The SCTM Phase Transition

ICTP / SAIFR 2015

Mateo García Pepin

In collaboration with:
Mariano Quirós

- **Motivation**
- **The Model**
- **The phase transition**
- **Summary**

EW Baryogenesis

A mechanism to explain the observed Baryon asymmetry of the Universe.

Electroweak Baryogenesis is an interesting mechanism that could explain the observed asymmetry between matter and antimatter in the universe.

It ties together Early Universe cosmology and physics at the Electroweak scale, specifically the process of Electroweak symmetry breaking.

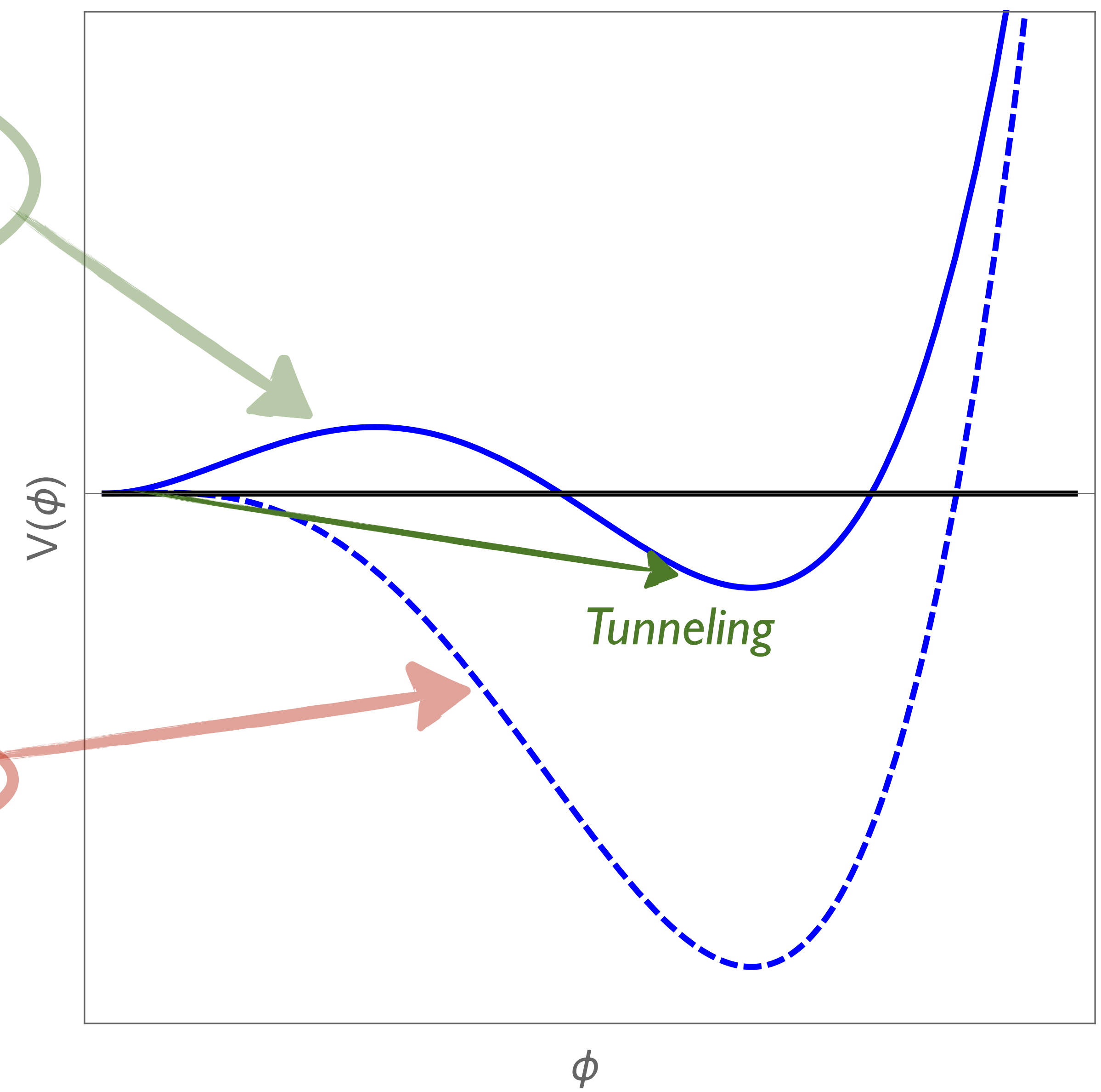
However, for this scenario to work, the phase transition needs to be of a certain kind, a <u>strongh enough first order phase transition</u>.

<u>1st Order</u> phase transition, barrier between the origin (symmetric phase) and the EW minimum (broken phase).

The phase transition proceeds through bubble nucleation.

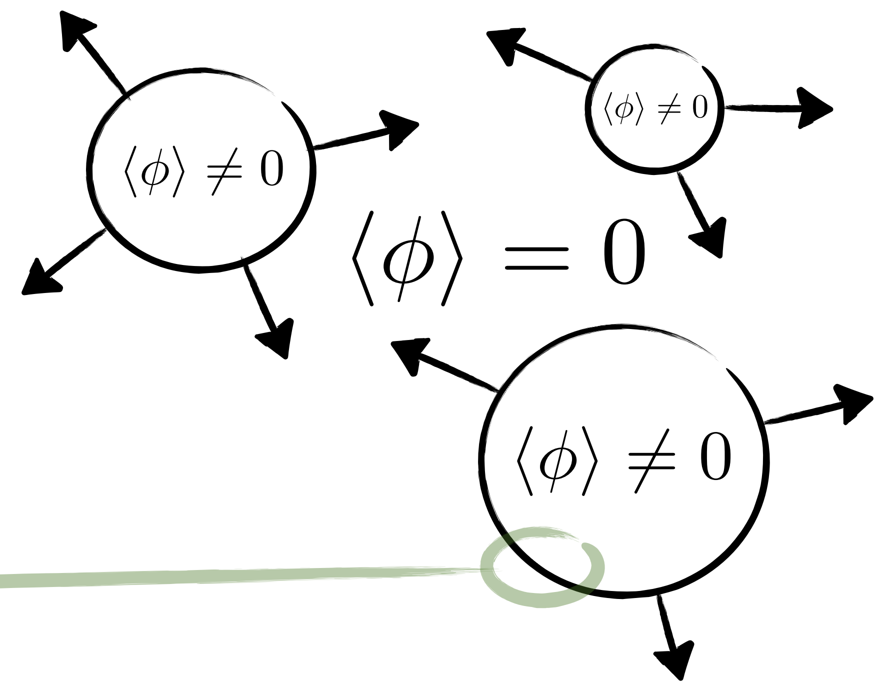
<u>2nd Order</u> phase transition, no barrier.

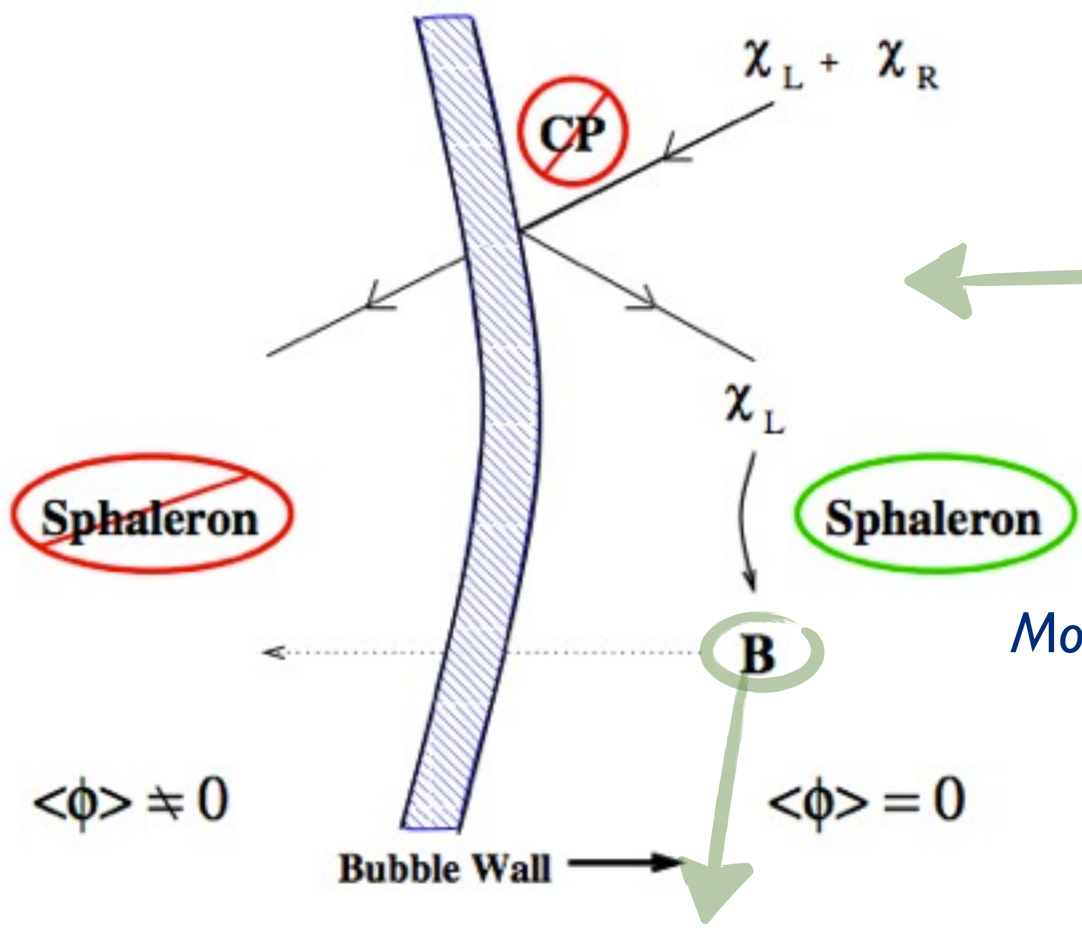
No tunneling, no bubble nucleation.



EW baryogenesis at work:

Through tunneling, bubbles of the broken phase start to appear and expand.





Morrisey & Ramsey-Musolf's review
hep-ph/1206.2942

The net Baryon number that is created in the symmetric phase is swept up by the expanding bubble of the broken phase where sphaleron transitions are supressed if the first order phase transition is strong enough.

We need to make sure that sphalerons do not wash out the asymmetry generated in the symmetric phase!

In the broken phase:

$$\Gamma \sim e^{-\frac{E_{\rm sph}(T)}{T}}$$

$$E_{\rm sph}(T) \sim a\,\langle\phi(T)\rangle$$

The SM is not able to generate such a strong first order phase transition.

The MSSM could do it through new light states entering loop corrections that would generate a barrier between the origin and the EW minimum, however, bounds on supersymmetric particles kill this scenario since one would need stops below the current experimental limits.

Even if stops somehow avoid direct searches the Higgs rates kill the MSSM scenario since light stops would contribute to gluon fusion and the $h\gamma\gamma$ coupling.

To circumvent this problem and get succesfull EW baryogenesis in supersymmetry one has to go beyond the MSSM.

What if the nature of the phase transition is not determined by the thermal corrections but already from the tree level shape of the potential?
Through the introduction of new degrees of freedom one could modify this shape and generate a barrier already at tree level.

The SCTM

Other extensions could also work. NMSSM, R-Symmetric SUSY...

The Supersymmetric Custodial Triplet Model

Why we consider it interesting?

Unlike other triplet extended Higgs sectors the model has Custodial Symmetry, this keeps $\rho = 1$ at tree level and suppresses loop contributions.

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$$

Due to this, sizeable vevs for the triplets are permitted.

It will raise the tree level mass while keeping stops light, alleviating the possible little hierarchy problem of the MSSM.

L. Cort, MGP, M. Quirós '13.
hep-ph/1308.4025

Solves some consistency problems that models of this kind have in non supersymmetric versions.

MGP, S. Gori, T-T Yu, R. Vega, R. Vega-Morales, M. Quiros '14.
hep-ph/1409.5737

When embedeed in Gauge Mediation it generates a spectrum with interesting collider phenomenology.

A. Delgado, MGP, M Quirós '15.
hep-ph/1505.07469

New DM phenomenology.

A. Delgado, MGP, B. Ostdiek, M Quirós '15.
hep-ph/1504.02486

See Antonio's talk!

Triplets + Custodial symmetry is not something new

H. Georgi, M. Machacek '85

DOUBLY CHARGED HIGGS BOSONS

Howard GEORGI

Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA

and

Marie MACHACEK

Department of Physics, Northeastern University, Boston, MA 02115, USA

Received 8 July 1985

We explore through two simple models, the first in which scalars are treated as fundamental and the second in which they are composite objects, the possibility that representations containing doubly charged scalars may participate in the spontaneous breakdown of the SU(2) × U(1) symmetry of electroweak interactions. We show that such exotic Higgs bosons may possess unsuppressed couplings to pairs of gauge vector bosons and comment on the observability of these charged Higgs bosons through the Cahn-Dawson mechanism in high-energy hadron colliders

Higgs doublet + one complex and one real SU(2)L scalar triplets ordered in such a way that custodial symmetry is preserved.

$$\phi = \begin{vmatrix} \phi^+ \\ \phi^0 \end{vmatrix}$$

be the usual Higgs doublet, and

$$\chi = \begin{vmatrix} \chi^0 & \zeta^+ & \chi^{++} \\ \chi^- & \zeta^0 & \chi^+ \\ \chi^{--} & \zeta^- & \chi^{0*} \end{vmatrix}$$

be Higgs triplet fields that satisfy

and three real neutral particles. Electroweak symmetry breaking occurs when both ϕ and χ develop non-zero vacuum expectation values (VEVs):

$$\langle\phi\rangle = \begin{vmatrix} 0 \\ a\sqrt{\tfrac{1}{2}} \end{vmatrix}, \qquad \langle\chi\rangle = \begin{vmatrix} b & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{vmatrix}. \qquad (8)$$

As written, the VEV of the χ preserves an approximate custodial SU(2) symmetry which ensures that the phenomenologically successful relation $M_W = M_Z \cos\theta$ is

Due to new degrees of freedom getting vevs, tadpole diagrams not present in the minimal SM picture are present here and make the naturalness issue of the SM even worse.

J. F. Gunion, R. Vega, J. Wudka '91

Supersymmetrizing the GM model will cancel the tadpoles and solve other issues that the model presents when studied in depth.

The Supersymmetric Custodial Triplet Model (SCTM)

L. Cort, M. Quirós, MGP '13
hep-ph/1308.4025

Just the MSSM with an extended Higgs sector:

MSSM DOUBLETS

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \quad Y = -\frac{1}{2}$$

$$H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \quad Y = +\frac{1}{2}$$

+

3 EXTRA TRIPLETS

$$\Sigma_1 = \Sigma_1^i \sigma_i = \begin{pmatrix} \frac{\psi^+}{\sqrt{2}} & \psi^{++} \\ \psi^0 & -\frac{\psi^+}{\sqrt{2}} \end{pmatrix} \quad Y = +1$$

$$\Sigma_0 = \Sigma_0^i \sigma_i = \begin{pmatrix} \frac{\phi^0}{\sqrt{2}} & \phi^+ \\ \phi^- & -\frac{\phi^0}{\sqrt{2}} \end{pmatrix} \quad Y = 0$$

$$\Sigma_{-1} = \Sigma_{-1}^i \sigma_i = \begin{pmatrix} \frac{\chi^-}{\sqrt{2}} & \chi^0 \\ \chi^{--} & -\frac{\chi^-}{\sqrt{2}} \end{pmatrix} \quad Y = -1$$

Custodial symmetry at work:

$$SU(2)_L \otimes SU(2)_R$$

At the Lagrangian level

The Custodial vacuum

$$SU(2)_V$$

To get a custodial vacuum we need first to make sure that our Lagrangian is SU(2)L x SU(2)R invariant. In order to do so we arrange the degrees of freedom into objects for which we know the transformation rules.

We do so by inserting the SU(2)L multiplets into SU(2)R multiplets.

Bidoublet $\mathbf{2} \otimes \bar{\mathbf{2}}$

bitriplet $\mathbf{3} \otimes \bar{\mathbf{3}}$

$$\bar{H} = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$$

$$\bar{\Delta} = \Sigma^i \bar{\sigma}_i = \begin{pmatrix} -\frac{\Sigma_0}{\sqrt{2}} & -\Sigma_{-1} \\ -\Sigma_{-1} & \frac{\Sigma_0}{\sqrt{2}} \end{pmatrix}$$

$$SU(2)_L \otimes SU(2)_R$$

The transformation rules for these objects under SU(2)L x SU(2)R

$$\bar{H} \to (\bar{U}_R \otimes U_L)\,\bar{H}$$

$$\bar{\Delta} \to (\bar{U}_R \otimes U_L)\,\bar{\Delta}\,(U_L^\dagger \otimes \bar{U}_R^\dagger)$$

where

$$U_L = e^{i\theta_L T}$$

$$U_R = e^{i\theta_R T}$$

The vacuum will be Custodially invariant if the following identities are satisfied,

$$\langle\bar{H}\rangle = (\bar{U}_R \otimes U_L)\,\langle\bar{H}\rangle$$

$$\langle\bar{\Delta}\rangle = (\bar{U}_R \otimes U_L)\,\langle\bar{\Delta}\rangle(U_L^\dagger \otimes \bar{U}_R^\dagger)$$

This only happens if,

$$\underbrace{\theta_R = \theta_L \quad + \quad \begin{array}{l} H_1^0|_{v_1} = H_2^0|_{v_2} \equiv v_H \\ \phi^0|_{v_\phi} = \psi^0|_{v_\psi} = \chi^0|_{v_\chi} \equiv v_\Delta \end{array}}$$

$$\boxed{\text{The vacuum is SU(2)V invariant and the } \rho \text{ parameter is protected}}$$

How is the ρ parameter protected?

From the kinetic terms we can derive what is going to be the contribution to the masses of the W and the Z coming from these new triplet states.
Without assuming that we are in the custodial vacuum the ρ parameter is:

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 + \frac{2(2v_\phi^2 - v_\psi^2 - v_\chi^2)}{v_1^2 + v_2^2 + 4(v_\chi^2 + v_\psi^2)}$$

this ρ parameter is kept to one when,

$$v_1 = v_2 \equiv v_H$$
$$v_\chi = v_\phi = v_\psi \equiv v_\Delta$$

Contribution to the W mass:

$$v^2 \equiv (174\ \mathrm{GeV})^2 = 2v_H^2 + 8v_\Delta^2$$

Note that the direction that keeps ρ =1 is more general, this will be critical in the study of the loop situation of this model:

$$v_\phi^2 = \frac{1}{2}(v_\psi^2 + v_\chi^2)$$

Superpotential:

$$W_0 = \lambda\,\bar{H}\cdot\bar{\Delta}\bar{H} + \frac{\lambda_3}{3}\,\mathrm{tr}\,\bar{\Delta}^3 + \frac{\mu}{2}\bar{H}\cdot\bar{H} + \frac{\mu_\Delta}{2}\,\mathrm{tr}\,\bar{\Delta}^2$$

$$+h_t\bar{Q}_L\cdot H_2 t_R + h_b\bar{Q}_L\cdot H_1 b_R + \cdots$$

Breaks explicitly SU(2)L x SU(2)R and will modify the potential but only at loop level.

The soft terms will be given by the SUSY breaking mechanism which may be SU(2)L x SU(2)R invariant or not, we will consider the former case for now,

$$V_{\rm Soft} = m_H^2|\bar{H}|^2 + m_\Delta^2\mathrm{tr}\,|\bar{\Delta}|^2$$

$$+\left\{\frac{1}{2}m_3^2\bar{H}\cdot\bar{H} + \frac{1}{2}B_\Delta\mathrm{tr}\bar{\Delta}^2 + A_\lambda\bar{H}\cdot\bar{\Delta}\bar{H} + \frac{1}{3}A_{\lambda_3}\mathrm{tr}\bar{\Delta}^3 + h.c.\right\}$$

The scalar potential is then:

$$V = V_F + V_D + V_{\rm Soft}$$

The Spectrum at tree level:

$$SU(2)_L \otimes SU(2)_R \to SU(2)_V$$

$$\bar{H} = \mathbf{2} \otimes \bar{\mathbf{2}} = \mathbf{h_1} \oplus \mathbf{h_3}$$

$$\bar{\Delta} = \mathbf{3} \otimes \bar{\mathbf{3}} = \delta_{\mathbf{1}} \oplus \delta_{\mathbf{3}} \oplus \delta_{\mathbf{5}}$$

Singlets (H^0)
$S_i,\ P_i$

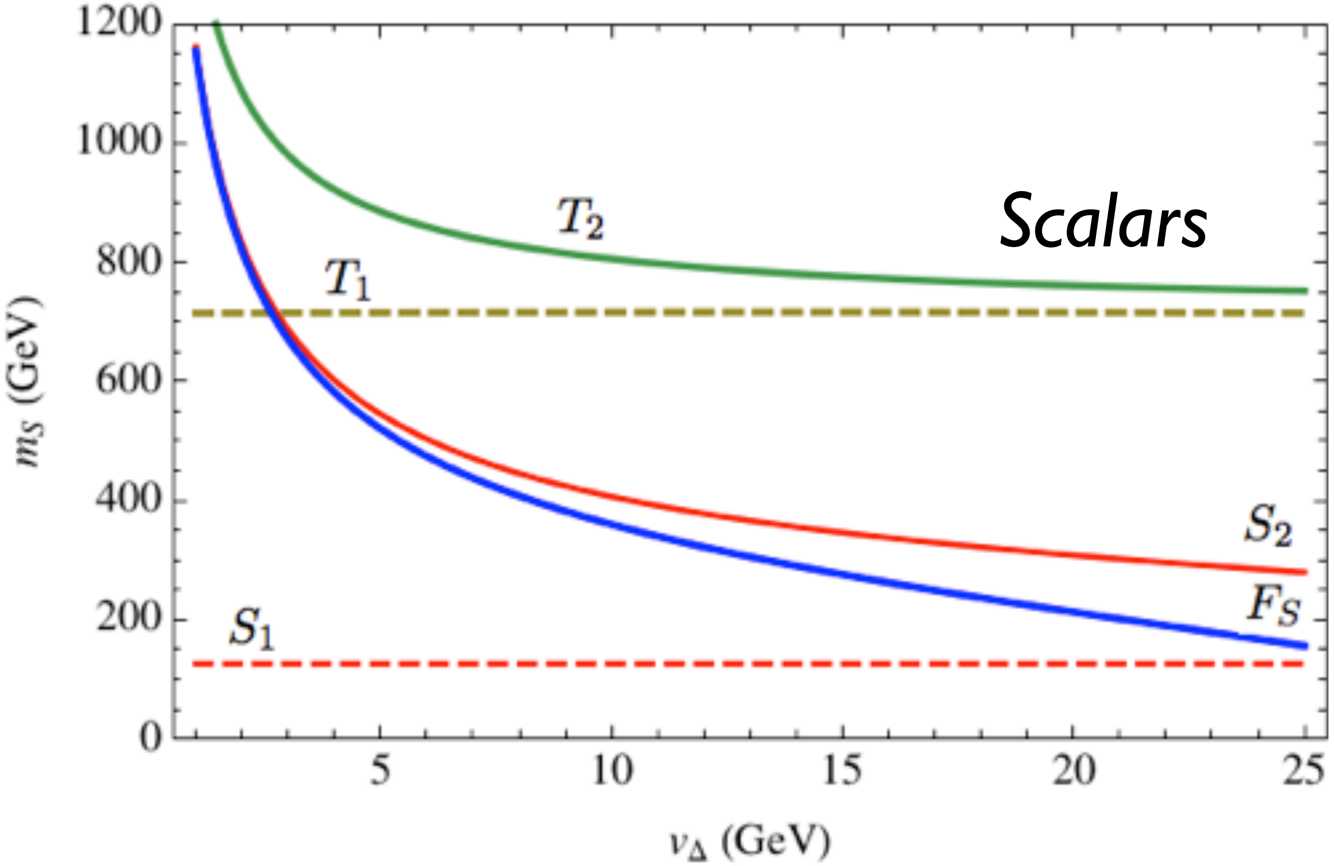
Triplets $(H^0, H^\pm)$
$T_i,\ A_i$

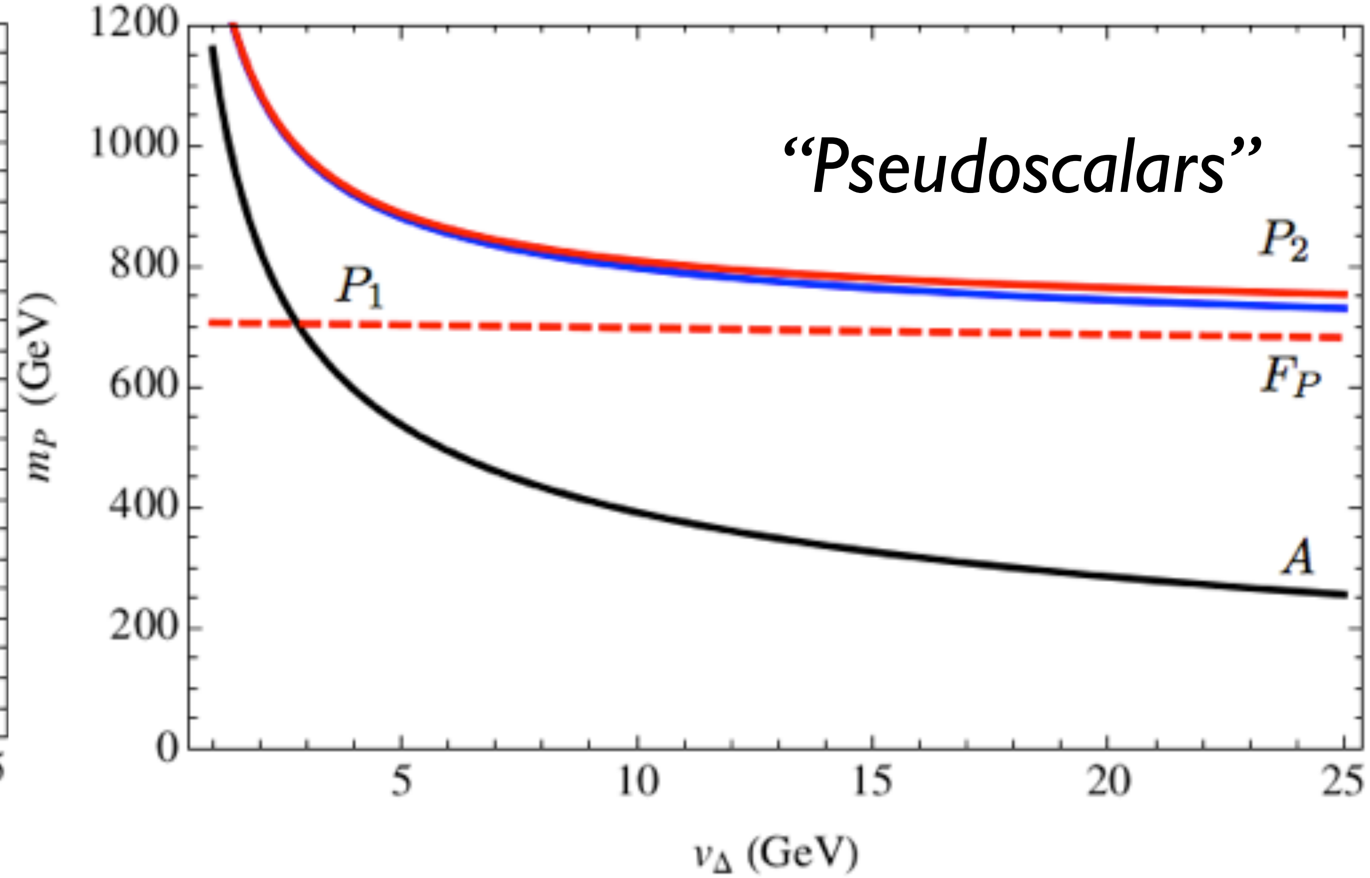
Fiveplets $(H^0, H^\pm, H^{\pm\pm})$
$F_S,\ F_P$

The Goldstone modes:

$$G \sim \sin\theta\, \mathbf{h_3} + \cos\theta\, \delta_{\mathbf{3}} \quad \begin{cases} \cos\theta = \dfrac{\sqrt{2} v_H}{v} \\ \sin\theta = \dfrac{2\sqrt{2} v_\Delta}{v} \end{cases} \qquad v_H \gg v_\Delta$$

Dependence with vDelta and decoupling of the triplet like sector:





$$S_1 \equiv h_{\rm MSSM} \quad T_1 \equiv H_{\rm MSSM},\, H^{\pm}_{\rm MSSM} \quad P_1 \equiv A_{\rm MSSM}$$

The tree level picture will be modified by loop corrections

The SCTM at loop level

At loop level U(1)Y and Yukawa couplings will break the custodial symmetry inducing a non custodial situation.

How bad? → determined by → RG running

A picture of how we expect things to happen:

Soft SUSY scale

custodially preserving superpotential

+

Soft terms determined by the SUSY breaking mechanism

RG running →

EW scale

smooth deformation of a custodially preserving lagrangian

EW vacuum

aproximatelly custodial vacuum

As a result of the running the vacuum will not be SU(2)V invariant.

We need parametrize this breaking through a rotation from the custodial direction:

$$\left.\begin{aligned} v_2 &= \sqrt{2}\sin\beta v_H \\ v_1 &= \sqrt{2}\cos\beta v_H \end{aligned}\right\} \quad \tan\beta = \frac{v_2}{v_1}$$

Custodial direction,

$$\tan\beta = \tan\theta_0 = \tan\theta_1 = 1$$

$$\left.\begin{aligned} v_\psi &= 2\cos\theta_1\cos\theta_0 v_\Delta \\ v_\chi &= 2\sin\theta_1\cos\theta_0 v_\Delta \\ v_\phi &= \sqrt{2}\sin\theta_0 v_\Delta \end{aligned}\right\} \quad \tan\theta_0 = \frac{\sqrt{2}v_\phi}{\sqrt{v_\psi^2 + v_\chi^2}} \quad \& \quad \tan\theta_1 = \frac{v_\chi}{v_\psi}$$

Now the deviations from ρ=1:

$$\Delta\rho = -4\frac{\cos 2\theta_0 v_\Delta^2}{v_H^2 + 8\cos^2\theta_0 v_\Delta^2}$$

when,

$$\tan\theta_0 = 1 \longrightarrow \rho = 1$$

$$v_\phi^2 = \frac{1}{2}(v_\psi^2 + v_\chi^2)$$

Example of a UV completion, Gauge Mediated Susy Breaking

See Antonio Delgado's talk and hep-ph/1505.07469

Totally general soft terms:

$$V_{\rm SOFT} = m_{H_1}^2 H_1^\dagger H_1 + m_{H_2}^2 H_2^\dagger H_2 + m_{\Sigma_0}^2 \Sigma_0^\dagger \Sigma_0 + m_{\Sigma_1}^2 \Sigma_1^\dagger \Sigma_1 + m_{\Sigma_{-1}}^2 \Sigma_{-1}^\dagger \Sigma_{-1} - m_3^2 H_1 \cdot H_2$$
$$+ \left\{ \frac{B_{\Delta_a}}{2} {\rm tr}\Sigma_0^2 + B_{\Delta_b} {\rm tr}\Sigma_1 \Sigma_{-1} - A_{\lambda_a} H_1 \cdot \Sigma_1 H_1 + A_{\lambda_b} H_2 \cdot \Sigma_{-2} H_2 \right.$$
$$\left. + \sqrt{2} A_{\lambda_c} H_1 \cdot \Sigma_0 H_2 + \sqrt{2} A_{\lambda_3} {\rm tr}\, \Sigma_1 \Sigma_0 \Sigma_{-1} + a_t \, \tilde{Q}_L \cdot H_2 \tilde{t}_R + a_b \, \tilde{Q}_L \cdot H_1 \tilde{b}_R + h.c. \right\}$$

Gauge Mediation will fix the values of the Soft parameters.

Thanks to the ρ =1 direction we can fix a point where falls within the allowed T parameter band

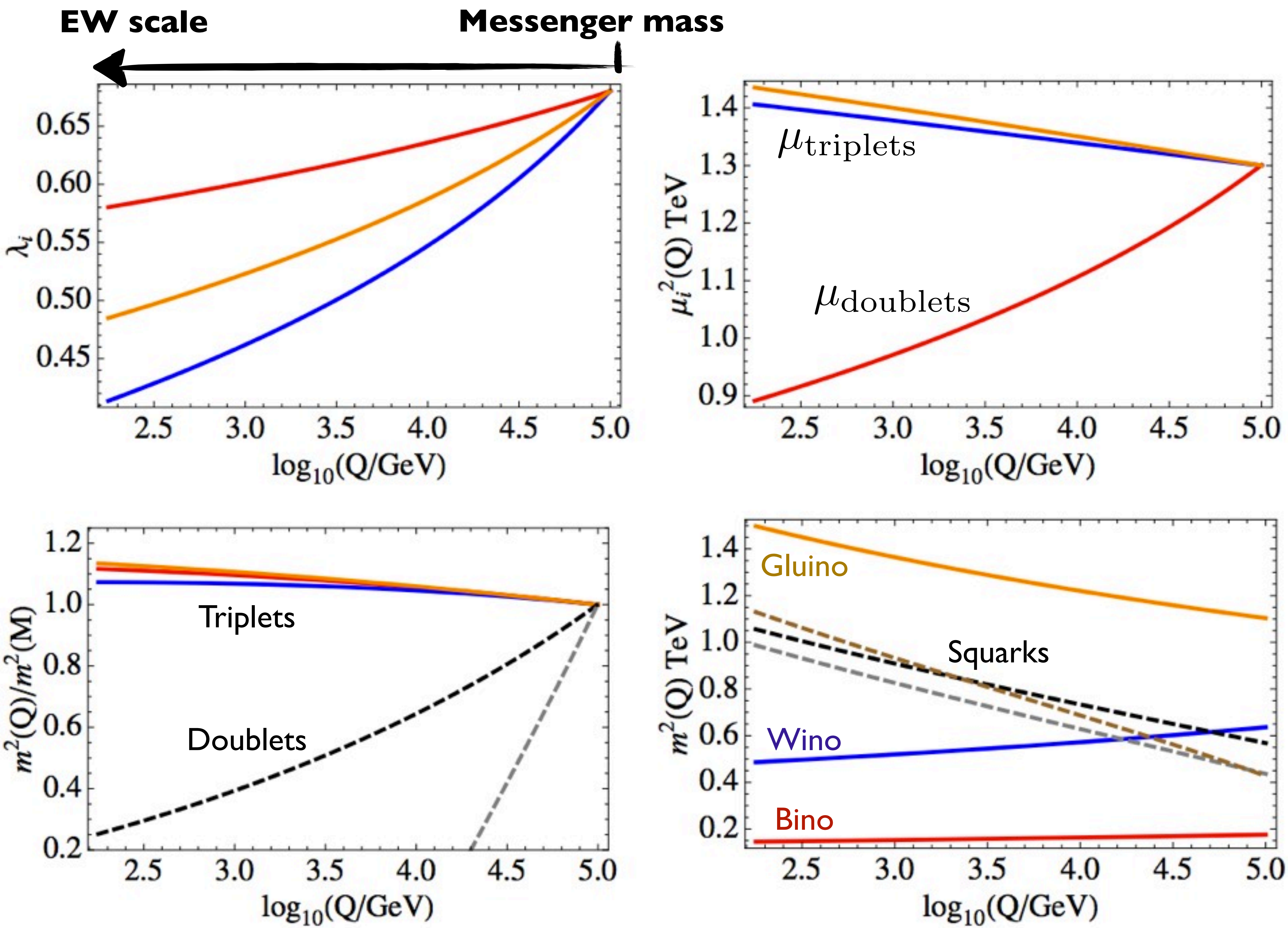
$$\mu = \mu_\Delta = 1.3\,{\rm TeV}$$

$$\tan\theta_0 = 1 \longrightarrow \rho = 1$$

We choose a low value of the messenger masses so that the custodial breaking by the RGE running is minimized:

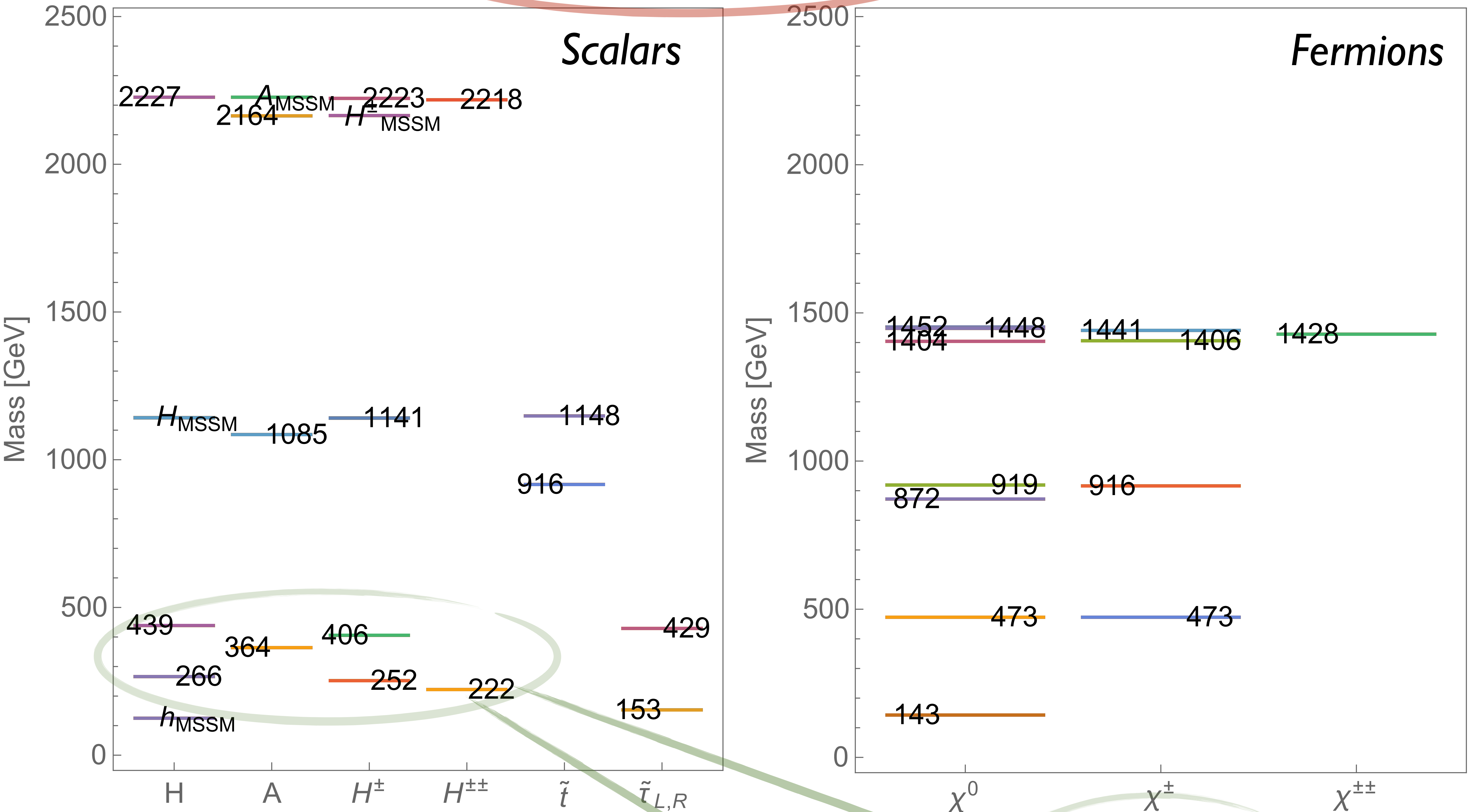
$$\mathcal{M} = 100\,{\rm TeV}$$

Loop corrections to the ρ parameter that are related to the custodial breaking, are proportional to $\tan\alpha_i - 1$, with $\alpha_i = \beta, \theta_0, \theta_1$


EW scale
Messenger mass
λ_i
$\log_{10}(Q/GeV)$
μ_{triplets}
μ_{doublets}
$\mu_i^2(Q)$ TeV
$m^2(Q)/m^2(M)$
Triplets
Doublets
$m^2(Q)$ TeV
Gluino
Squarks
Wino
Bino

The spectrum:

$\tan\beta = 1.38$ $\quad$ $v_\Delta = 20\,\mathrm{GeV}$



Triplet-like scalars, couplings supressed by:

$$\sim \frac{2\sqrt{2}v_\Delta}{v}$$

Higgs couplings:

Scenario #1	WW	ZZ	$b\bar{b}$	$t\bar{t}$	$\gamma\gamma$
r_{hXX}	1.05	1.04	1.01	1.01	1.22

The SCTM phase transition

The potential in this model is five dimensional

$$V(H_1^0, H_2^0, \psi^0, \phi^0, \chi^0)$$

To study the full RGE running and the structure of the vacuum is already a computational task by itself.

To perform a study of the strenght of the phase transition is better if we use a simplified approach that is able to parametrize the custodial breaking caused by the RG running but still keeps some of the calculabity of the custodial situation.

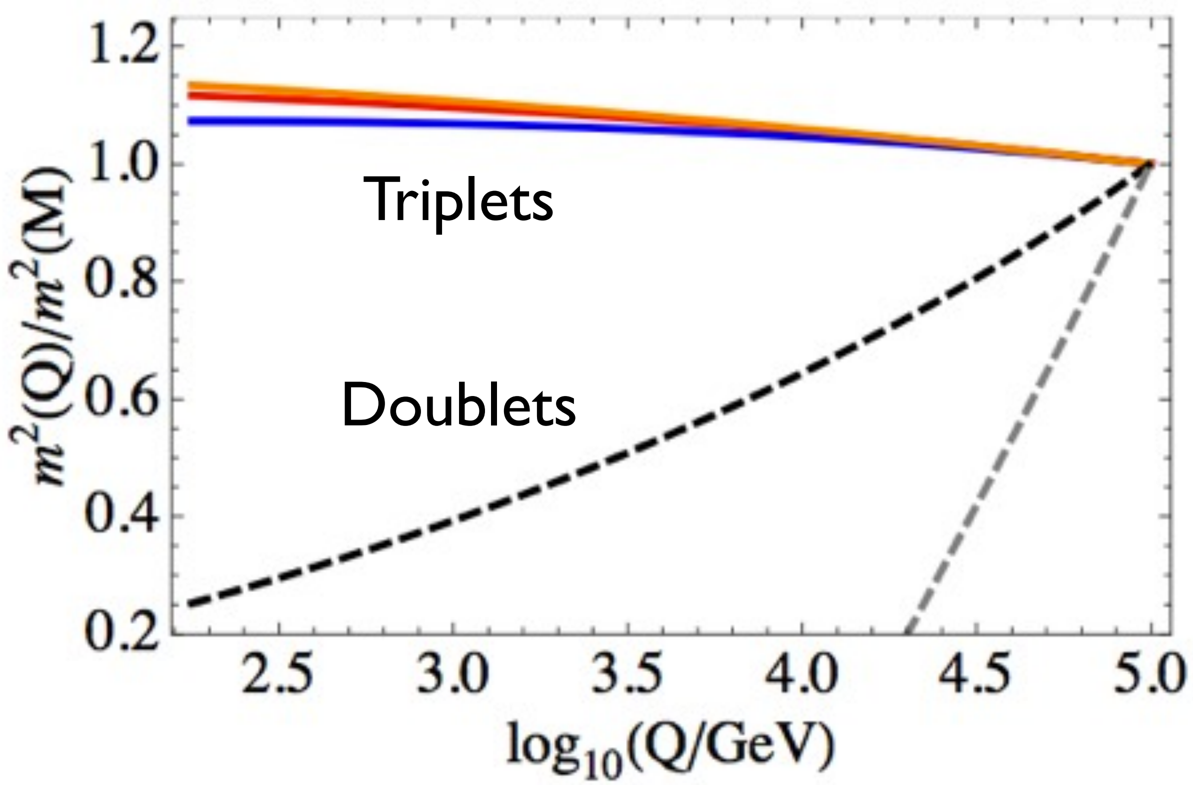
Already used for a DM study: hep-ph/1504.02486

We will use tan β as the parameter that will parametrize the breaking.

Why?

The triplet sector does not couple directly to fermions, due to this, they only feel the breaking of the yukawa coupling at the 2-loop order.

The running difeferentiates the two soft doublet masses from each other much more than the three triplet ones among themselves.



The vacuum misalignment in the triplet sector will be one loop supressed with respect to the doublets

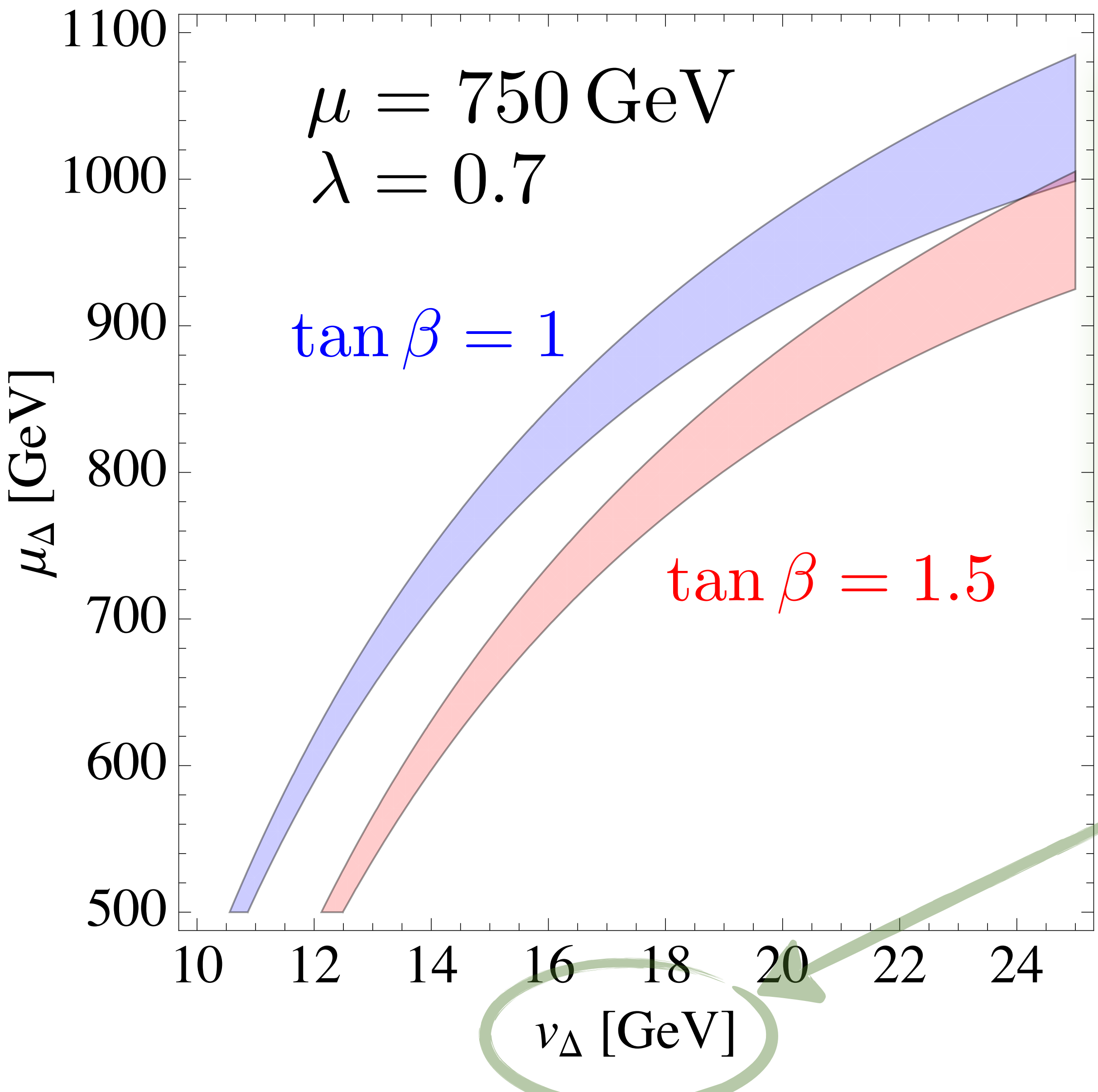
$\boxed{\text{It is a good approximation to consider only vacuum misalignment in the doublet sector.}}$

$$\tan\beta \neq 1$$
$$\tan\theta_0 = \tan\theta_1 = 1$$

Due to this vacuum structure, only the soft masses for the Higgs multiplets will lose their custodial SU(2)L x SU(2)R structure.

Within this approach we study the properties of the phase transition:

The reason why the phase transition of this model is interesting is because, already at tree level the potential shows a barrier between the origin and the Electroweak minumum for an important part of the parameter space.



Benchmark values:

$$A_\lambda = A_{\lambda_3} = A_t = 0, \quad \lambda_3 = 0.35,$$
$$m_3 = 750\text{ GeV}, \quad B_\Delta = -(750\text{ GeV})^2,$$
$$m_{\tilde{Q}_3} = 800\text{ GeV}, \quad \text{and} \quad m_{\tilde{u}_3^c} = 800\text{ GeV}.$$

This barrier is directly related to the presence of the triplets and the non negligible contribution to EWSB that they generate.

We still have to study the T=0 1-loop potential and the thermal corrections to determine the presence and strengh of a first order phase transition.

To analyze the nature of the phase transition we need to analyze the background dependent potential

T=0

Coleman-Weinberg Formula

$$V_1(\phi_i) = V_{\rm tree}(\phi_i) + \Delta V_1^{T=0}(\phi_i)$$

Where,

$$\phi_i \equiv H_1^0,\ H_2^0,\ \phi^0,\ \psi^0,\ \chi^0$$

What do we introduce in the T=0 Coleman piece?

$$W,\ Z,\ t,\ \tilde{t}_1,\ \tilde{t}_2$$

We consider the dominant contributions to the Higgs mass, other triplet-like scalar states will also be around but their couplings are small.

We solve the minimization conditions for the one-loop effective potential,

$$\left.\frac{\partial V_1(\phi_i)}{\partial H_1^0}\right|_{\phi_i=v_i} = \left.\frac{\partial V_1(\phi_i)}{\partial H_2^0}\right|_{\phi_i=v_i} = \left.\frac{\partial V_1(\phi_i)}{\partial \psi^0}\right|_{\phi_i=v_i} = \left.\frac{\partial V_1(\phi_i)}{\partial \phi^0}\right|_{\phi_i=v_i} = \left.\frac{\partial V_1(\phi_i)}{\partial \chi^0}\right|_{\phi_i=v_i} = 0$$

and force the vacuum to be at,

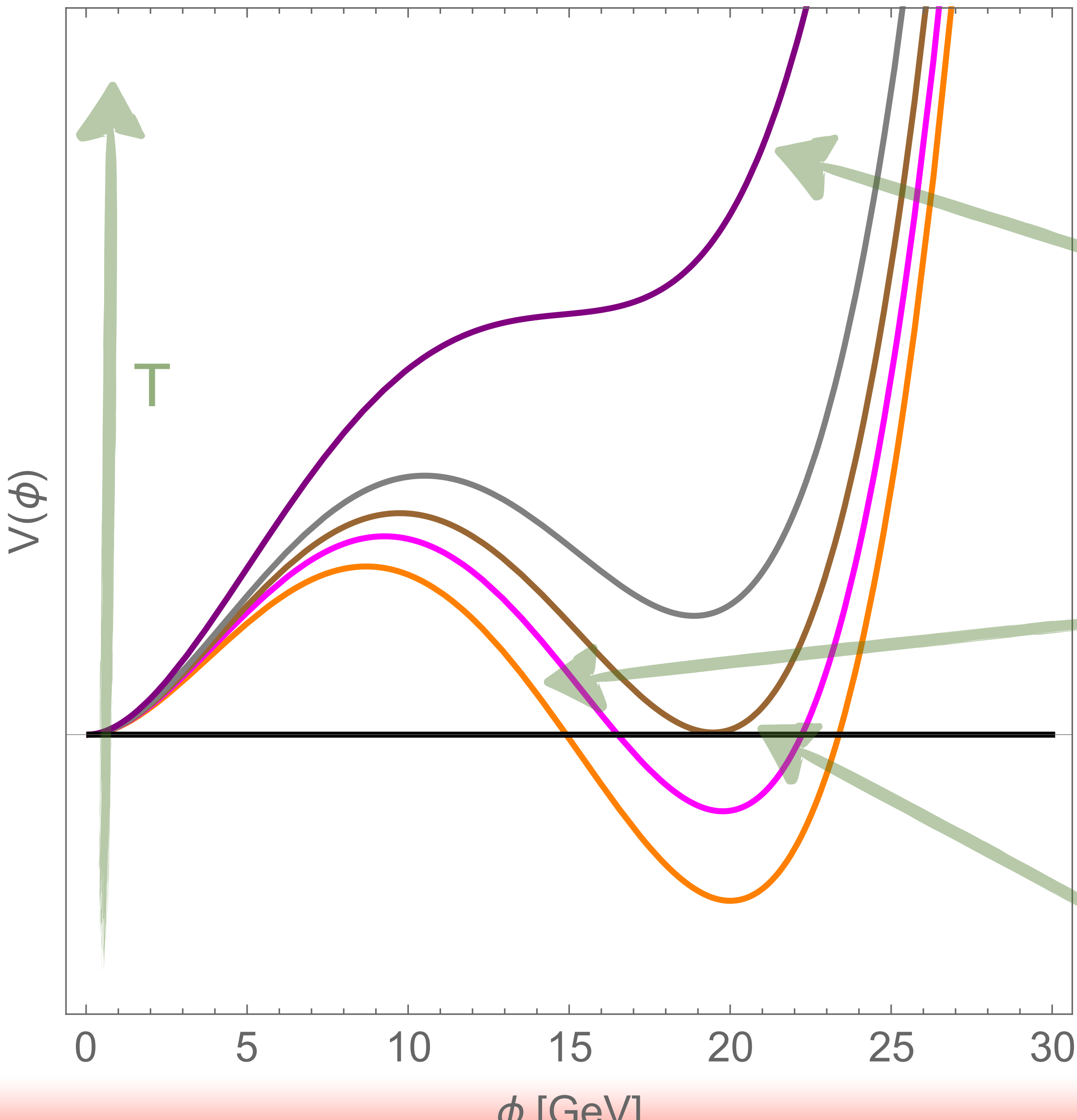
$$v_2 = \sqrt{2}\sin\beta v_H \qquad v_\psi = v_\chi = v_\phi \equiv v_\Delta$$
$$v_1 = \sqrt{2}\cos\beta v_H$$

The finite temperature part:

$$V_1(\phi_i, T) = V_{\text{tree}}(\phi_i) + \Delta V_1^{T=0}(\phi_i) + \Delta V_1(\phi_i, T) + \Delta V_{\text{daisy}}(\phi_i, T)$$

SM

This is the effect of the temperature corrections on the potential:



Thermal corrections are controlled by SM states (W, Z and t), new states of the model are either decoupled or too weakly coupled to generate a sensible contribution.

Symmetry restoration

T=0 and there is already a barrier!

Our first order phase transition is not generated thermally, it comes from a tree level effect.

The critical temperature is the temperature at which both minima are degenerate

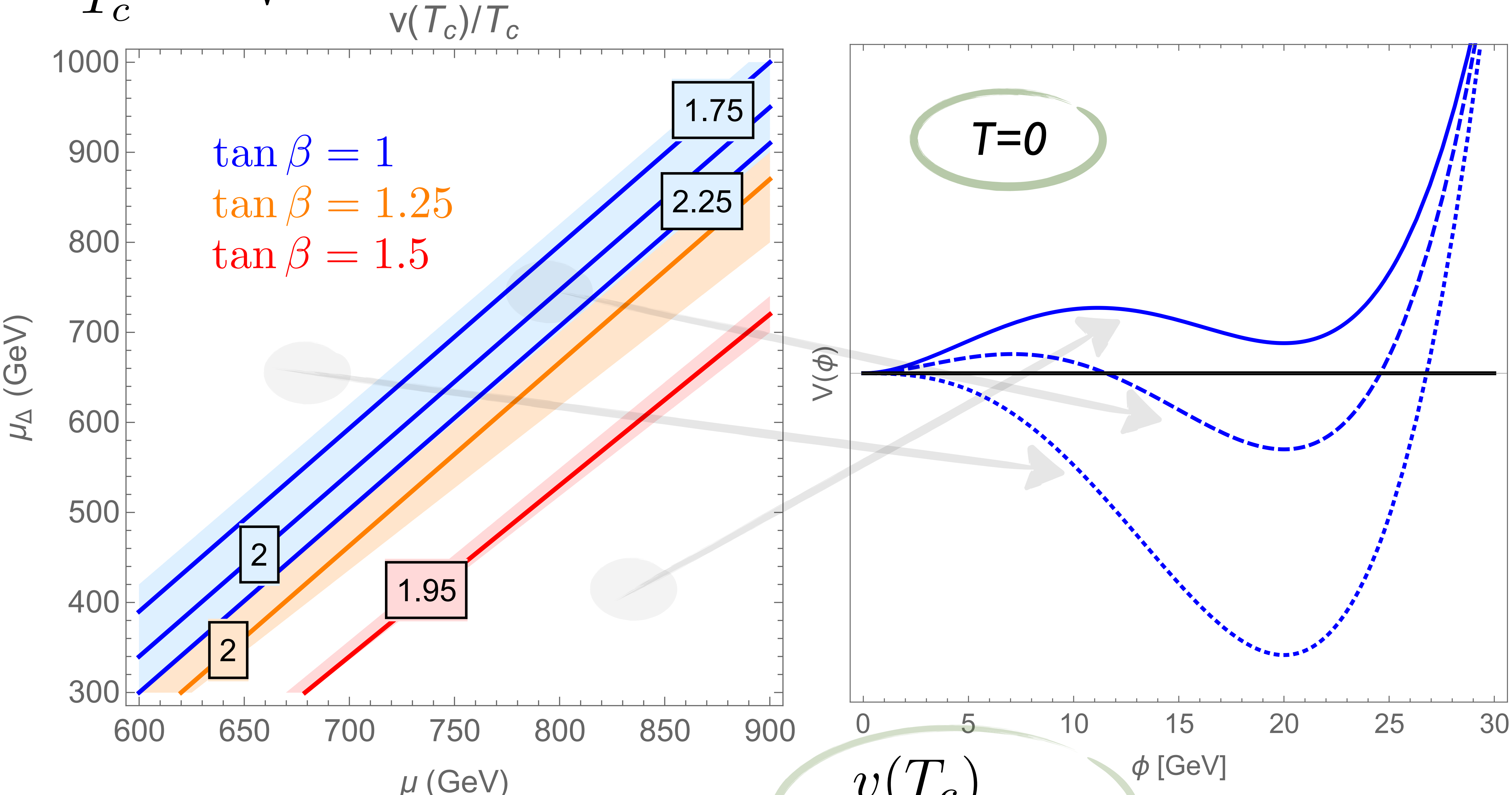
$$T = T_c$$

This picture 1-Dimensional simplification, it is the direction that joins the origin and the EW minimum

The strenght of the phase transition:

We scan over temperatures to get the degeneracy temperature and then calculate the order parameter of the phase transition:

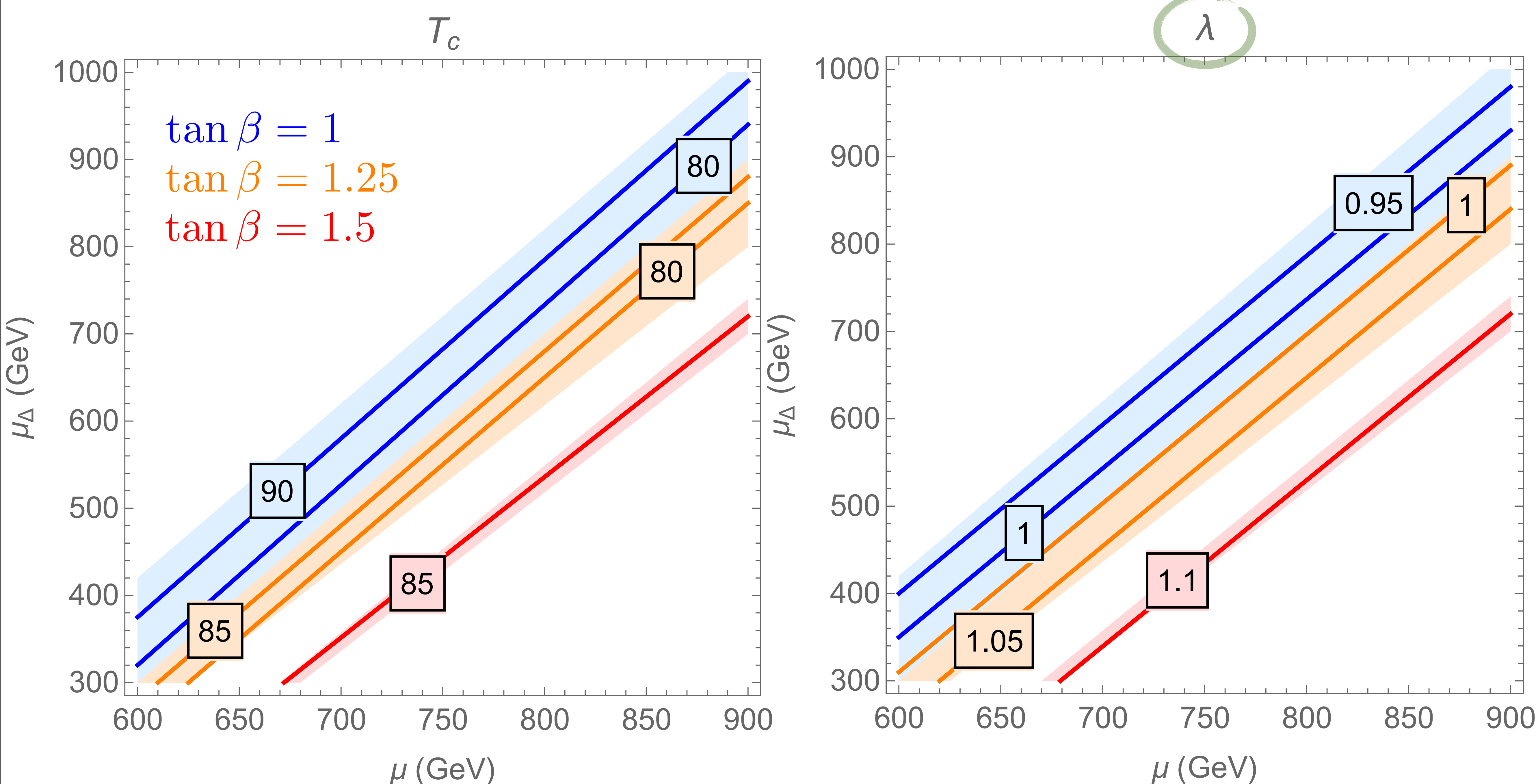
$$\frac{v(T_c)}{T_c} = \sqrt{H_1^0(T_c)^2 + H_2^0(T_c)^2 + 2\psi^0(T_c)^2 + 4\phi^0(T_c)^2 + 2\chi^0(T_c)^2}$$



Bound on the strenght $\rightarrow$ $\frac{v(T_c)}{T_c} > 1$

More results:

Fixes the Higgs mass



PRELIMINARY

Nucleation temperature:

The temperature at which bubbles of broken phase start to appear and fill the Universe.

$$T = T_n$$

Since we are generating the first order phase transition at tree level and not thermally, we should check that the tree level effect is not too strong and the tunneling really happens at some point.

We have not derived results on the exact nucleation temperature yet, however, for different points in which there is a strong first order phase transition and for temperatures below the critical temperature we find,

$$B \equiv \frac{S_3}{T} \ll \mathcal{O}(130 - 140)$$

Ensuring that bubble nucleation really happens at some point.

Tunneling probability,

$$\Gamma \sim a\, e^{-B}$$

Summary

- The SCTM is a supersymmetric generalization of the GM model that is interesting for a few reasons (Higgs mass, Gauge mediation, DM pheno...).
- It features an extended Higgs sector which can be of great help when trying to introduce EW Baryogenesis in the picture, a strong first order phase transition is present for a sizeable part of the parameter space.
- The presence of this strong first order phase transition is directly related to the triplets acquiring a sizeable vev and therefore to a non standard EWSB process.
- Direct searches of the new triplet like states are possible but challenging, another way to constrain this scenario is through the $h\gamma\gamma$ coupling.
- Remains to study if gravitational waves could be generated as a result of the phase transition.

Thank you!

Back-up Slides

Group theory relations:

$$U\phi^a = (\exp i\theta_c T^c)^{ab}\phi_b \qquad (T^a)^\dagger = T^a$$

$$\bar{U}\bar{\phi}_a = (\exp i\theta_c \bar{T}^c)_{ab}\bar{\phi}^b \qquad \bar{T}^a = T^a_{\bar{\mathbf{n}}} = -(T^a_{\mathbf{n}})^* = -(T^a_{\mathbf{n}})^t$$

$$[(\exp i\theta_c \bar{T}^c)_{ab}\bar{\phi}^b]^t = \bar{\phi}^b(\exp i\theta_d \bar{T}^d)_{ba} = \bar{\phi}^{b\,t}U^\dagger$$

Gauge Mediation formulas:

Messenger sector:

$$\Phi_8 = (\mathbf{8},\mathbf{1})_0, \quad \Phi_3 = (\mathbf{1},\mathbf{3})_0 \quad \text{and} \quad \left[\Phi_1 = (\mathbf{1},\mathbf{1})_1,\, \bar{\Phi}_1 = (\mathbf{1},\mathbf{1})_{-1}\right] \; .$$

$$W = \left(\tilde{\lambda}_8^{ij} X + \mathcal{M}_8^{ij}\right) \Phi_{8i}\Phi_{8j} + \left(\tilde{\lambda}_3^{ij} X + \mathcal{M}_3^{ij}\right) \Phi_{3i}\Phi_{3j} + \left(\tilde{\lambda}_1^{ij} X + \mathcal{M}_1^{ij}\right) \bar{\Phi}_{1i}\Phi_{1j}$$

Point:

$$n_1 = 1,\; n_3 = 2,\; n_8 = 6 \quad \text{and} \quad \tilde{\lambda}_1 = 0.9,\; \tilde{\lambda}_3 = 0.5,\; \tilde{\lambda}_8 = 0.1$$

Gaugino masses

$$M_3 = \frac{\alpha_3(\mathcal{M})}{4\pi} 3 n_8 g(\Lambda_8/\mathcal{M})\Lambda_8 \, ,$$

$$M_2 = \frac{\alpha_2(\mathcal{M})}{4\pi} 2 n_3 g(\Lambda_3/\mathcal{M})\Lambda_3 \, ,$$

$$M_1 = \frac{\alpha_1(\mathcal{M})}{4\pi} \frac{6}{5} n_1 g(\Lambda_1/\mathcal{M})\Lambda_1$$

Sfermion masses:

$$m_{\tilde{f}}^2 = 2[C_3^f \left(\frac{\alpha_3(\mathcal{M})}{4\pi}\right)^2 3 n_8 f(\Lambda_8/\mathcal{M})\Lambda_8^2 + C_2^f \left(\frac{\alpha_2(\mathcal{M})}{4\pi}\right)^2 2 n_3 f(\Lambda_3/\mathcal{M})\Lambda_3^2$$

$$+ C_1^f \left(\frac{\alpha_1(\mathcal{M})}{4\pi}\right)^2 \frac{1}{2}\left(\frac{6}{5}\right)^2 n_1 f(\Lambda_1/\mathcal{M})\Lambda_1^2] \, .$$

Pheno approach:

Soft terms:

$$V_{\rm soft} = m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + m_{\Sigma_1}^2 \,{\rm tr}\,|\Sigma_1|^2 + m_{\Sigma_{-1}}^2 \,{\rm tr}\,|\Sigma_{-1}|^2 + m_{\Sigma_0}^2 \,{\rm tr}\,|\Sigma_0|^2$$
$$+ \left\{ \frac{1}{2} m_3^2 \bar{H} \cdot \bar{H} + \frac{1}{2} B_\Delta {\rm tr} \bar{\Delta}^2 + A_\lambda \bar{H} \cdot \bar{\Delta} \bar{H} + \frac{1}{3} A_{\lambda_3} {\rm tr} \bar{\Delta}^3 + h.c. \right\}$$

In the tan β =1 limit,

$$m_{H_1} = m_{H_2} \equiv m_H$$

$$m_{\Sigma_1} = m_{\Sigma_{-1}} = m_{\Sigma_0} \equiv m_\Delta$$

we recover the fully custodial situation.

Effective potential formulas:

Coleman-Weinberg Formula:

$$\Delta V_1^{T=0}(\phi_j) = \sum_i \frac{n_i}{64\pi^2} m_i^4(\phi_j) \left(\log \frac{m_i^2(\phi_j)}{Q^2} - C_i \right)$$

Thermal piece:

$$\Delta V_1(\phi_i, T) = \frac{T^4}{2\pi^2} \left(\sum_i n_i J_i \left[\frac{m_i^2(\phi_i)}{T^2} \right] \right)$$

Thermal integrals:

$$J_\pm(y) \equiv \int_0^\infty dx\, x^2 \log\left(1 \mp e^{-\sqrt{x^2+y}}\right)$$

$$J_i = J_+(J_-)$$

Bosons (Fermions)

Daisy part:

$$\Delta V_{\rm daisy}(\phi_j, T) = -\frac{T}{12\pi} \sum_i n_i \left[\mathcal{M}_i^3(\phi_j, T) - m(\phi_j)^3 \right]$$

Nucleation temperature:

The temperature at which bubbles of broken phase start to appear and fill the Universe.

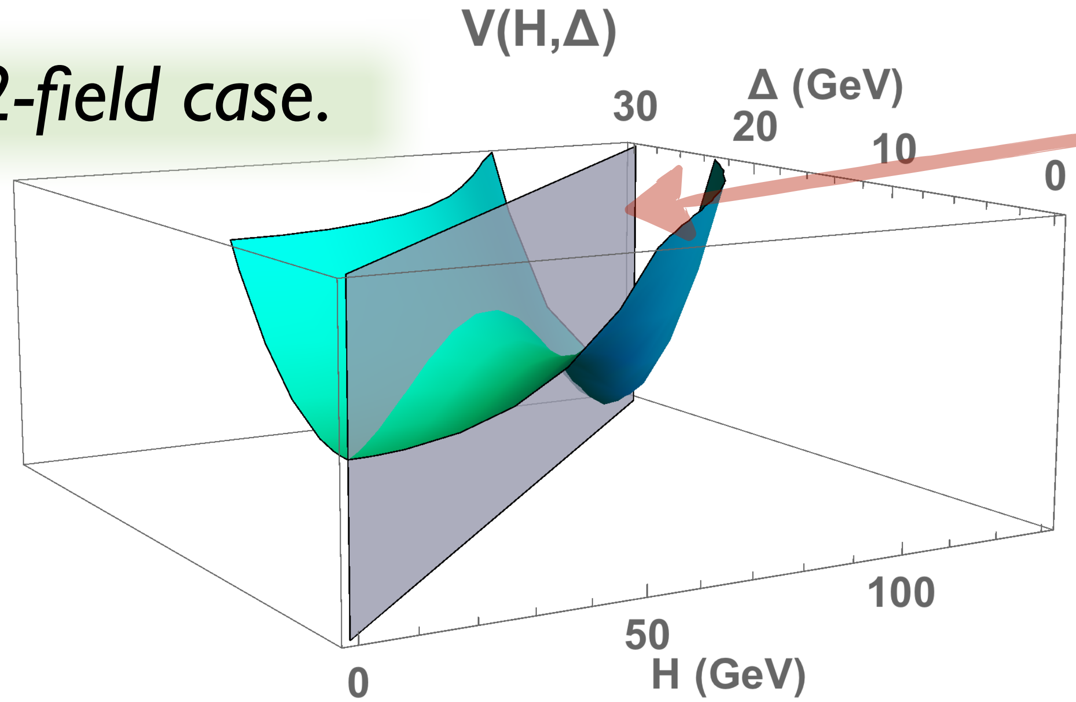
$$T = T_n$$

Since we are generating the first order phase transition at tree level and not thermally, we should check that the tree level effect is not too strong and the tunneling really happens at some point.

Technical issues make a computation of the nucleation temperature in the 5-field case a very challenging endeavor. Instead of doing so, we can work on a 1-field approximation and see what information can we get from there.

To show this clearly we study what happens in the case where we go from a 2-field case to the 1-field aproximation:

2-field case.



1-field approx:

The direction that joins the origin and the EW minimum

$$H \to \frac{v_H(T)}{v_\Delta(T)}\Delta \quad \text{and} \quad \Delta \to \Delta.$$

$$T_n^{1\ \text{field}} \lesssim T_n^{2\ \text{field}}$$

For any general field configuration, the tunneling direction will be always the one where the phT is weaker, therefore we can generalize the argument to the 5 field case and get a lower bound for the nucleation temperature.

The true nucleation temperature will be in between values that we can compute:

$$T_n^{1\,\text{field}} \lesssim T_n^{5\,\text{field}} \lesssim T_c^{5\,\text{field}}$$

Where the 1-field case corresponds to

$$H_1^0 \to \frac{v_1(T)}{v_\phi(T)}\phi^0,\ H_2^0 \to \frac{v_2(T)}{v_\phi(T)}\phi^0 \quad \text{and} \quad \psi^0 \to \frac{v_\psi(T)}{v_\phi(T)}\phi^0,\ \chi^0 \to \frac{v_\chi(T)}{v_\phi(T)}\phi^0.$$