SAIFR - ICTP Summer School 2018 Early Universe Laura Covi

Exercise Sheet 1: Standard Cosmology

Exercise 1

Compute the classical background evolution of a Friedmann-Robertson-Walker Universe with metric

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-\kappa/r^2} + r^2 d\Omega\right) \tag{1}$$

for a single fluid component. Denote $H(t) = \dot{a}(t)/a(t)$.

a) Consider first the continuity equation for a barotropic fluid with pressure $p = w\rho$, with constant w, in an expanding Universe and solve for $\rho(a)$. Recall from the lecture that the continuity equation is

$$\dot{\rho} + 3H(\rho + p) = 0 \; . \tag{2}$$

b) Insert the solution from a) into the Friedmann equation

$$H^2(t) = \frac{8\pi G_N}{3}\rho(a) - \frac{\kappa}{a^2} \; , \tag{3}$$

with $\kappa = 0$ and obtain $a(t), H(t)$ and $\rho(t)$ for general $w \neq 1$.

c) Compute the evolution of the scale factor, the Hubble parameter and the density for the special cases $w = 1/3$ (radiation), $w = 0$ (matter) and $w = -1/3$ (curvature). What happens for $w < -1/3$?

d) Solve the Friedmann equation also for the special case $w = -1$ (cosmological constant), both for the flat case and for the generic case $\kappa \neq 0$.

Exercise 2

The Planck 2015 results have measured the curvature to be consistent with $\kappa = 0$, the matter density to be $\Omega_M = 0.308 \pm 0.012$ and the redshift of matter-radiation equality to be $z_{eq} = 3365 \pm 44$. Consider a ΛCDM model with matter and radiation components and a cosmological constant as a reference to fit the Planck 2015 results.

a) Estimate the present radiation density, assuming that only the three active SM neutrinos are present at the CMB decoupling time and that they are massless.

b) Compute the present Ω_Λ and the redshift corresponding to matter-cosmological constant equality z_Λ.

c) How much does the presence of an additional relativistic neutrino with the same temperature as the present photons change z_{eq} ? How does it compare to the error on z_{eq} ? Recall that for fermionic degrees of freedom we have $\rho_F = \frac{7}{8} g_F \frac{\pi^2}{30} T^4$ and for bosonic degrees of freedom $\rho_B = g_B \frac{\pi^2}{30} T^4$, where $g_{F/B}$ are the number of internal degree of freedom.

d) How much is z_Λ affected by an additional relativistic neutrino ?