Sommerfeld enhancement in the double Higgs boson production by e^+e^- annihilation.

Andres Vasquez

Instituto de Física Teórica - UNESP

First Joint ICTP-Trieste/ICTP-SAIFR School on Particle Physics.

IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

Introduction

The cross-section for the process $e^+e^- \to hh$ in the SM is too small compared to other electroweak processes. A detectable cross-section, bigger than the predicted value from the SM would be a hint for new physics, which makes interesting the study of the enhancement in such process due to physics BSM.

The main goal is to determine the enhancement of the cross-section for the double Higgs production by the non-perturbative Sommerfeld effect.

It is studied the threshold behavior of the cross-section for the double Higgs production when a hidden sector couples to the Higgs boson, yielding bound-states below the threshold energy due to non-perturbative effects.

Why Physics Beyond the Standard Model?

Observational Facts

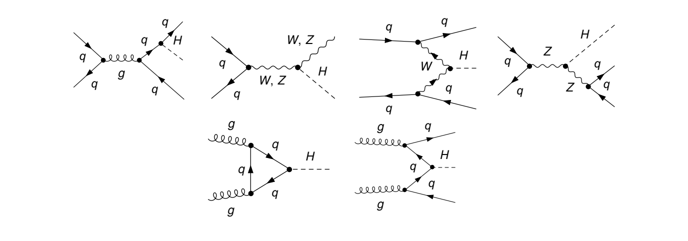
1. Gravity
2. Cosmological Constant Problem
3. Dark Matter
4. Neutrino Masses
5. Strong CP Problem
6. Matter-Antimatter Asymmetry

Theoretical Motivation

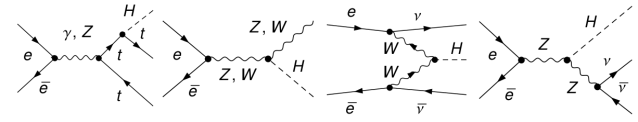
1. Naturalness and Hierarchy problem

Higgs Physics
Higgs boson production.

Relevant processes for the Higgs boson production in proton-proton colliders:



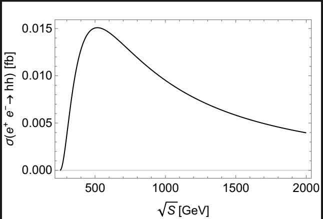
and in electron-positron colliders:

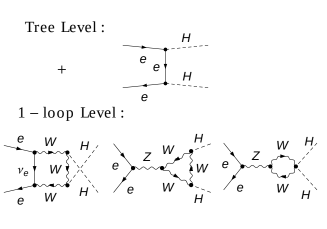


$\sigma\left(e^+e^- \to hh\right)$ in the Standard Model

Remarks

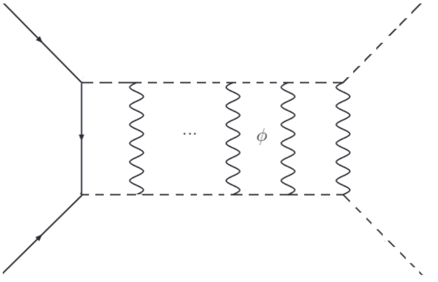
1. The leading-order of this process is found at 1-loop.
2. Too small in the SM ($\sigma \sim 10^{-2}\,\mathrm{fb}$ at $\sqrt{s} = 500\,\mathrm{GeV}$). At tree level, the cross-section is proportional to the electron mass.
3. Studied in the MSSM and 2HDM.





Threshold behaviour of the $\sigma\left(e^+e^- \rightarrow hh\right)$
Sommerfeld effect from a hidden sector

The bound-state would have negative binding energy, allowing its creation below $\sqrt{s} = 250\,\mathrm{GeV}$. The mediator field that would bind the Higgs bosons was taken to be a hidden scalar ϕ.



Sommerfeld Effect

The probability of transition is proportional to $|\psi_k^0(\mathbf{0})|^2$, with

$$\psi_i^0(\mathbf{r}) = e^{i\mathbf{k}_i\cdot\mathbf{r}}, \qquad \psi_f^0(\mathbf{r}) = e^{i\mathbf{k}_f\cdot\mathbf{r}}, \tag{1}$$

the initial and final wavefunctions.

In a central potential, for a particle near the origin with a small velocity it could happen that the wavefunction suffers strong distortions.

In the s-channel for a 2-2 scattering, we can have

$$\mathcal{M}_s = \mathcal{M}_s^{pert.} \cdot \frac{\psi_i(\mathbf{0})\,\psi_f^*(\mathbf{0})}{\psi_i^0(\mathbf{0})\,\psi_f^{0*}(\mathbf{0})}, \tag{2}$$

This new factor gives corrections to the usual planar-wave in the perturbative amplitude. Hence,

$$|\mathcal{M}|^2 = |\mathcal{M}_s^{pert.}|^2 |\psi_f(\mathbf{0})|^2, \qquad \text{or} \qquad , \quad \sigma = \sigma^{pert} S_k. \tag{3}$$

where the corrections to the amplitude come from the factor $S_k \equiv |\psi_f(\mathbf{0})|$.

Let us see the non-perturbative effects below threshold energy ($\sqrt{s} = 250\,\mathrm{GeV}$) from

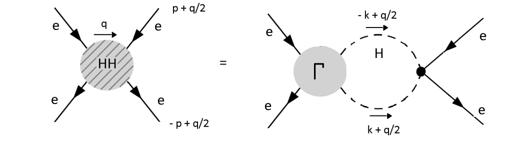
$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \frac{1}{2} m_\phi^2 \phi^2 + guhh\phi. \tag{4}$$

The analysis is based on the Peskin and Strassler work (1990). The cross-section is found by using the optical theorem

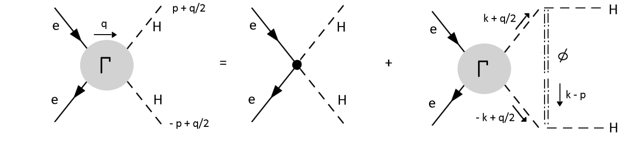
$$\sigma\left(e^+e^- \to hh\right) = \frac{1}{s}\mathrm{Im}\mathcal{M}\left(e^+e^- \underset{hh}{\longrightarrow} e^+e^-\right). \tag{5}$$

Then, one is concentrated in the amplitude $\mathcal{M}\left(e^+e^- \underset{hh}{\longrightarrow} e^+e^-\right)$.

Near threshold, the leading-order contributions can be found as



where the gamma coefficient satisfies



In the end

$$\sigma\left(e^+e^- \to hh\right) = \sigma_0\left(e^+e^- \to hh\right) R\left(E\right), \qquad (6)$$

$$R\left(E\right) = \frac{\mathrm{Im}\, G\left(0, E\right)}{\mathrm{Im}\, G_0\left(0, E\right)}. \qquad (7)$$

By solving the Schrödinger equation for the possible bound-state made up of two Higgs bosons,

$$\left[-\frac{\nabla^2}{m_h} - E - i\Gamma_h + V(\mathbf{r})\right] G\left(\mathbf{r}, \mathbf{r}'; E + i\Gamma_h\right) = \delta^3\left(\mathbf{r} - \mathbf{r}'\right), \qquad (8)$$

with

$$V(r) = -\kappa \frac{e^{-m_\phi r}}{r}, \qquad \kappa \equiv \frac{g^2 u^2}{4\pi m_h^2}, \qquad (9)$$

one finds that

$$\mathrm{Im}\, G(0,0) = -\frac{m_h}{4\pi} \mathrm{Im}\, B, \qquad (10)$$

where $\mathrm{Im}B$ is found numerically.

How was performed this study?

1. FeynArts: Generation of Feynman diagrams.
2. FormCalc: Creation and simplifation of the amplitudes.
3. LoopTools: Computation of the Passarino-Veltman integrals and generation of the cross-sections as a function of the energy and other important parameters.
4. Interpolation of the pure SM cross-section.
5. Numerical solution of the imaginary part of the Green's function at the origin, $G(0,0;E)$.
6. Creation of a data list containing $\mathrm{Im}G(0,0;E)$ for different values of energy. This has to be done taking special care of the narrow peaks.
7. Generation of the final cross-section from the multiplication of the different factors obtained in the previous steps.

In the formula

$$\sigma\left(e^{+}e^{-}\rightarrow hh\right) = \sigma_0\left(e^{+}e^{-}\rightarrow hh\right)R\left(E\right), \tag{11}$$

with

$$R\left(E\right) = \frac{\mathrm{Im}\,G\left(0,E\right)}{\mathrm{Im}\,G_0\left(0,E\right)}, \tag{12}$$

the SM cross-section enters as

$$\sigma_0\left(e^{+}e^{-}\rightarrow hh\right) = \frac{1}{64\pi s}\sqrt{1-\frac{4m_h^2}{s}}\left|\mathcal{M}\left(e^{+}e^{-}\rightarrow hh\right)\right|^2. \tag{13}$$

Moreover, one can use that

$$G_0\left(r,r';E+i\Gamma\right) = -\frac{m_h}{4\pi}\frac{\sin\lambda r}{\lambda r}\frac{e^{i\lambda r'}}{r'}, \tag{14}$$

to find

$$\mathrm{Im}\,G_0\left(0,E\right) = -\frac{m_h}{4\pi}\sqrt{m_h\left(\sqrt{s}-2m_h\right)}. \tag{15}$$

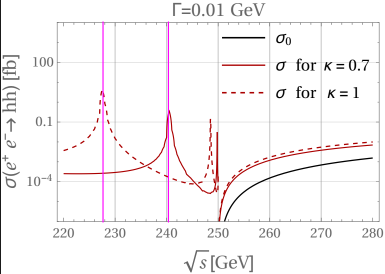
Thus, he cross-section takes the form

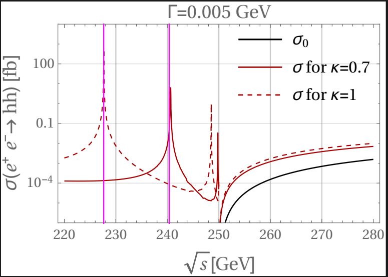
$$\begin{aligned}\sigma\left(e^+e^- \to hh\right) &= \frac{1}{64\pi s}\sqrt{\left(1-\frac{2m_h}{\sqrt{s}}\right)\left(1+\frac{2m_h}{\sqrt{s}}\right)}\overline{|\mathcal{M}\left(e^+e^- \to hh\right)|^2} \\ &\quad\times\frac{\mathrm{Im}B}{\sqrt{m_h\left(\sqrt{s}-2m_h\right)}}, \\ &= \frac{1}{64\pi s}\sqrt{1+\frac{2m_h}{\sqrt{s}}}\overline{|\mathcal{M}\left(e^+e^- \to hh\right)|^2}\times\frac{\mathrm{Im}B}{\sqrt{m_h\sqrt{s}}}. \quad (16)\end{aligned}$$

Threshold behaviour of the $\sigma\left(e^+e^- \to hh\right)$

Sommerfeld effect from a hidden sector

For $m_\phi = 10, \mathrm{GeV}$, it is found





Remarks

1. The location of the resonances does not depend on the Width Γ_h, but it does on the coupling strength κ.
2. The enhancement of the cross-section depends on the Width and the coupling strength.
3. The study of the threshold behaviour of $\sigma\left(e^+e^- \rightarrow hh\right)$ takes relevance nowadays, when it has been confirmed that the ILC would be a Higgs-boson factory operating at a centre-of-mass energy of 250 GeV with a luminosity goal of 2 ab^{-1}. (K. Fujii et al., 2017)

Threshold behaviour of the $\sigma(e^+e^- \to hh)$

Sommerfeld effect from a hidden sector

Outlooks

The above results stand for the situation in which the amplitude in the SM is s-wave dominated. For p-wave dominated processes, the Schrödinger equation gets an inhomogeneous term that depends on the SM amplitude. The $e^+e^- \to hh$ is p-wave dominated.

Thanks!!