A Brief Introduction to Neutron Stars and Quark Stars

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Sumary

Stellar Evolution

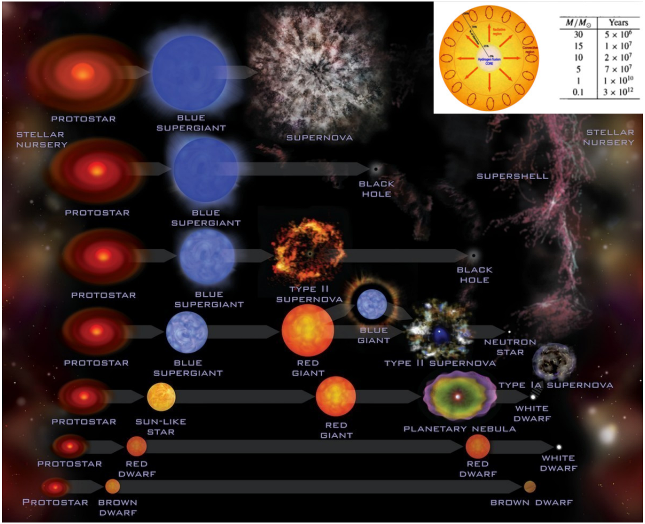


Figura: Stellar Evolution and their lifetime.

Main Features[5, 2]

- The Neutron Star is formed when a large star which before collapse had a total of 10-29 $M_\odot$. The radius is of order $\approx$ 10 km and a mass between 1.4-2.6 $M_\odot$;
- The NS was proposed by Baade and Zwicky in 1933 at CALTECH and discovered by Joselyn Bell and Anthony Hewish at Cambridge in 1967. The first theoretical model was proposed by J. Robert Oppenheimer and G. M. Volkoff in 1939;
- The NS are determined by the EoS (equations of state) at densities ranging from $\rho_0 \approx 3x10^{14}$ g/cm^3 (nuclear density) to 10 ρ_0;
- The EoS (bulk matter: when mass number $A \rightarrow \infty$) for NS is constrained by empirical symmetric[1] nuclear properties near or equal at ρ_0 and extended for non-symmetric matter;
- In these stars, causality and relativistic propagation are important dynamical considerations;
- The NS is not made only by neutrons, but also by electron, protons, muons and other particles that can be created at a specific energy. The inclusion of these particles is needed to maintain charge neutrality and the chemical potential in equilibrium;
- The processes driven by the weak force (β decay, its inverse and quarks decay) must be taken into account;
- The stability of neutron star is maintained by degeneracy[2] pressure of nucleons plus repelling force (short distances) between them. The same occurs for quark star, however, we only have the contribution of degeneracy pressure from the quarks.

[1] Neutrons = Protons.

[2] $E_F \gg T$, where $E_F = \sqrt{k_F^2 + m^2}$.

Einstein's Equations[5, 2]

The Einstein's equations relate matter-energy to space geometry. The left side tells how the space is curved in presence of matter-energy by the tensor $G^{\mu\nu}$. This tensor is equal to $-8\pi T^{\mu\nu}$ and is 0 in the absence of matter.

$$R_{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \equiv G^{\mu\nu}, \tag{1}$$

We consider a perfect fluid (the pressure is isotropic in each element of the fluid at the rest frame). The shear stress and heat transport are absent in it.

$$T^{\mu\nu} = p g^{\mu\nu} + (p + \varepsilon) u^{\mu} u^{\nu} \tag{2}$$

where p is the pressure, $g^{\mu\nu}$ is the metric, ε is the energy density and u^{μ} is the quadri-velocity of the fluid. We want to study the case of a static star and spherically symmetric given by the Schwarzschild metric. The invariant line interval is given by:

$$d\tau^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} = e^{2\nu(r)} dt^2 - e^{2\lambda(r)} dr^2 - r^2 d\theta^2 - r^2 sen^2\theta d\phi^2, \tag{3}$$

The Schwarzschild metric is diagonal, thus its components are

$$g_{00} = e^{2\nu(r)},\ g_{11} = -e^{2\lambda(r)},\ g_{22} = -r^2,\ g_{33} = -r^2 sen^2\theta \text{ and } g_{\mu\nu} = 0\ (\mu \neq \nu) \tag{4}$$

Solving the Einstein's equation for this metric and considering a perfect fluid, we obtain the TOV's equations[1].

The Tolman-Oppenheimer-Volkov Equation [1, 3]

These equations are solutions of the Einstein's equations for static and spherically symmetric star filled with a perfect fluid.

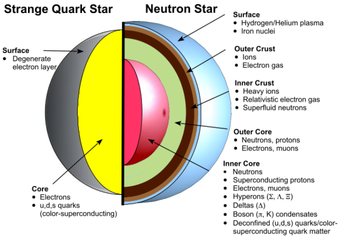
$$\frac{dp}{dr} = -\frac{G\varepsilon(r)\mathcal{M}(r)}{c^2 r^2}\left[1+\frac{p(r)}{\varepsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{\mathcal{M}(r)c^2}\right]\left[1-\frac{2G\mathcal{M}(r)}{c^2 r}\right]^{-1} \tag{5}$$

$$\frac{d\mathcal{M}}{dr} = 4\pi r^2 \rho(r) = \frac{4\pi r^2 \varepsilon(r)}{c^2} \tag{6}$$

Where G is the gravitational constant and c is the speed of light. The terms in square brackets are relativistic corrections. These differential coupled equations can be solved using Runge-Kutta method. The evolution of the system occurs in r and the initial conditions for r=0 are: m(r)=0, $\rho(r)$= some value and stops every time when p(r$\neq$ 0) =0 and jumps to the next $\rho(r)$ value.

The input for solving these equations are the EoS, which may describe nuclear matter, dark matter or quark matter.

Structure of Compact Objects



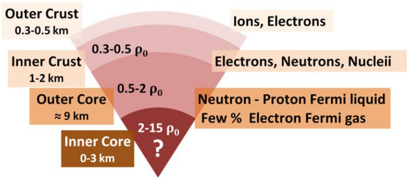


Figura: The internal and external layers of neutron star and strange star.

In this work, we are considering the entire star made of proton and neutrons for nuclear matter. For quark matter, made of quarks up, down and strange. Charge neutrality and potential chemical equilibrium are not considered here.

$\sigma - \omega$ Nuclear Model [4]

- Reproduces the nuclear potential features(attractive and repulsive), the binding energy and symmetry energy at saturation point;
- Failures to reproduce the compressibility factor (K) and the effective mass per nucleon (m^*/M_n);
- The approach used by this model is the Relativistic Mean Field, i.e, the meson field operators are constants in position and time (or, in other words, they are classical field operators).

The equations of states (EoS) are:

Baryonic density ($\rho_B \equiv B/V$)

$$\rho_B = \frac{\gamma}{6\pi^2} k_F^3$$

Energy Density

$$\varepsilon = \frac{g_v^2}{2m_V^2}\rho_B^2 + \frac{m_s^2}{2g_S^2}(M - M^*)^2 + \frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k(\mathbf{k}^2 + M^{*2})^{1/2}$$

Pressure

$$P = \frac{g_v^2}{2m_V^2}\rho_B^2 - \frac{m_s^2}{2g_S^2}(M - M^*)^2 + \frac{1}{3}\frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k \frac{\mathbf{k^2}}{(\mathbf{k}^2 + M^{*2})^{1/2}}$$

Effective Mass

$$M^* = M - \frac{g_S^2}{m_S}\rho_S$$

$\sigma - \omega$ Nuclear Model [4] - Results

The couplings are: $C_s^2 \equiv g_s^2(\frac{M^2}{m_s^2}) = 357.4$ and $C_v^2 \equiv g_v^2(\frac{M^2}{m_v^2}) = 273.8$. The nucleon mass is 938 MeV.

Nuclear Properties of Symmetric Matter ($\gamma = 4$)						
	m^*/M_n	K[MeV]	B/A [MeV]	α_4 [MeV]	ρ_0 (fm^{-3})	K_{F_0} (fm^{-1})
Waleckas's Model	0.557	535.28	-15.749	22.01	0.193	1.419
Experimental values	0.564-658	200-300	-15.75	21.96	0.193	1.41

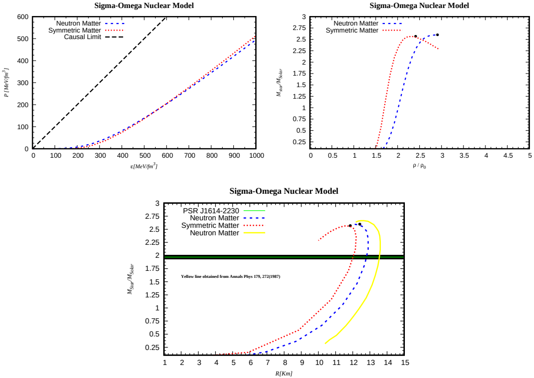


Figura: EoS, Densities at Maximum Mass star and M-R relation, respectively.

M.I.T Bag Model

- It was proposed[9] in order to take into account hadronic masses in terms of their constituents;
- Here we are just considering u, d e s quarks;
- The quarks are inside a bag which reproduces the asymptotic freedom (inside the bag, quarks behave as being free) and confinement;
- The lower and upper limits of B constant is obtained by considering two quark or three quarks inside it. Assuming this quark matter as stable as iron atom(the most stable atom). The B constant assuming this assumption[10] 145.0 MeV $< B^{1/4} <$ 162.76 MeV;
- The masses of u, d and s quarks are 5 MeV, 7 MeV and 150 MeV[5], respectively;
- We consider $\rho_0 = 0.25$ fm^{-3} which is the actual value used in the literature;

The equations of states (EoS) are :
Baryonic Density

$$\rho = \frac{1}{2\pi^2} k_F^3$$

Energy Density

$$\varepsilon = B + \sum_{u,d,s} \frac{3}{4\pi^2} [\mu_f k_f (\mu_f^2 - \frac{1}{2} m_f^2) - \frac{1}{2} m_f^4 \ln(\frac{\mu_f + k_f}{m_f})]$$

M.I.T Bag Model

Pressure

$$P = -B + \sum_{u,d,s,} \frac{1}{4\pi^2}[\mu_f k_f(\mu_f^2 - \frac{5}{2}m_f^2) + \frac{3}{2}m_f^4 \ln(\frac{\mu_f + k_f}{m_f})]$$

where the $B^{1/4}$=145 MeV. The chemical potential is defined as $\mu_f = \sqrt{m_f^2 + k_f^2}$. The difference in signs for B and fermions for the pressure equation, show us this counterbalance between them.

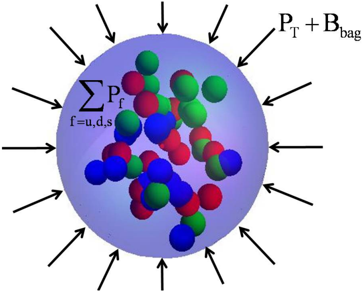


Figura: An ilustrative way to look the quarks and the bag.

M.I.T Bag Model - Results

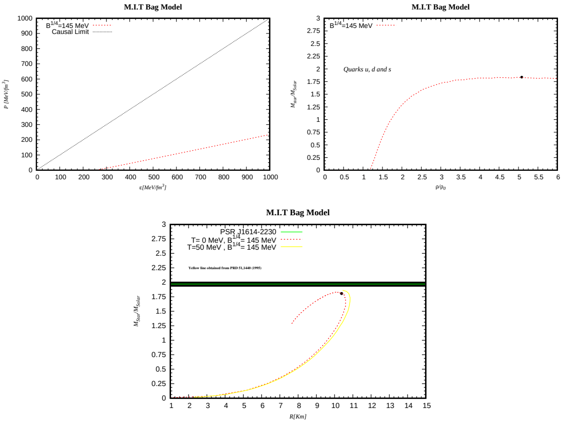
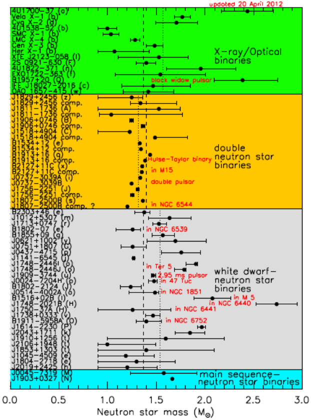


Figura: EoS, Densities at Maximum Mass Star and M-R relation, respectively.

Experimental Observations and Realistic EoS Results



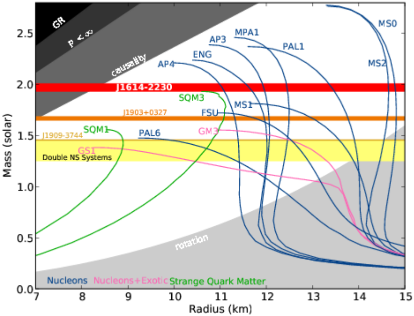


Figura: Neutron Mass observations and M-R relations from realistic EoS[12].

Conclusions

- We disregarded some essential physics as chemical equilibrium and the local charge neutrality.
- Our M-R relation for Sigma-Omega model is almost the same obtained in [7]. The latter considered all physics mentioned before.
- For the M.I.T model, our M-R results are the same as obtained in [6].
- We are working on numerical calculations to include all aspects we mentioned previously in order to be physicaly consistent. .
- However, the sigma-omega model could describe, in principle, the observation of PS1614-2230. The next step of this work is use some realistic model for the nuclear (outer crust) like Boguta-Bodmer and consider the core of star made of quarks.
- Use free codes like RNS and Lorene to compare the same models with and without rotation, magnetic field and tidal effects.
- We quote[13, 14] as excellent articles for undergraduate students and for those who search for introductory articles.

"A man provided with paper, pencil, and rubber, and subject to strict discipline, is in effect

Backup Slides - Sigma-Omega Model

For the nuclear model studied here, we have the following decay:

$$n \rightarrow p + e^{-} + \bar{\nu}_{e^-} \tag{7}$$

The chemical equilibrium for this decay is:

$$\mu_n = \mu_p + \mu_e^{-} + \mu_{\bar{\nu}_{e^-}} \tag{8}$$

The condition of electric charge neutrality reads[3]:

$$k_{F_e^-} = k_{F_p} \tag{9}$$

Since the time of weak decays is very short when compared to neutron star formation time, all neutrinos leaked from star ($\mu_{\bar{\nu}_{e^-}} = 0$). In this way, the chemical equilibrium reads

$$\sqrt{k_{F_m}^2 + m_n^2} = \sqrt{k_{F_e}^2 + m_p^2} + \sqrt{k_{F_e}^2 + m_e^2} \tag{10}$$

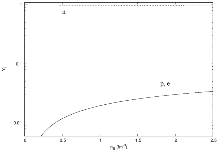


Figura: Relative particle population in term of baryonic density [7].

[3] $\sum_i Qb_i \rho_{b_i} - Qe^{-} \rho_{e^-} = 0$

Backup Slides - M.I.T Bag Model

For the M.I.T bag model, we have the following decays[8]:

$$d \to u + e^{-} + \bar{\nu}_{e^-} \tag{11}$$
$$s \to u + e^{-} + \bar{\nu}_{e^-} \tag{12}$$
$$s \to c + e^{-} + \bar{\nu}_{e^-} \tag{13}$$
$$\mu^{-} \to + e^{-} + \bar{\nu}_{e^-} + \nu_{\mu^-} \tag{14}$$
$$\tag{15}$$

And the reactions:

$$s + d \longleftrightarrow d + u \tag{16}$$
$$c + d \longleftrightarrow u + d \tag{17}$$
$$\tag{18}$$

contribute to the equilibrium of flavors. Hence, from the relation above one gets:

$$\mu_d = \mu_u + \mu_{e^-},\ \mu_c = \mu_u,\ \mu_d = \mu_s,\ \mu_{e^-} = \mu_- \tag{19}$$

As we have done before, using the condition of electric charge neutrality[4] and the chemical equilibrium relations above.

[4] $\sum_i Qb_i \rho_{b_i} - Qe^{-} \rho_{e^-} - Q\mu^{-} \rho_{\mu^-} = 0$

Backup Slides - M.I.T Bag Model

The equations above can be simultaneously solved(numerically) for each baryonic density to get the relative particle population. The relative population for the M.I.T bag model with u,d,s and c quarks plus electrons and muons are shown in the below figure.

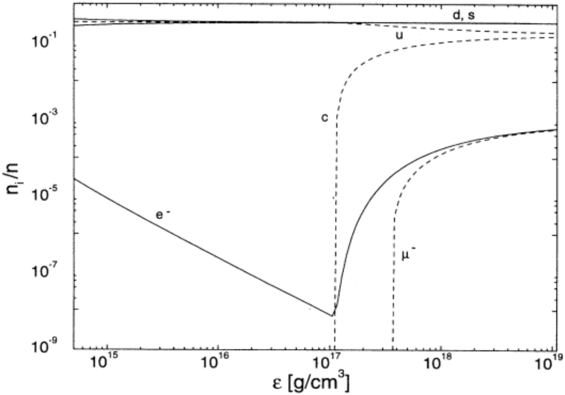


Figura: Relative particle population in terms of density energy[8].

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