Delta Baryon in the Nuclear Medium in QCD Sum Rules

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Work done in collaboration with SuHoung Lee, KieSang Jeong, Ricardo Matheus and Aaron Park (arXiv: 1806.01668 [nucl-th])

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Clarifications

- What is the Δ Baryon?
    - A cousin of the nucleons with spin and isospin 3/2.
    - Decays very quickly, most of the time to a nucleon and a pion.
    - 4 species with a Breit-Wigner mass of about 1232 MeV.

- How do you picture and describe the nuclear medium?
    - Infinite nuclear medium composed of protons and neutrons with the particle one wants to describe as an expectator. Characterized by (4-vector medium velocity) "u_μ" , density of nucleons "ρ" and isospin asymmetry factor "I".

Clarifications

- What is QCD Sum Rules
  - Non-perturbative method relating QCD and Hadron observables (masses, etc.).
  - Advantages with respect to other techniques (Dyson-Schwinger, Lattice QCD, AdS/QCD, hadronic effective theories, ...).
  - Disadvantages and limitations.
- Why is this study of the Δ interesting?
  - Of interest to neutron stars.
  - Experimental results are in contradiction and theoretically no clear picture yet. New results will be available soon and we will be able to check if QCD Sum Rules in the medium is introducing the medium effects correctly.
  - Studies of nuclear medium provide a theoretical laboratory of chiral symmetry restoration.

QCD Sum Rules Basics

Proposed in 1978 Shifman, Vainshtein and Zakharov (Nucl. Phys. B147 385, 448 (1979))

Based on the Operator Product Expansion for the correlator

$$\lim_{q^2\to-\infty} \int dx e^{iqx} \langle\Omega|T\{J(x)J(0)\}|\Omega\rangle = \Pi(q^2)_{q^2\to-\infty} = \Sigma_n C_n(q)\langle\Omega|O_n(0)|\Omega\rangle$$

Applied to the currents constructed with QCD degrees of freedom. This is the OPE Side.

For the Δ^{++}

$$J(x) = \varepsilon_{abc}(u_a(x)^T C\gamma_\mu u_b(x))u_c(x)$$

QCD Sum Rules Basics

OPE for the Correlator?

$$\lim_{q^2\to-\infty}\int dx e^{iqx}\langle\Omega|T\{J(x)J(0)\}|\Omega\rangle = \Pi(q^2)_{q^2\to-\infty} = \Sigma_n C_n(q)\langle\Omega|O_n(0)|\Omega\rangle$$

For the Δ^{++}: $J(x) = \varepsilon_{abc}(u_a(x)^T C\gamma_\mu u_b(x))u_c(x)$

The $\langle\Omega|O_n(0)|\Omega\rangle$ are the condensates. Constructed from QCD fields and organized by their mass dimensions.

$$\langle\bar{u}u\rangle, \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle, \langle\bar{u}u\bar{u}u\rangle.$$

QCD Sum Rules Basics

The current with the correct quantum numbers can excite hadrons from the QCD vacuum and this information is contained in the correlator for If $q^2>0$, as in the spectral representation of Källen-Lehmann for the hadron under consideration. This is called the phenomenological Ansatz.

$$\mathrm{Im}[\Pi(q^2)] = f^2\delta(q^2 - m_\Delta^2) + \rho_{\mathrm{cont}}(q^2)\theta(q^2 - s_0)$$

The Matching

$$\Pi(q^2)_{q^2\to-\infty} = \Sigma_n C_n(q)\langle\Omega|O_n(0)|\Omega\rangle$$

$$\mathrm{Im}[\Pi(q^2)] = f^2\delta(q^2 - m_\Delta^2) + \rho_{\mathrm{cont}}(q^2)\theta(q^2 - s_0)$$

There is some hope for a match between both sides for $\Pi(q^2)$. This will be achieved through a dispersion relation.

$$\Pi(q^2) = \int_{s0}^{\infty} \frac{\mathrm{Im}[\Pi(s)]}{s-q^2} ds$$

The Matching

$$\Pi(q^2)_{q^2\to-\infty} = \Sigma_n C_n(q)\langle\Omega|O_n(0)|\Omega\rangle$$

$$\mathrm{Im}[\Pi(q^2)] = f^2\delta(q^2 - m_\Delta^2) + \rho_{\mathrm{cont}}(q^2)\theta(q^2 - s_0)$$

$$\Pi(q^2) = \int_{s0}^{\infty} \frac{\mathrm{Im}[\Pi(s)]}{s-q^2} ds$$

Still not the whole story, because to study the hadron at hand we would like to pick only the contribution at $q^2=m_\Delta^2$, but on the other hand we only know how to calculate the left hand side at $q^2\to-\infty$. So we introduce the Borel Transform.

The Matching - Borel Transform

$$\Pi(M^2) \equiv \mathcal{B}_{M^2}\Pi(q^2) = \lim_{\substack{-q^2, n\to\infty \\ -q^2/n = M^2}} \frac{(-q^2)^{(n+1)}}{n!}\left(\frac{d}{dq^2}\right)^n \Pi(q^2)$$

$$\mathcal{B}_{M^2}(q^2)^k = 0,\ \mathcal{B}_{M^2}\left(\frac{1}{(m^2-q^2)^k}\right) = \frac{1}{(k-1)!}\frac{\exp(-m^2/M^2)}{M^{2(k-1)}}$$

$$\Pi(M^2) = f^2\, e^{-m^2/M^2} + \int_{s_0}^{\infty} ds\ \rho(s) e^{-s/M^2}$$

Borel Transformed Sum Rule

This has the following effects

- Eliminate all polynomials in q^2 from the OPE calculation and the subtraction on the phenomenological side,
- Suppression of the continuum states and possible resonances
- Improves the convergence of the OPE slightly,
- But the price we pay is the introduction of the unphysical parameter M, called the Borel mass. But we can deal with this searching for stability (flatness of mass plot against M).

Medium modification of the Sum rule

Sum Rules in the Nuclear Medium is a theoretical lab to explore partial restoration of chiral symmetry.

When we consider the Sum Rule for a particle moving in nuclear matter, at finite density, we have to modify some aspects of the sum rule, but the main ideas are maintained.

On the OPE side:

- Condensates will depend of the density ρ and isospin asymmetry I;
- New condensates, which were forbidden due to the Lorentz symmetry are now possible, like $\langle \bar{u}\gamma_{\mu} u \rangle$,
- The coefficients which depended only on the invariant q^2, now can also depend on q.u and we must do the sum rule with a fixed q vector;
- There is now a new structure and a new invariant, proportional to u slash.

In the case of Baryons, the phenomenological side now reads

$$\Pi(q^2, q_0) = \lambda^2 \frac{\not{q} + m_\Delta^* - \not{u}\Sigma_v}{(q_0 - E_q)(q_0 - \bar{E}_q)} + \rho_{cont}(q^2, q_0)$$

$$E_q = \Sigma_v + \sqrt{\vec{q}^2 + {m_\Delta^*}^2} \qquad \bar{E}_q = \Sigma_v - \sqrt{\vec{q}^2 + {m_\Delta^*}^2}$$

The Δ Sum Rule - The Borel Transformed OPE

Scalar part:

$$\bar{\mathcal{B}}[\Pi^{e}_{\Delta,s}(q_0^2,|\vec{q}|)]_{\mathrm{subt.}} - \bar{E}_{\Delta}\bar{\mathcal{B}}[\Pi^{o}_{\Delta,s}(q_0^2,|\vec{q}|)]_{\mathrm{subt.}}$$
$$= -\frac{1}{3\pi^2}(M^2)^2\langle\bar{u}u\rangle_{\rho,I}\tilde{E}_1 L^{\frac{16}{27}} + 2\bar{E}_{\Delta,q}\langle u^{\dagger}u\rangle_{\rho,I}\langle\bar{u}u\rangle_{\mathrm{vac}}L^{\frac{4}{27}}$$

The Δ Sum Rule - The Borel Transformed OPE

Structure proportional to q slash:

$$
\begin{aligned}
&\bar{\mathcal{B}}[\Pi^e_{\Delta,q}(q_0^2,|\vec{q}|)]_{\text{subt.}} - \bar{E}_\Delta \bar{\mathcal{B}}[\Pi^o_{\Delta,q}(q_0^2,|\vec{q}|)]_{\text{subt.}} \\
&= -\frac{1}{80\pi^4}(M^2)^3 \tilde{E}_2 L^{\frac{4}{27}} + \frac{1}{9} M^2 \tilde{E}_0 \langle u^\dagger i D_0 u\rangle_{\rho,I} L^{\frac{4}{27}} \\
&\quad + \frac{5}{288\pi^2} M^2 \tilde{E}_0 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rho,I} L^{\frac{4}{27}} - \frac{1}{36\pi^2} M^2 \tilde{E}_0 \left\langle \frac{\alpha_s}{\pi}((u\cdot G)^2 + (u\cdot\tilde{G})^2) \right\rangle_{\rho,I} L^{\frac{4}{27}} \\
&\quad - \frac{3}{4}\langle \bar{u}u\bar{u}u\rangle L^{\frac{4}{27}} + \frac{3}{4}\langle \bar{u}\gamma_5 u\bar{u}\gamma_5 u\rangle L^{\frac{4}{27}} - \frac{5}{4}\langle \bar{u}\gamma u\bar{u}\gamma u\rangle_{\text{tr.}} L^{\frac{4}{27}} + \frac{5}{4}\langle \bar{u}\gamma_5\gamma u\bar{u}\gamma_5\gamma u\rangle_{\text{tr.}} L^{\frac{4}{27}} \\
&\quad - \frac{1}{8}\langle \bar{u}\gamma u\bar{u}\gamma u\rangle_{\text{s.t.}} L^{\frac{4}{27}} + \frac{9}{8}\langle \bar{u}\gamma_5\gamma u\bar{u}\gamma_5\gamma u\rangle_{\text{s.t.}} L^{\frac{4}{27}} + \frac{3}{4}\langle \bar{u}\sigma u\bar{u}\sigma u\rangle_{\text{s.t.}} L^{\frac{4}{27}} \\
&\quad + \bar{E}_\Delta \frac{1}{6\pi^2} M^2 \tilde{E}_0 \langle u^\dagger u\rangle_{\rho,I} L^{\frac{4}{27}}
\end{aligned}
$$

The Δ Sum Rule - The Borel Transformed OPE

Structure proportional to u slash:

$$
\begin{aligned}
&\bar{\mathcal{B}}[\Pi^{e}_{\Delta,u}(q_0^2,|\vec{q}|)]_{\text{subt.}} - \bar{E}_\Delta \bar{\mathcal{B}}[\Pi^{o}_{\Delta,u}(q_0^2,|\vec{q}|)]_{\text{subt.}} \\
&= \frac{1}{4\pi^2}(M^2)^2 \tilde{E}_1 \langle u^\dagger u\rangle_{\rho,I} L^{\frac{4}{27}} \\
&\quad + \bar{E}_\Delta \Bigg[\frac{8}{9\pi^2} M^2 \tilde{E}_0 \langle u^\dagger i D_0 u\rangle_{\rho,I} L^{\frac{4}{27}} - \frac{1}{72\pi^2} M^2 \tilde{E}_0 \left\langle \frac{\alpha_s}{\pi}((u\cdot G)^2 + (u\cdot \tilde{G})^2)\right\rangle_{\rho,I} L^{\frac{4}{27}} \\
&\qquad\quad - \langle \bar{u}\gamma u \bar{u}\gamma u\rangle_{\text{s.t.}} L^{\frac{4}{27}} - \langle \bar{u}\gamma_5\gamma u \bar{u}\gamma_5\gamma u\rangle_{\text{s.t.}} L^{\frac{4}{27}} \Bigg],
\end{aligned}
$$

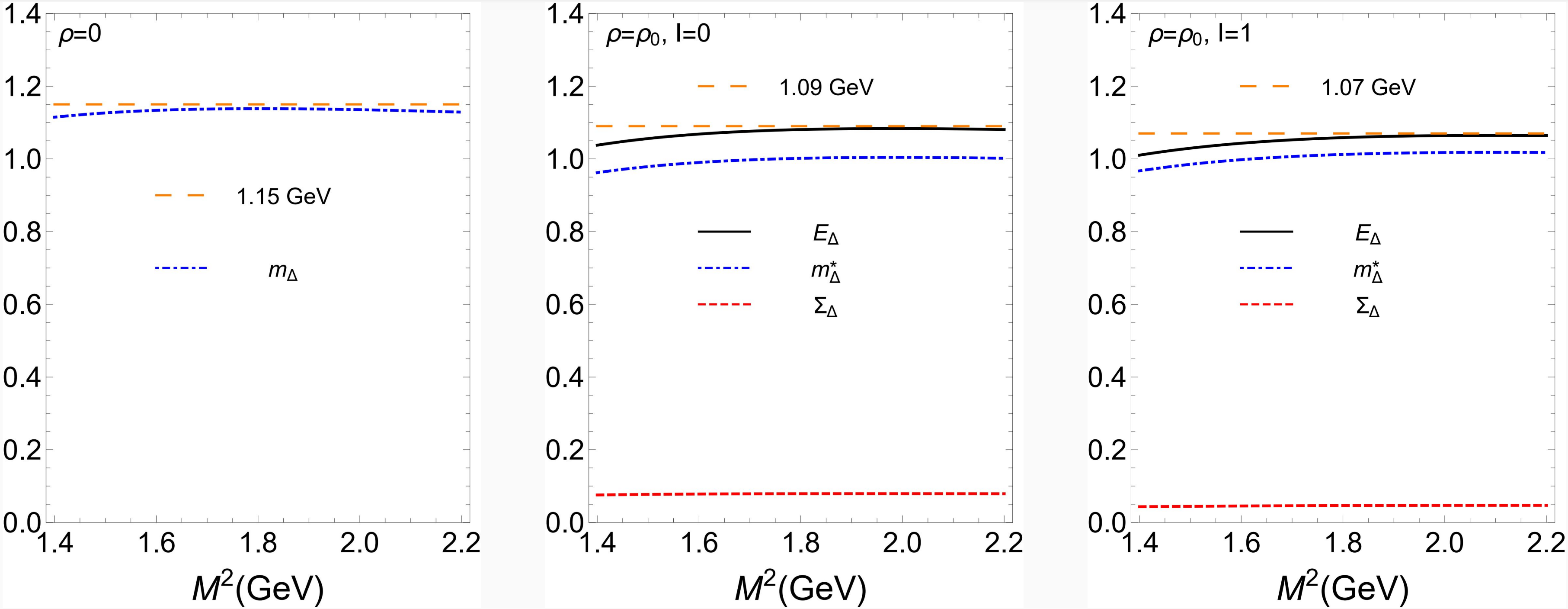
Obtaining the Mass

By simply dividing one structure by the other we can obtain the pole mass E_q

$$-\lambda_s^{*2} m_\Delta^* e^{-(m_\Delta^*+\Sigma_v)^2/M^2} = \overline{\mathcal{W}}_M^{\text{subt.}}[\Pi_{\Delta,s}(q_0^2,|\vec{q}|)],$$
$$\lambda_u^{*2} \Sigma_v e^{-(m_\Delta^*+\Sigma_v)^2/M^2} = \overline{\mathcal{W}}_M^{\text{subt.}}[\Pi_{\Delta,u}(q_0^2,|\vec{q}|)],$$
$$-\lambda_q^{*2} e^{-(m_\Delta^*+\Sigma_v)^2/M^2} = \overline{\mathcal{W}}_M^{\text{subt.}}[\Pi_{\Delta,q}(q_0^2,|\vec{q}|)].$$

$$E_q = \Sigma_v + \sqrt{\vec{q}^2 + {m_\Delta^*}^2}$$

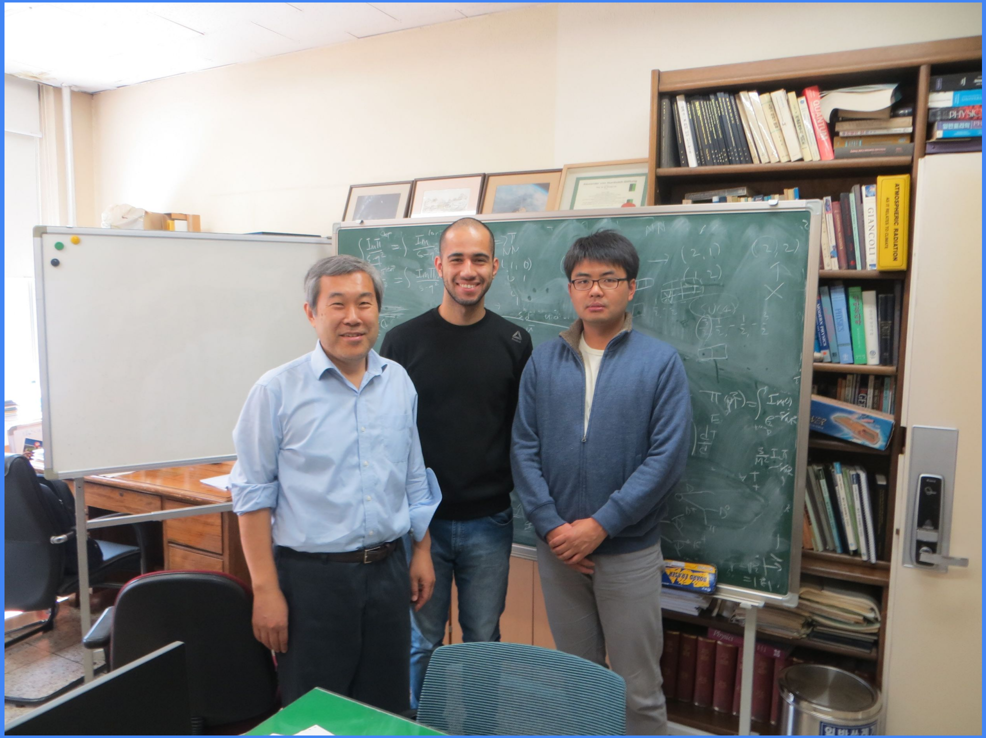
Mass Plots (in GeV)



Conclusion

- We obtained a mass for the Δ of about 1.15 GeV in the vacuum. The theoretical uncertainties are typically of 20% in QCD Sum Rules calculations, so our result is relatively good.
- We predict a reduction of 60 MeV of this mass at nuclear density for symmetric nuclear medium and a further reduction of 20 MeV in the case of neutronic matter.

Acknowledgments



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