Naturalness of supersymmetric dark matter

Program on Particle Physics at the Dawn of the LHC 13

Collab. with

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Work in progress

Fine-tuning is a slippery subject

... but physically relevant

Some of the most intriguing questions of theor. physics are naturalness problems

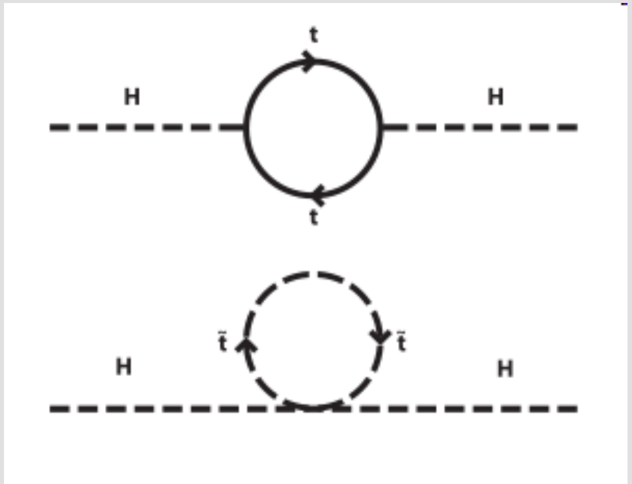
Why v^2 so small ?? (hierarchy problem)

Why ρ_Λ so small ?? (D.E. problem)

SUSY is still one of the preferred candidated for BSM physics:

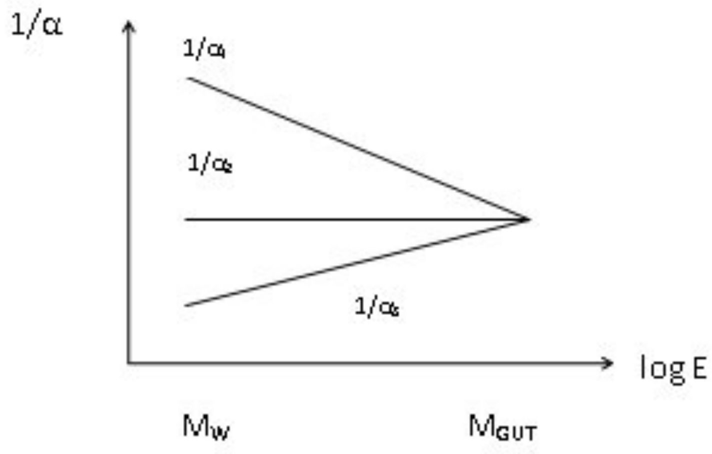
Motivations:

- Beautiful symmetry, strongly suggested by string theories
- Elegant solution to the Hierarchy Problem



Bonus:

- Higgs looks fundamental, and $m_h < 135 \text{ GeV}$
- Radiative EW breaking
- Good DM (WIMPs) candidates
- Gauge Unification



BUT

- $m_h \simeq 125 \text{ GeV}$ a bit too heavy for naive SUSY expectations

- No signal of SUSY from LHC-8 TeV

These two facts imply $m_{\text{SUSY particles}} \gtrsim 1 \text{ TeV}$

fine-tuning to get the correct EW scale

(as all BSM scenarios)

There are possible exceptions, if SUSY leaves in special corners of the parameter space,

e.g. if the SUSY spectrum is "compressed", so that visible particles in the events have small p^T.

Such situation would fool the LHC to some extent. It is certainly possible, but it sounds artificial (a "trick" to save low-energy SUSY)

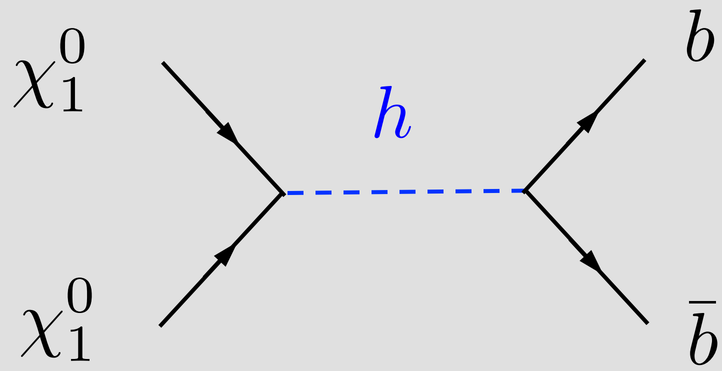
There are further possiblities going beyond the MSSM: NMSSM, BMSSM, etc.

In any case, we cannot just “forget” about the fine-tuning problem, since the main reason to consider Weak-Scale SUSY was to avoid the Hierarchy Problem (fine-tuning of EW breaking in the SM)

In addition, if we wish to fulfill the expectation of SUSY dark matter, usually some kind of fine-tuning is necessary

Typically SUSY leads to too much DM. In order to reduce $\Omega_{\rm DM}$ a mechanism to increase $\sigma_{\rm ann}$ is needed.

E.g. resonant annihilation through a Higgs
$(h - \text{funnel})$



requires $m_{\chi_1^0} \simeq \dfrac{m_h}{2}$

Bonus:

naturalness problems

- Higgs looks fundamental, and $m_h < 135\ \mathrm{GeV}$
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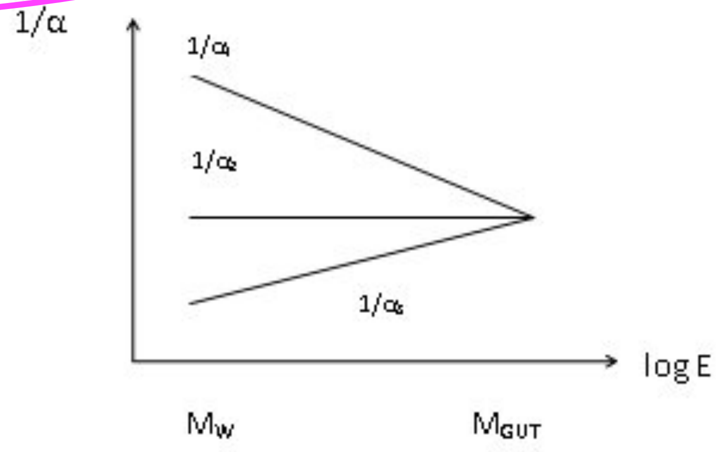
- Gauge Unification



The EW fine-tuning

At tree-level and large $\tan\beta$

$$\underbrace{-\frac{1}{8}(g^2+g'^2)v^2}_{\frac{1}{2}M_Z^2} = \boxed{\mu^2 + m_{H_u}^2}$$

$$m_{H_u}^2(LE) = \quad c_{M_3}M_3^2 + c_{M_2}M_2^2 + c_{M_1}M_1^2 + c_{A_t}A_t^2 + c_{A_tM_3}A_tM_3 + c_{M_2}M_2^2 + \cdots$$
$$+c_{M_3M_2}M_3M_2 + \cdots + c_{m_{H_u}}m_{H_u}^2 + c_{m_{\tilde{Q}_3}}m_{\tilde{Q}_3}^2 + c_{m_{\tilde{t}_R}}m_{\tilde{t}_R}^2 + \cdots$$

How to measure of the EW fine-tuning

Most used and popular (= standard) criterion:

$$\Delta_i = \frac{d \log v^2}{d \log \theta_i} \,, \qquad\qquad \Delta \equiv \max\{|\Delta_i|\}$$

Ellis, Enqvist, Nanopoulos & Zwirner' 86

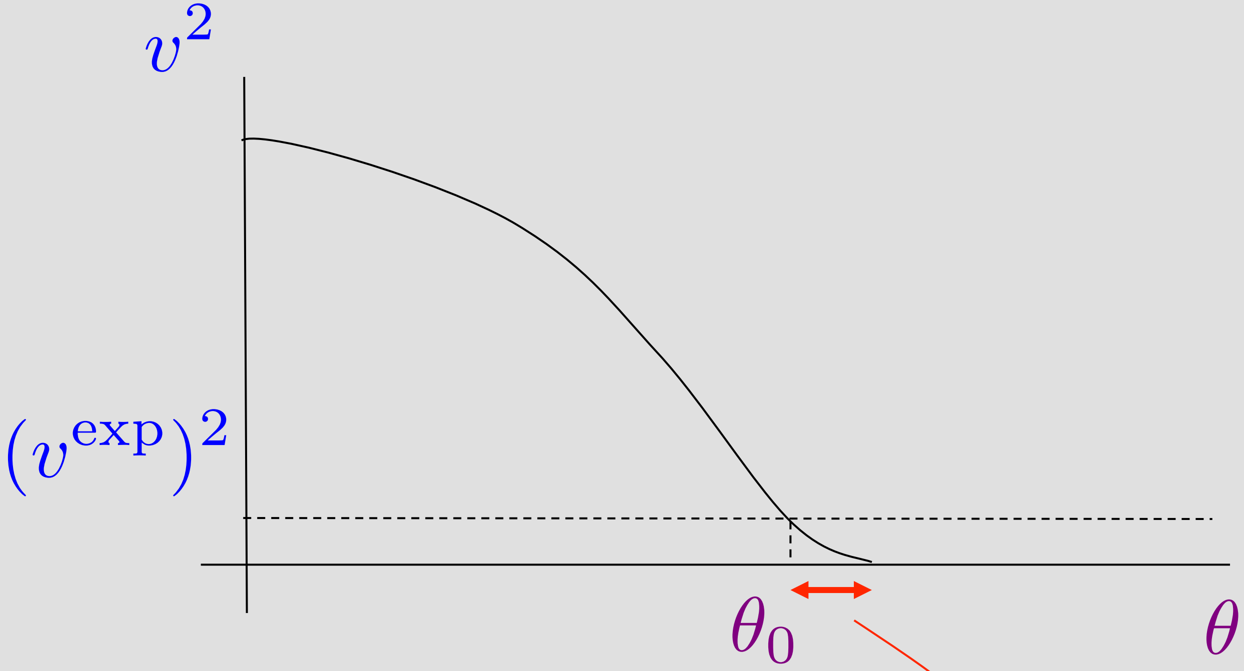
Barbieri & Giudice' 88

$\theta_i \quad \equiv$ independent parameters of the model

$\Delta = 100$ means $\sim 1\%$ fine-tuning, etc.

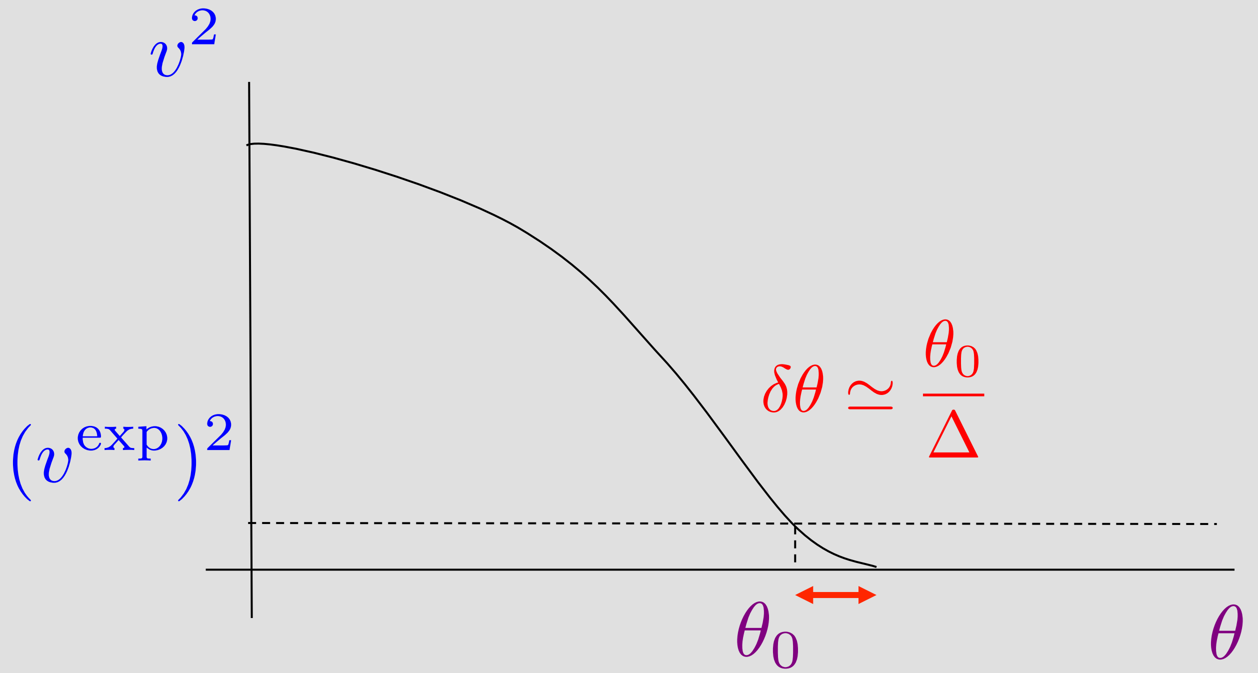
Δ_{θ_i} admits an statistical interpretation

Ciafaloni & Strumia' 97



only in this $\delta\theta_0$ range, $v^2 \leq (v^{\exp})^2$

Δ_{θ_i} admits an statistical interpretation



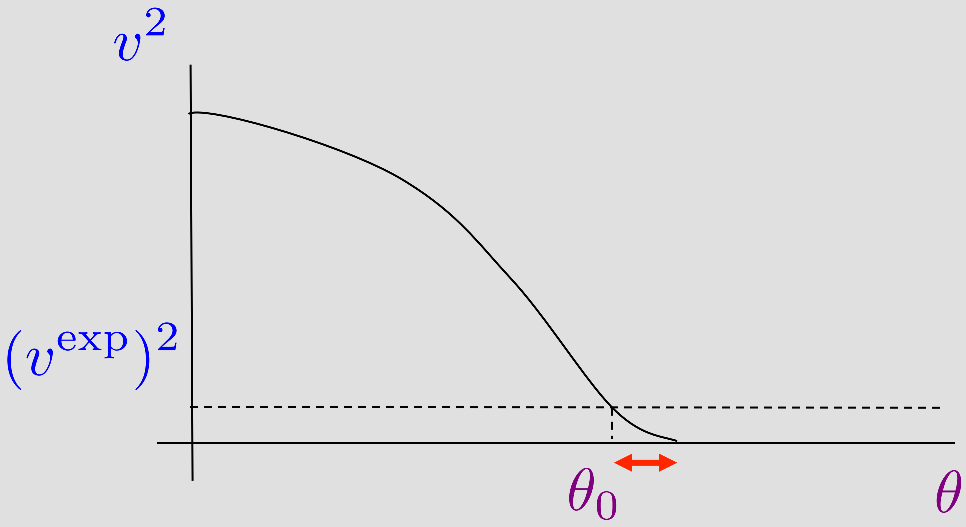
$$\mathcal{P}[v^2 \leq (v^{\exp})^2] = \frac{\delta\theta_0}{\theta_0} \simeq \boxed{\Delta^{-1} \equiv \quad p - \text{value}}$$

There are tree implicit assumptions behind this statistical interpretation

- Range of $\theta \sim [0,\ \theta_0]$
- Prior $p(\theta) = \text{flat}$
- The expansion of $v^2(\theta)$ at first order captures its behavior in the neighborhood of interest.

Reasonable, but can be inappropriate in particular theoretical scenarios

Comment 1



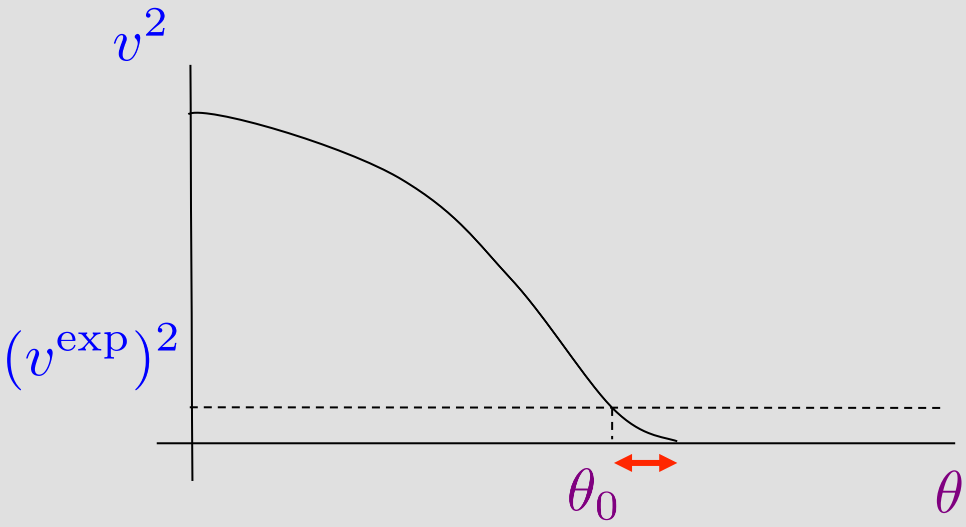
Changing the θ - range, $[0,\ \theta_0] \to [\frac{1}{2}\theta_0, \frac{3}{2}\theta_0]$

would apparently change the p-value.

But using m^2 instead of v^2 remains stable:

$$\mathcal{P}(|m^2| \leq |m^{\exp}|^2)$$

Comment 2

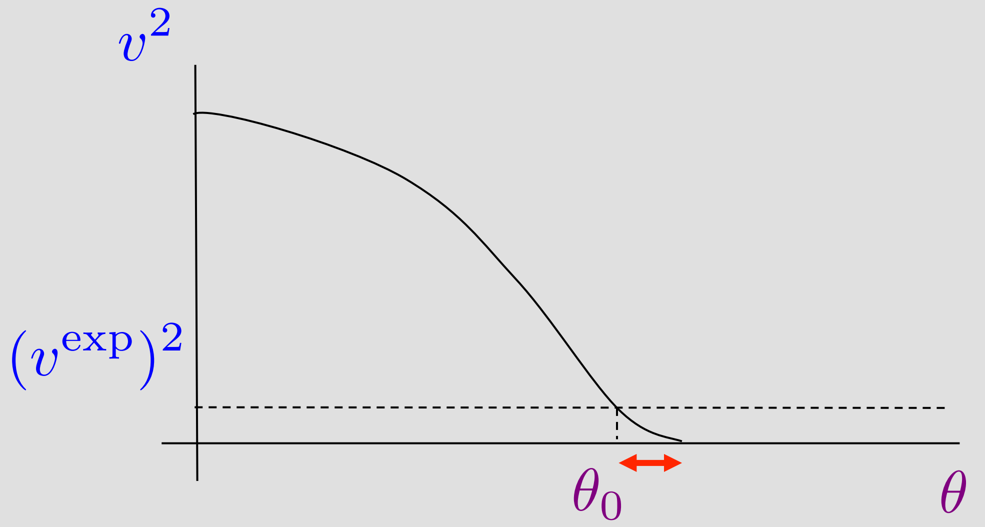


Doubling the θ- range, $[0,\ \theta_0] \to [0,\ 2\theta_0]$

makes $\Delta \to 2\Delta$

$\mathcal{O}(1)$ factor of arbitrariness in the fine-tuning measure

Comment 3



The standard criterion does also work with other choices for the θ prior.

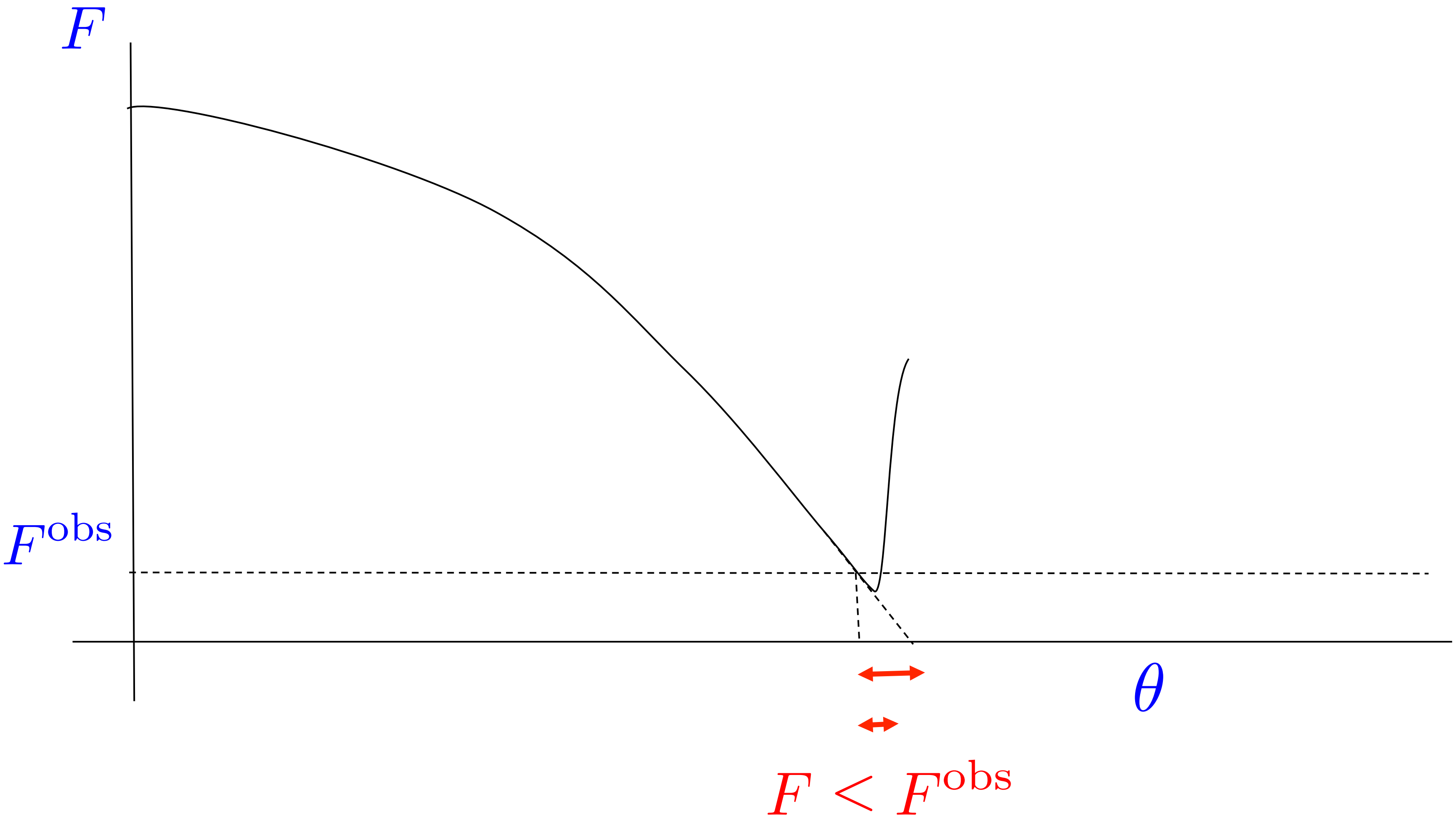
E.g for a logarithmic prior:

$$\mathcal{P}(\theta) \propto \frac{1}{\theta}\ , \quad \text{with} \quad \log\left|\frac{\theta_{\max}}{\theta_{\max}}\right| = 1$$

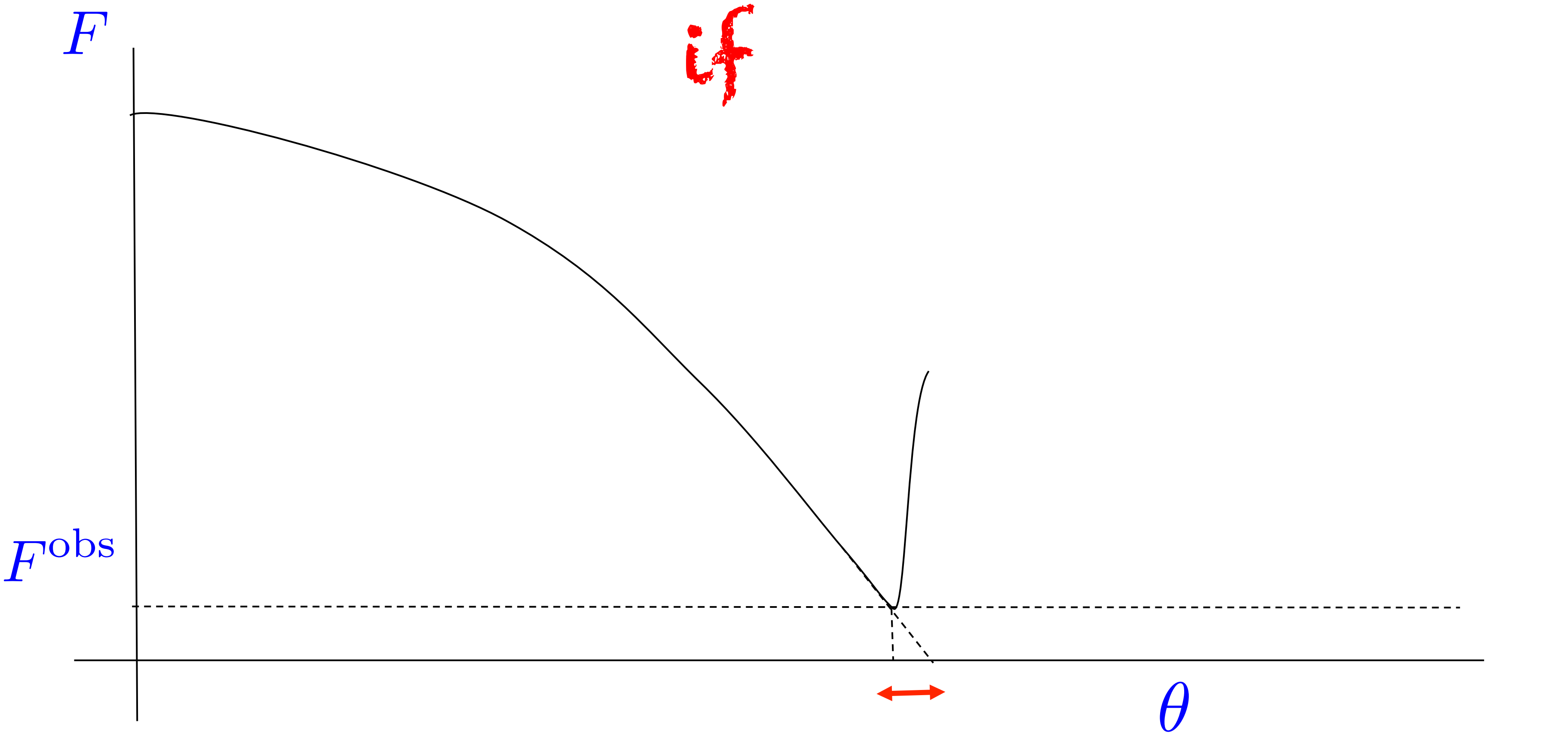
There are tree implicit assumptions behind this statistical interpretation

- Range of $\theta \ \sim [0,\ \theta_0]$
- Prior $p(\theta) \ = \ \mathrm{flat}$
- The expansion of $v^2(\theta)$ at first order captures its behavior in the neighborhood of interest.

Reasonable, but can be inappropriate in particular theoretical scenarios



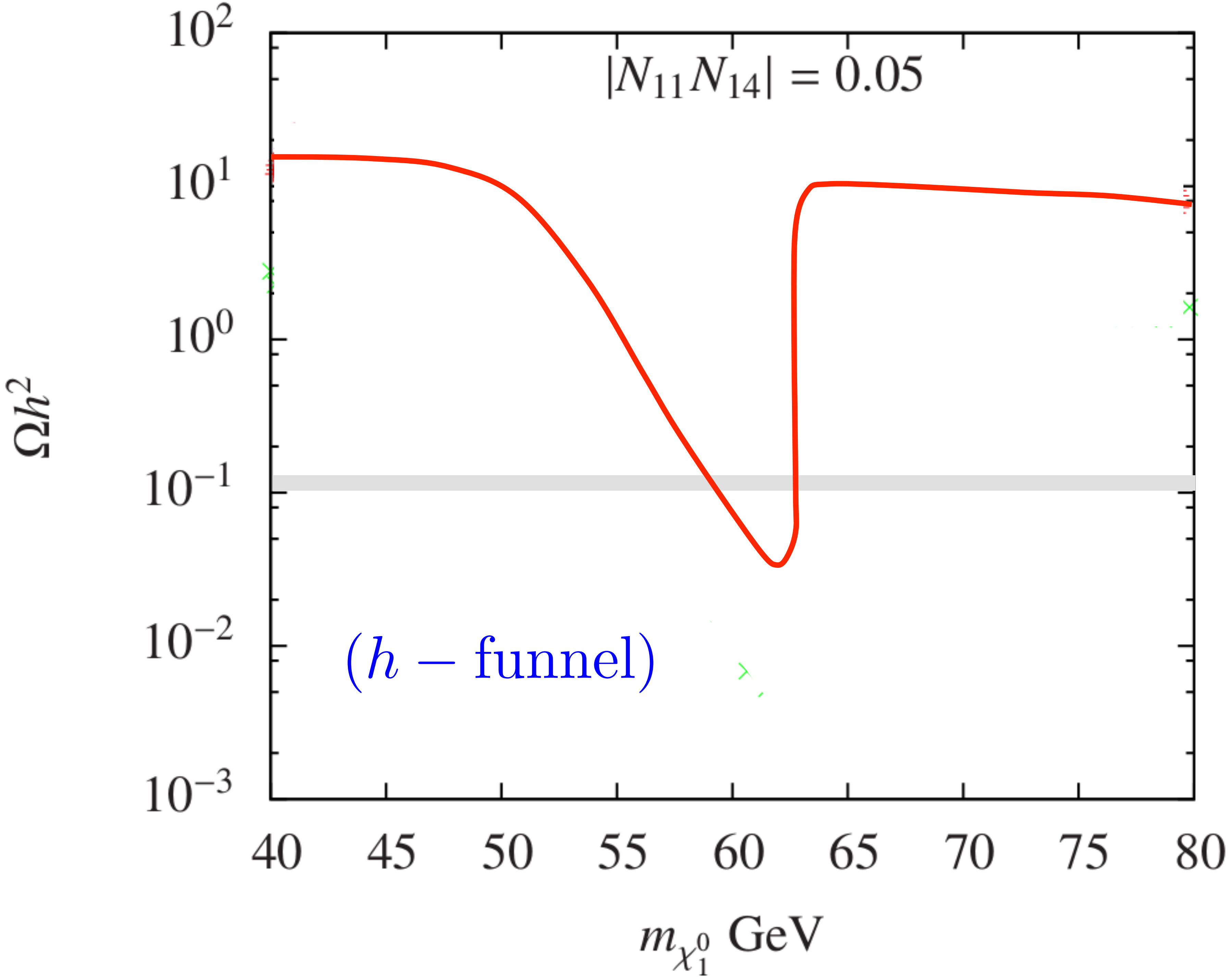
$$\Delta^{\text{true}} > \Delta^{\text{st. crit.}}$$



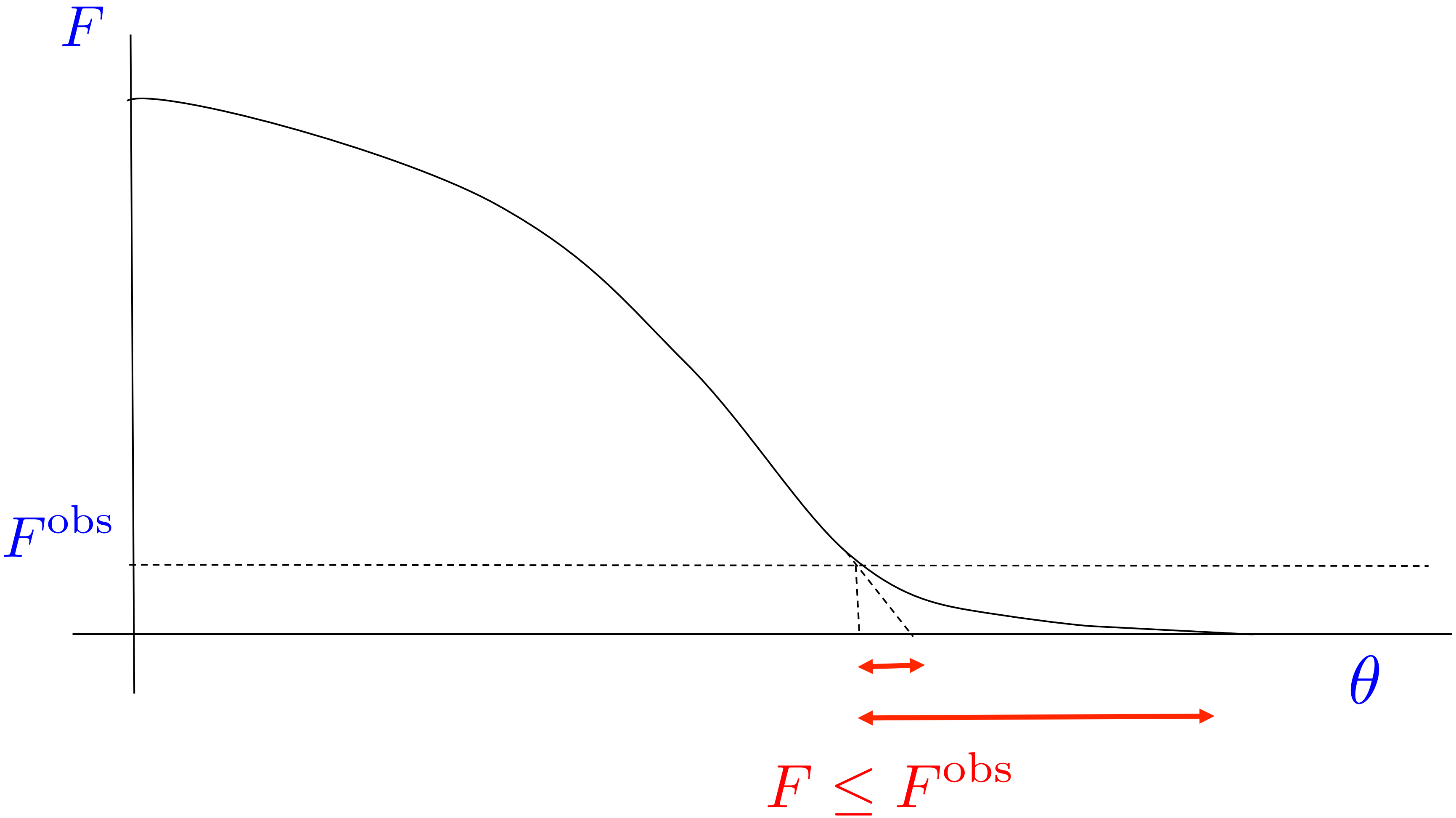
$$\Delta^{\text{true}} \to \infty$$

$$\Delta^{\text{st. crit.}} \to 0$$

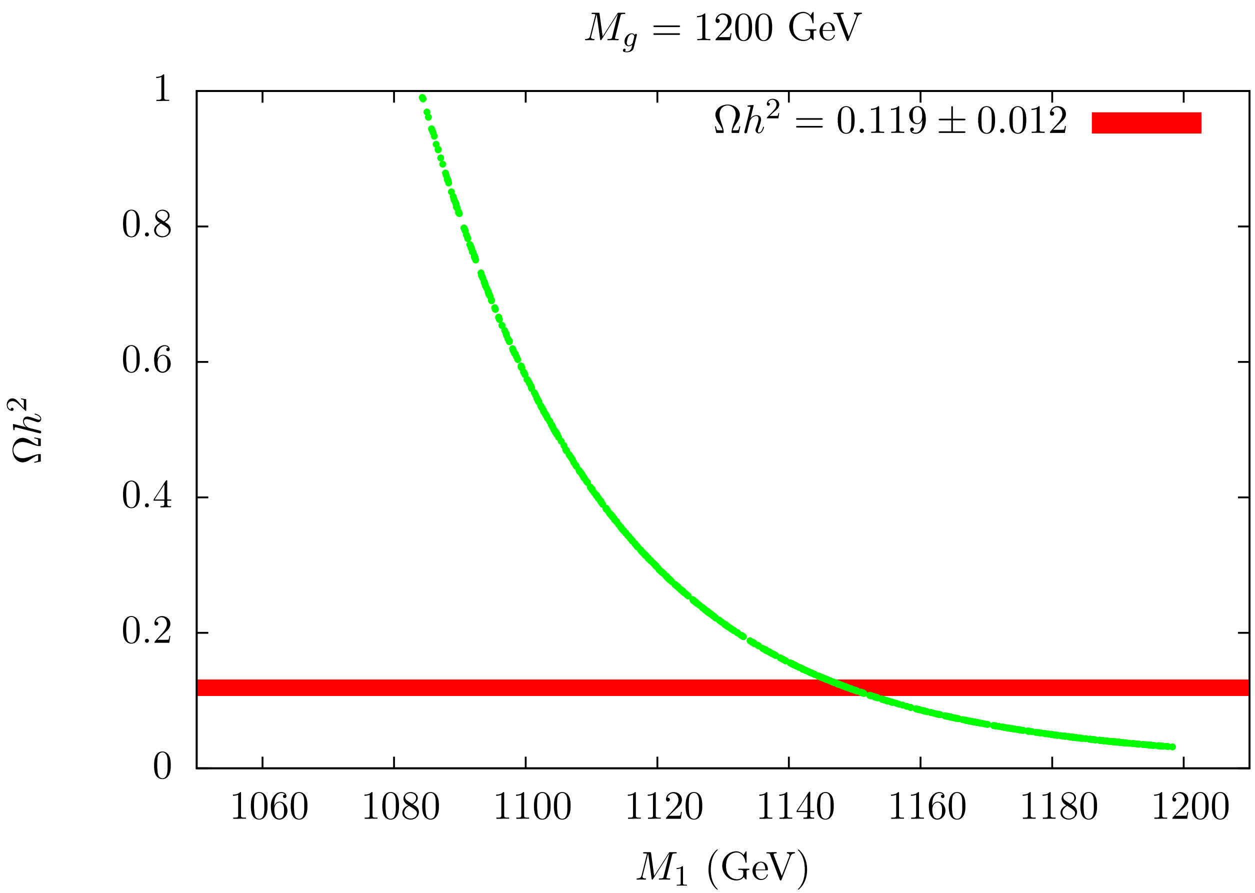
(!!)



10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
Ωh^2
$|N_{11}N_{14}| = 0.05$
$(h - \text{funnel})$
40
45
50
55
60
65
70
75
80
$m_{\chi_1^0}$ GeV



$$\Delta^{\mathrm{true}} < \Delta^{\mathrm{st.\ crit.}}$$



$\tilde{g}$ − coannihilation

Supersymmetric Dark Matter

An excellent candidate for DM particle is the MSSM-LSP, typically the lightest neutralino,

$$\chi_1^0$$

The mass and character of χ_1^0 arises from the diagonalization of the neutralino mass matrix

$$\mathcal{M}_{\chi^0} = \begin{array}{c} \begin{pmatrix} M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0 \end{pmatrix} \\ \begin{array}{cccc} \quad\tilde{B} \quad & \quad\tilde{W}_0 \quad & \quad\tilde{H}_d^0 \quad & \quad\tilde{H}_u^0 \quad \end{array} \end{array}$$

In general: χ_1^0 is a comb. of $\tilde{B}/\tilde{W}_0/\tilde{H}_u^0/\tilde{H}_d^0/$

Usually: χ_1^0 is dominated by one of the components

DM relic density

$$\Omega h^2 = \frac{8.7 \times 10^{-11}\,\mathrm{GeV}^{-2}}{\sqrt{g_*}\int_{x_f}^{\infty} dx\ \langle \sigma_{eff} v\rangle x^{-2}}.$$

We need $\Omega^{\rm obs} h^2 = 0.119 \pm 0.012$

Typically $\langle \sigma v \rangle$ is too small (but not always)

$\chi_1^0 =$ pure-state case

- $\chi_1^0 = \tilde{B}$ (bino) $\Longrightarrow$ non-viable
- $\chi_1^0 = \tilde{H}^0$ (Higgsino)

  $M_{\chi_1^0} \simeq \mu \simeq 1 \text{ TeV}$

- $\chi_1^0 = \tilde{W}_0$ (wino)

  $M_{\chi_1^0} \simeq M_2 \simeq 3 \text{ TeV}$

heavy SUSY spectrum

On the other hand the pure-Higgsino and pure-wino cases are not fine-tuned (regarding DM):

$$\langle \sigma v \rangle \sim \mu^{-2}$$

$$\langle \sigma v \rangle \sim M_2^{-2}$$

χ_1^0 can be lighter than 1 TeV or 3 TeV if

☆ χ_1^0 has a significant component of $\tilde{B}$

☆ There is an additional mechanism to increase $\langle \sigma_{\rm ann} v \rangle$

- ☆ Well-tempered χ_1^0: $\chi_1^0 = \tilde{B}/\tilde{H}^0$ or $\tilde{B}/\tilde{W}_0/\tilde{H}^0$

  $$M_1 \simeq \mu \quad \text{or} \quad M_1 \simeq M_2$$

- ☆ h - , Z - and A - funnels

  $$\chi_1^0 \chi_1^0 \to h, Z, A \to \text{SM SM}$$

  $$M_1 \simeq \frac{m_h}{2}, \ \frac{M_Z}{2} \quad \text{or} \quad \frac{m_A}{2}$$

- ☆ Co-annihilation with another fast-annihilating particle, e.g. a stop

  $$M_1 \simeq m_{\tilde{t}}$$

fine-tuning

However, not many studies of DM fine-tuning in the literature

Cheung, Hall, Pinner and Ruderman 2012

Fichet 2012

Grothaus, Lndner and Takanish 2012

Cohen and Wacker 2013

....

Well-tempered Bino-Higgsino

$$\mathcal{M}_{\chi^0} = \begin{pmatrix} M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0 \end{pmatrix}$$

Relevant parameters: $M_1, \ \mu$

Remark:

The degree of naturalness must be evaluated by examining the behavior of the fine-tuned quantities with respect of the independent parameters of the theory, e.g.

$$\Delta_i = \frac{d \log \Omega_{\rm DM}}{d \log \theta_i}$$

fortunate fact:

From the 4 parameters involved in the game,

$$\mathcal{M}_{\chi^0} = \begin{pmatrix} M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0 \end{pmatrix}$$

$\tan\beta$ is a derived parameter but it is (almost always) irrelevant for fine-tuning issues

M_1, M_2, μ are essentially in one-to-one multiplicative correspondence with the three initial (high-energy) parameters

$$M_i|_{LE} = c_{M_i}\; M_i|_{HE}\,, \qquad \mu|_{LE} = c_\mu\; \mu|_{HE}$$

depend on the HE scale

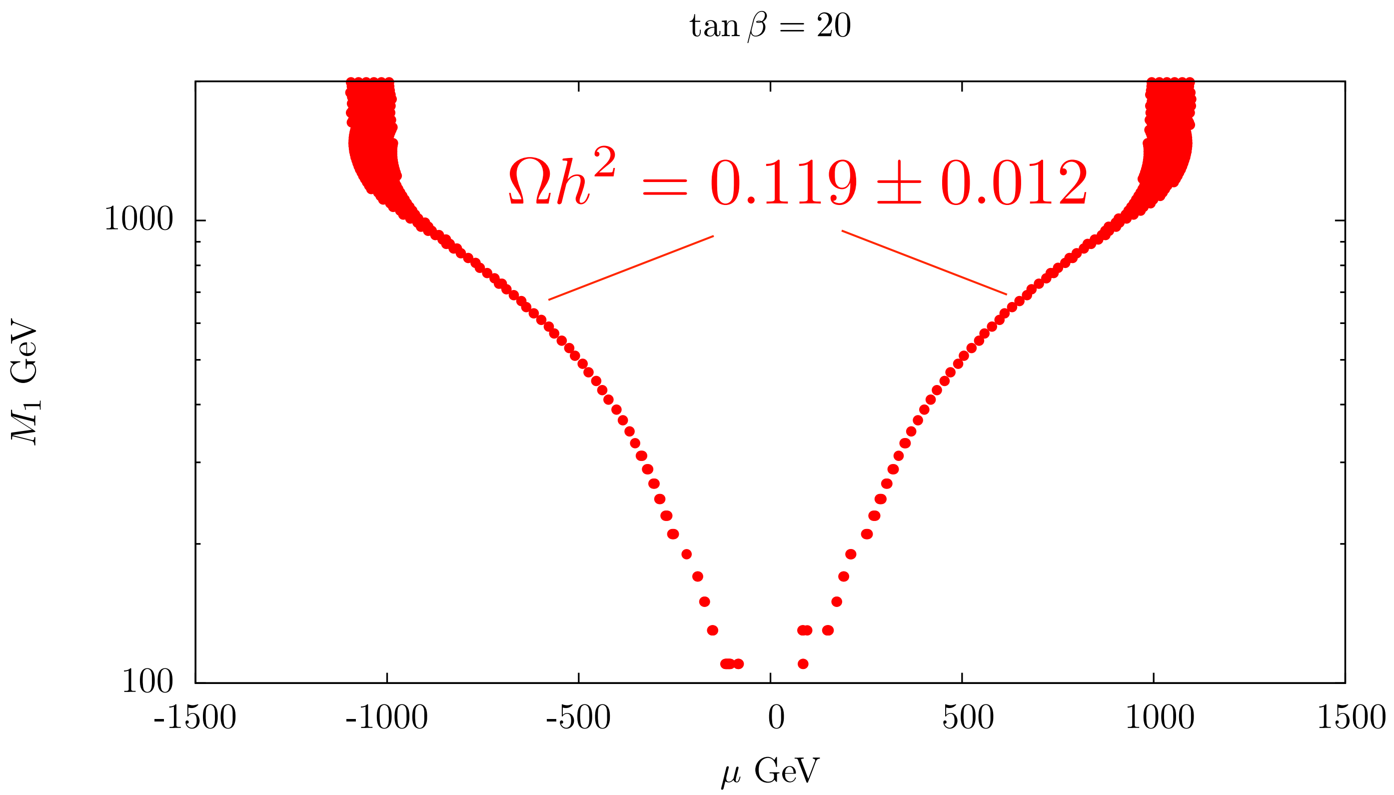
But, for the fine-tuning $c_{M_i}\,, c_\mu$ are irrelevant. E.g.

$$\Delta^{\rm DM}_{M_1} = \frac{d\log\Omega_{\rm DM}}{d\log\, M_1|_{HE}} = \frac{d\log\Omega_{\rm DM}}{d\log\, M_1|_{LE}}$$

The results for $\Delta^{\rm DM}$ do not depend on the HE scale or on the values of the remaining MSSM parameters, which is notable.

(End of remark)

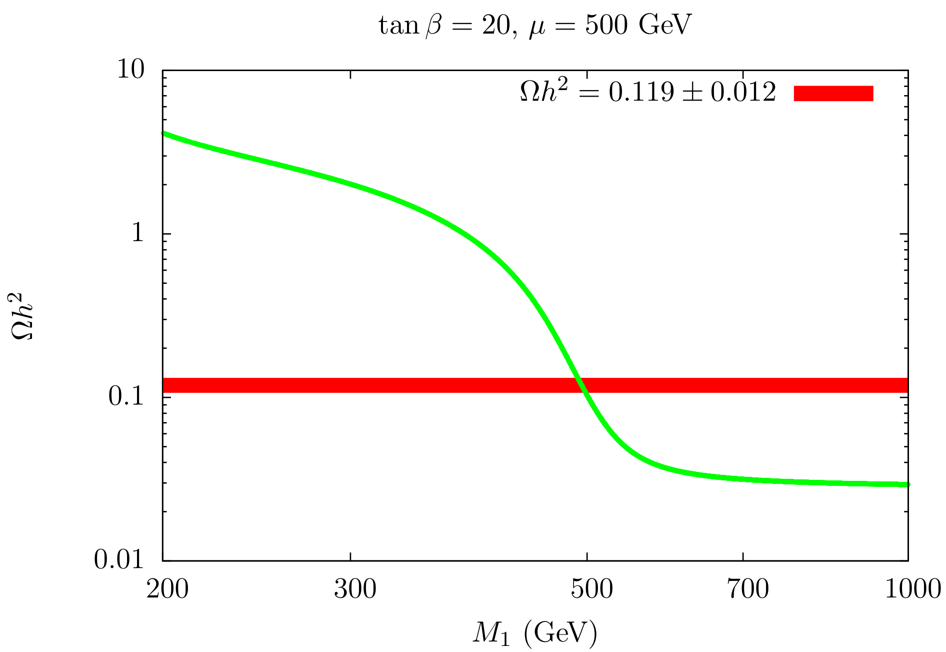
Well-tempered Bino-Higgsino

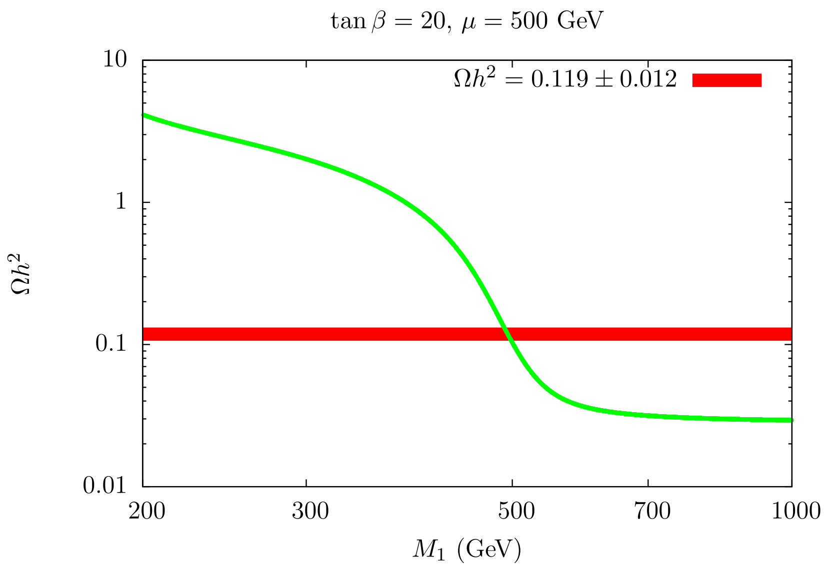


shows $M_1 \simeq \mu$, except in the limit of pure Higgsino ($\mu \simeq 1$ TeV)

Evaluation of the fine-tuning

Does $\Omega^{\mathrm{DM}}(M_1)$ satisfy the conditions for the standard criterion to be valid?

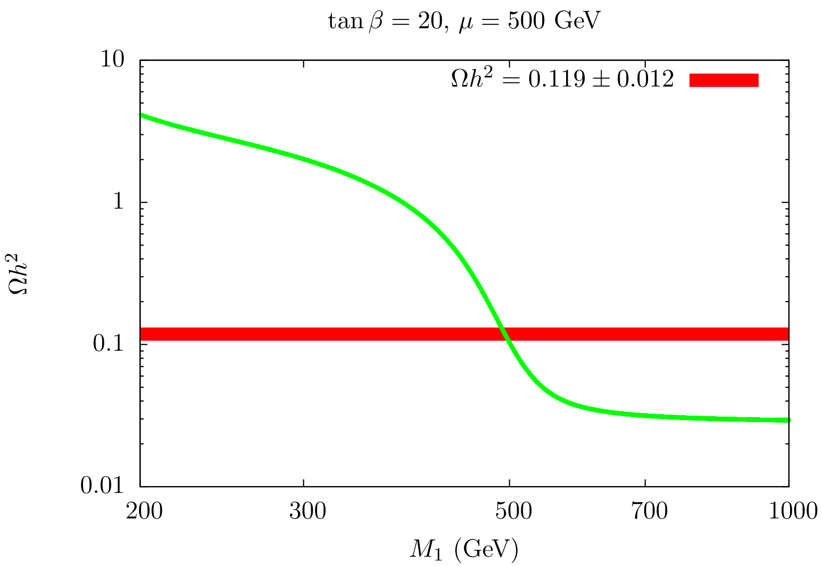




If the p-value is defined as $\mathcal{P}(\Omega \leq \Omega_{\rm DM}^{\rm obs})$

then, apparently, p-value $\to \mathcal{O}(1)$
(and the standard criterion fails)

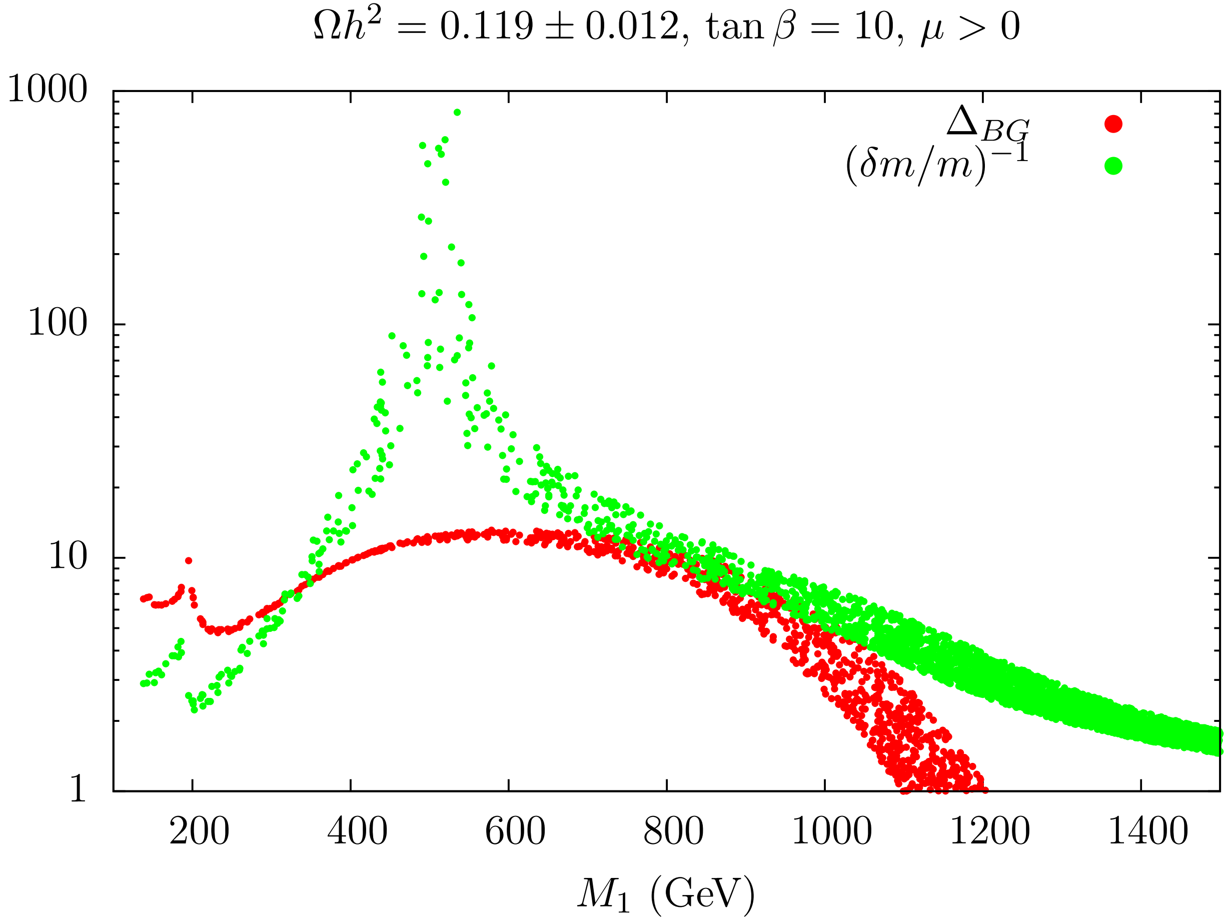
But this result depends drastically upon the limits shosen for the M_1 range



A much more satisfactory result is obtained by changing the choice of the fine-tuned quantity

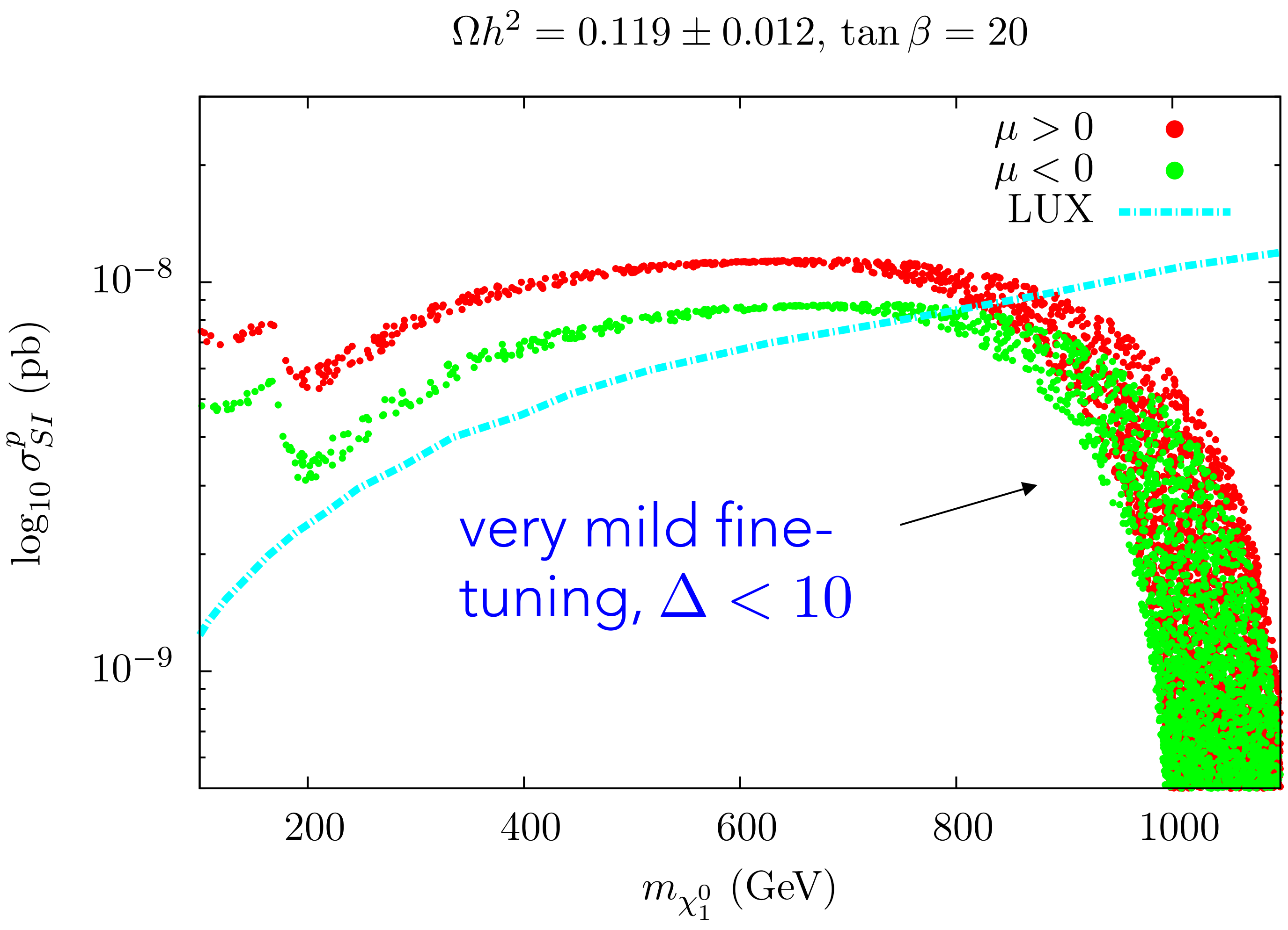
$\Omega \to \tan 2\theta$ with $|\tan 2\theta| \simeq \dfrac{\sqrt{2} s_W M_Z}{|\mu - M_1|}$

$$p - \text{value} = \mathcal{P}(|\tan 2\theta| > |\tan 2\theta^{\text{obs}}|) = \boxed{\frac{2|\mu - M_1|}{M_1}}$$



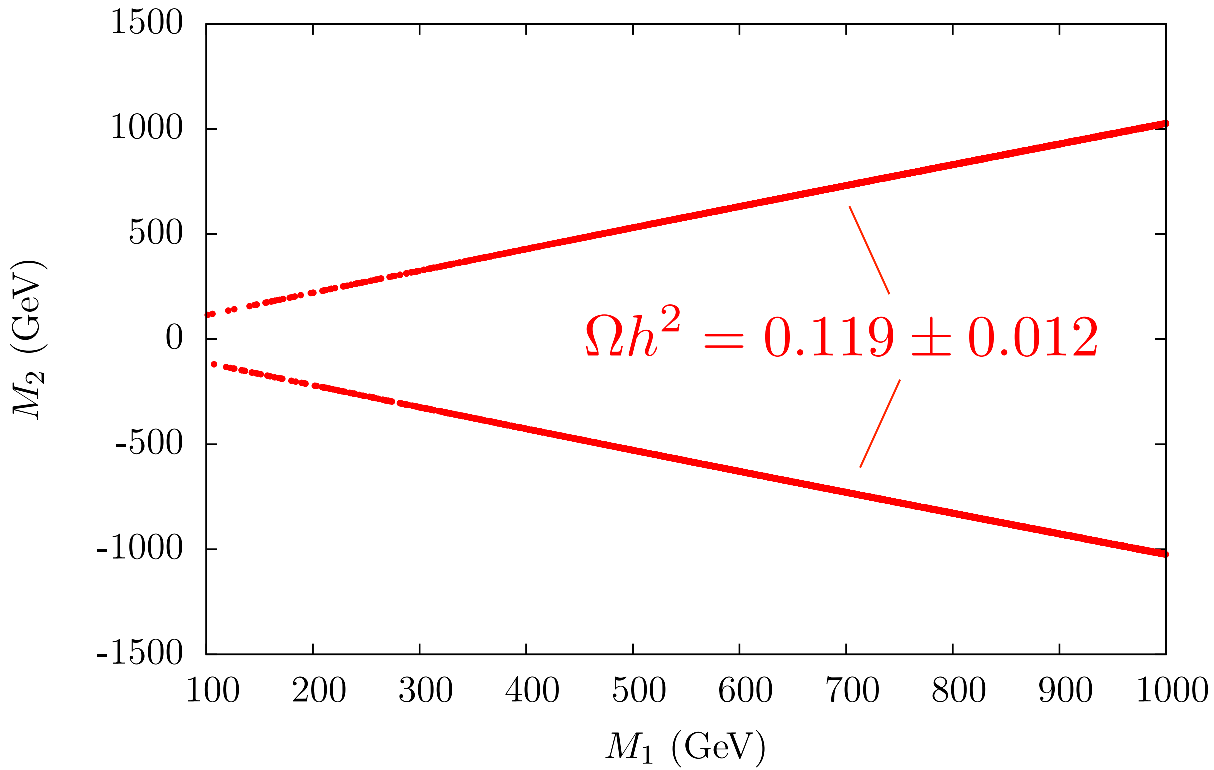
$\Omega h^2 = 0.119 \pm 0.012$, $\tan\beta = 10$, $\mu > 0$
1000
100
10
1
Δ_{BG}
$(\delta m/m)^{-1}$
200
400
600
800
1000
1200
1400
M_1 (GeV)

$\tilde{B}/\tilde{H}^0$ severely constrained by DD



Well-tempered Bino-Wino

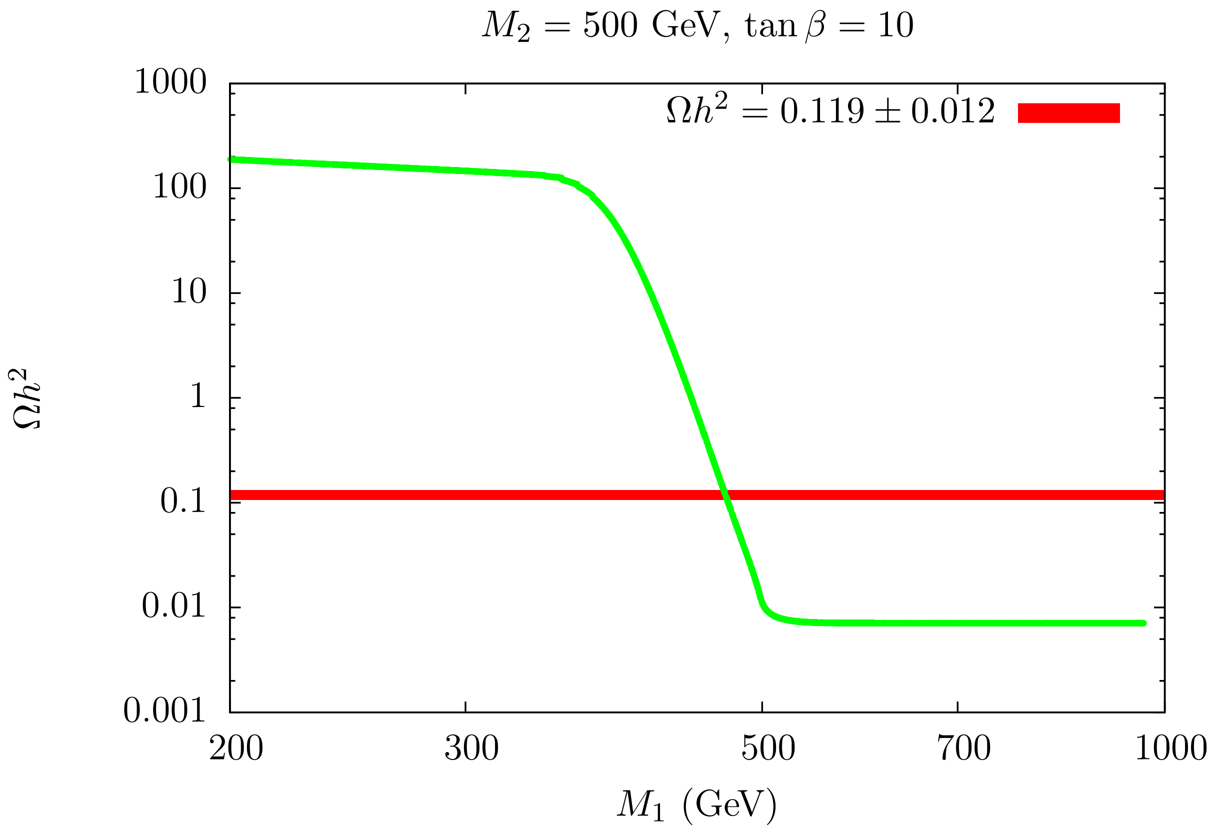
For large enough μ the mixing between $\tilde{B}/\tilde{W}_0$ is small and the annihilation is dominated by co-annihilation of winos (and is independent of the value of μ).

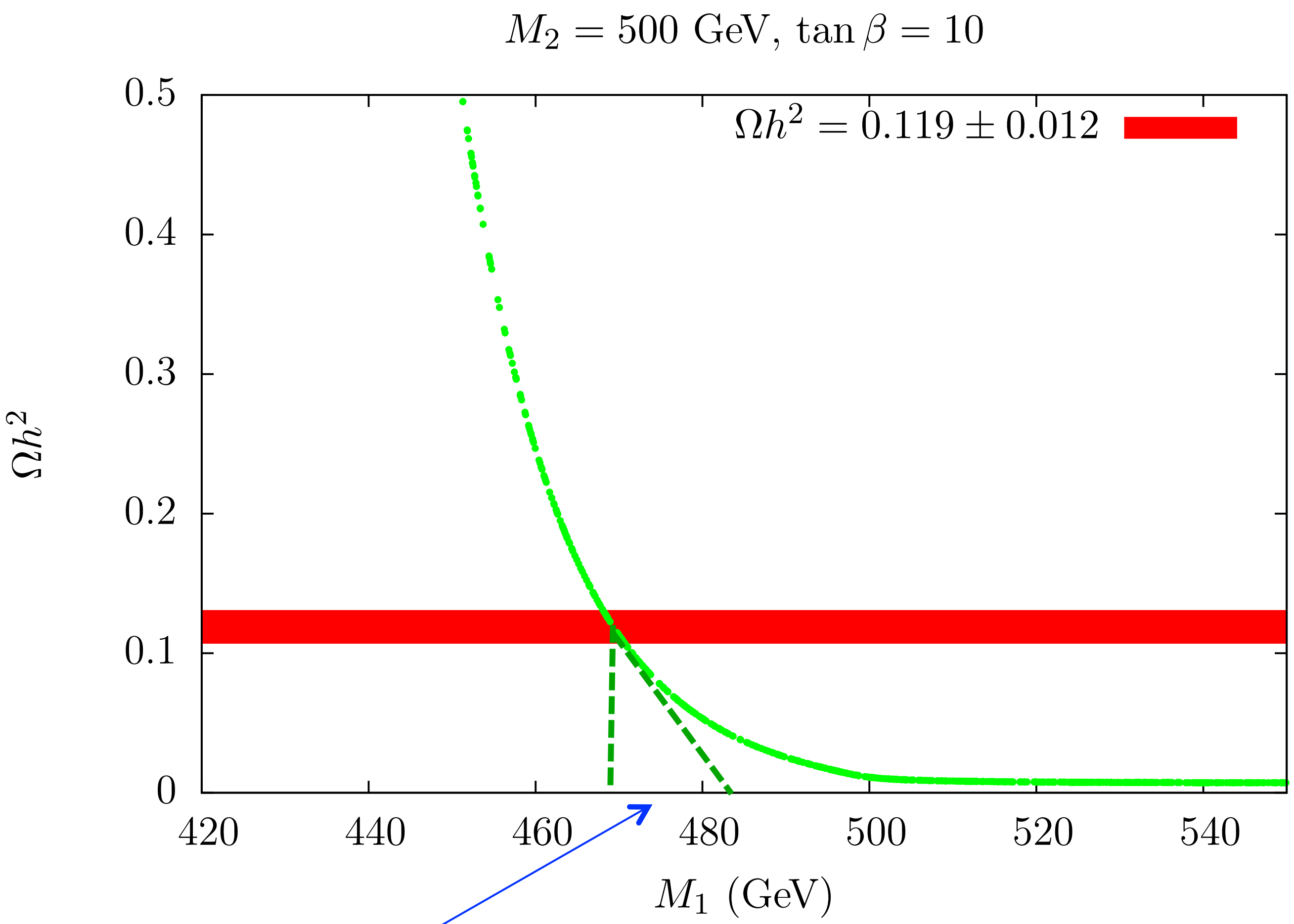


note $M_1 \simeq M_2$

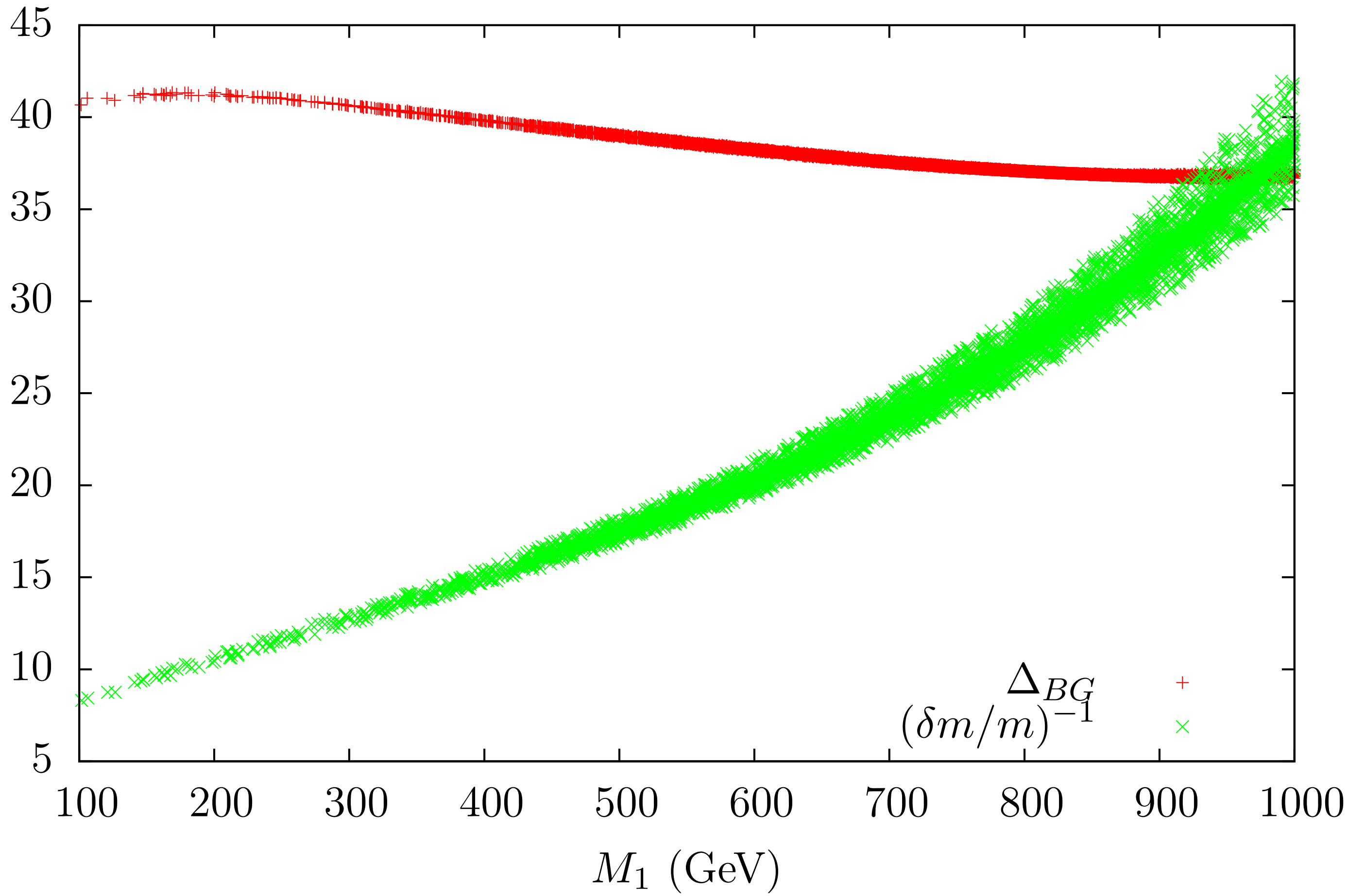
Evaluation of the fine-tuning

$\Omega^{\mathrm{DM}}(M_1)$





Standard criterion overestimates the fine-tuning

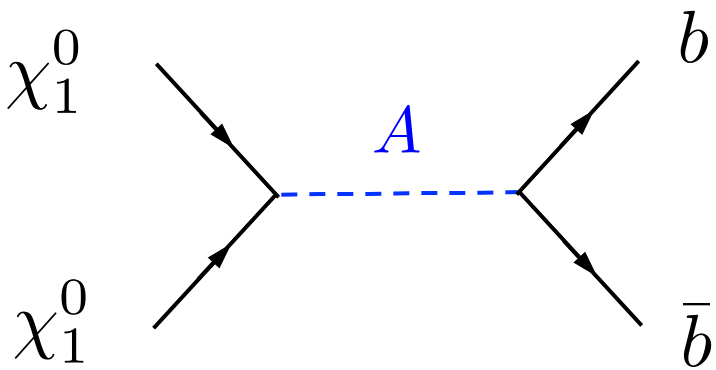

45
40
35
30
25
20
15
10
5
100
200
300
400
500
600
700
800
900
1000
M_1 (GeV)
Δ_{BG}
$(\delta m/m)^{-1}$

Funnels

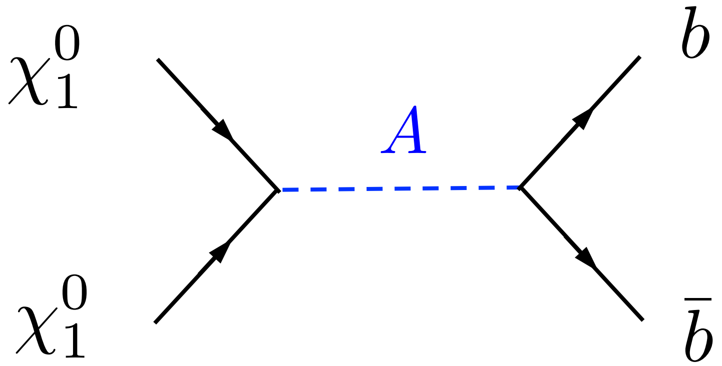
$$\Omega h^2 = \frac{8.7 \times 10^{-11}\,\mathrm{GeV}^{-2}}{\sqrt{g_*}\int_{x_f}^{\infty} dx\ \langle \sigma_{eff} v\rangle x^{-2}}.$$

$\langle \sigma v \rangle$ may increase thanks to resonant annihilation, e.g.

$(A - \text{funnel})$

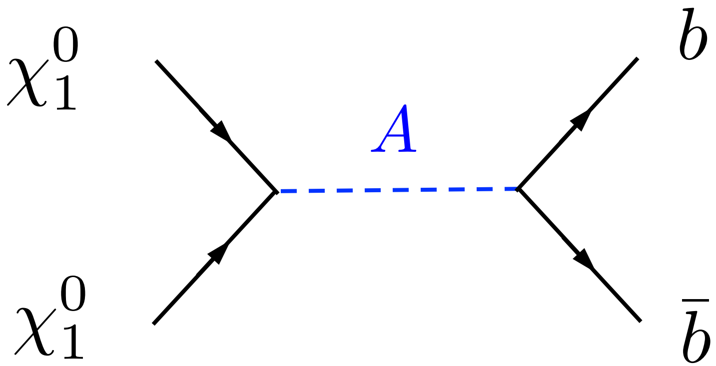


Note that even if $M_1 < \frac{m_A}{2}$ there can be res. annihilations, since the kinetic energy of the neutralinos can be large, thanks to the thermal bath



$M_1 < \frac{m_A}{2}$ but $s \simeq m_A$ for some collisions

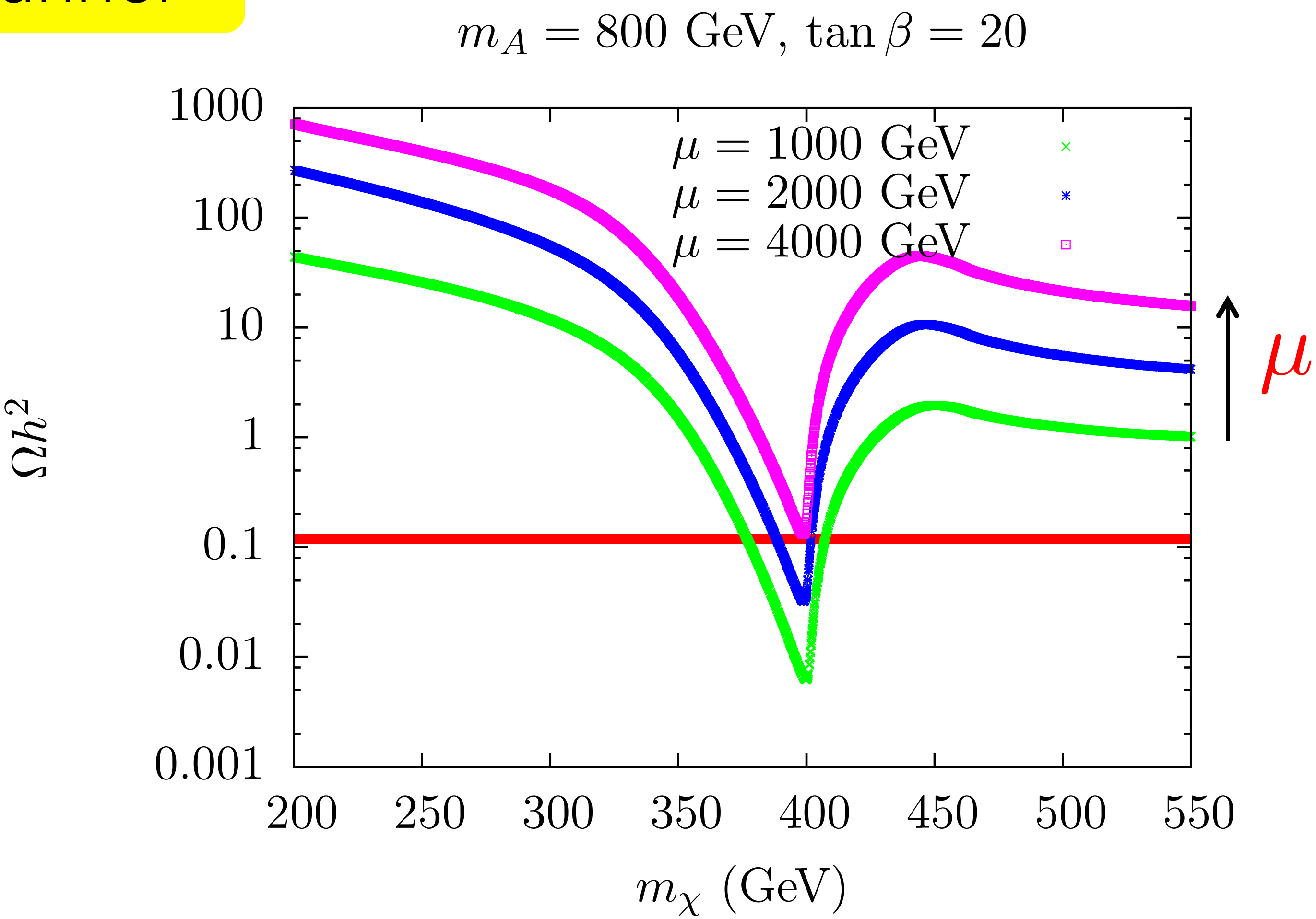
The process



requires $\chi_1^0 = \tilde{B}/\tilde{H}^0$

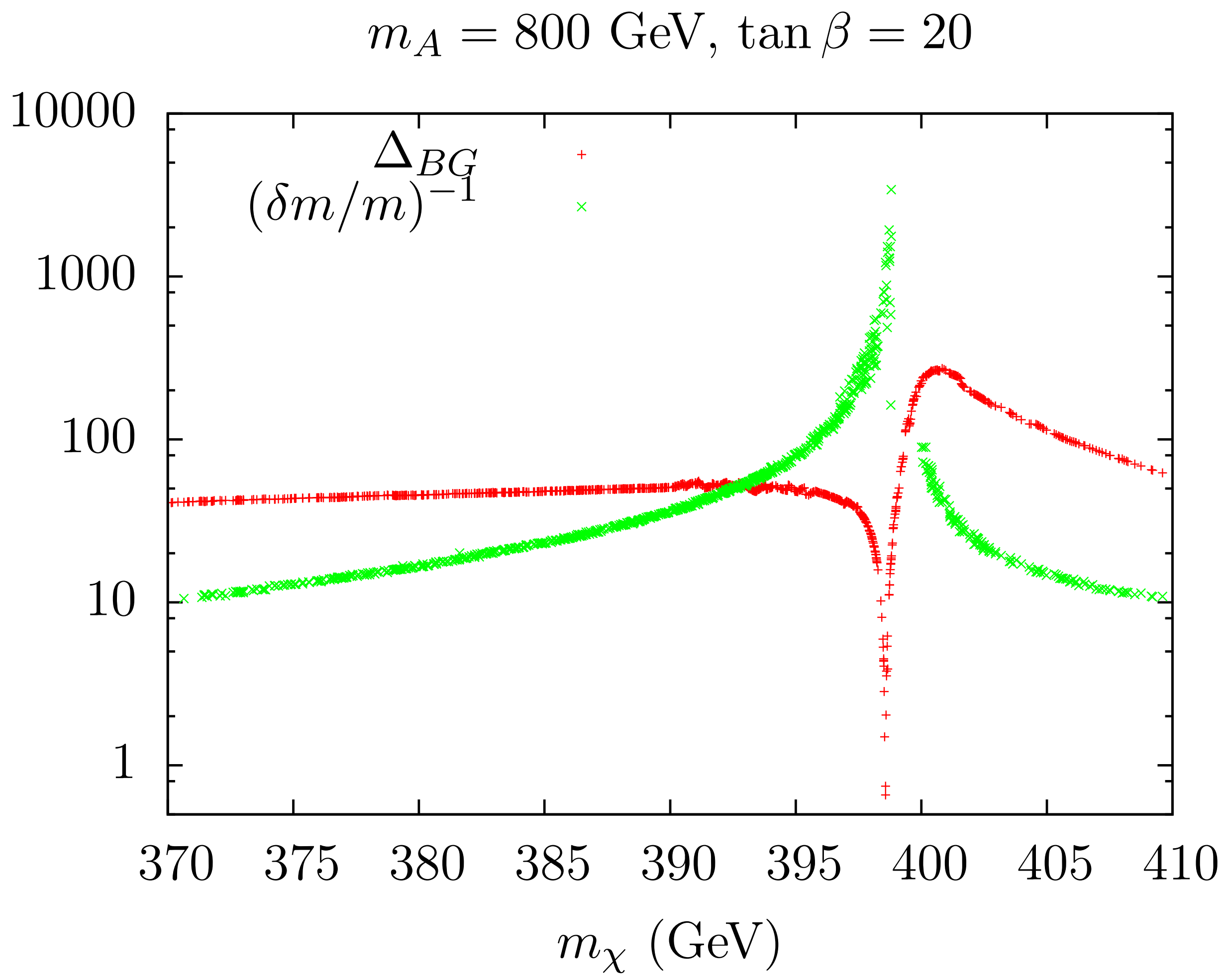
The higher the Higgsino component (and thus the smaller μ), the more efficient the annihilation

A-funnel



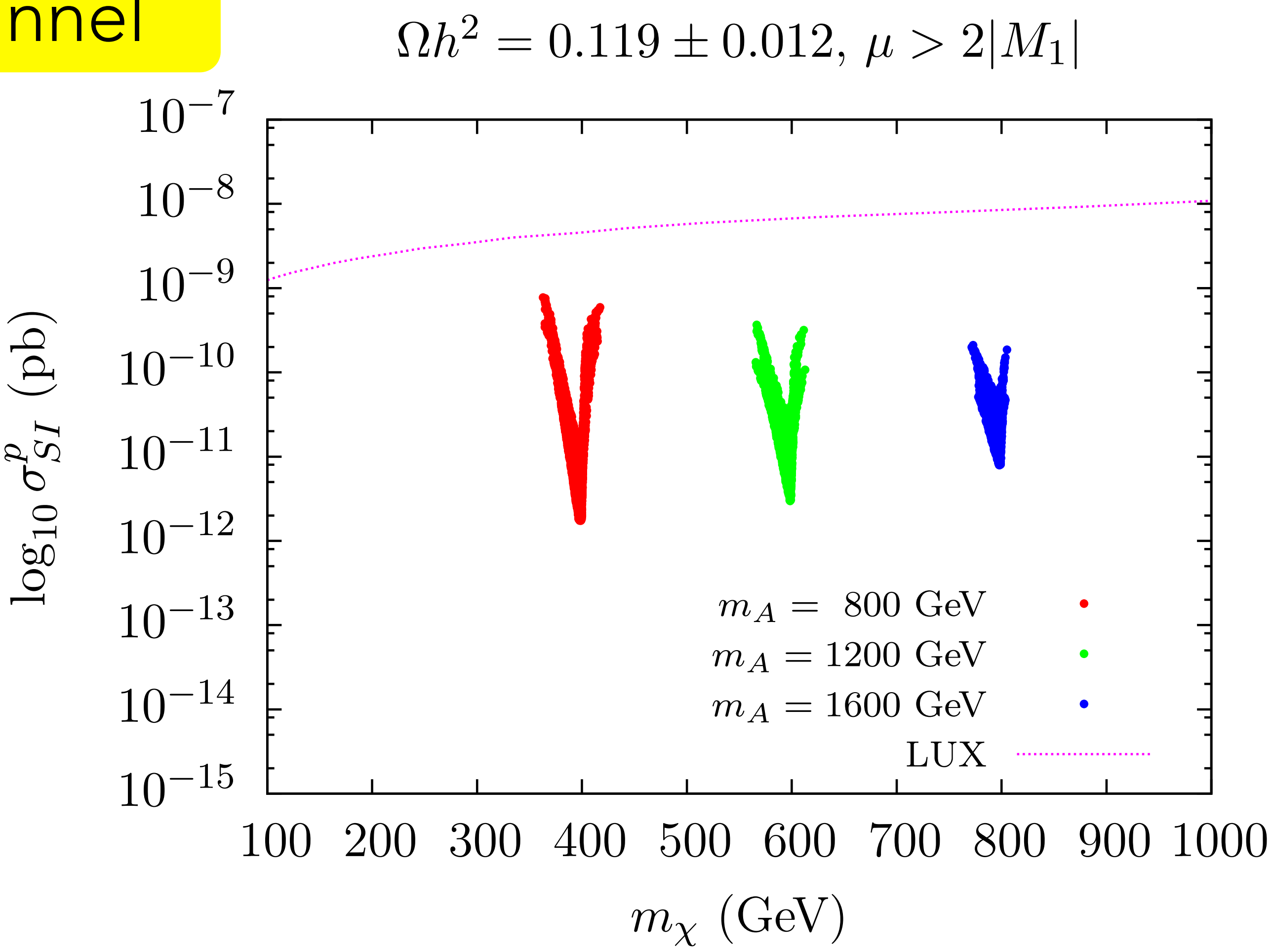
$$\mathcal{P}(\Omega \leq \Omega_{\rm DM}^{\rm obs}) \simeq \frac{|\delta m|}{m}$$

A-funnel



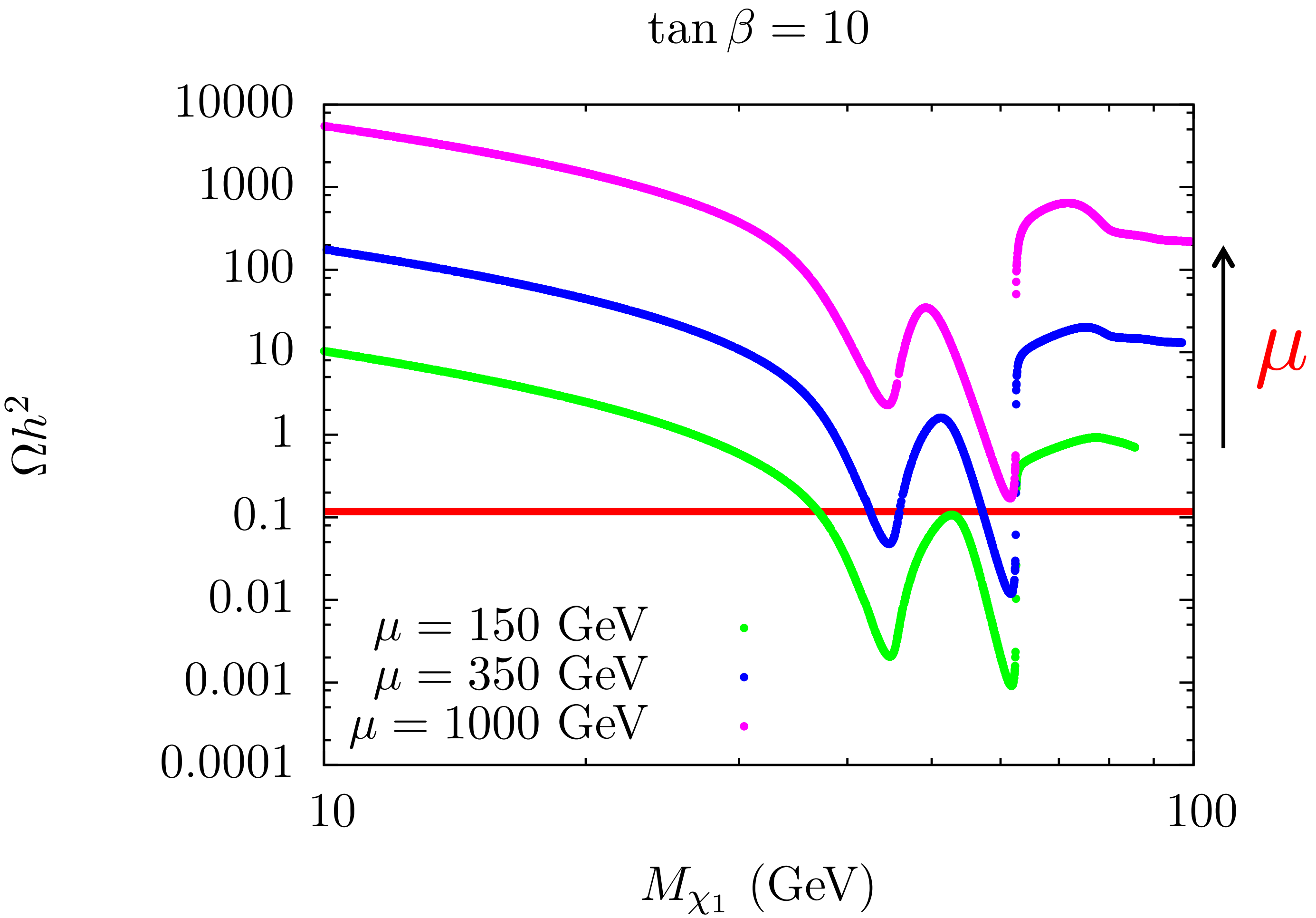
$m_A = 800$ GeV, $\tan\beta = 20$
10000
1000
100
10
1
Δ_{BG}
$(\delta m/m)^{-1}$
370
375
380
385
390
395
400
405
410
m_χ (GeV)

A-funnel



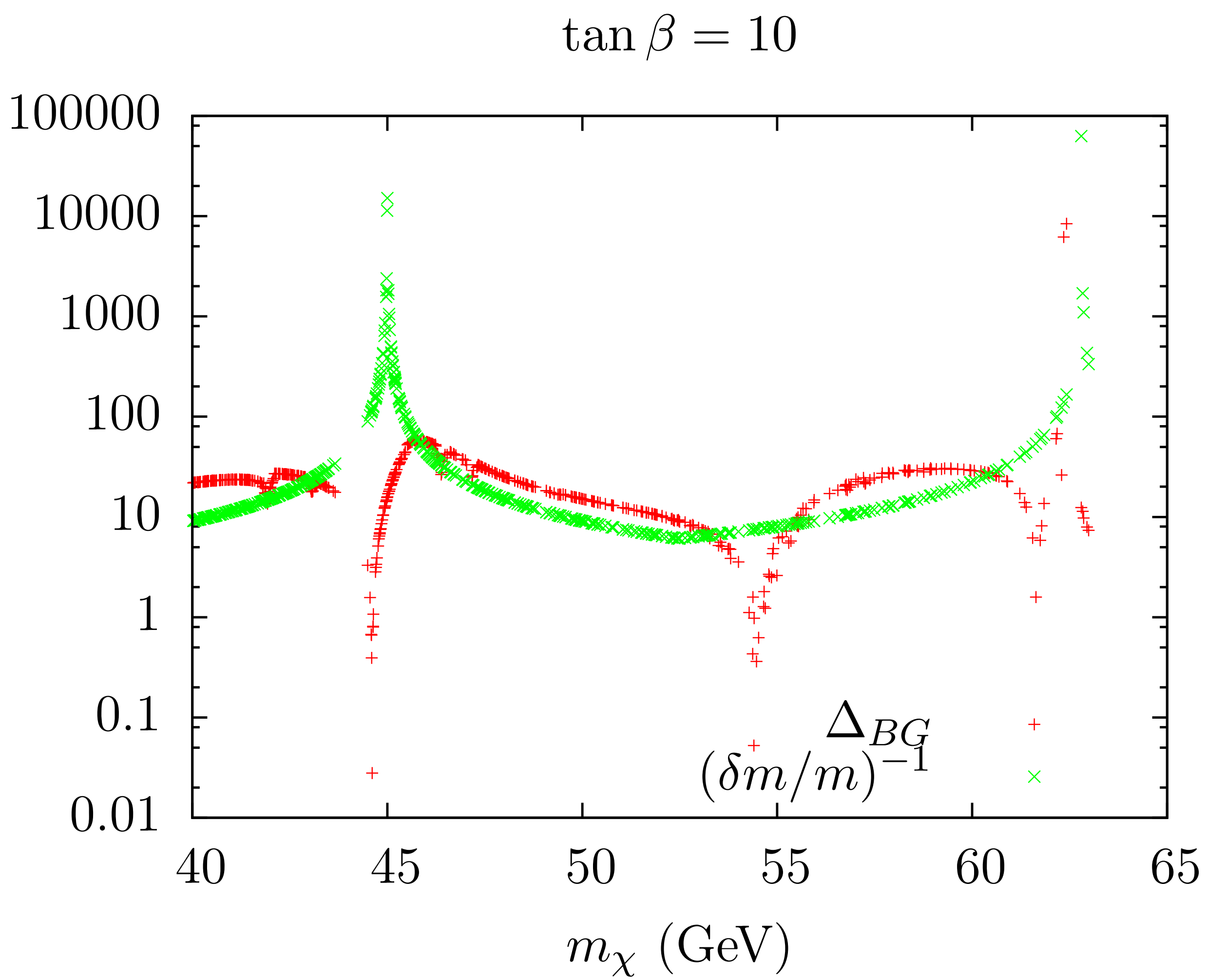
The scenario is safe respect to DD, unless μ goes too small.

h & Z - funnels



$\tan\beta = 10$
10000
1000
100
10
1
0.1
0.01
0.001
0.0001
10
100
Ωh^2
M_{χ_1} (GeV)
$\mu = 150$ GeV
$\mu = 350$ GeV
$\mu = 1000$ GeV
μ

h & Z - funnels



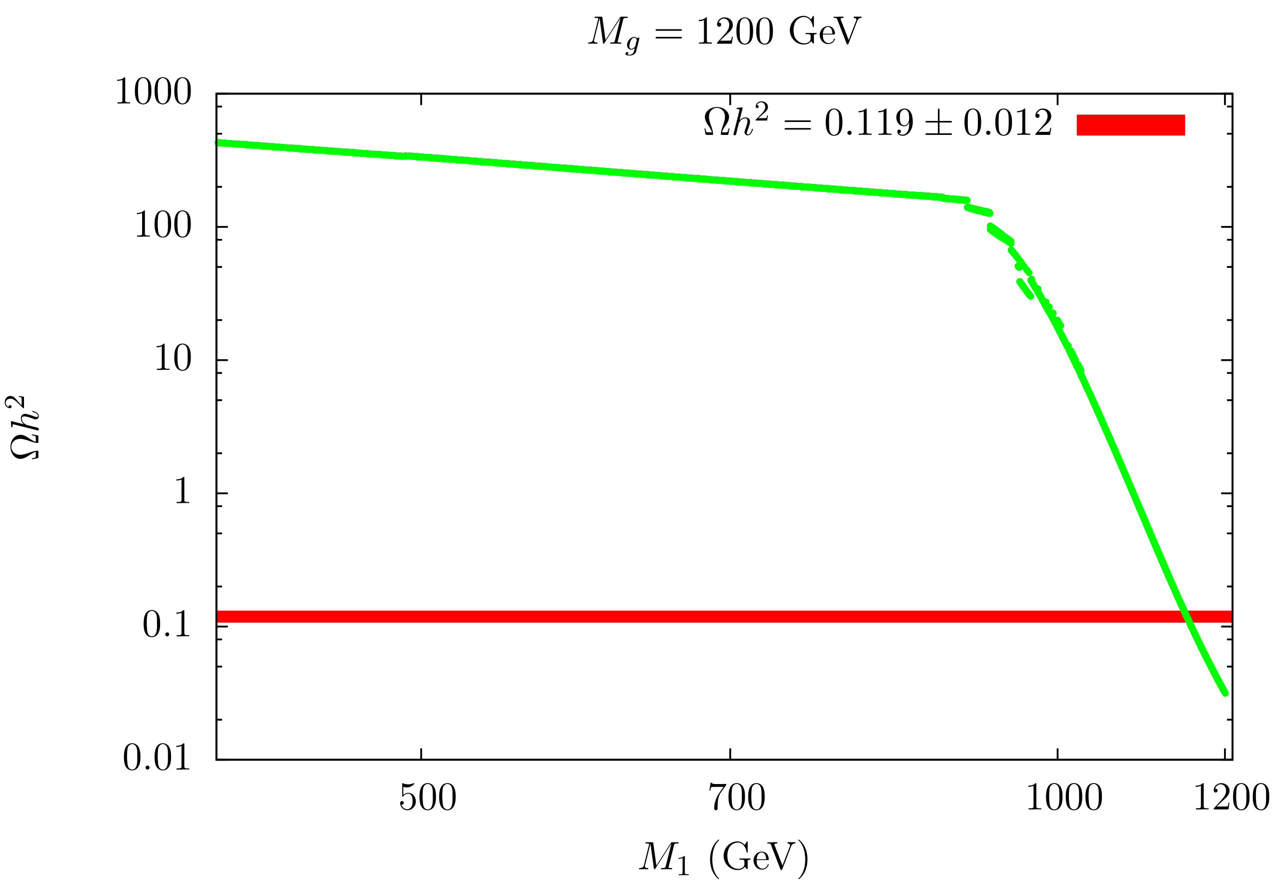
$\tan\beta = 10$
100000
10000
1000
100
10
1
0.1
0.01
40
45
50
55
60
65
m_χ (GeV)
Δ_{BG}
$(\delta m/m)^{-1}$

Co-annihilation

Co-annihilation occurs whe one or several species with masses close to the LSP annihilate efficiently

Co-annihilation is exponentially sensitive to the mass difference between the DM and its neighboring states.

$$\langle \sigma_{eff} v \rangle = \frac{\sum_{i,j=1}^{N} w_i w_j \sigma_{ij} x^{-n}}{\left(\sum_{i=1}^{N} w_i\right)^2}, \quad w_i = \left(\frac{m_i}{m_1}\right)^{3/2} e^{-x\left(\frac{m_i}{m_1}-1\right)}.$$



$M_g = 1200$ GeV
1000
100
10
1
0.1
0.01
$\Omega h^2 = 0.119 \pm 0.012$
Ωh^2
500
700
1000
1200
M_1 (GeV)

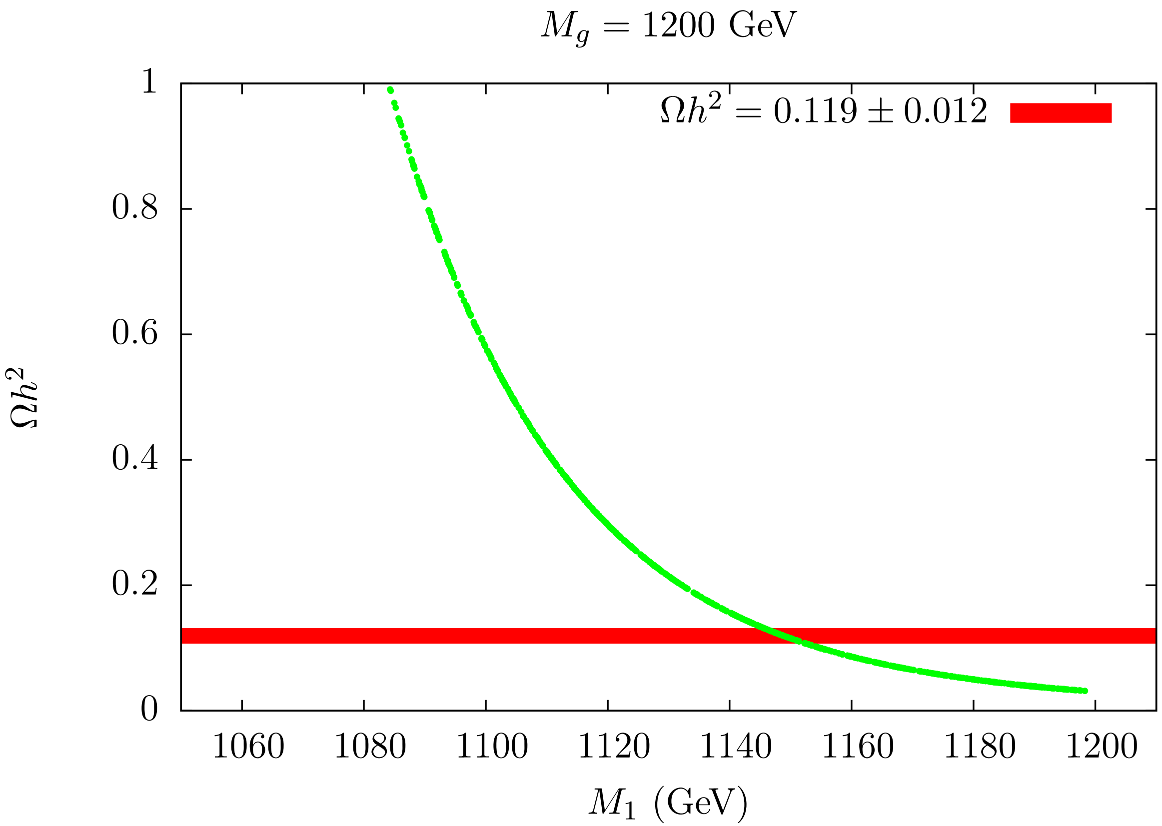
$$\Omega h^2 \sim \exp(-\mathcal{O}(1)\frac{\Delta m}{T_f})$$

$$\Delta m \simeq m_{\tilde{g}} - M_1$$

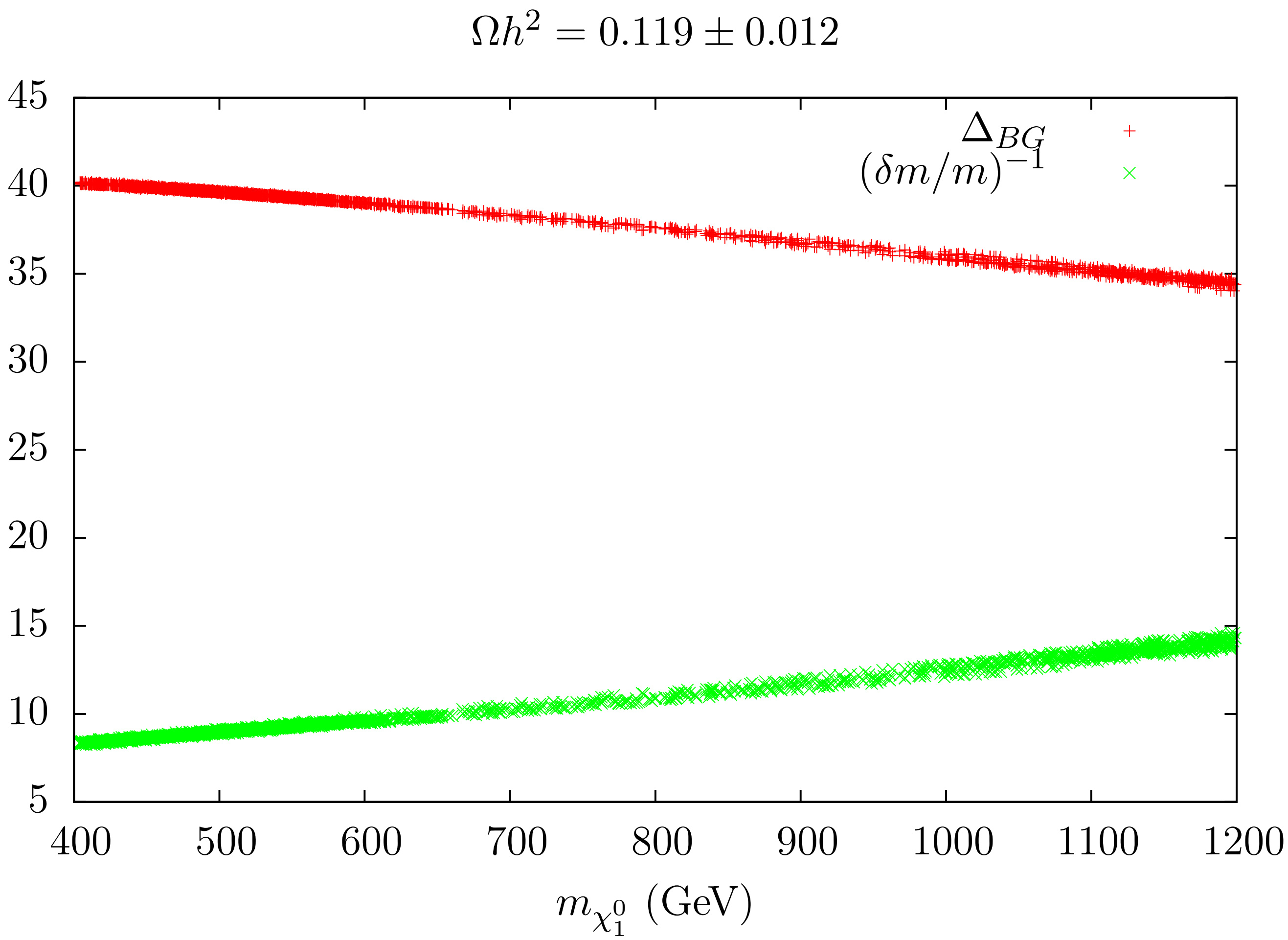
$$T_f \sim \frac{M_{\chi_1^0}}{20}$$

$$\rightarrow \Delta_{M_1} \simeq \mathcal{O}(20-40)\frac{m_{\tilde{g}}}{M_1} \simeq \mathcal{O}(20-40)$$

(independent of Δm !)



$M_g = 1200$ GeV
$\Omega h^2 = 0.119 \pm 0.012$
Ωh^2
M_1 (GeV)
0
0.2
0.4
0.6
0.8
1
1060
1080
1100
1120
1140
1160
1180
1200



As usual for co-annihilations the standard criterion overestimates the fine-tuning

Conclusions

- Typically the mechanisms for (thermal) DM production in the MSSM are fine-tuned.

- Some exceptions: pure Higgsino, pure Wino; but they lead (especially the pure Wino) to heavy SUSY spectrum

  EW fine-tuning & problems to see SUSY at the LHC

- Co-annihilation cases are (much) less fine-tuned than indicated by the standard criterion for fine-tuning

Conclusions II

- This is also true for funnels when the mass of the neutralino is not too close to (twice) the resonance

- In several cases the fine-tuning is mild ($\lesssim 10$)
  - Well-tempered bino-Higgsino
  - Higgs-funnel
  - Some regions of co-annihilation

- It is important to mantain the DM fine-tuning in mild levels, since it must be combined with the EW one

Conclusions III

- The most robust prediction from Natural SUSY is, by far,

$$m_{\tilde{H}} \lesssim 0.7\ \mathrm{GeV}$$

- SUSY is in good shape, though somewhat fine-tuned

  “Natural” SUSY (the less fine-tuned version of the MSSM without “fooling” the LHC) is 1%-10% fine-tuned

  Going beyond the MSSM, i.e. NMSSM, BMSSM, RPV,... could reduce the fine-tuning as well

- If naturalness arguments are sound and SUSY is true, we could be about seeing SUSY (or perhaps other BSM) in LHC-14