NILPOTENT SUPERGRAVITY, INFLATION AND MODULI STABILIZATION

Based on :

- I.Antoniadis, E.D., S.Ferrara and A. Sagnotti, Phys.Lett.B733 (2014) 32 [arXiv:1403.3269 [hep-th]].
- E.D., S.Ferrara, A.Kehagias and A.Sagnotti, JHEP 1509 (2015) 217 [arXiv:1507.07842 [hep-th]].

dec. 15, 2015

IFT, Sao Paulo

Outline

1) Nonlinear SUSY realizations

- constrained superfields

2) Nonlinear supergravities

- Minimal nonlinear supergravity

3) Simplest models of inflation in supergravity

- Chaotic inflation
- Starobinsky models

4) Inflation with nilpotent superfields

5) Moduli stabilization with nilpotent uplift

5) Conclusions and perspectives

Large literature on SUSY non-linear realizations and low-energy goldstino interactions

- Volkov-Akulov, Ivanov-Kapustinov, Siegel, Samuel-Wess, Clark and Love...
- Casalbuoni, Dominicis, de Curtis, Feruglio, Gatto
- Luty, Ponton
- Brignole, Feruglio, Zwirner; Brignole
- Casas, Espinosa, Navarro
- Komargodski and Seiberg...

Most pheno studies based on a component formalism, tedious computations. With constrained superfield formalism, easier computations.

Today applications to inflation.

Application to (MS)SM: Antoniadis,E.D.,Ghilencea,Tziveloglou: E.D.,Gersdorff, Ghilencea,Lavignac,Parmentier; Petersson,Romagnoni…

1) Non-linear SUSY realizations

- In supergravity, the gravitino Ψ becomes massive by absorbing a spin ½ fermion: the goldstino G

$$\Psi_\mu \begin{pmatrix} 3/2 \\ - \\ - \\ -3/2 \end{pmatrix} + G \begin{pmatrix} - \\ 1/2 \\ -1/2 \\ - \end{pmatrix} = \Psi_\mu \begin{pmatrix} 3/2 \\ 1/2 \\ -1/2 \\ -3/2 \end{pmatrix}$$

Goldstino is part of a multiplet $X = (x, G, F_X)$

The gravitino mass is $m_{3/2} \sim \frac{f}{M_P}$, where

$F_X = f + \cdots$ is the scale of SUSY breaking

In the decoupling limit $f << M_P$, the partner of the goldstino, the sgoldstino x decouples. Goldstino couplings to matter scale as $1/f$

The first written SUSY lagrangian had nonlinearly realized SUSY, Volkov-Akulov (VA).

$$\mathcal{L}_X = \int d^4\theta \; X^\dagger X + \left\{ \int d^2\theta \, f \; X + h.c. \right\}$$

$$= det \; (E_\mu^a) \; , \quad \text{where} \quad E_\mu^a = e_\mu^a + (\frac{i}{2f^2} G \sigma^a \partial_\mu \bar{G} + h.c.)$$

is the VA "vierbein". In the standard VA prescription, couplings to matter proceed as in gravity

$$G^{\mu\nu} \; T_{\mu\nu,M} \; = \; g^{\mu\nu} \; T_{\mu\nu,M} \; + \; (\frac{i}{2f^2} G \sigma^\mu \partial^\nu \bar{G} + h.c.) \; T_{\mu\nu,M}$$

- Constrained superfields

- VA action can be constructed in superspace (Rocek,78) introducing a constrained, nilpotent superfield

$$X^2 = 0$$

whose solution is

no fundamental scalar

$$X = \frac{GG}{2 F_X} + \sqrt{2}\,\theta\, G + \theta^2 F_X$$

The full VA action is $\mathcal{L}_{VA} = \Big[X \, \overline{X} \Big]_D + \Big[f X + h.c. \Big]_F$

There are two different cases to consider:

i) non-SUSY matter spectrum

$$E << m_{sparticles} \ , \ \sqrt{f}$$

non-linear SUSY in the matter sector

ii) SUSY matter multiplets : $(\tilde{q}, q)$, etc

$$m_{sparticles} \leq E << \sqrt{f}$$

linear SUSY in the matter sector

Case i) (non-linear matter) $\Rightarrow$ additional constraints

- light fermions : X Q = 0 eliminates complex scalars

$$Q = \frac{1}{F_X}(\Psi_q - \frac{F_q G}{2F_X})G + \sqrt{2}\theta\Psi_q + \theta^2 F_q$$

- light complex scalars : $X\bar{H} =$ chiral, eliminates fermions

$$H = h + i\sqrt{2}\theta\sigma^m\partial_m h\frac{\bar{G}}{\bar{F}_X} + \theta^2[-\partial_n(\frac{\bar{G}}{\bar{F}_X})\bar{\sigma}^m\sigma^n\partial_m h\frac{\bar{G}}{\bar{F}_X} + \frac{1}{2\bar{F}_X^2}\bar{G}^2\partial^2 h]$$

In this case, there is no more auxiliary field.

- light real scalar (inflaton ?) : $X(\Phi - \bar{\Phi}) = 0$ eliminates a scalar (sinflaton ?) and the fermion (inflatino ?)

2) Non-linear supergravities

In SUGRA, the most general couplings of the nilpotent X are described by (ADFS)

$$K = -3 \log (1 - X \overline{X}) \equiv 3 X \overline{X} , \qquad W = f X + W_0$$

and as a result

$$\mathcal{L}_{mass} = - m_{3/2} \left(\psi_m + \frac{i}{\sqrt{6}} \sigma_m \overline{G} \right) \sigma^{mn} \left(\psi_n + \frac{i}{\sqrt{6}} \sigma_n \overline{G} \right) + \text{h.c.}$$

The SUGRA lagrangian contains the proper goldstino couplings and

$$V = \frac{1}{3} |f|^2 - 3 |W_0|^2 , \qquad m_{3/2}^2 = |W_0|^2$$

Recently the complete lagrangian was written by
Bergshoeff, Freedman,Kallosh, Van Proeyen and Hasegawa,Yamada (2015)

Consider the gravity multiplet,

$$(e_m^a, \psi_m^\alpha, u, A_m)$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

vierbein gravitino auxiliary fields

coupled to the goldstino multiplet X, in the decoupling limit. The theory contains actually just
the graviton + one massive gravitino

It seems logical to anticipate that there should be a purely gravitational description, with a
modified/constrained gravity multiplet.

- Minimal nonlinear supergravity

(DFKS ; Antoniadis,Markou, 2015)

- This is described by

$$\mathcal{L} = \left[-S_0 \, \overline{S}_0 \right]_D + \left[W_0 S_0^3 \right]_F \qquad (1)$$

with the constraint

$$\left(\frac{\mathcal{R}}{S_0} - \lambda \right)^2 = 0 \qquad (2)$$

where :

- $\mathcal{R}$ is the chiral curvature multiplet,
- S_0 is the chiral compensator field
- λ is related to the cosmological constant.

whereas the goldstino is

$$G = -\frac{3}{2\lambda}\left(\gamma^{\mu\nu}\,\partial_\mu\,\psi_\nu - \frac{\lambda}{2}\,\gamma^\mu\,\psi_\mu\right)$$

This describes just the gravitational multiplet, with nonlinear SUSY. We will show that this is exactly dual to the simplest Volkov-Akulov SUGRA .

Introduce two lagrange multipliers X, Λ_1 that « linearize » the lagrangian

$$\mathcal{L} = \left[-\, S_0\,\overline{S}_0\right]_D + \left[\left\{X\left(\lambda - \frac{\mathcal{R}}{S_0}\right) - \frac{1}{4\Lambda_1}\,X^2 + W_0\right\} S_0^3\right]_F$$

which leads to

$$\mathcal{L} = \Big[- (1 + X + \bar{X}) \; S_0 \overline{S}_0 \Big]_D + \Big[\Big\{ \lambda X - \frac{1}{4\Lambda_1} X^2 + W_0 \Big\} S_0^3 \Big]_F$$

by using the identity

$$\Big[f(\Lambda) \, \mathcal{R} \, S_0^2 \Big]_F + \text{h.c.} = \Big[\big(f(\Lambda) + \overline{f}(\overline{\Lambda}) \big) S_0 \overline{S}_0 \Big]_D + \text{tot. deriv.}$$

One finally finds a standard SUGRA with

$$K = -3 \ln \big(1 + X + \overline{X} \big) \,, \qquad W = W_0 + \lambda X$$

plus the constraint $X^2 = 0$, or equivalently

$$K = 3 |X|^2 \,, \qquad W = W_0 + (\lambda - 3 W_0) X$$

Coupling to matter (chiral multiplets Q) can be implemented starting from

$$\mathcal{L} = \left[- e^{-\frac{1}{3} K_0(Q_i,\bar{Q}_{\bar{i}})} S_0 \overline{S}_0 \right]_D + \left[W_0(Q_i)\, S_0^3 \right]_F$$

supplemented by the nilpotency constraint

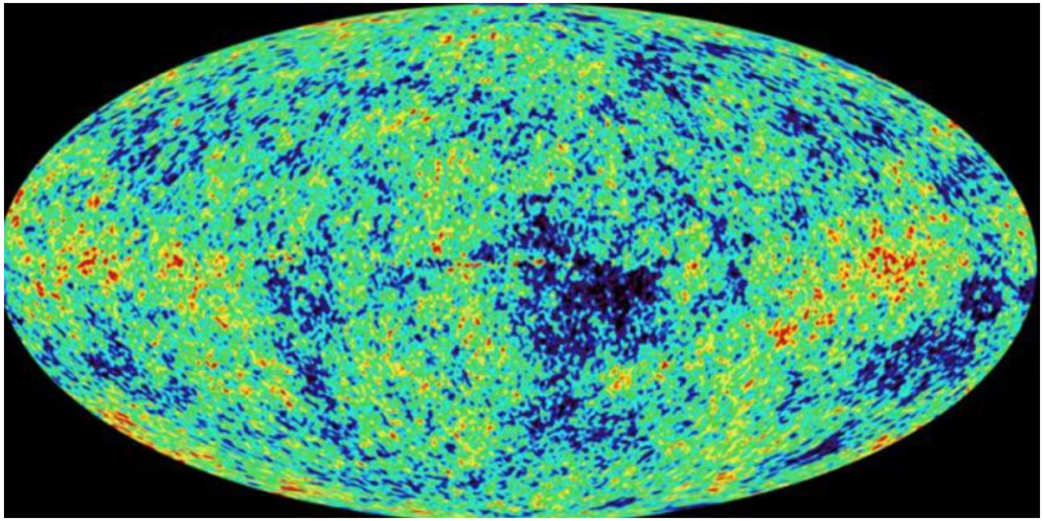
$$\left(\frac{\mathcal{R}}{S_0} - f(Q_i) \right)^2 = 0$$

It can be shown that there is a consistency condition

$$K_{0,\bar{i}}\, K_0^{\bar{i}j}\, K_{0,j} < 3$$

3) Simplest models of inflation in supergravity

Why Supergravity for early cosmology ?



- Inflation with super-Planckian field variations needs an UV completion → String Theory
- Supersymmetry crucial ingredient in String Theory, supergravity its low-energy effective action

- Chaotic inflation in supergravity

- Large-field chaotic inflation (Linde,1983) is an attractive scenario for explaining the initial conditions of the early universe.

Simplest realization: free massive scalar field

$$V = \frac{1}{2} m^2 \varphi^2$$

Chaotic inflation needs transplanckian values and small inflaton mass

$$\varphi \sim 10 - 15 \ M_P \qquad\qquad m \sim 10^{-5} \ M_P$$

Best explanation of smallness of inflaton mass (the η problem) is an approximate shift symmetry $\varphi \to \varphi + \alpha$

- Naively, the simplest example would be

$$W = \frac{m}{2}\phi^2 \quad , \quad K = \frac{1}{2}(\phi + \bar{\phi})^2$$

where the inflaton is $\varphi = \sqrt{2}\ \mathrm{Im}\ \phi$. This doesn't work, since for large φ the potential is unbounded from below

$$V(\varphi) \sim -3m^2\varphi^4$$

The problem can be avoided by introducing a « stabilizer » field S, with no shift symmetry (Kawasaki,Yamaguchi,Yanagida,2000)

$$W = mS\phi \quad , \quad K = \frac{1}{2}(\phi + \bar{\phi})^2 + |S|^2 - \xi|S|^4$$

The term in ξ is needed in order to give a large mass to S during inflation.

Obs:

- The mass m breaks softly the shift symmetry.

- As usual, chaotic inflation predicts

$$n_s \simeq 0.967\,, \qquad r \simeq 0.13$$

the value of the inflaton and the Hubble scale during inflation are

$$\varphi_\star \simeq 15 \qquad H \sim m\varphi_\star \sim 10^{14}\,\mathrm{GeV}$$

- The model was generalized to (Kallosh,Linde,Rube,2011)

$$W = S f(\phi)$$

- Starobinsky model

- First model of inflation, based on a higher-deriv gravity action (1980)

$$\mathcal{S}_2 = \int d^4x \sqrt{-g} \left[\frac{1}{2} R + \alpha R^2 \right]$$

It can be rewritten with a help of a lagrange multiplier scalar

$$\mathcal{S}_1 = \int d^4x \sqrt{-g} \left[\frac{1}{2}(1 + 2\chi) R - \frac{1}{4\alpha} \chi^2 \right]$$

Going in the Einstein frame and defining $1 + 2\chi = e^{\sqrt{\frac{2}{3}}\phi}$

one finds a standard dual two-derivative action

$$S = \int d^4x \sqrt{-g_E} \left[\frac{1}{2} R_E - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

with $$V(\phi) = \frac{1}{16\alpha} (1 - e^{-\sqrt{\frac{2}{3}}\phi})^2$$

The embedding of the model in SUGRA is (Cecotti,1987) :

real function

$$\mathcal{L} = \left[- S_0 \, \overline{S}_0 \; + \; h \left(\frac{\mathcal{R}}{S_0} , \frac{\overline{\mathcal{R}}}{\overline{S}_0} \right) S_0 \, \overline{S}_0 \right]_D + \left[W \left(\frac{\mathcal{R}}{S_0} \right) S_0^3 \right]_F \; ,$$

where :

- D and F subscripts are superspace densities
- $\mathcal{R}$ is the chiral scalar curvature superfield $\mathcal{R} = \frac{\Sigma(\bar{S}_0)}{S_0}$, defined with the curved chiral projector Σ , of components

$$\mathcal{R} = \left(\overline{u} \equiv S + iP \, , \; \gamma^{mn} \mathcal{D}_m \psi_n \, , \; - \frac{1}{2} \, R - \frac{1}{3} \, A_m^2 + i \, D^m A_m - \frac{1}{3} \, u \, \overline{u} \right) \, ,$$

and :

- u and A_m are « old minimal » auxiliary fields of N=1 SUGRA
- ψ_n is the gravitino

This action can be recast into a two-derivative dual form, using two chiral multiplets (Cecotti;Ferrara,Kallosh,Van Proeyen) :

$$\mathcal{L} = \left[- \, S_0 \overline{S}_0 + h(C,\overline{C}) \, S_0 \overline{S}_0 \right]_D + \left[\left\{ \Lambda \left(C - \frac{\mathcal{R}}{S_0} \right) + W(C) \right\} S_0^3 \right]_F$$

where Λ implements $\mathcal{R} = S_0 C$.

- It can be shown that $W(C)$ can be shifted away by a field redefinition.

- Defining $\Lambda' = T - \frac{1}{2}$, one finally finds a standard N=1 SUGRA with Kahler potential and superpotential

$$K = -3 \ln \left[T + \overline{T} - h(C,\overline{C}) \right] \, , \qquad W = C \left(T - \frac{1}{2} \right) + W_0$$

The inflaton is $Re(T) = \exp \left(\sqrt{2/3} \, \phi \right)$

Setting to zero the other three fields one recovers, for $W_0 = 0$ exactly the Starobinsky model.
However, for the minimal choice

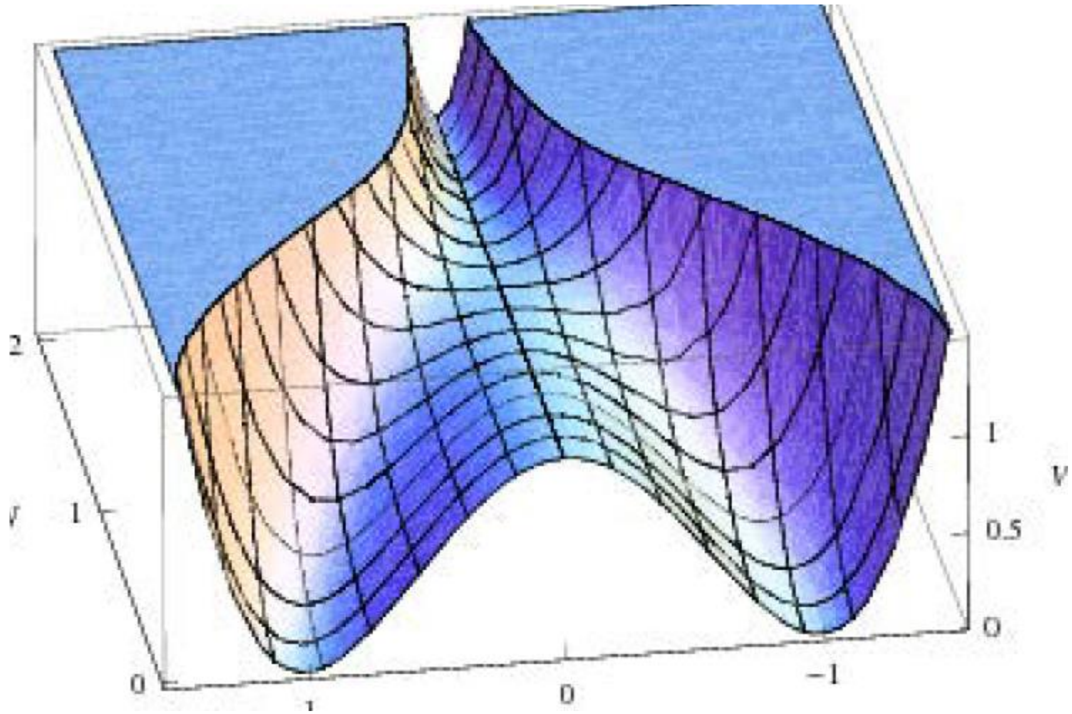
$$h(C,\overline{C}) = C\overline{C}$$

the field direction C is unstable during inflation (Kallosh-Linde).
Non-minimal Kahler potential is needed, for ex:

$$h(C,\overline{C}) = C\overline{C} - \zeta\,(C\,\overline{C})^2$$

Large list of recent papers on both Starobinsky and chaotic inflation:
SUGRA: Kallosh,Linde+coll.; Ferrara,Porrati + coll., Ellis,Nanopoulos,Olive; Yanagida et al., Ketov,Terada…
Strings: Palti,Weigand; Marchesano,Shiu,Uranga, Hebecker et al…(2014)

Cosmological data favors models with one light inflation



Simplest SUGRA models contain 2 complex scalars for inflation and 3 complex scalars if SUSY breaking. Desirable to have simpler models.

4) Inflation with nilpotent fields
- minimal Starobinsky model (ADFS)

- Consider the chiral formulation of Starobinsky

$$K = -3 \ln \left[T + \overline{T} - X \overline{X} \right] , \qquad W = M X T + f X + W_0$$

and impose $X^2 = 0$. There is only one scalar and one axion bosonic fields $T = e^{\phi \sqrt{\frac{2}{3}}} + i a \sqrt{\frac{2}{3}}$

$$\mathcal{L} = \frac{R}{2} - \frac{3}{(T + \overline{T})^2} |\partial T|^2 - \frac{|M T + f|^2}{3(T + \overline{T})^2}$$

$$= \frac{R}{2} - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} e^{-2\phi \sqrt{\frac{2}{3}}} (\partial a)^2 - \frac{M^2}{12} \left(1 - e^{-\sqrt{\frac{2}{3}} \phi} \right)^2 - \frac{M^2}{18} e^{-2\phi \sqrt{\frac{2}{3}}} a^2$$

This is exactly Starobinsky + a (much) heavier axion.

- Dual gravitational formulation

- The previous lagrangian is

$$\mathcal{L} = -\left[(T+\overline{T}-|X|^2)S_0\,\overline{S}_0\right]_D + \left[(MXT + fX + W_0)S_0^3 + \text{h.c}\right]_F$$

and can be rewritten

$$\mathcal{L} = \left[|X|^2\,S_0\,\overline{S}_0\right]_D + \left[\left(T\left(-\frac{\mathcal{R}}{S_0} + M\,X\right) + f\,X + W_0\right)S_0^3 + \text{h.c}\right]_F$$

using the identity $\left[(T+\overline{T})S_0\,\overline{S}_0\right]_D = \left[T\,\mathcal{R}\,S_0^2\right]_F + \text{h.c.}$

T enters now as a Lagrange multiplier, which imposes the constraint

$$X = \frac{1}{M}\frac{\mathcal{R}}{S_0} \Longrightarrow \mathcal{R}^2 = 0$$

Nonlinear Supergravity

There is however a problem : in the ground state
$F_X \to 0$ and the formalism breaks down.

There are therefore consistency conditions to be satisfied (Dall'Agata-Zwirner,2014) :

- $F_X \neq 0$ during inflation and afterwards
- unitarity. In flat background, unitarity valid for

$$E^2 < m_{3/2} M_P$$

During inflation, this should be replaced by

$$E_q^2 < m_{3/2}(\varphi) M_P$$

where $E_q \sim V_{\text{inf}}^{1/2}(\varphi)/M_P$ is the typical energy scale of quantum fluctuations during inflation.

Several inflationary models constructed afterwards

(Buchmuller, E.D., Heurtier, Wieck; (Ferrara) Kallosh,Linde, Dall'Agata-Zwirner)

Simple models constructed based on (Dall'Agata-Zwirner)

$$K = \frac{1}{2}(\Phi + \bar{\Phi})^2 + |X|^2 \quad , \quad W = f(\Phi)(1 + \sqrt{3}X)$$

with $X^2 = 0$, $f(0) \neq 0, f'(0) = 0$ and $\overline{f(x)} = f(-\bar{x})$

The inflationary potential is $V = \left| f'\left(i\,\frac{\varphi}{\sqrt{2}}\right)\right|^2$

Inflationary energy is decoupled from the gravitino mass: possible to have GUT vacuum energy with low (TeV) gravitino mass

Explicit examples :

$$f(\Phi) = f_0 - \frac{m}{2}\Phi^2$$ (chaotic inflation)

$$f(\Phi) = f_0 - i\sqrt{V_0}(\Phi + i\frac{\sqrt{3}}{2}e^{\frac{2i\Phi}{3}})$$ (Starobinsky)

Perturbative unitarity OK if

$$|f'(i\frac{\varphi}{\sqrt{2}})| < |f(i\frac{\varphi}{\sqrt{2}})|$$

This is satisfied in both examples and is essentially insensitive on the scale of SUSY breaking in the vacuum.

Coupling to matter can lead to positive definite potentials and inflation (DFKS)

$$K_0 \;=\; -\,3\;\ln\left(1\;-\;\frac{1}{2}\,(\Phi+\bar{\Phi})^2\;-\;|Q|^2\right)$$

$$W \;=\; W_0(\Phi,Q_i)\;+\;f(\Phi,Q_i)\,X$$

where $$W_0 \;=\; \alpha(\Phi)\;+\;\frac{1}{2}\,b_{ij}\,Q_i\,Q_j\;+\;\frac{1}{6}\,\lambda_{ijk}\,Q_i\,Q_j\,Q_k$$

$$f \;=\; 6\,\alpha(\Phi)\;+\;b_{ij}\,Q_i\,Q_j\;,$$

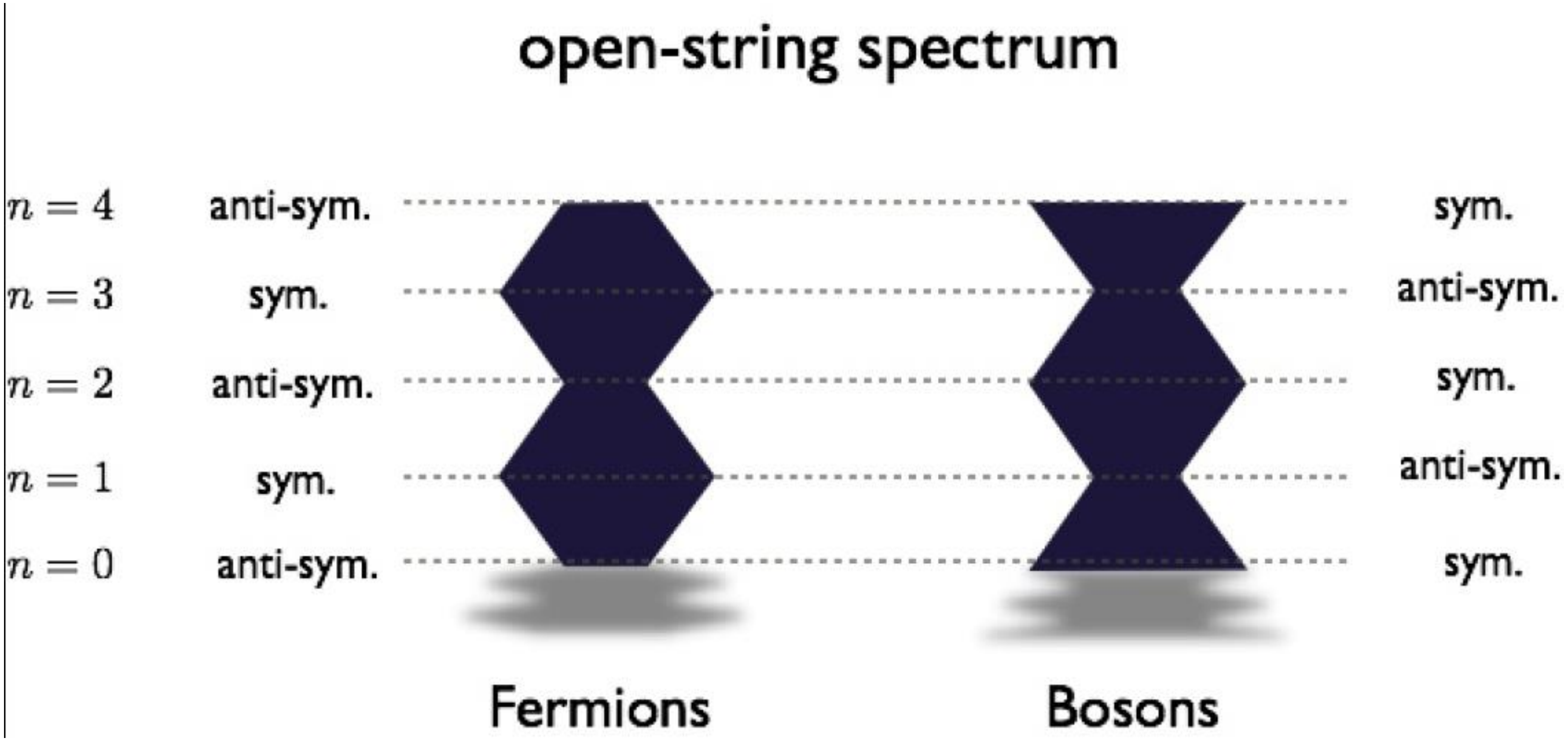
The scalar potential is a generalization of Dall'Agata-Zwirner

$$V \;=\; \frac{1}{3Y^2}\left\{\sum_i\left|\frac{\partial W_0}{\partial Q_i}\right|^2+\left|\frac{\partial W_0}{\partial \Phi}\right|^2+\frac{1}{2}\,(\Phi+\bar{\Phi})^2\,|f|^2+(\Phi+\bar{\Phi})\left(\bar{f}\,\frac{\partial W_0}{\partial \Phi}\;+\;\mathrm{h.c}\right)\right\}$$

Recent ex. of chaotic inflation with $V=\frac{1}{2}m^2\varphi^2$; $m_{3/2}\sim m^2\sim 10^5 TeV$ and $m_{\rm soft}\sim m^3\sim TeV$ MSSM soft masses (Hasegawa, Yamada)

5) Moduli stabilization with nilpotent uplift

- The string theory realization of the nilpotent superfield is by the $\overline{D}3/O3$ system (Kallosh, Quevedo,Uranga, 2015). Explicit similar string vacua known (Brane SUSY breaking: (Sugimoto;Antoniadis,E.D.,Sagnotti, 1999; $\overline{D}p/Op_+$)



-SUSY is non-linearly realized on the antibranes (E.D.,Mourad, 2000).
-For one $\overline{D}3$ on top of an $O3_-$ the only degree of freedom is the goldstino.
-The $\overline{D}3$ action can be written with the goldstino nilpotent superfield (Bandos,Martucci,Sorokin,Tonin,2015.)

$\overline{D}3/O3$ also used in the KKLT moduli stabilization. Its effective SUGRA description is (Kallosh-Linde,2014)

$$W = W_0 + Ae^{-a\rho} - \mu^2 S \,, \qquad K = -3\ln(\rho + \overline{\rho}) + S\bar{S} \qquad \text{at} \qquad S^2 = 0$$

- The scalar potential is

$$V_{New\,O'KKLT} = V_{KKLT}(\rho,\bar{\rho}) + \frac{\mu^4}{(\rho+\bar{\rho})^3}$$

- The uplift term is actually as the $\overline{D}3$ tension in flat space.

- More detailed effective action analysis reveals the emergence of the constraint

$$\left(\frac{\mathcal{R}}{S_0} - \lambda\right)^2 = 0$$ (Bandos,Martucci,Sorokin,Tonin,2015.)

Conclusions and perspectives

- The standard models of inflation in SUGRA contain (at least) three more scalars in addition to the inflaton (more if we include SUSY breaking).
- Using constrained superfields we can now construct simpler models (only the inflaton !) ; SUSY is automatically broken.
- There are consistency conditions to be satisfied. Are they really sufficient ?
- Interesting to analyze further the gravity dual of Volkov-Akulov nonlinear SUSY $\Longrightarrow$ nonlinear supergravity. Constraints on couplings to matter. Component action ?

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- Right framework to include moduli stabilization, reheating.
- Maybe time to put together inflation, moduli stabilization and matter sector into this framework. Seems possible to obtain realistic scales with few parameters

Thank you

Backup slides