The Dynamical Strong-Field Regime of General Relativity

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Outline

- General Relativity @100
  - the dynamical, strong-field regime
- In the wake of GW150914
  - why this was such a remarkable event
  - consequences for fundamental physics
  - Briefly mention consequences for astrophysics
- Conclusions

General Relativity

- Einstein's General Relativity, published in 1915, is a theory about the nature of *space and time*, claiming it can be described as a dynamical, 4-dimensional geometry
  - curvature in the geometry is responsible for what we think of as the force of gravity
  - matter/energy is responsible for producing curvature

General Relativity

- Beyond the numerous “corrections” to Newtonian gravity, general relativity predicts three profoundly different class of solutions
  - Gravitational waves
  - Black holes
  - Solutions describing the large scale structure and evolution of the cosmos

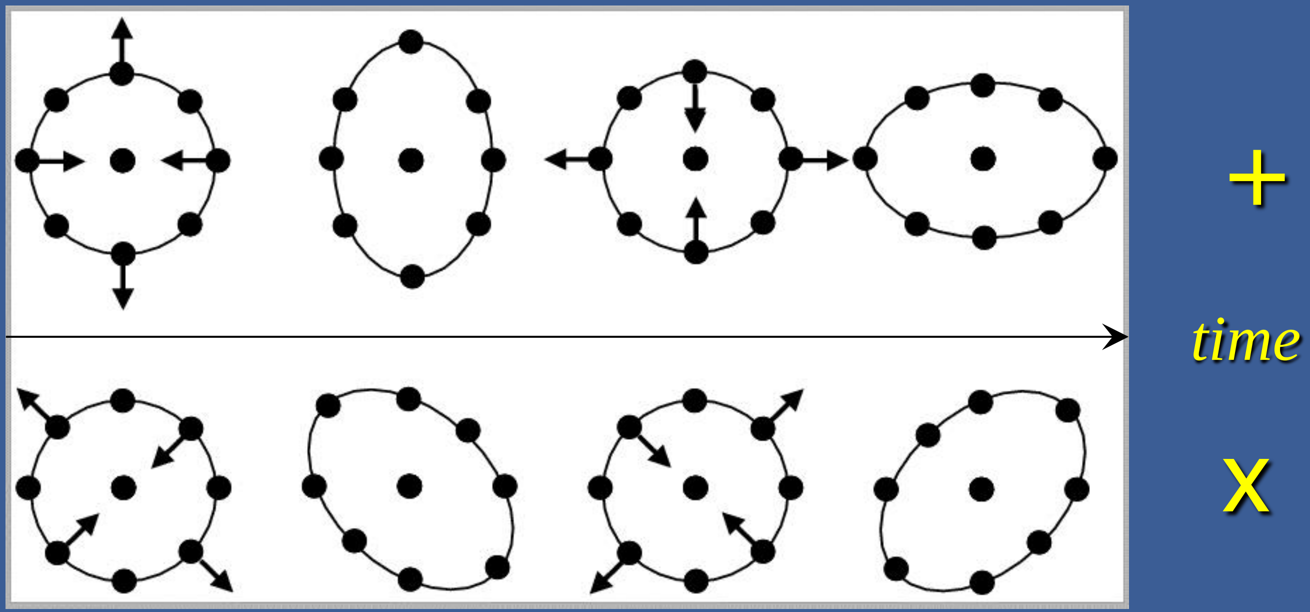
Gravitational Waves

- Gravitational waves are localized “disturbances” in the spacetime geometry that propagate at the speed of light
- Asymmetric, bulk accelerations of dense concentrations of matter/energy produce gravitational waves that may be strong enough to detect
  - when observed gravitational waves are a weak-field phenomenon, but the most promising sources for the advanced LIGO generation of detectors emit in strong field

Weak field nature of gravitational waves

- Far from the source, the effect of a gravitational wave is to cause distortions in distance transverse to the direction of propagation

- Two linearly independent polarizations (+ and x)
  - schematic effect of a wave, traveling into the slide, on the distances between an initially circular ring of particles:



  - this basic property of gravitational waves underlies all direct detection efforts

The dynamical strong-field regime of GR

- No characteristic scales in the field equations
- Define the dynamical strong field regime as that governed by solutions to the field equations
  - that exhibit highly non-linear spacetime kinematics/dynamics
  - where the radiative degrees of freedom are strongly excited

The strong-field regime of GR

- Non-linear regime: introduce a length scale R containing a total mass M, expect strong non-linearity when

$$\boxed{\frac{GM}{Rc^2} \approx 1}$$

The dynamical strong-field regime of GR

- In the radiative regime, to leading order the power emitted in gravitational waves is

$$\boxed{P \propto \frac{G}{c^5}\left(\dddot{Q}\right)^2}$$

for a source with quadrupole moment Q:

$$\boxed{Q \propto MR^2;\ \dddot{Q} \propto \frac{MR^2}{T^3}}$$

with T a characteristic time scale on which the source varies.

- If $T=R/c$, the light crossing time of the system, and it is in the non-linear regime where $GM/Rc^2 \sim 1$, then the characteristic power approaches the Planck luminosity:

$$\boxed{P \propto \frac{c^5}{G}}$$

Black holes

- Black holes are solutions to the Einstein Field equations describing regions of spacetime that are undergoing *gravitational collapse*
  - the exterior of a black hole is causally disconnected from the exterior; once you cross the event horizon, you can never escape
  - once inside, the spacetime is intrinsically dynamical, and time flows toward a singularity in the geometry of spacetime
  - this singularity is generically crushing – infinite tidal forces are exerted on objects approaching it

Black holes

- Black hole spacetimes exhibit many remarkable properties
  - In 4D, the geometry exterior to the event horizon of a stationary black hole is uniquely characterized by the 2 parameter (mass M and angular momentum J) Kerr family of solutions : *the uniqueness, or so-called "no hair" theorems*
  - Dynamics of quasi-stationary evolution given by laws akin to the laws of thermodynamics (in particular black hole area is the analogue of entropy)
  - When quantum effects are considered, black holes truly become thermodynamic objects, radiating as thermal black bodies

Binary Black holes

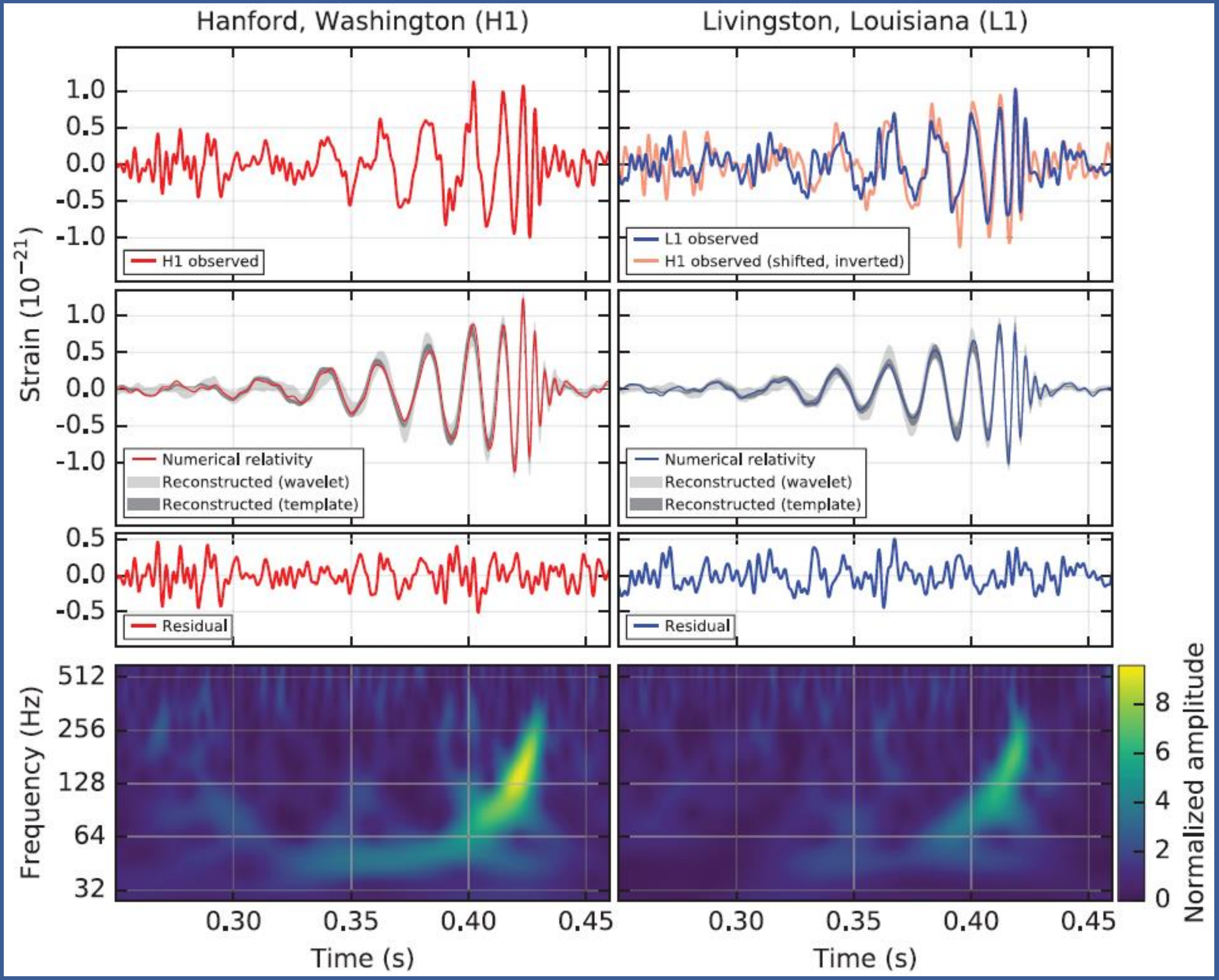
- In general relativity a binary orbit is unstable, decaying due to gravitational wave emission
  - Black holes are as dense as compact objects can be, and close to merger their orbital motion approaches a sizeable fraction of the speed of light
  - A binary black hole merger is thus expected to be the strongest expected source of GWs in the post-big-bang universe

Do Black Holes Exist?

- Remarkable predictions require remarkable evidence
  - until now, the evidence has been strong, but entirely circumstantial :

    dark, massive objects exist that are consistent with being black holes, and there is no other theoretical explanation within general relativity and verified standard model physics

GW150914 : Leaping into a new era in observational astrophysics



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GW150914

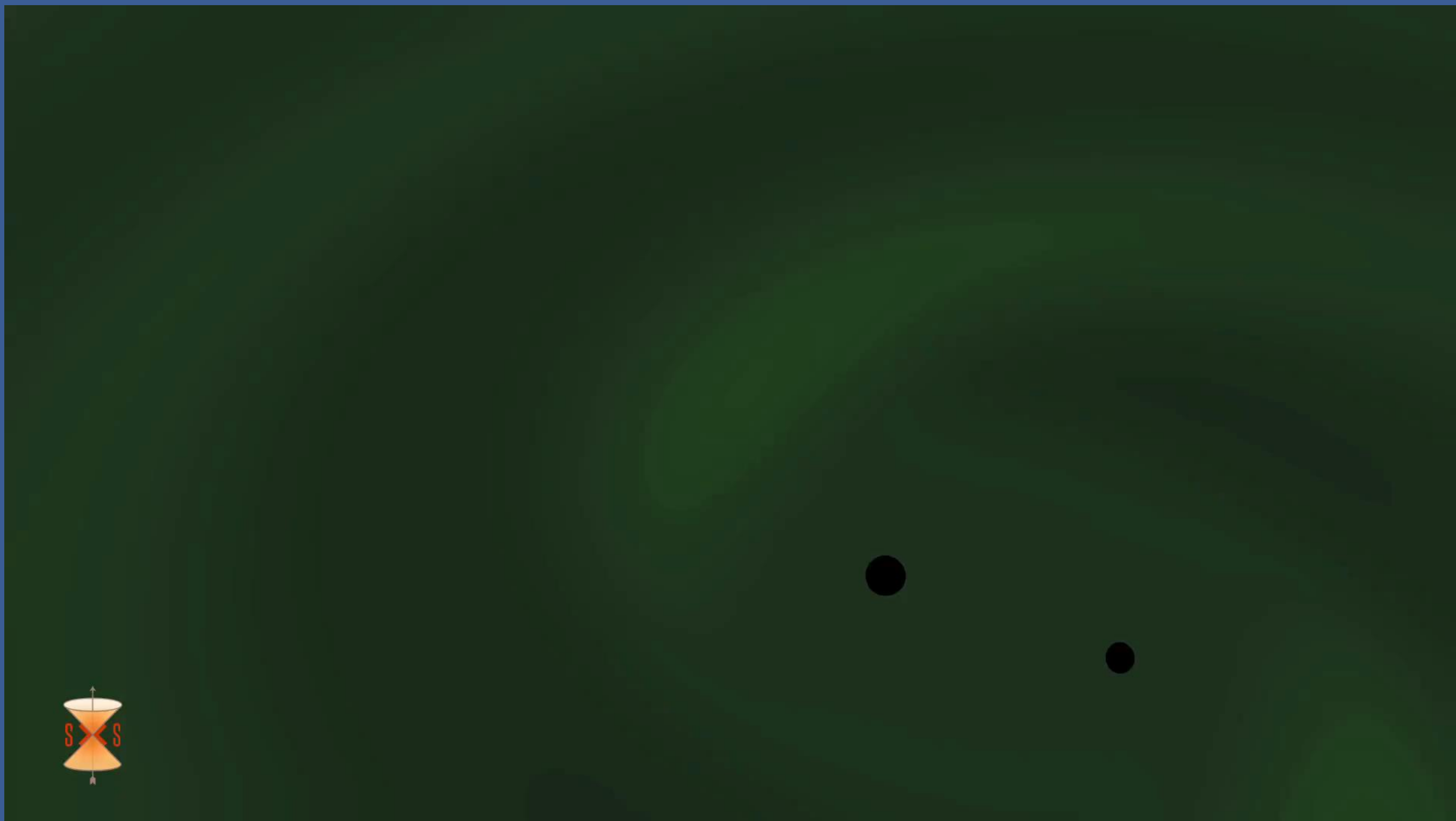
- The observed signal is consistent with the late inspiral of two black holes, the coalescence of their individual event horizons into a common, larger horizon, and the subsequent ringdown of this to a Kerr black hole remnant

Primary black hole mass	$36^{+5}_{-4} M_{\odot}$
Secondary black hole mass	$29^{+4}_{-4} M_{\odot}$
Final black hole mass	$62^{+4}_{-4} M_{\odot}$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$

- Initial spins poorly constrained; most consistent with either low spins, or anti-aligned spins

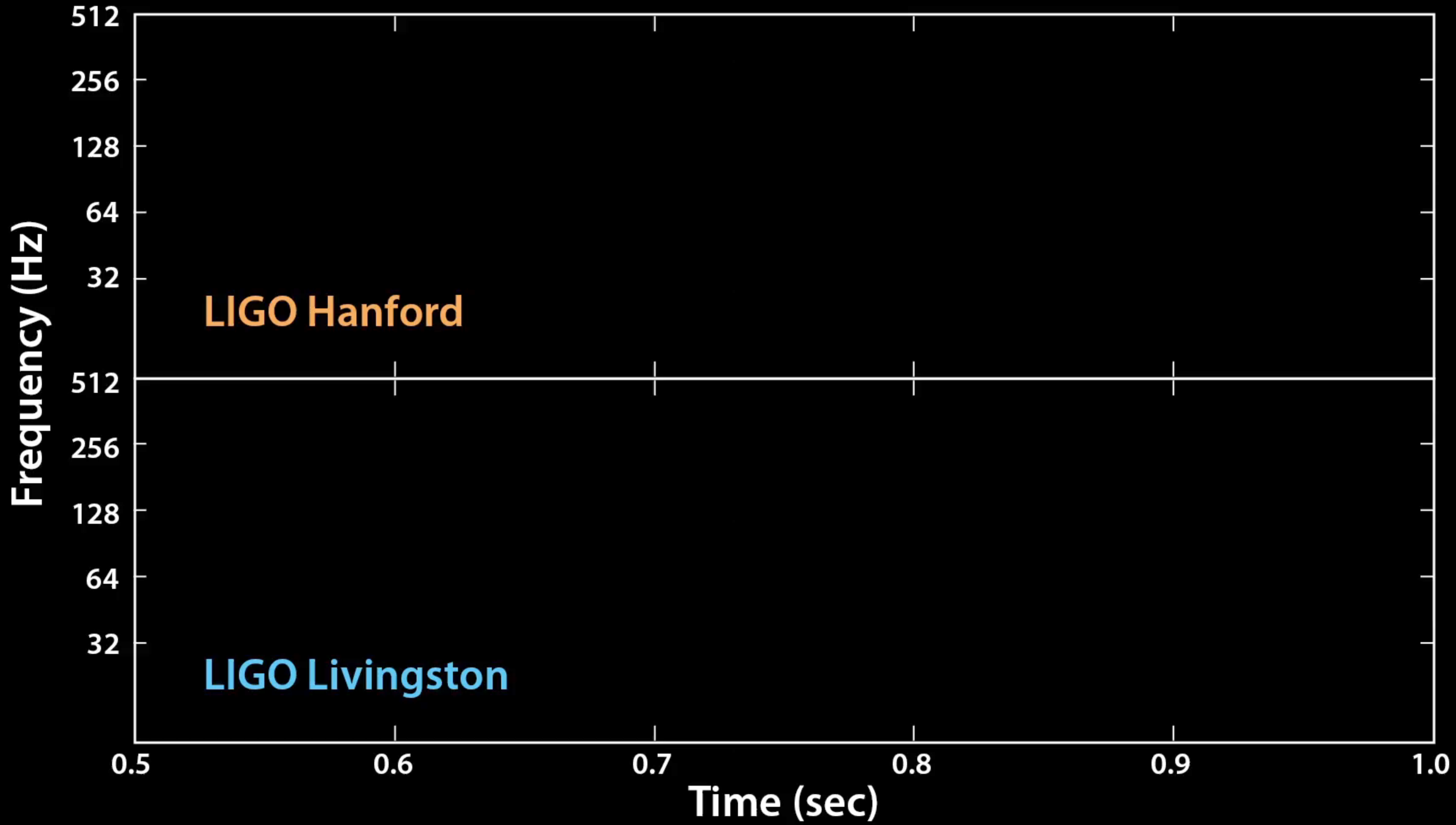
- The Signal-to-noise ratio (SNR) was 24, with an implied false-positive rate of less than 1 every 220,000 years

GW emission from a merger simulation



From the SXS collaboration (Caltech, Cornel, CITA, …)

GW150914



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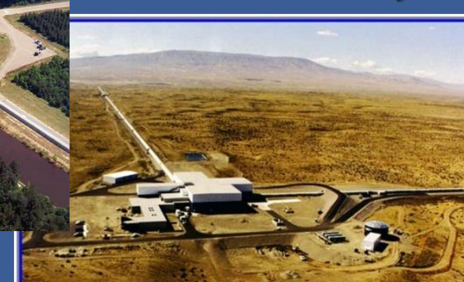
Why is this a remarkable event?

- ***Reason 0:*** *advanced LIGO's sensitivity*
  - The strain, or fractional distortion in distance induced by the GWs, has a maximum amplitude ~0.1 at the source (at a scale of the Schwarzchild radius ~ 10^5m)
  - It's ~10^{25}m away, so by the time it reached Earth, the 1/R decay reduced the peak strain amplitude to ~10^{-21}
  - aLIGO has 4km long arms : maximum total stretching/squeezing of these arms is ~ 4 x 10^{-18} m ~ 0.004 fm < 1/100th radius of a proton!

LIGO Livingston

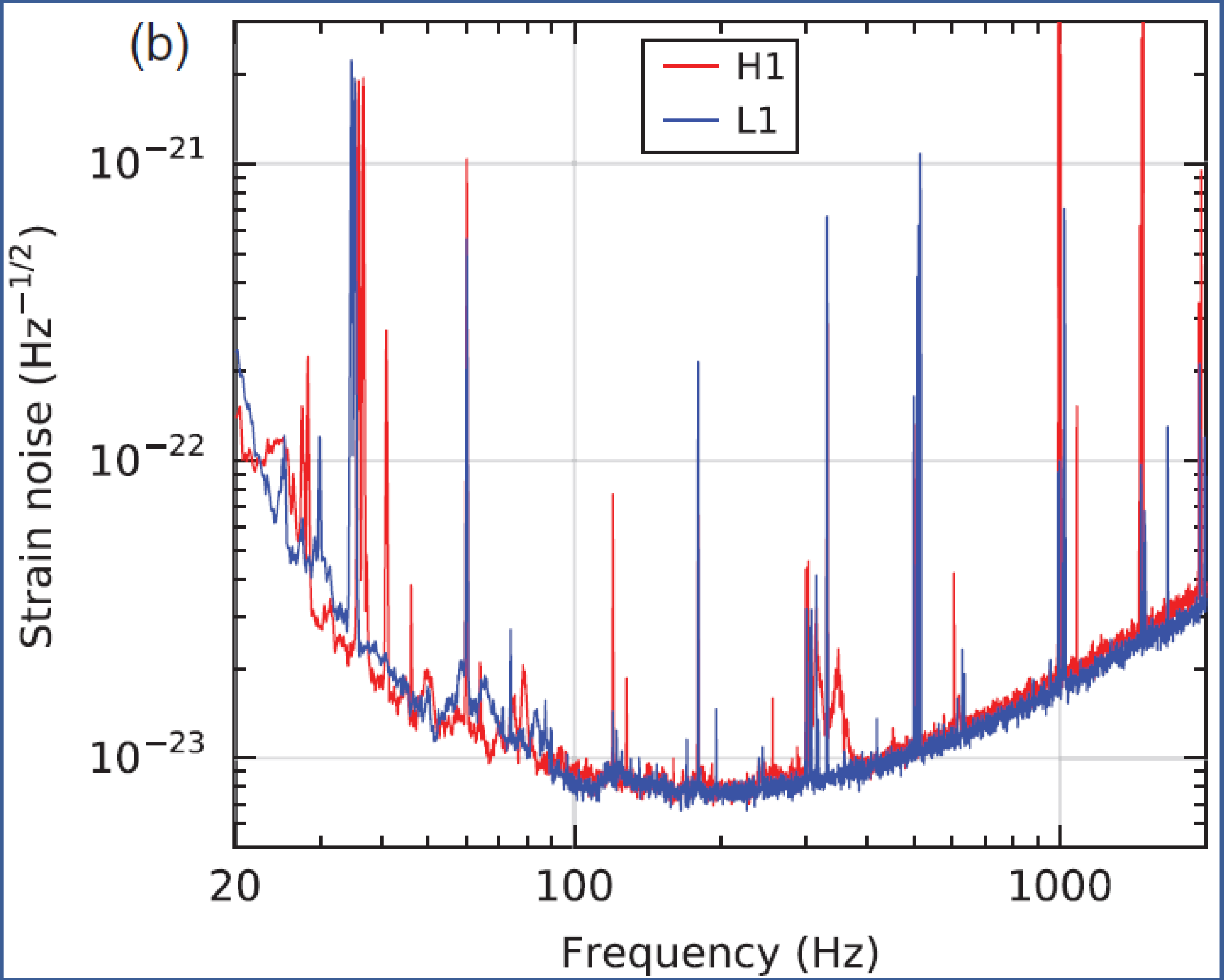


LIGO Hanford

Why is this a remarkable event?

- ***Reason 1** : the first detection is a loud, heavy binary black hole system*
  - Though binary black holes have always been one of the standard expected sources, “off the record” it was often said LIGO was built to be a binary neutron star merger detection machine
    - Event rates for all sources uncertain, but at least there are observed binary neutron stars in our galaxy that will merge in less than a Hubble time
    - Pre-2005 there was a worry that binary black hole templates would never be ready for optimal detection, which if coupled with low event rates could mean they would go undetected
  - Loud enough to see (hear) signal without a template
    - Did not expect this would happen until a space-based detector such as LISA
    - Compared to a similar SNR binary neutron star or (expected) lower mass binary black hole system, this is thanks to the high total mass of the event

LIGO Detector Noise Curves



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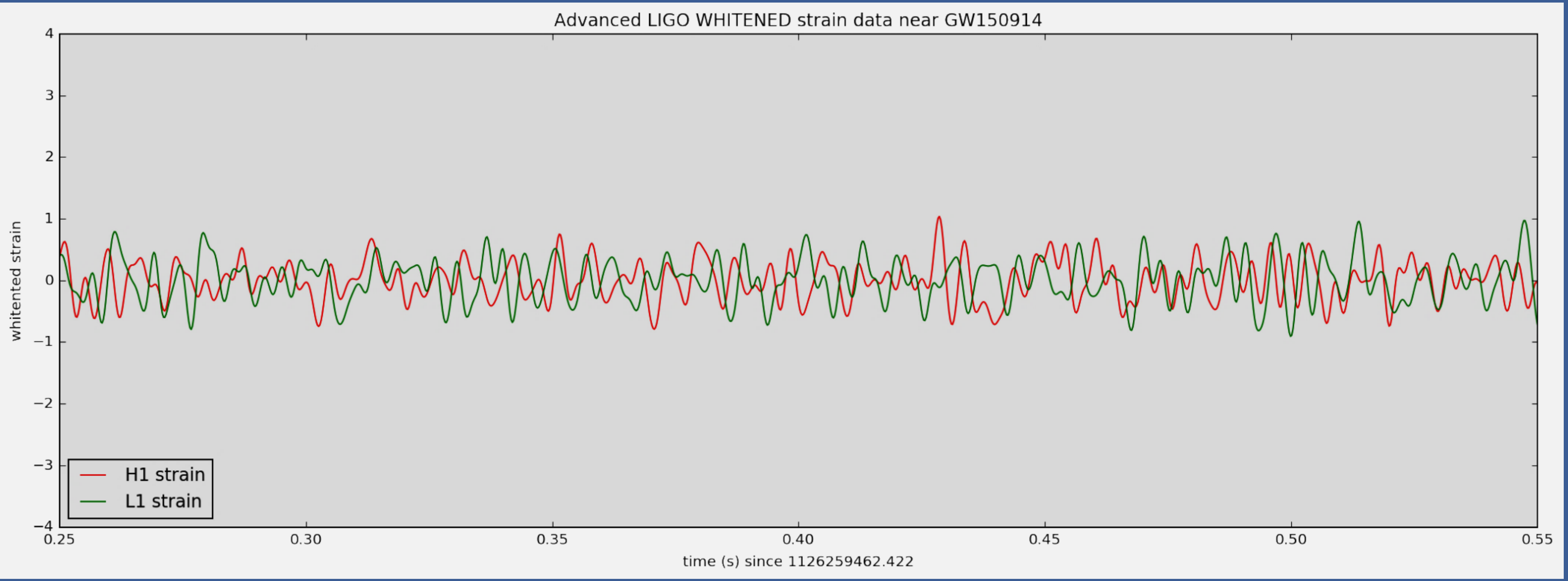
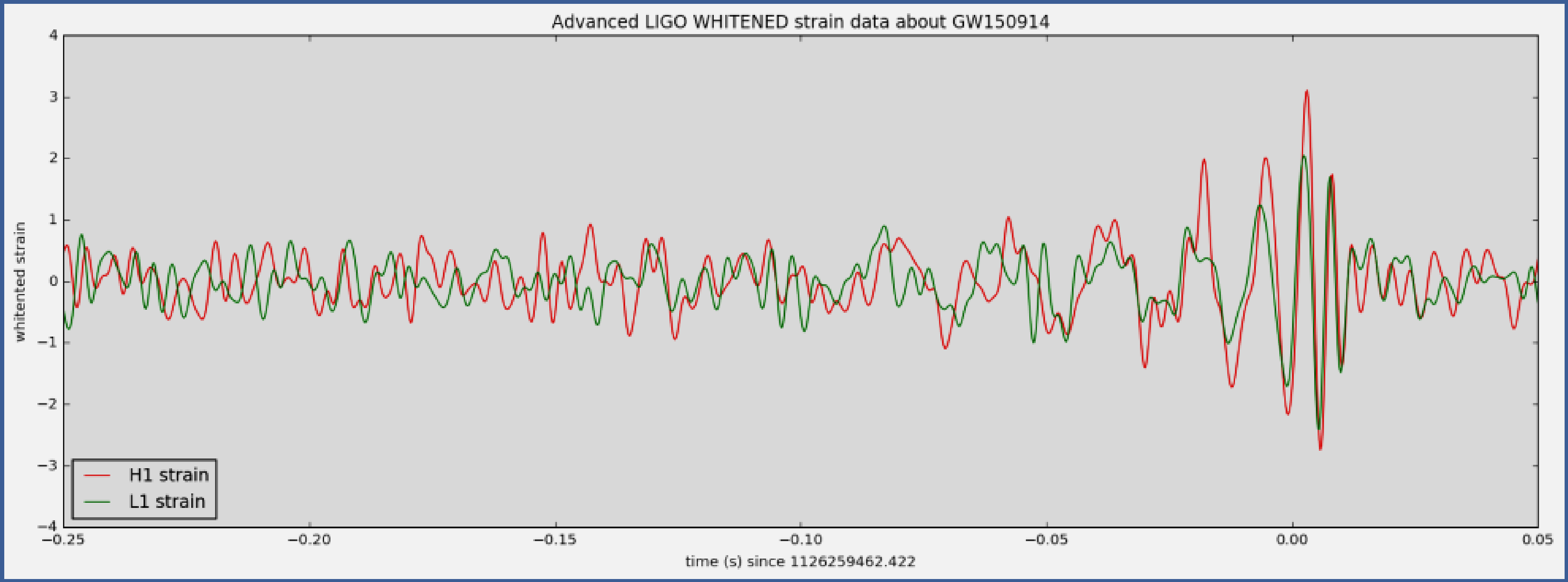
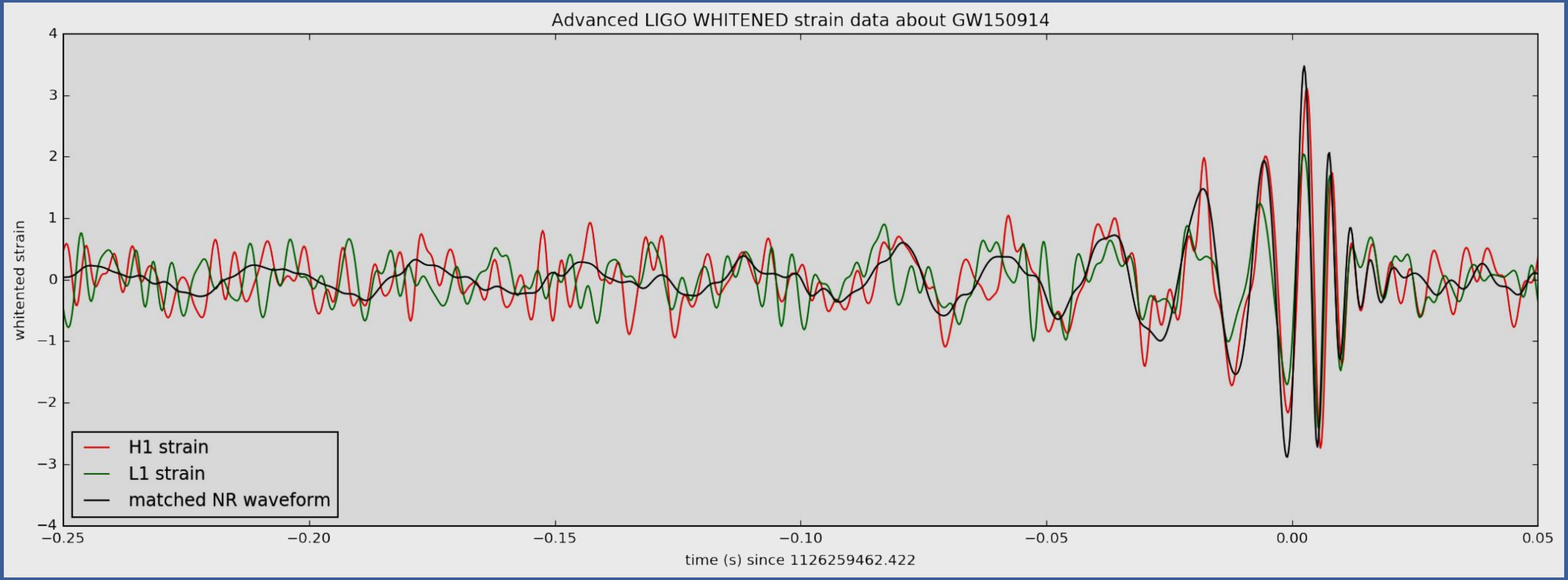
Whitened Signal, ~0.4s after GW150914



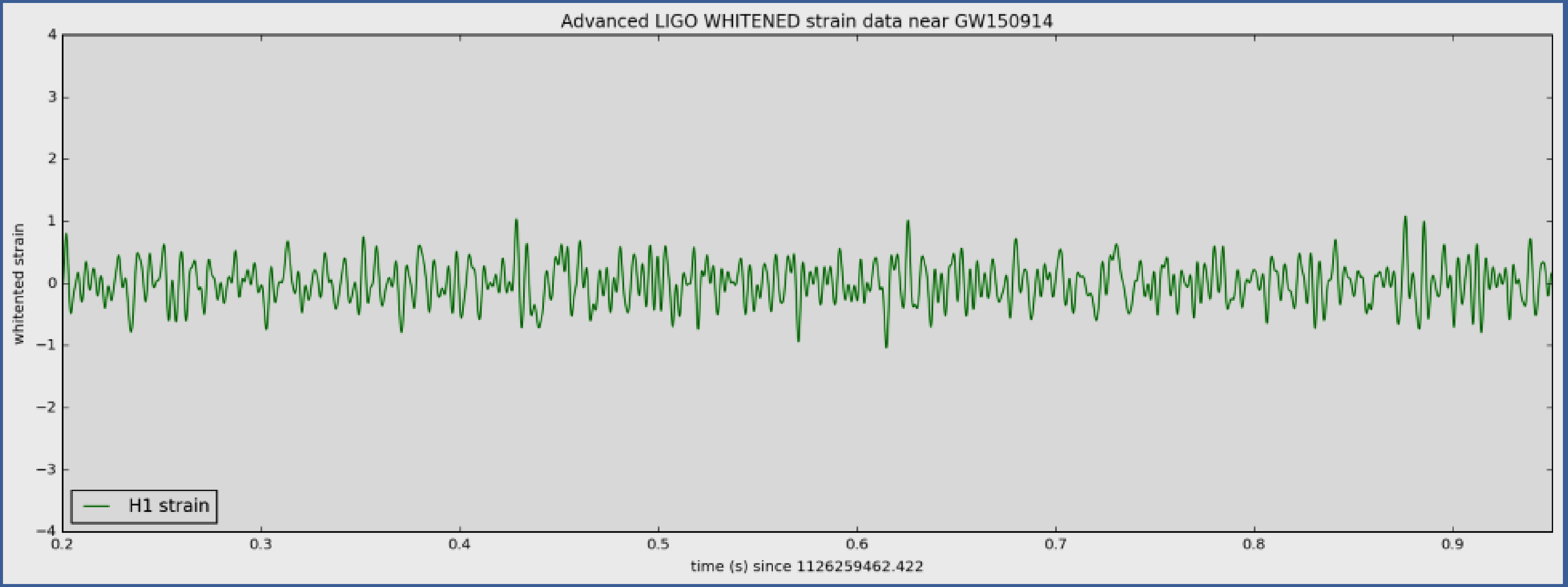
Whitened Signal, around GW150914



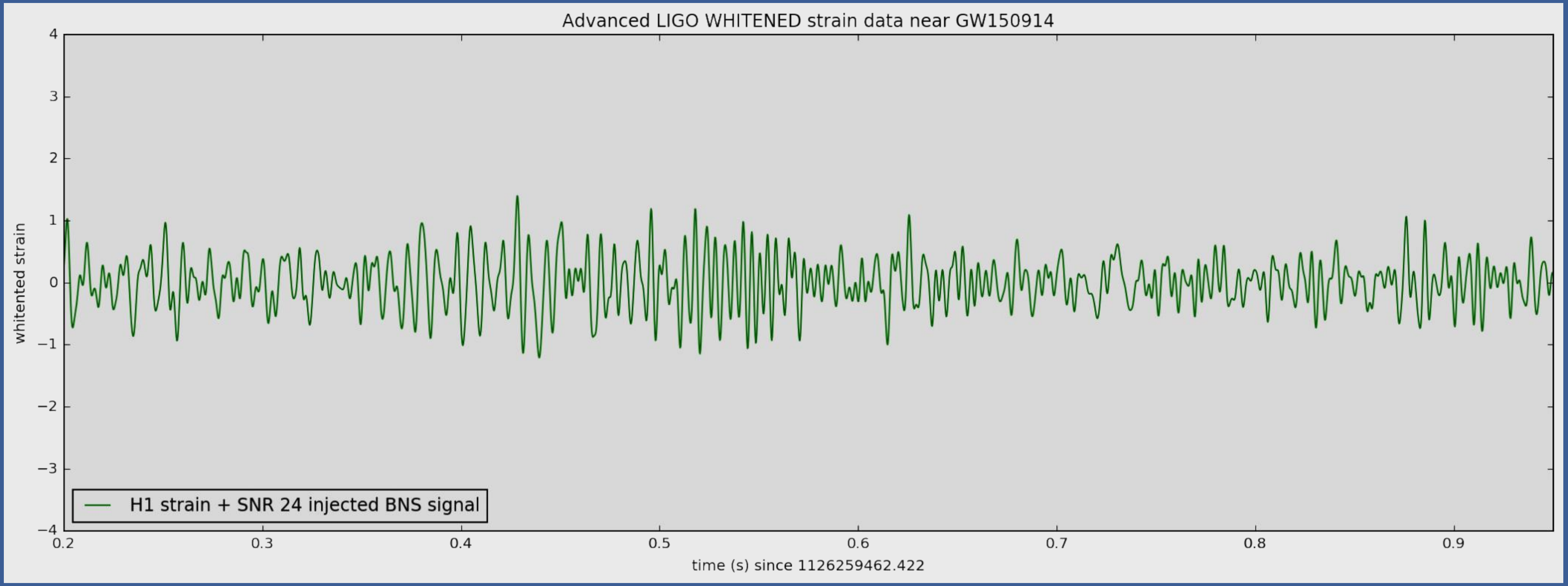
Whitened Signal plus Best-Fit Template



H1 Whitened Signal, ~0.6s after GW150914



H1 Whitened Signal plus Injected SNR 24 Binary Neutron Star Merger

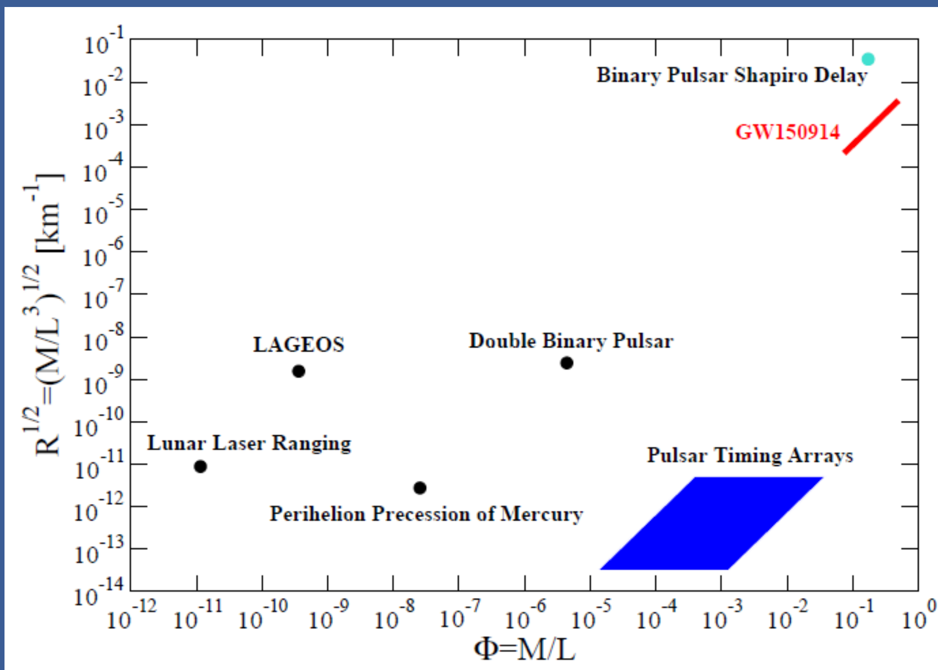


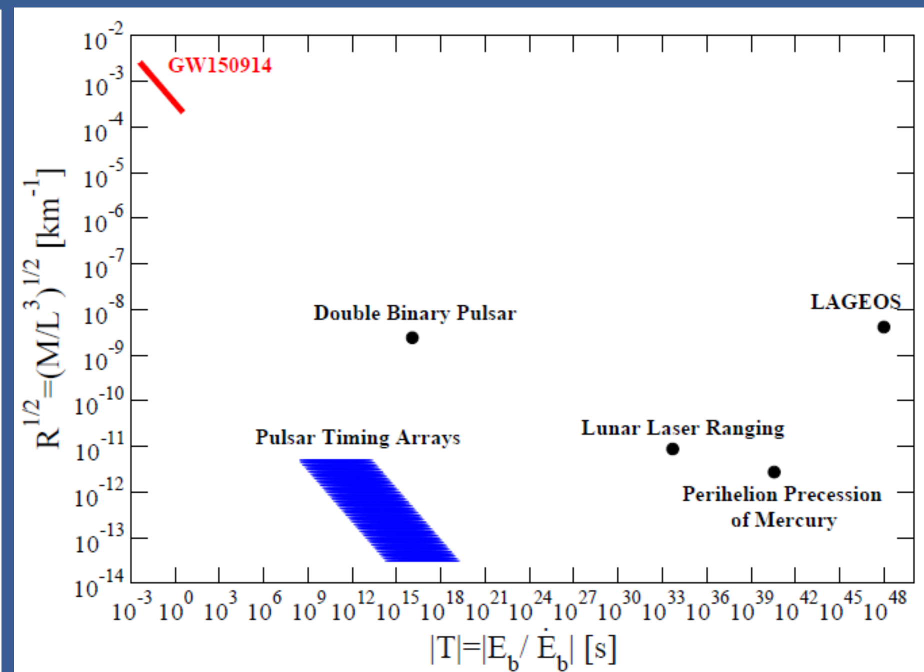
Why is this a remarkable event?

- ***Reason 2:*** *Immediate consequences for dynamical, strong-field gravity*
  - BH’s exist, and can start placing quantitative (albeit weak) constraints on GR in this regime
- ***Reason 3:*** *Immediate consequences for astrophysics*
  - Stellar mass binary BH systems can form and merge in a Hubble time (most pessimistic models ruled out)
  - These black holes are surprisingly heavy if one had assumed masses of candidate stellar mass black holes in our galaxy were representative of the population that would end up in binary black hole systems
- In the remainder of the talk give a few examples related to tests of GR in the dynamical strong-field

GW150914 and the dynamical strong-field

Work with N. Yunes (MSU) and K. Yagi (Princeton)

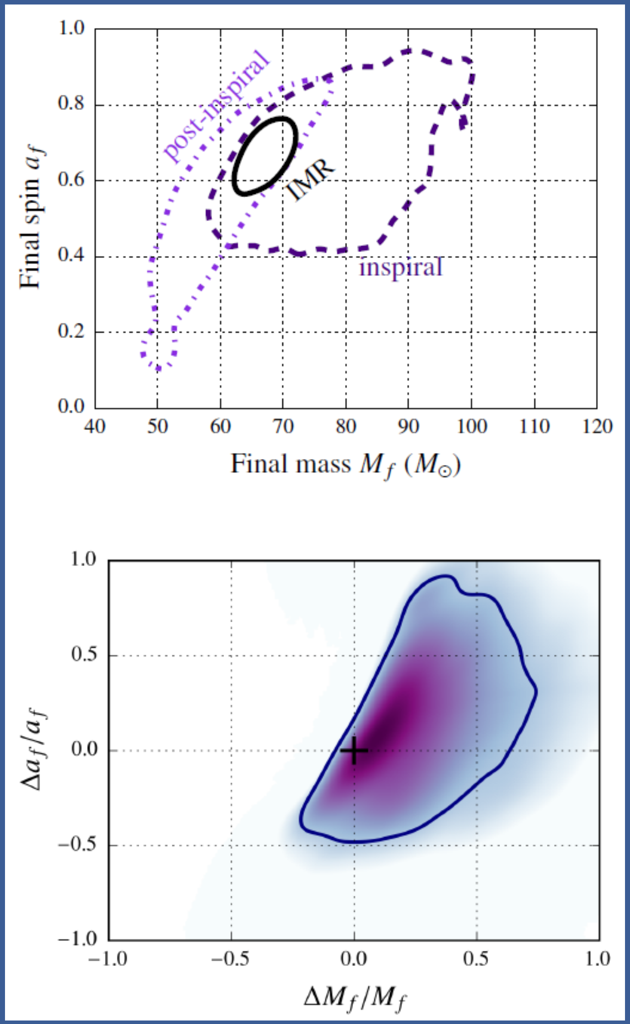




- Left : characteristic curvature scale vs effective Newtonian potential of several existing observations
- Right : characteristic curvature vs radiation-reaction time scale

Testing dynamical, strong-field GR

- Residual consistent with noise
  - Though best-fit template could have a parameter bias
- Can measure consistency within GR; for example (right) the final mass and spin estimated from
  1. numerical relativity calculations of the merger given initial binary parameters measured from the inspiral portion of the signal
  2. calculations of the quasi-normal mode spectra of Kerr black holes and mapping that to the measured least damped ringdown mode



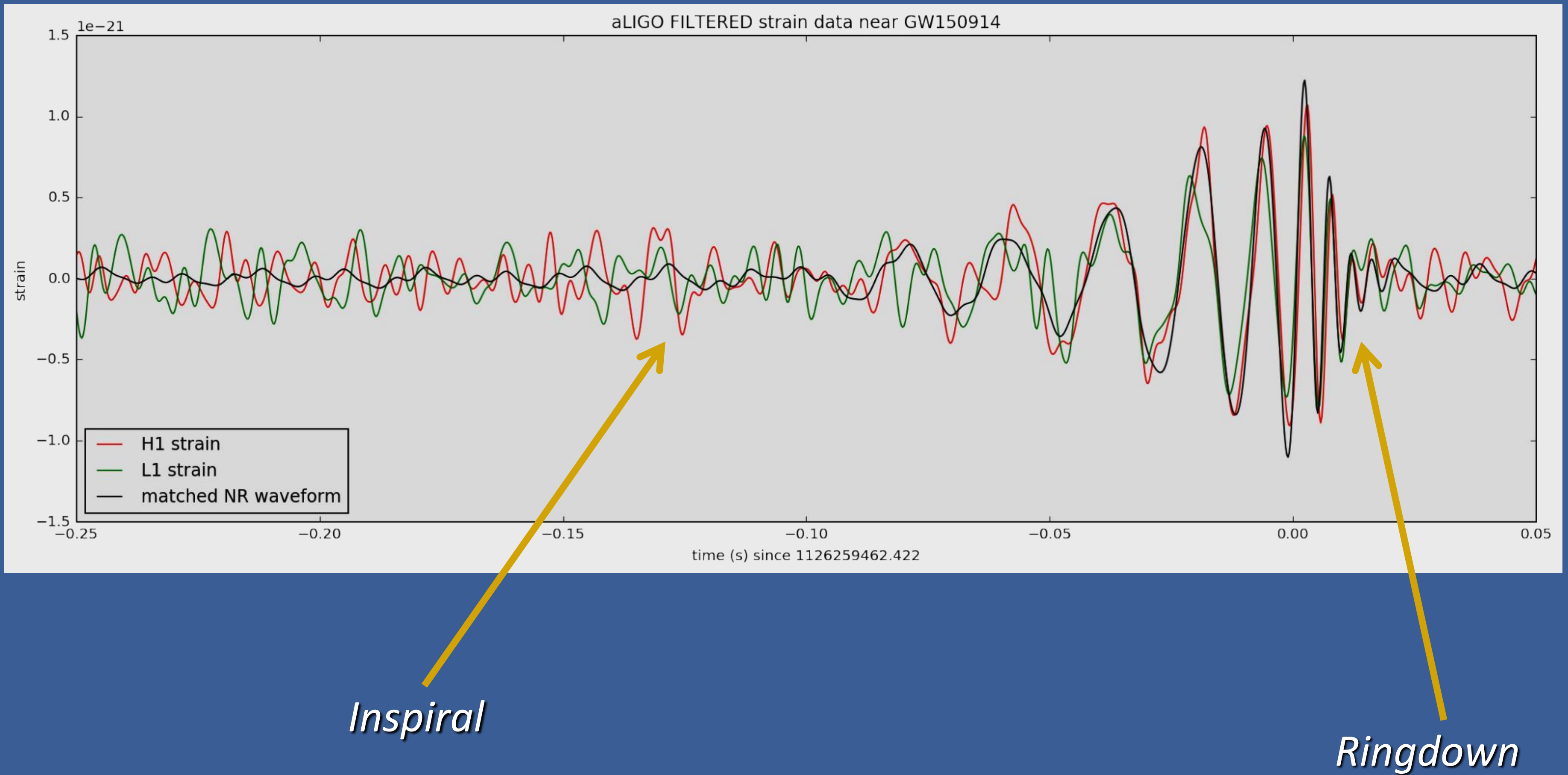
arxiv:1602.03841, *LIGO & Virgo Collaboration*

Testing dynamical, strong-field GR

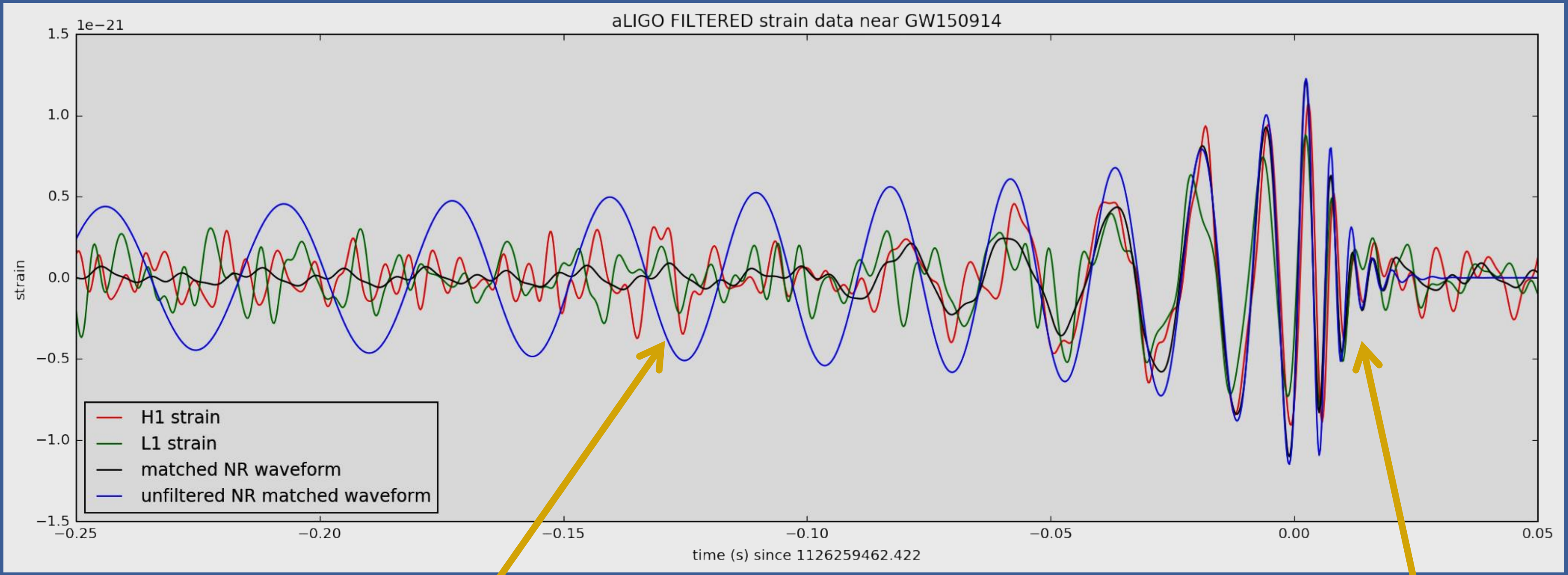
- To constrain specific alternative theories or exotic compact object alternatives, estimate parameter bias, calculate quantitative statics for how GR is is favored over another hypothesis, etc., need *predictions* for strong-field gravity beyond GR

- In this sense GW150914 has blindsided studies of alternative theories
  - What should be an exquisite piece of data to constrain/rule-out alternatives is hampered by the fact that we do not understand this regime of the vacuum two body problem in *any* alternative theory
    - What is known to date is either the early inspiral from PN-like perturbative expansions, or the linear quasi-normal mode structure of single stationary modified black holes
    - With GW150914 most of the SNR is coming from the regime between where these two approximations are valid (again, didn't expect such heavy stellar mass black holes)

Filtered Signal plus Best-Fit Template



Filtered Signal plus Filtered & Unfiltered Best-Fit Template



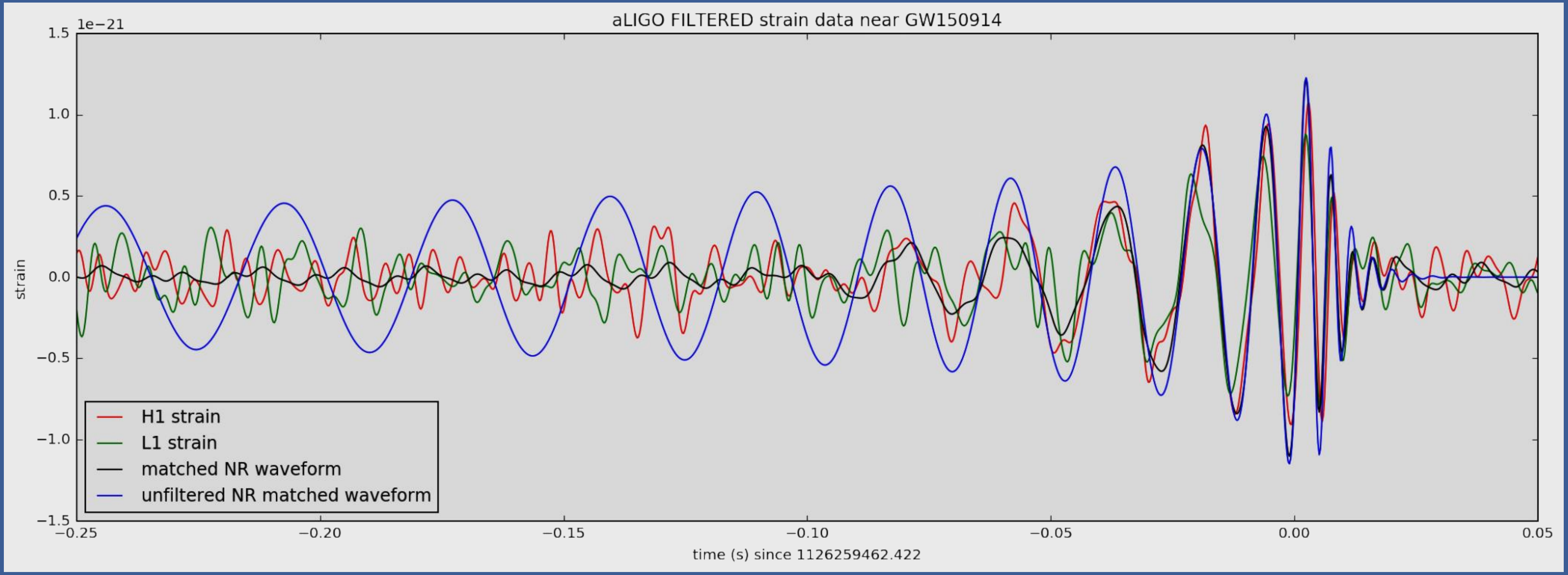
Inspiral

Ringdown

Constraints on exotic compact object alternatives to black holes *within* GR

- Similar issues plague exotic compact object alternatives such as boson stars, gravastars, traversable wormholes, etc.

- However, even if in principle the inspiral can be made consistent with exotica, the rapid damping of the signal after peaking would be very difficult to explain *unless* prompt collapse to a black hole occurred (i.e., at the very least, this event shows strong evidence for black hole formation)



Constraints on exotic compact object alternatives to black holes *within* GR

- One way to think of this is to quantify the material properties the exotic compact object need to explain the signal :
  - Here we assume GR is the correct theory of gravity, and this event is a binary merger.
  - The signal characteristics of the inspiral thus tells us that merger occurs on a spatial scale of ~400km, and the rapid ringdown says the GW emission drops below noise threshold on a time-scale that's *on order the light-crossing time of the remnant* (~4ms)
  - Therefore, to not source observable GWs, the matter dynamics excited by the collision must damp down on this time scale as well

Effective Hydrodynamic Model

- Many conceivable ways to do this; for a simple, order of magnitude idea, consider the bulk and shear viscosities that would be needed to damp the dominant ($l=2$) mode in an incompressible Newtonian fluid star *[Cutler and Lindblom, 1987]*

$$\bar{\eta}_{\rm eff} \sim 4 \times 10^{28} \frac{\rm g}{\rm cm \cdot s} \left(\frac{m}{65 M_\odot}\right) \left(\frac{370 \rm km}{R}\right) \left(\frac{4 \rm ms}{\tau_{\bar{\eta}}}\right)$$

$$\bar{\zeta}_{\rm eff} \sim 3 \times 10^{30} \frac{\rm g}{\rm cm \cdot s} \left(\frac{m}{65 M_\odot}\right) \left(\frac{370 \rm km}{R}\right) \left(\frac{4 \rm ms}{\tau_{\bar{\zeta}}}\right)$$

i.e. this says, given the observed properties on the RHS, we can interpret the dynamics of the exotic object as an effective, viscous fluid with viscosity coefficients as given on the LHS.

Effective viscosities of a few known compact objects

- Black holes (via the membrane paradigm, *Thorne, Price and McDonald 1986*) :

$$\bar{\eta}_{\rm BH} = -\bar{\zeta}_{\rm BH} \sim 1.3\times 10^{30}\frac{\rm g}{\rm cm\cdot s}\left(\frac{m}{65M_\odot}\right)^{-1}$$

- Neutron stars, where the damping comes from neutron scattering and strong magnetic fields

$$\bar{\eta}^{(n)}_{\rm NS} \sim 2\times 10^{14}\rho_{15}^{9/4}T_{11}^{-2}\frac{\rm g}{\rm cm\cdot s}$$

$$\bar{\zeta}^{(n)}_{\rm NS} \sim 2.4\times 10^{29}\rho_{15}^{2}T_{11}^{6}\omega_{10}^{-2}\frac{\rm g}{\rm cm\cdot s}$$

$$\bar{\eta}^{(B)}_{\rm NS} \sim 1.3\times 10^{28}B_{16}R_{12}\sqrt{\rho_{15}}\frac{\rm g}{\rm cm\cdot s}$$

- Solitonic boson stars (*Macedo, Pani, Cardoso and Crispino and Cardoso, 2013*)

$$\bar{\eta}_{\rm BS} \sim 7\times 10^{26}{\rm g/cm/s}$$

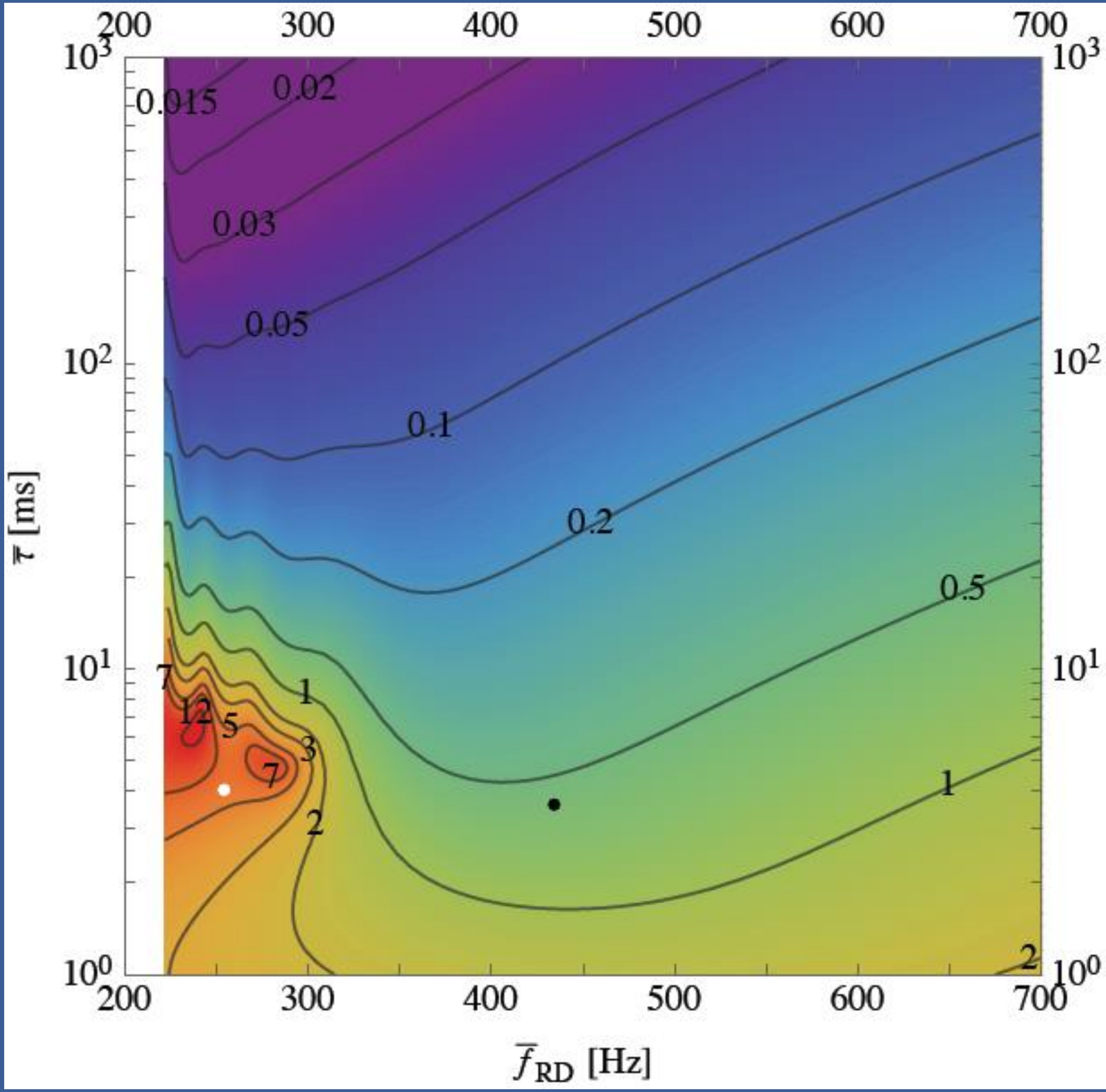
$$\bar{\zeta}_{\rm BS} \sim 5\times 10^{28}{\rm g/cm/s}$$

(bosonic material has very low intrinsic viscosity; this is all coming from GW damping of the mode)

Exotic compact objects

- LIGO essentially rules out an exotic remnant with bosonic or neutron star-like material, *if* the $l=2$ matter mode was excited with an initial amplitude to give an signal as large as that observed

- Right: can invert the question, and place limits on the initial amplitude, for a given frequency and damping time, above which LIGO should have seen the mode

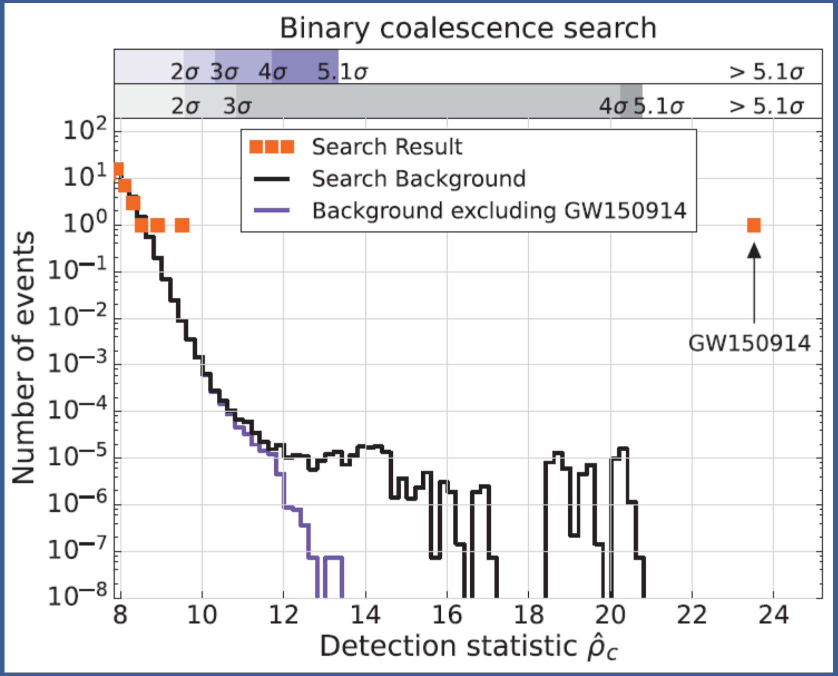


Concluding Comments: Why the first aLIGO Observing Run Was so Remarkable

- ***Reason 4:*** *The additional triggers imply GW150914 was not an unusually rare event*

- Within 5 years may have O(100) binary black hole merger events
  - much will be learnt from statistical properties of the population, the loudest events, and rare events (or lack of them)

- The future looks bright for learning about the universe through the lens of dynamical, strong-field gravity



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