the $m = 0$ replica.
- gap depends on polarization.
- 'bulk' insulator
  (if polarization is not linear, broken TRS).



Oka & Aoki, PRB (2009); Calvo, *et al.*, APL (2011)

Topological insulators vs. Floquet TI's

Can we extend the idea of topological invariants

TI

$$C_n = \frac{1}{\pi}\mathrm{Im}\int_{\mathrm{BZ}} \langle \partial_{k_y}\phi_{n\boldsymbol{k}} | \partial_{k_x}\phi_{n\boldsymbol{k}} \rangle \, d^2k$$

Chern number Bloch bands

→ Number edge states (ES)

→ All ES are equally important

→ Hall (spin) conductance

Hasan & Kane *et al*, RMP (2010)

FTI

Classification in terms of :

- $U(\boldsymbol{k}, T)$?
- H_F?

Chern number of Floquet bands

Same physical consequences?

Oka & Aoki, PRB (2009); Kitagawa *et al*, PRB (2010); Lindner *et al*, Nat Phys. (2011); Rudner *et al*, PRX (2013)

Floquet TIs

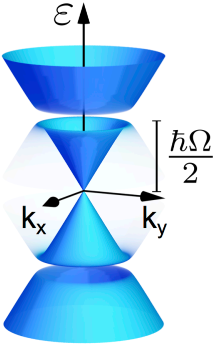
Bulk gap

$$H_F = \begin{pmatrix} \hbar\Omega - \hbar v_F k & \frac{v_F e}{2c} A_0 e^{i\theta_{\boldsymbol{k}}} \\ \frac{v_F e}{2c} A_0 e^{-i\theta_{\boldsymbol{k}}} & \hbar v_F k \end{pmatrix} = \boldsymbol{h}(\boldsymbol{k})\cdot\boldsymbol{\sigma}$$

$$C = \frac{1}{4\pi}\int \hat{\boldsymbol{h}}\cdot\left(\partial_{k_x}\hat{\boldsymbol{h}} \times \partial_{k_y}\hat{\boldsymbol{h}}\right) d^2k$$

Gives a contribution to the Floquet Chern number of one (per cone)

⇒ Do we observe edge states?



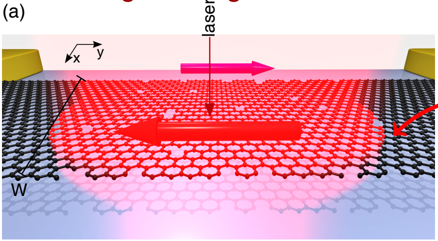
Gap: $\Delta = \hbar\Omega\eta$

$$\eta = \frac{e v_F A_0}{c\hbar\Omega} \ll 1$$

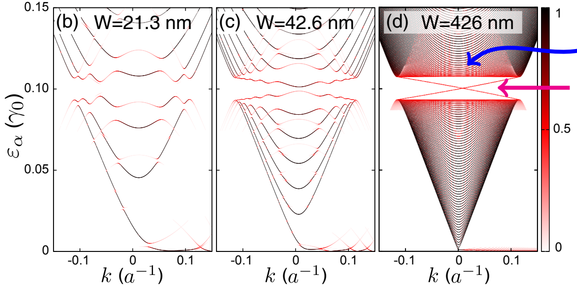
small parameter

Floquet edge states in zigzag nanoribbons

Numerical solution tight binding model



ribbon's width W



Dynamical gap

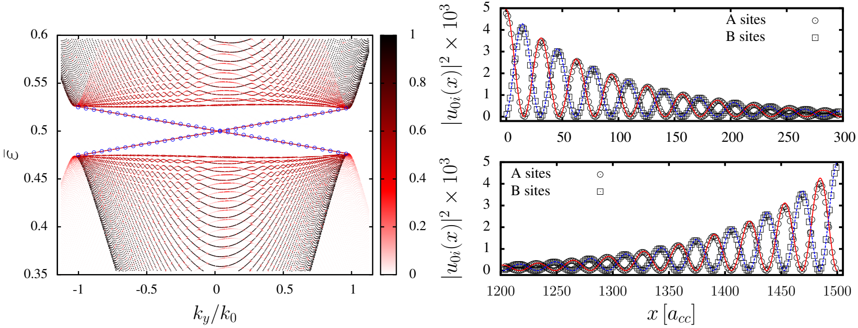
Gap states

$W \gg \xi$, critical width

Perez-Piskunov, GU, Balseiro, Foa Torres, PRB (2014)

Floquet edge states in nanoribbons!

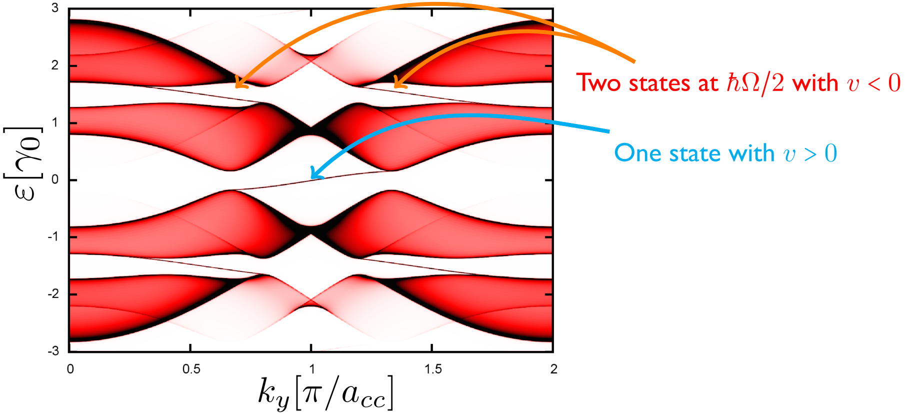
Numerical and analytical solution (zigzag edge)



- Numerics done with several Floquet replicas; analytical solution only two $\Rightarrow$ high order effects are irrelevant if $\eta \ll 1$.
- Decay length, $\xi = \hbar\Omega/eE_0$, does not depend on graphene's parameters (roughly: $\xi \sim \hbar v_F/\Delta$)

FES semi-infinite sheet

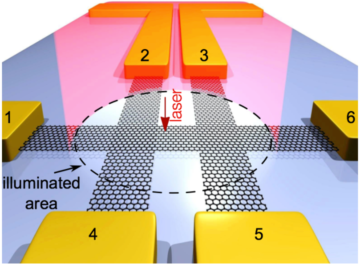
Zigzag edge

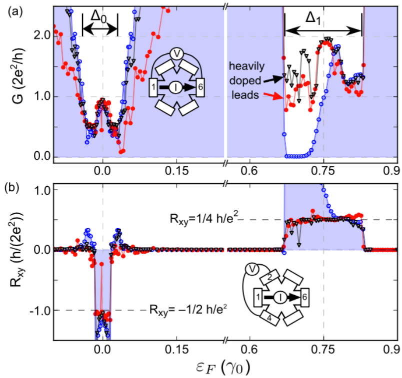


⇒ a Hall signal should appear in a Hall bar geometry (with opposite signs at the neutrality point and at the dynamical gap)

Transport: two terminal and Hall signal

Numerical calculation (multi-terminal tight binding/scattering approach)





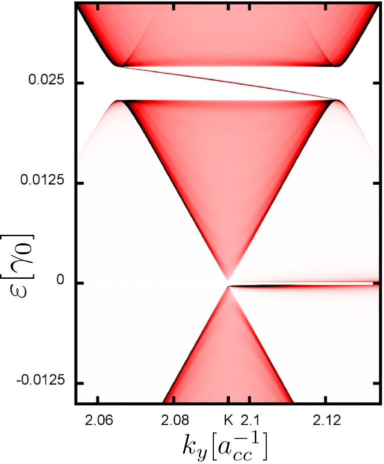
- Clear Hall signal.
- Sign changes between gaps (chirality).
- Lead-system interface matching is important

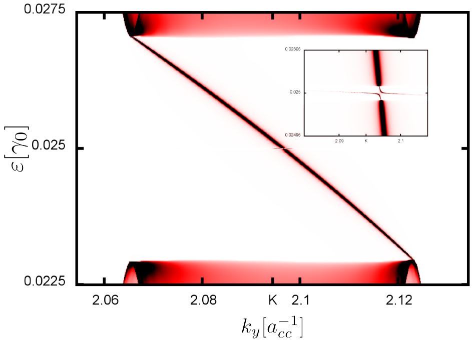
Is the conductance $\sim G_0$ related to the Floquet Chern number?

Foa Torres, Perez-Piskunow, Balseiro, GU PRL (2014)

Floquet edge states hierarchy

Many edge states



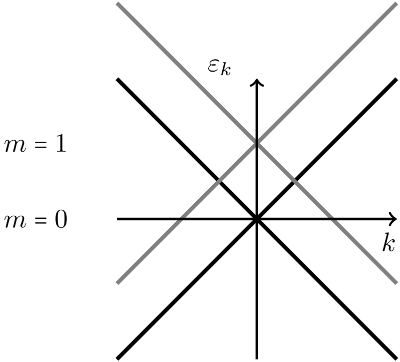


There are more states inside the gap!

GU, Perez-Piskunow, Foa Torres, Balseiro, PRB (2014)

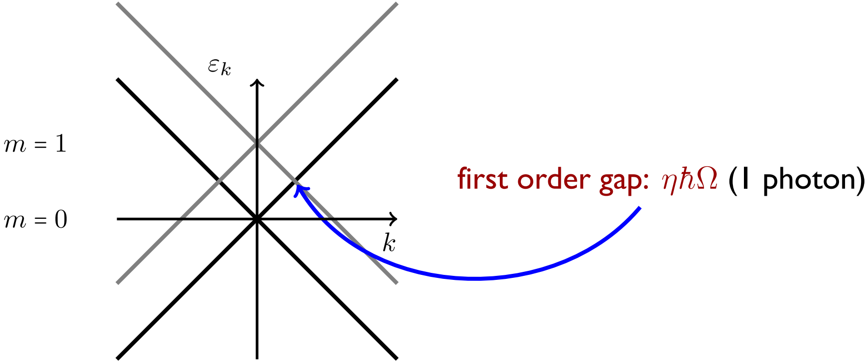
Floquet edge states hierarchy

Many edge states



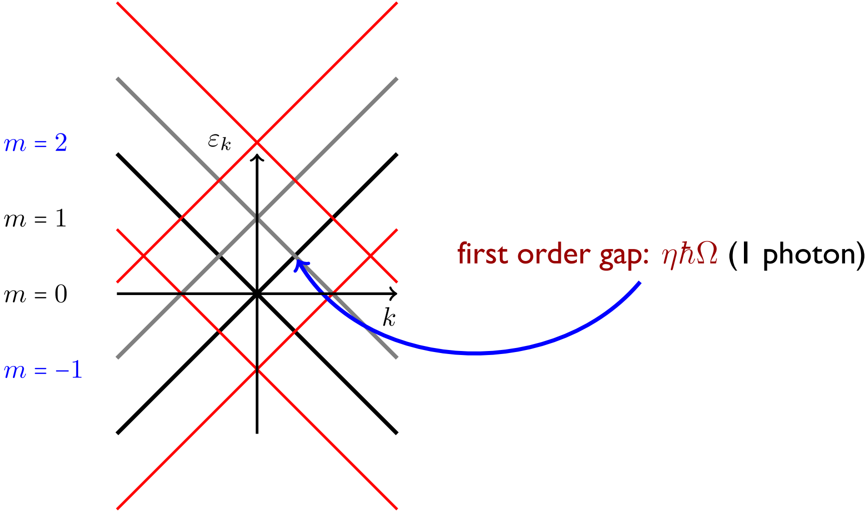
Floquet edge states hierarchy

Many edge states



Floquet edge states hierarchy

Many edge states



Floquet edge states hierarchy

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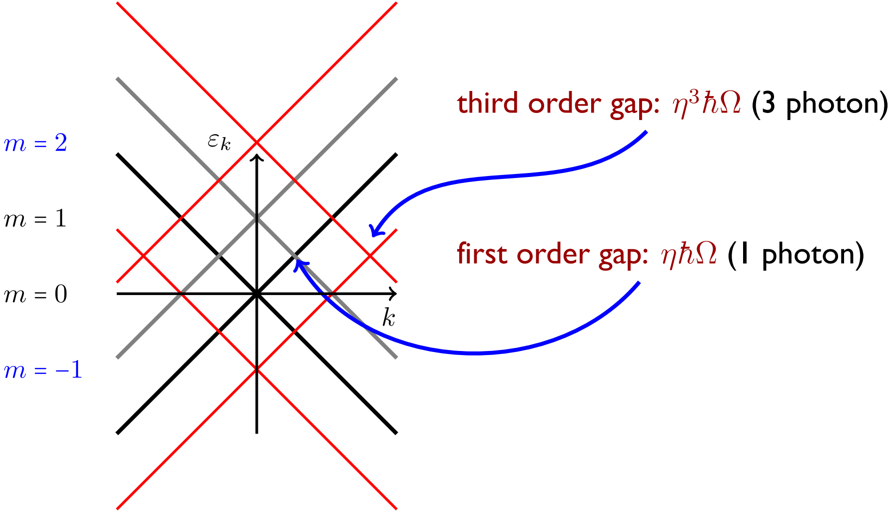
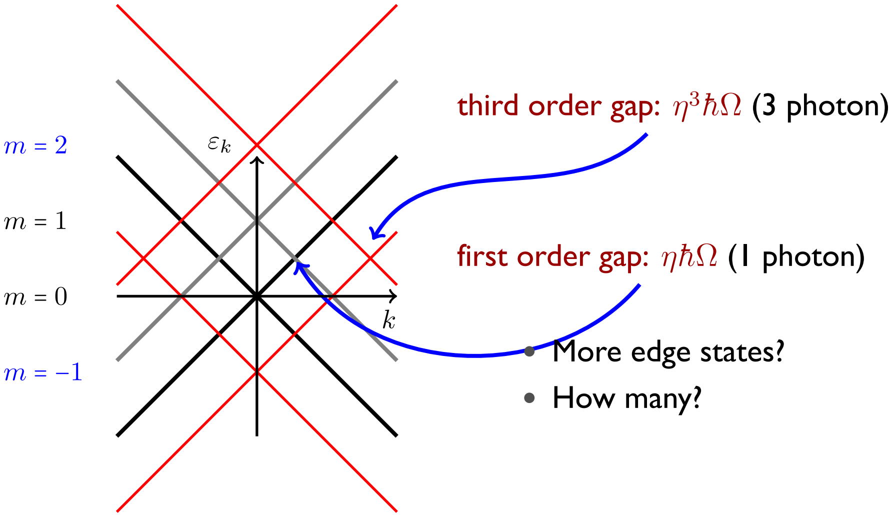


Floquet edge states hierarchy

Many edge states



- More edge states?
- How many?

Floquet edge states hierarchy

Chern number

$$C_n = \frac{1}{2\pi} \int_{\mathrm{BZ}} \mathbf{\Gamma}_{n\boldsymbol{k}} \cdot d\boldsymbol{S}_{\boldsymbol{k}} \,,$$

with

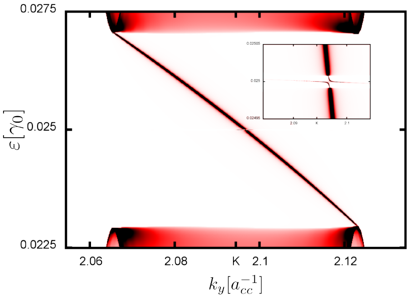
$$\mathbf{\Gamma}_{n\boldsymbol{k}} \quad = \quad \mathrm{Im} \sum_{m \neq n} \frac{\langle u_{n\boldsymbol{k}} | \nabla_{\boldsymbol{k}} H_{\boldsymbol{k}} | u_{m\boldsymbol{k}} \rangle \times \langle u_{m\boldsymbol{k}} | \nabla_{\boldsymbol{k}} H_{\boldsymbol{k}} | u_{n\boldsymbol{k}} \rangle}{(\varepsilon_{n\boldsymbol{k}} - \varepsilon_{m\boldsymbol{k}})^2} \,.$$

Near each degeneracy point between pairs of replicas: $H_{\boldsymbol{k}} = \boldsymbol{h}_p \cdot \boldsymbol{k} \to c_p$

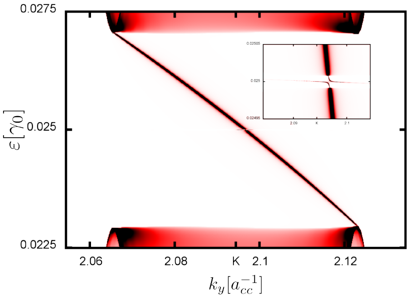
$$\#\text{edge states} = N = \sum_p c_p = \sum_i |\delta m_i| \sim (D/\hbar\Omega)^2$$

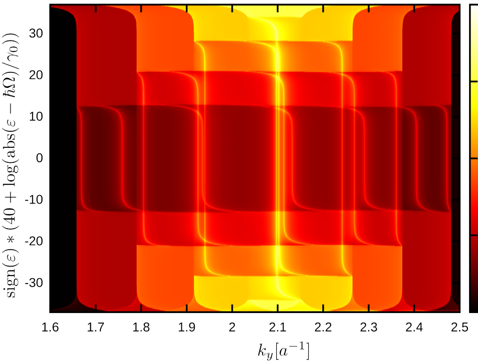
- δm_i difference of the Floquet indexes of degenerated states (number of photons involved in the process).

Floquet edge states hierarchy



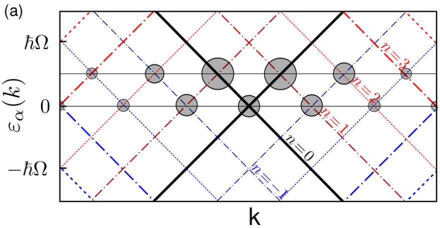
Floquet edge states hierarchy





- First order gap $(\eta\hbar\Omega) \Rightarrow c_1 = 1 \Rightarrow N = 1.$
- Third order gap $(\eta^3\hbar\Omega) \Rightarrow c_3 = 3 \Rightarrow N = c_1 + c_3 = 4.$
- Fifth order gap $(\eta^5\hbar\Omega) \Rightarrow c_5 = 5 \Rightarrow N = c_1 + c_3 + c_5 = 9.$
- Note the intensity, NOT all states are equally important!

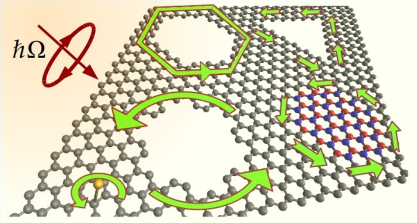
A matryoshka Floquet structure



Perez-Piskunow, Foa Torres, GU, PRA (2015)

‘Bulk’ effects

Bound states around defects



- Chiral bound states?
- Shape or edge termination matters?

'Bulk' effects

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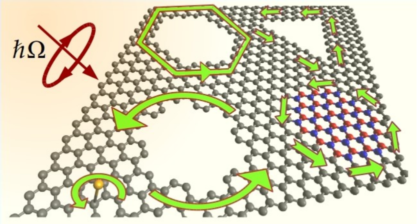

- Chiral bound states?
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'Bulk' effects

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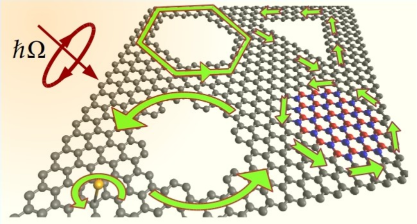

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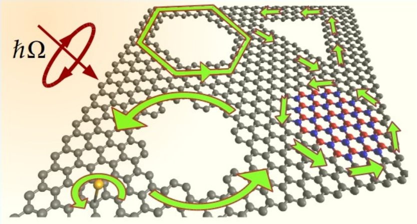

- Chiral bound states? YES
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- staggered potential (IMBC)

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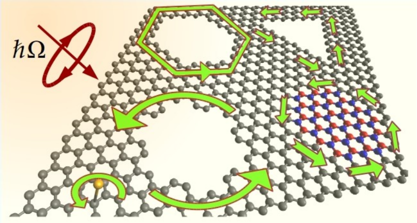

- Chiral bound states? YES
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  Topological argument says ...
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- We found analytical solutions for simple geometries.
- Boundary conditions $\Psi(\boldsymbol{r}) = M\Psi(\boldsymbol{r})$, $\quad M(\varphi) = (\hat{\boldsymbol{\nu}} \cdot \boldsymbol{\tau}) \otimes (\hat{\boldsymbol{n}} \cdot \boldsymbol{\sigma})$.

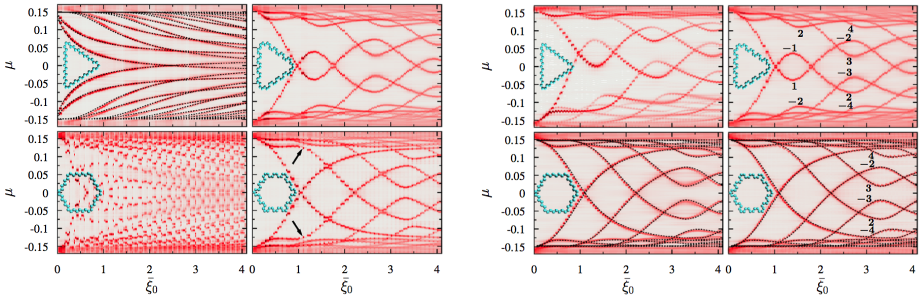
McCann&Fal'ko, JPCM (2014); Akhmerov&Beenakker, PRL (2007)

- All states are chiral (current probability calculations).
- For arbitrary geometries use numerics (TB model, Chebyshev method).

Lovey, GU, Foa Torres, Balseiro, arxiv:1603.04398 (2016)

‘Bulk’ effects

Bound states around defects



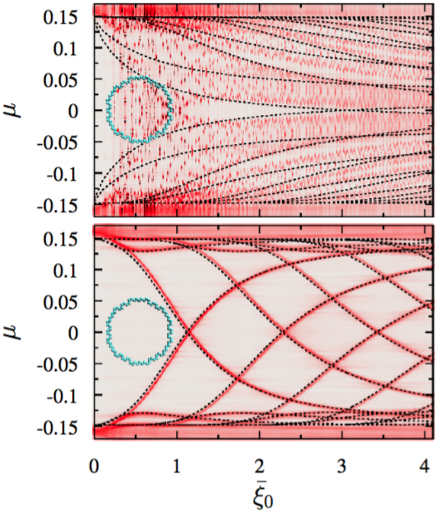
holes

staggered potential

- Good general agreement with analytical solution for some geometries.
- Zigzag hexagonal hole is different (?)
- Analytical solutions correspond to circular geometry + finite number of sides $\mathcal{N}$ included perturbatively $(1/\mathcal{N})$ → anticrossings.
- For large R, we get discrete states with $k_l \sim l/R$ as expected.

'Bulk' effects

Bound states around 'circular' defects

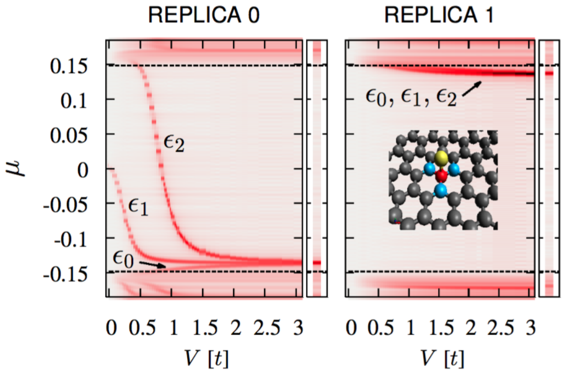


- Circular hole presents a complex spectrum (not understood).
- Good agreement in the staggered potential case.
  Chiral states only in one cone.
- What about de vacancy/adatom limit?

Lovey, GU, Foa Torres, Balseiro, arxiv:1603.04398 (2016)

‘Bulk’ effects

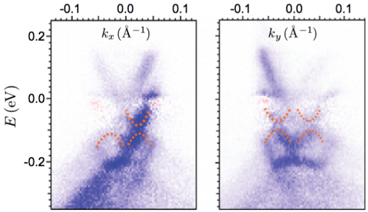
Adatom and vacancy limit



- There is a bound state for any hybridization (V).
- Exact position depends on adatom’s orbital energy.
- Manifestation on spectroscopic and transport properties?

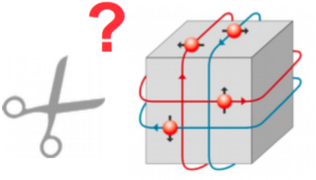
Another example

Irradiated TI (Bi_2Se_3)



Wang *etal*, Science (2013)

$$H = \hbar v(k_y\sigma_x - k_x\sigma_y)$$



- How to cut it? → domain wall

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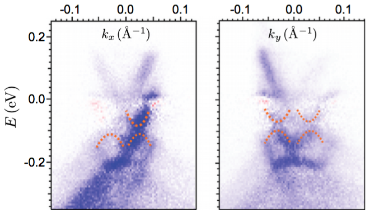

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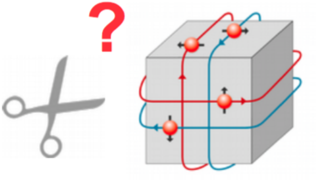

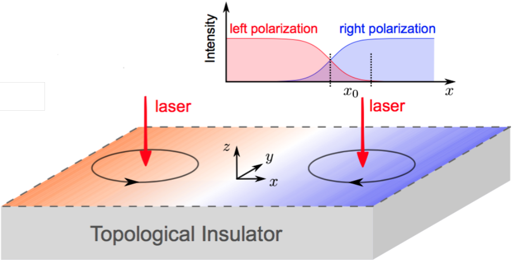

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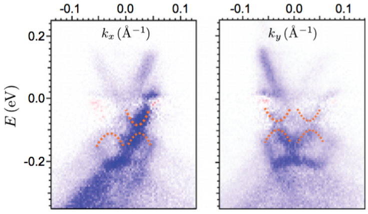

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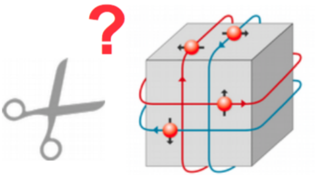

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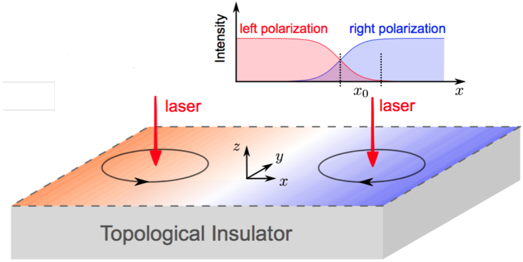
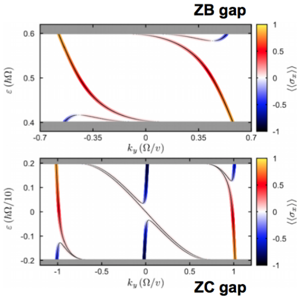
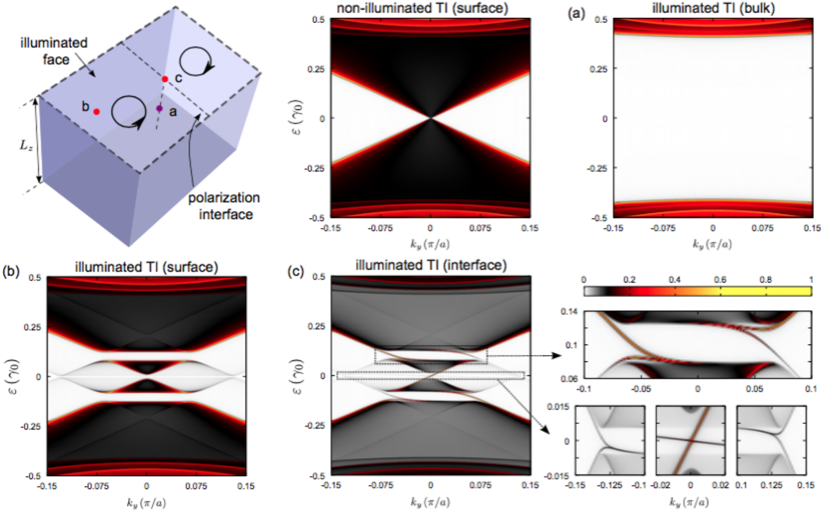




Irradiated TI



H. Calvo, L. E. F. Foa Torres, P. M. Perez-Piskunow, C. A. Balseiro and GU, PRB (2015)

SUMMARY

- EM radiation induces chiral edge states in graphene: chirality $\Leftrightarrow$ EM polarization
- There is a hierarchy of such states (parameter: $\eta = \frac{ev_F A_0}{c\hbar\Omega}$).
- Chiral states at the first order gap dominate the dc properties.
- Clear Hall signal due to these states.
- Chern numbers of the Floquet bands are not directly related to the magnitude of the Hall signal.
- Chiral bound states appear around defect, even in the adatom or vacancy limit

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Thanks!

P. M. Perez-Piskunow, GU, C. A. Balseiro and L. E. F. Foa Torres, PRB **89**, 121401 (R) (2014)
GU, P. M. Perez-Piskunow, L. E. F. Foa Torres and C. A. Balseiro Phys. Rev. B **90**, 115423 (2014)
L. E. F. Foa Torres, P. M. Perez-Piskunow, C. A. Balseiro and GU, PRL **113**, 266801 (2014)
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Lovey, GU, Foa Torres, Balseiro, arxiv:1603.04398 (2016)