DIARRHEA IS HEREDITARY: IT RUNS IN ONE'S GENES

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Abstract. Simple notes on special relativistic fluids in 2D

1. Introduction

We consider special relativistic hydrodynamics in $2D$. We concentrate on a perfect fluid with stress energy tensor : $T_{ab} = p\eta_{ab} + (e+p)u_a u_b$ satisfying the conformal fluid, thus

$$T^a_a = 0 \rightarrow e = -2p \tag{1.1}$$

The hydrodynamic equations $\partial_a T^{ab} = 0$, determine the evolution of the system. With a goal of studying this system numerically, we express the stress energy components as

$$T^{00} = \frac{e}{2}\left(-1 + \frac{3}{W^2}\right) \equiv D/2 \tag{1.2}$$

$$T^{0i} = \frac{3e}{2}\frac{v^i}{W^2} \equiv S^i/2 \tag{1.3}$$

$$T^{ij} = \frac{e}{2}\delta^{ij} + \frac{3e}{2}\frac{v^i v^j}{W^2}\,; \tag{1.4}$$

where we have defined (the inverse of the Lorentz factor) $W = \sqrt{1 - v^i v_i}$ and introduced the fields D and S^i to express the system in conservation form,

$$\partial_t D + \partial_i S^i = 0 \tag{1.5}$$

$$\partial_t S^i + \partial_j (e\delta^{ij} + S^i v^j) = 0 \tag{1.6}$$

Such a form is convenient for a numerical implementation. As a result one hast two sets of variables, the *conservative* variables $U \equiv (D, S^i)$ and *primitive* variables $P \equiv (e, v^i)$.

1.1. Conservative to primitive transformation.

Going from $P \rightarrow U$ is straightforward, the inverse is only slightly involved. This transformation is achieved by first noting that,

$$S^i S_i / D^2 = 9(1 - W^2)/(3 - W^2)^2 \tag{1.7}$$

Thus, defining $\Xi \equiv S^i S_i/D^2$, and solving the quadratic equation above, one obtains:

$$W^2 = \frac{3}{2\Xi}\left(2\Xi - 3 + \sqrt{9 - 8\Xi}\right) \tag{1.8}$$

With this, the primitive variables are obtained trivially

$$e = \frac{D}{-1 + 3W^{-2}}\ ,\ v^i = \frac{W^2 S^i}{3e} \tag{1.9}$$

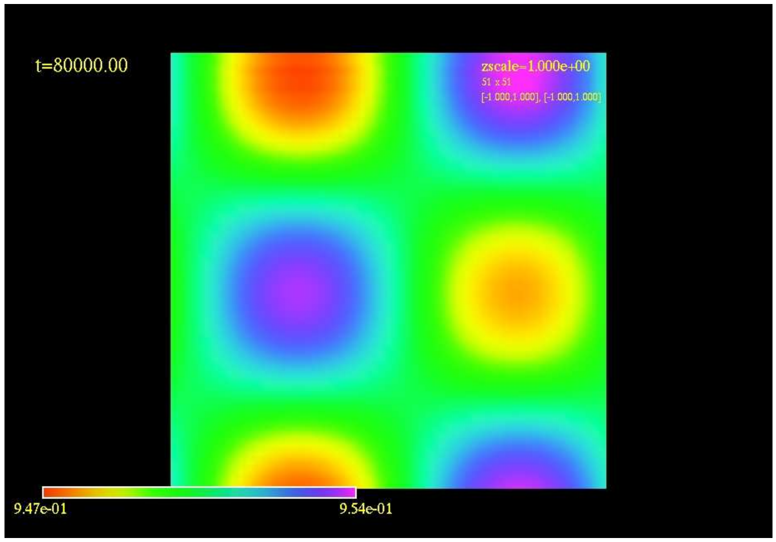

FIGURE 1. e at $t = 810^4$.).

1.2. **Initial data.** We set up an initial configuration defined by $e = 1, v^y = 0$ and $v^x = \kappa sin(2\pi/Ly)$ in the torus $[0, L] \times [0, L]$. We adopt $\kappa = 0.1$ and evolve the system with $L = 10$. After some transient behavior, an interesting pattern develops, figure 1 illustrates the primitive variables e after $8\,10^3$ light crossing times.