ICTP-TS/ICTP-SAIFR Summer School 2018

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Particle Physics & the Early Universe

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$[q,p]=i\hbar$

elusiVes-inVisiblesPlus
neutrinos, dark matter & dark energy physics

MARIE CURIE ACTIONS

Outline

- Lecture 1: Standard Cosmology & the cosmological parameters
- Lecture 2: Inflation & the CMB
- Lecture 3: Thermal Universe and Big Bang Nucleosynthesis
- Lecture 4: Structure Formation & Dark Matter
- Lecture 5: Baryogenesis

Lecture 3: Outline

- The inflationary paradigm
- Single field inflationary models,
  the power spectrum and the Lyth bound
- CMB & the Planck data release 2013-15
- Polarization of the CMB and primordial

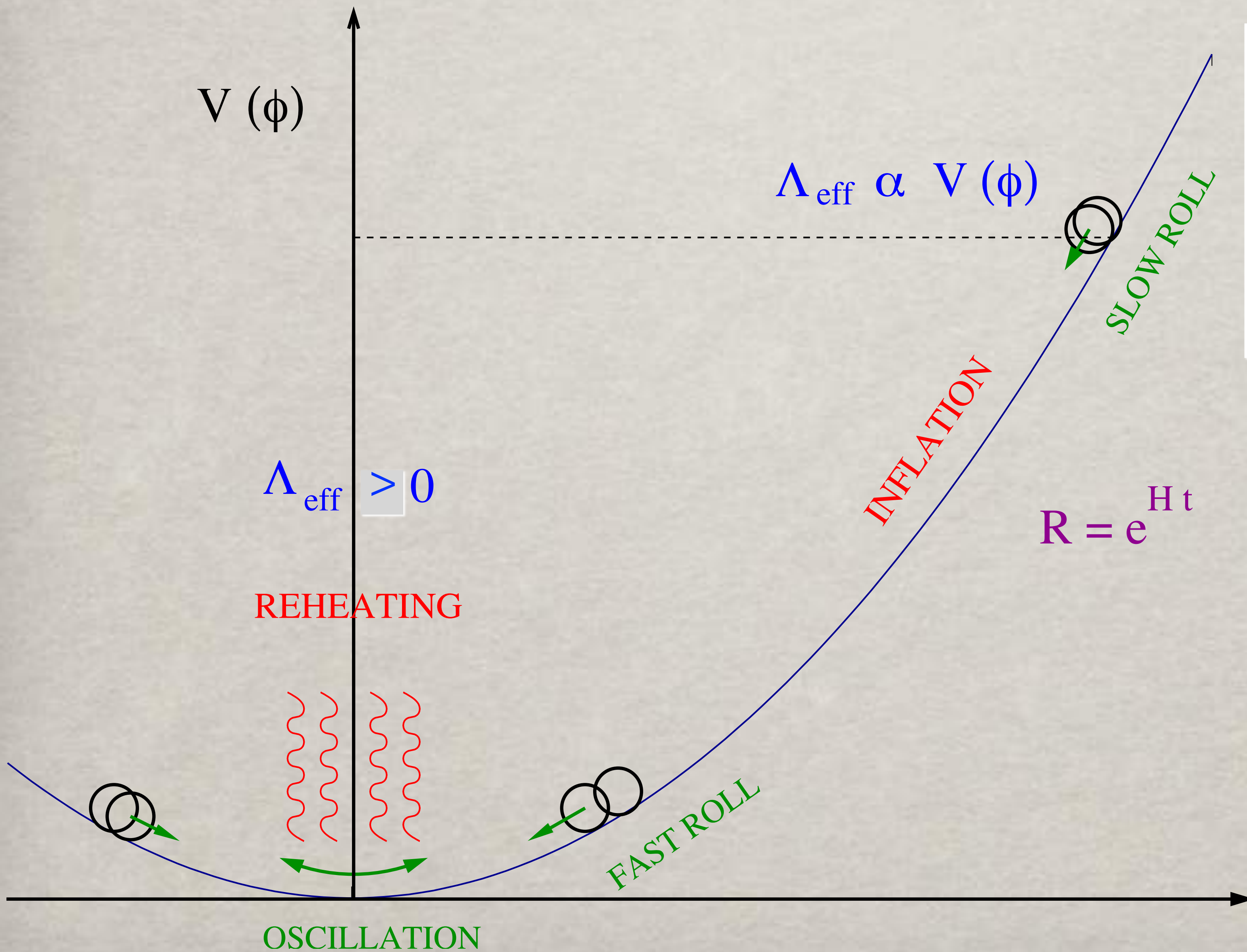
The inflationary paradigm

Where do the ripples come from?
why is the universe flat,
homogeneous & isotropic ?

INFLATION

early phase of exponential expansion

Inflation: driven by a scalar field ϕ



$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

$$|\eta| = \left|\frac{V''}{V}\right| \ll 1$$

$R = e^{H t}$ Quasi - de Sitter

de Sitter ?

Scalar Field in Cosmology

Energy-momentum tensor in Quantum Field Theory

For a general lagrangian $\mathcal{L}$ for a quantum field φ we have that the energy-momentum tensor is given by

$$T_{\mu\nu} = \frac{d\mathcal{L}}{d\partial^\mu \varphi}\partial_\nu \varphi - g_{\mu\nu}\mathcal{L}$$

so we have for example for a scalar field, when we can neglect space-gradients:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

so we have $$-1 \leq w = -1 + \frac{\dot{\phi}^2}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \leq 1$$

In particular $w = -1$ for $\dot{\phi} \simeq 0$ (slow roll) or $w \sim 0$ for $\dot{\phi}^2 \sim 2V(\phi)$ (oscillatory regime).

For a massless gauge field like the photon instead we have always $w = \frac{1}{3}$.

N.B. The Einstein equation is classical, so we should actually consider in the r.h.s. the vacuum expectation value of $T_{\mu\nu}$, but for a quantum field $\langle T_{\mu\nu} \rangle$ gets contribution from all vacuum fluctuations and *diverges* !

One part of the cosmological constant problem...

Einstein's Equations for a homogeneous scalar field

$$H^2 = \frac{1}{3M_P^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \qquad \dot{H} = \frac{\ddot{a}}{a} - H^2 = -\frac{\dot{\phi}^2}{2M_P^2}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

In general one has to solve this system of coupled equations, but things greatly simplify if we assume SLOW ROLL...: i.e. $\dot{\phi}^2 \ll V(\phi)$ and $\ddot{\phi} \ll 3H\phi$.

Then the simple solution is

$$H^2 = \frac{V(\phi)}{3M_P^2} \simeq \text{constant} \qquad \dot{H} \simeq 0 \to a \sim e^{Ht} \quad \text{INFLATION !}$$

While for the scalar field we have

$$\dot{\phi} = -\frac{V'(\phi)}{3H}\ .$$

Note that this solution is a *late-time attractor* as long as the potential is flat !

SLOW ROLL: $\epsilon = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2, |\eta| = \left|\frac{M_P^2 V''}{V}\right| \ll 1$

BUT IT IS A QUANTUM FIELD !

Apart for the classical motion, there are fluctuations:

$$\boxed{\phi = \varphi_c + \delta\varphi}$$

In an inflationary (de Sitter) phase these are given by

$$\boxed{\delta\varphi = \frac{H}{2\pi}}$$

THEY REMAIN IMPRINTED IN THE METRIC AND ARE STRETCHED TO COSMOLOGICAL SCALES !!!

Scalar field in de Sitter

For the quantum fluctuation of the field $\phi = \varphi_c + \delta\varphi$

in an inflationary (de Sitter) phase the field equation is

$$\delta\ddot{\varphi} - \nabla\delta\varphi + 3H\delta\dot{\varphi} + V''(\phi)\delta\varphi = 0$$

In conformal time & Fourier space, rescaling the field, one has

$t \to \eta = -1/aH \qquad ' \to d/d\eta \qquad u_k = a\,\delta\phi_k$

$$\boxed{u_k'' + \left(k^2 + V''(\phi)a^2 - \frac{a''}{a}\right)u_k = 0}$$

Usually the potential term is negligible and $\dfrac{a''}{a} \sim H^2 a^2$

Harmonic oscillator with negative time-dependent mass !

Scalar field in de Sitter

Later on for

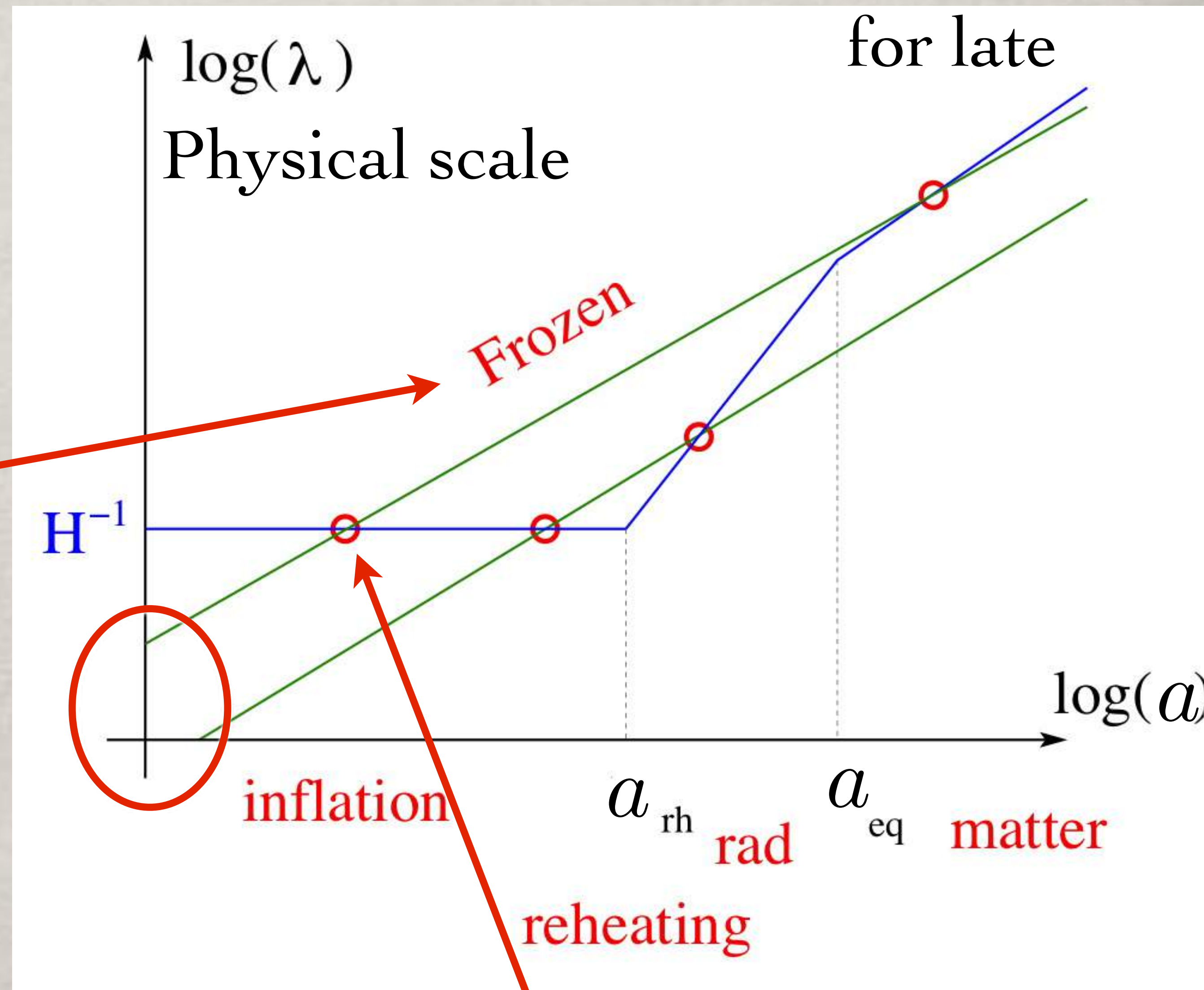
$\lambda \gg 1/H \ (k \ll aH)$

the dominant solution is the growing mode

$$u_k \propto a$$

$\rightarrow \delta\varphi_k = $ constant

Initial condition for late



Initial conditions for

$\lambda \ll 1/H \ (k \gg aH)$

as in Minkowski

Bunch-Davies vacuum

Quantum to classical transition

Primordial power spectrum

Testing inflation: Single field inflation $\Longleftrightarrow$ Flat Potential $V(\phi)$

The scalar power spectrum is given by

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{1}{12\pi^2 M_P^6} \frac{V^3}{V'^2}\bigg|_{k=aH} \quad \propto k^{n-1}$$

and its spectral index is:

$$n(k) - 1 = \frac{d\log(\mathcal{P}_{\mathcal{R}})}{d\log(k)}\bigg|_{k=aH} = 2\eta - 6\epsilon + \dots$$

For gravity waves the situation is simpler since in fact they are perfectly massless... : the gravity waves are generated by fluctuations in the metric, i.e. $h_{ij} = \delta g_{ij}$.

The tensor power spectrum is given by

$$\mathcal{P}_{grav}(k) = \frac{1}{6\pi^2} \frac{V}{M_P^4}\bigg|_{k=aH}$$

Scale of inflation

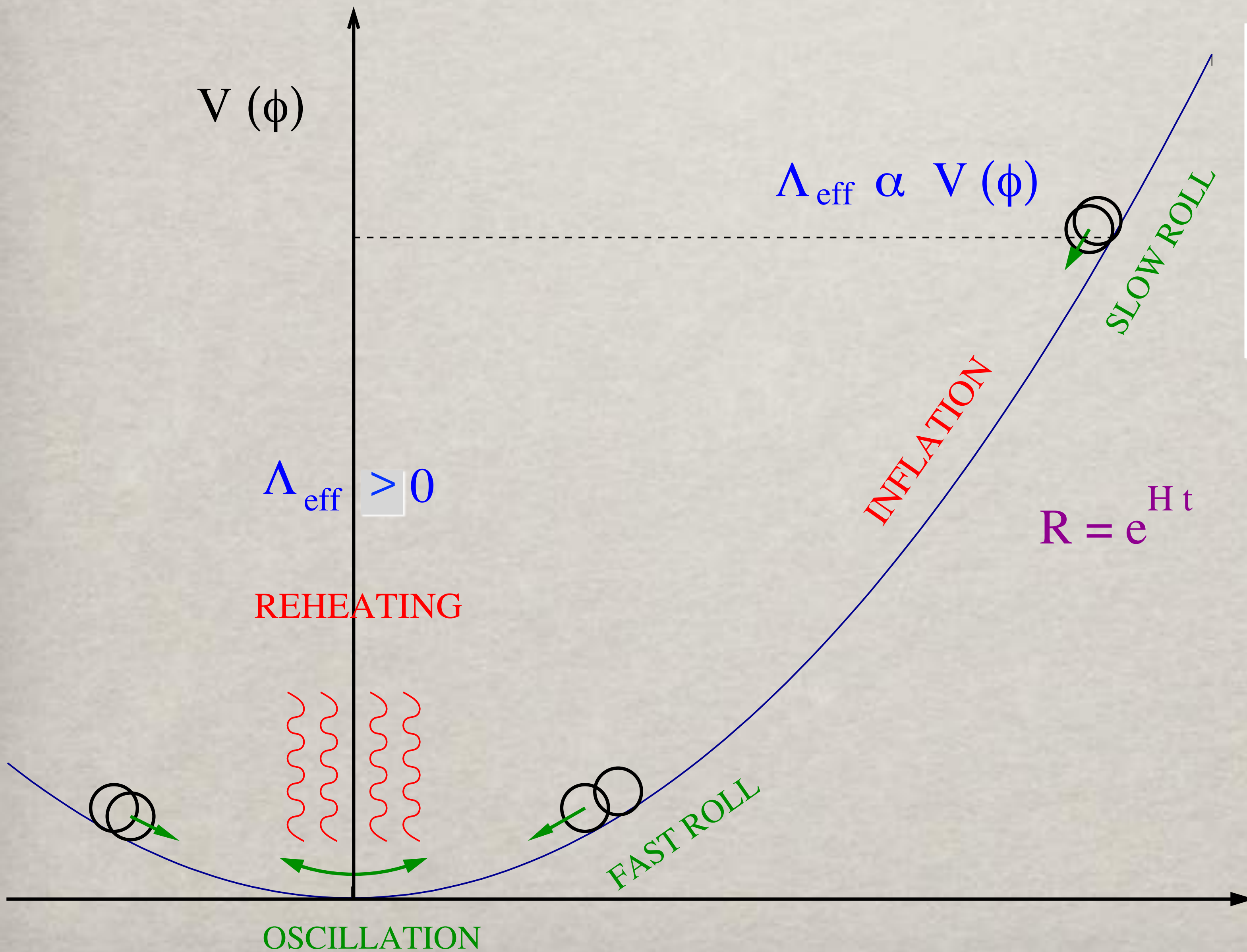
and its spectral index is

$$n_{grav}(k) = \frac{d\log(\mathcal{P}_{grav})}{d\log(k)}\bigg|_{k=aH} = -2\epsilon + \dots$$

In the simplest models the power spectrum is gaussian.

Single field inflationary models

Inflation: driven by a scalar field ϕ



$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

$$|\eta| = \left|\frac{V''}{V}\right| \ll 1$$

$R = e^{H t}$

Quasi - de Sitter

de Sitter ?

Simple monomial models

Consider the simplest possible scalar field potential:

$$V(\phi) = \lambda\phi^\alpha$$

Then the slow-roll parameters are simply:

$$\epsilon = \frac{\alpha^2}{2}\frac{M_P^2}{\phi^2} \qquad \eta = \alpha(\alpha - 1)\frac{M_P^2}{\phi^2}$$

Need large field values to realize slow roll: $\phi >> M_P$.

$$N = \int_{\phi_e}^{\phi} \frac{d\phi}{M_P}\frac{\phi}{\alpha M_P} \sim \frac{1}{2\alpha}\frac{\phi^2}{M_P^2}$$

$$\epsilon(N) = \frac{\alpha}{4N}$$

$$\eta(N) = \frac{\alpha - 1}{2N}$$

So naturally large N !

Simple monomial models

The scalar power spectrum is then given by

$$P_{\mathcal{R}} = \frac{\lambda N}{6\pi^2\alpha} M_P^{\alpha-4} (2\alpha N)^{\alpha/2} \qquad n - 1 = -\frac{\alpha+2}{2N}$$

red-tilted

Therefore to have fluctuations at the order 0.00001 we need

$$\lambda \sim 10^{-10} M_P^{4-\alpha} \frac{6\pi^2\alpha}{N(2\alpha N)^{\alpha/2}}$$

The spectrum of gravitational waves is instead given by

$$P_{grav} = \frac{\lambda}{6\pi^2} M_P^{\alpha-4} (2\alpha N)^{\alpha/2} \qquad n_{grav} = -\frac{\alpha}{2N}$$

satisfying the consistency relation $r = \dfrac{P_{grav}}{P_{\mathcal{R}}} = 16\epsilon = \dfrac{4\alpha}{N}$

The Lyth bound

The power spectrum of the gravitational waves gives fundamental informations on the inflationary model:

$$V^{1/4} \sim \left(\frac{r}{0.01}\right)^{1/4} 10^{16}\ \mathrm{GeV}$$

Scale of inflation

$$\Delta\phi \geq \left(\frac{r}{0.002}\right)^{1/2} \left(\frac{N}{60}\right) M_{Pl}$$

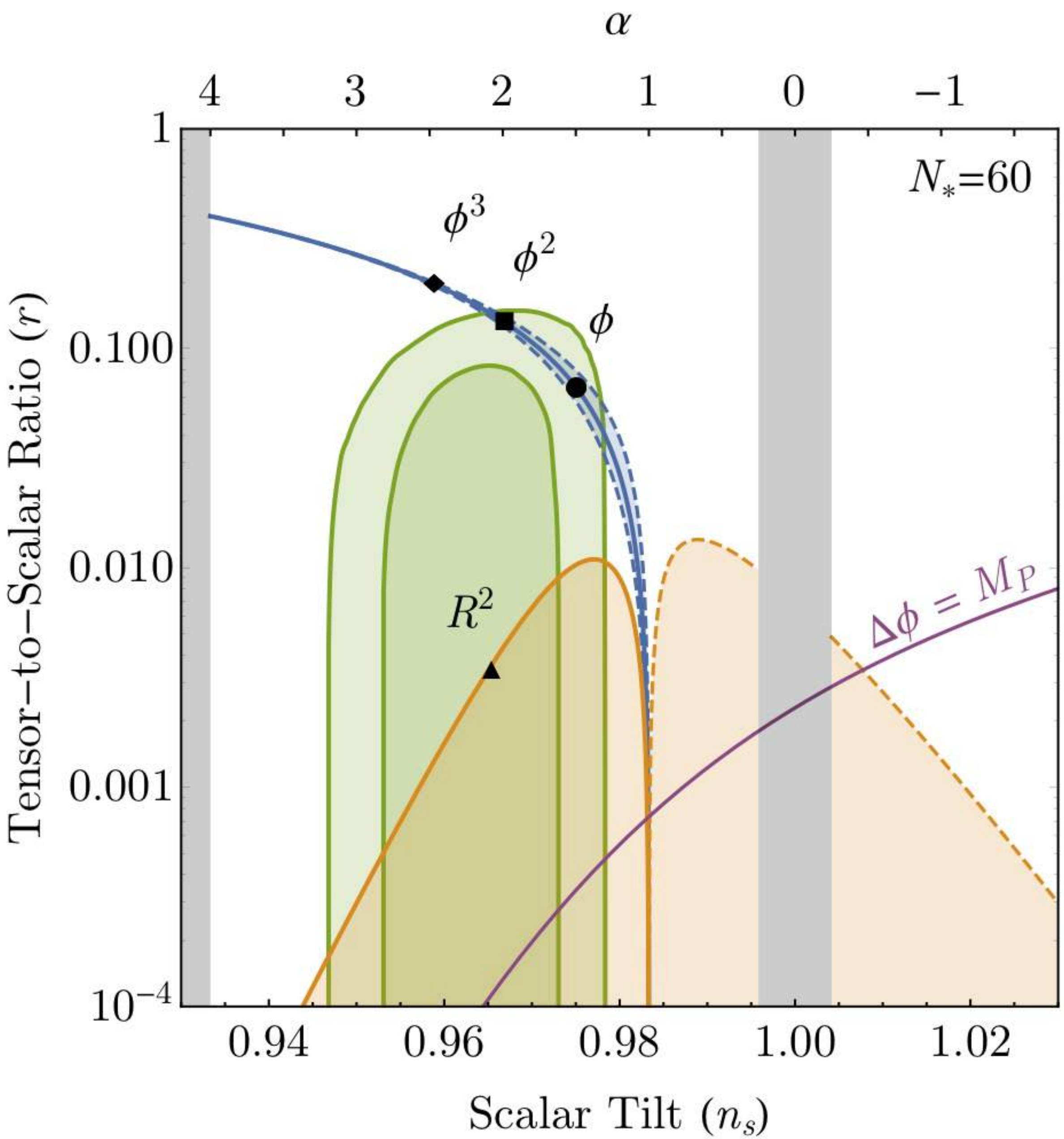
Lyth bound on the field change during N e-folds

If the tensor-to-scalar ratio is in the measurable range, the inflaton range was of the order of the Planck scale.

Zaldarriaga, Ferrara 2014

The Lyth Bound

$$\frac{d\phi}{dN} = \sqrt{2\epsilon(N)}M_{pl} = \sqrt{\frac{r(N)}{16}}M_{pl}$$



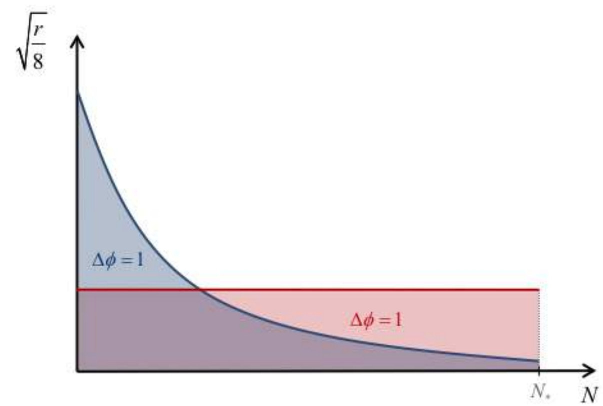


FIG. 1. *Two curves indicating* $\sqrt{r(N)/8}$. *The central idea is that both have identical areas and lead to* $\Delta\phi = 1$. *The flat curve depicts the Lyth bound, while the tilted curve indicates the improvement when taking the spectral index into account.*

1408.6839
Garcia-Bellido et al.

Inflation: driven by a scalar field ϕ

Can the Higgs do the job ? It seems a pretty obvious choice, it is the only scalar in the SM with a simple $\lambda\,\phi^4$ potential !

Unfortunately it does not work...: the normalization of the spectrum requires a coupling of the order $\lambda \sim 10^{-13}$.

From the LEP Higgs searches we know that the Higgs mass is larger than 114 GeV, so the coupling has to be larger than

$$\lambda = \frac{m_H^2}{2v^2} \geq \frac{1}{2}\left(\frac{125}{256}\right)^2 \sim 0.1$$

But this holds only for a field minimally coupled to gravity !

Higgs Inflation

[Bezukov & Shaposhnikov 09]

Couple the Higgs field non-minimally to gravity:

$$\mathcal{L}_\xi = -\frac{\xi}{2}\phi^2 R$$

The term combines with the usual Einstein-Hilbert term and changes the strength of gravity at large field:

$$(M_P^{eff})^2 = M_P^2 + \xi\,\phi^2$$

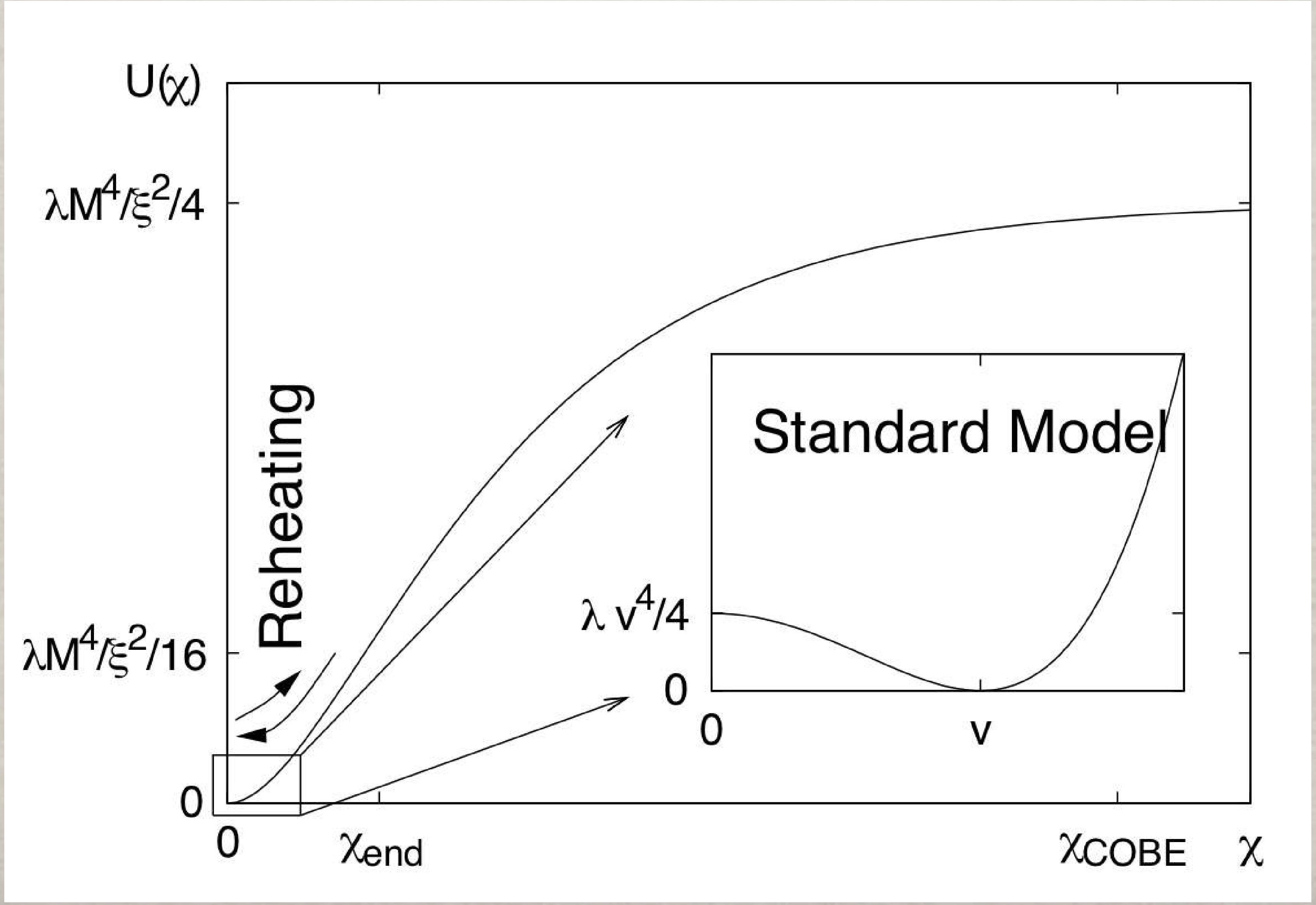
At large field values all the mass scales are proportional to the field and this can be "rescaled" away >> flat direction !
Indeed in the Jordan frame (via conformal transformation)

$$\tilde{g}_{\mu\nu} = \left(1 + \frac{\xi\phi^2}{M_P^2}\right) g_{\mu\nu} \qquad \frac{d\chi}{d\phi} = \frac{1}{\Omega}\sqrt{1 + \frac{6\xi^2\phi^2}{\Omega^2 M_P^2}}$$

Higgs Inflation

[Bezukov & Shaposhnikov 09]

In the redefined canonically normalized field the potential is:



Higgs Inflation

[Bezukov & Shaposhnikov 09]

Inflation is possible, BUT

- the normalization of the CMB power spectrum requires $\xi \sim 5 \times 10^4 \sqrt{\lambda} \gg 1$

  Very large non-minimal coupling to gravity !

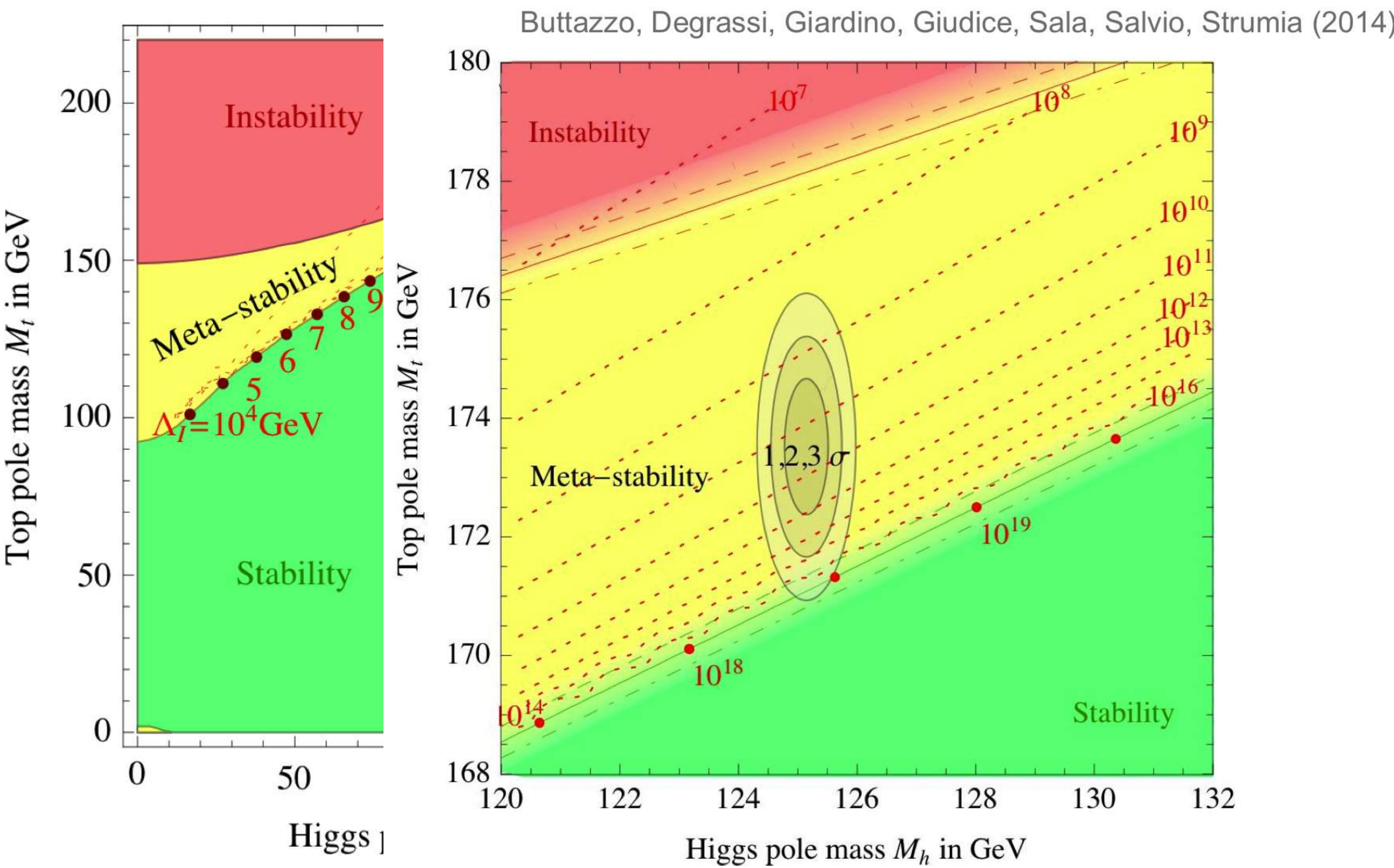
- connection to the Higgs coupling and therefore the Higgs mass as well by requiring consistency to the inflationary scale: $130\,\mathrm{GeV} \leq m_H \leq 194\,\mathrm{GeV}$
  ... now a bit on the boundary due to Higgs mass !

- Possible trouble: unitarity bound saturated at a scale

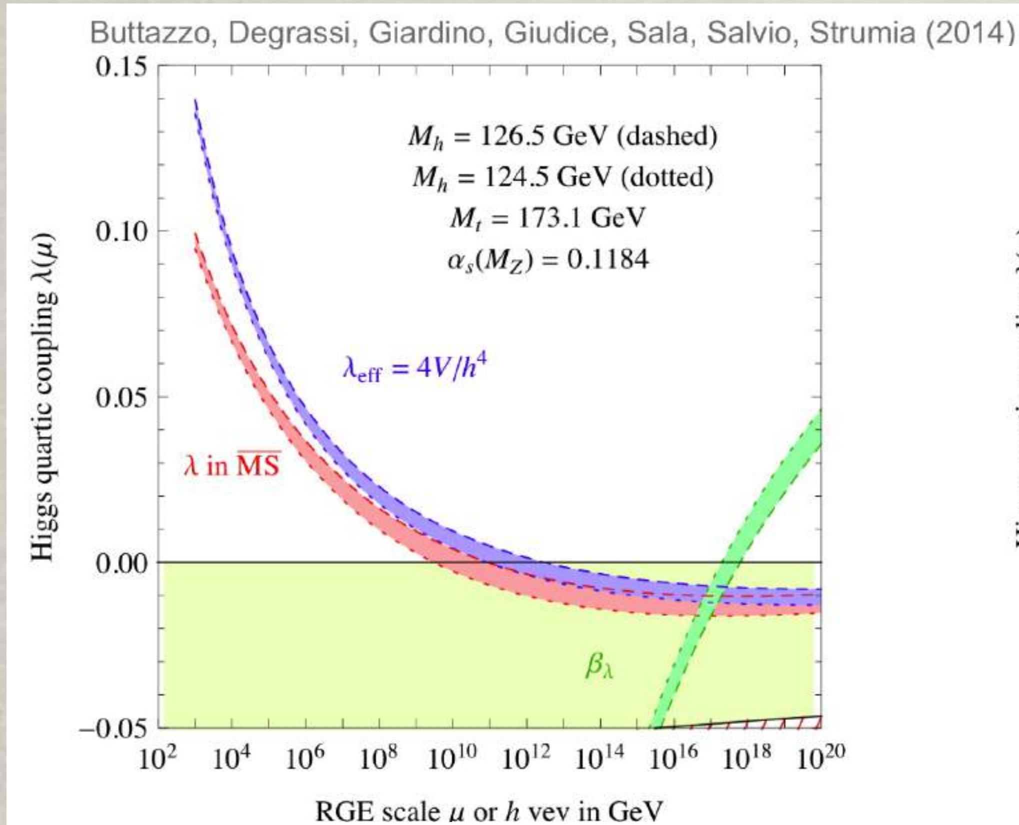
$$M_P/\sqrt{\xi} < M_P$$

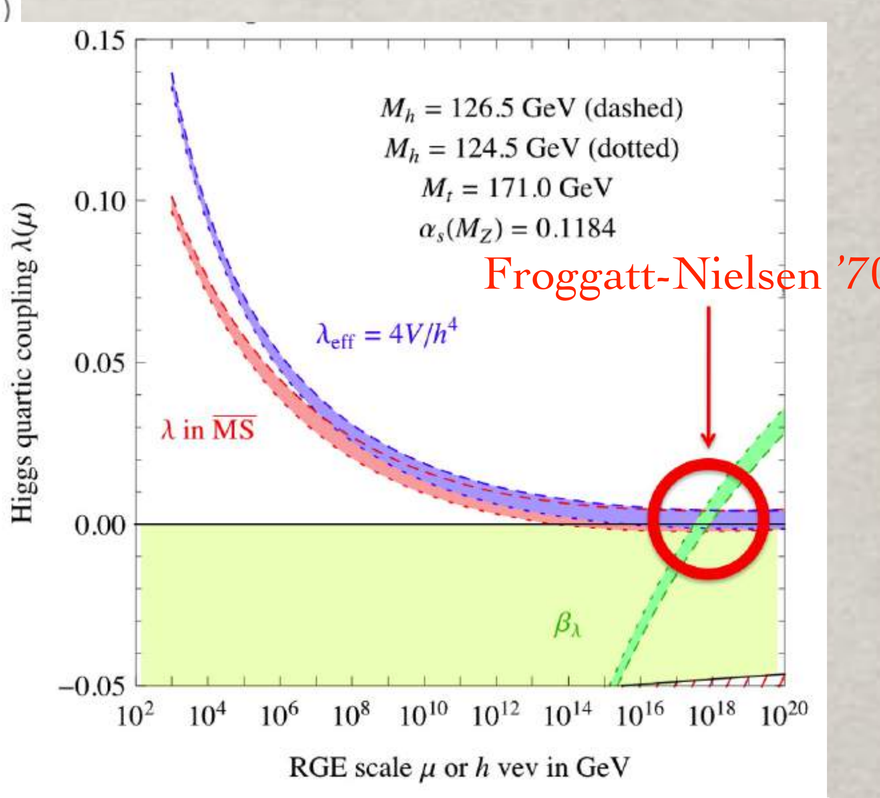
Higgs Potential at M_Pl?

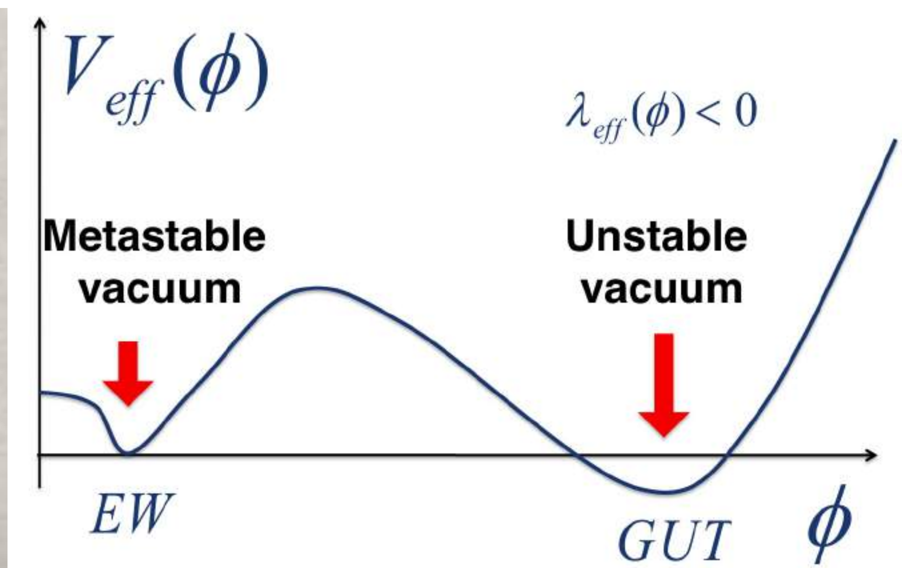
[Buttazzo & al. 14]

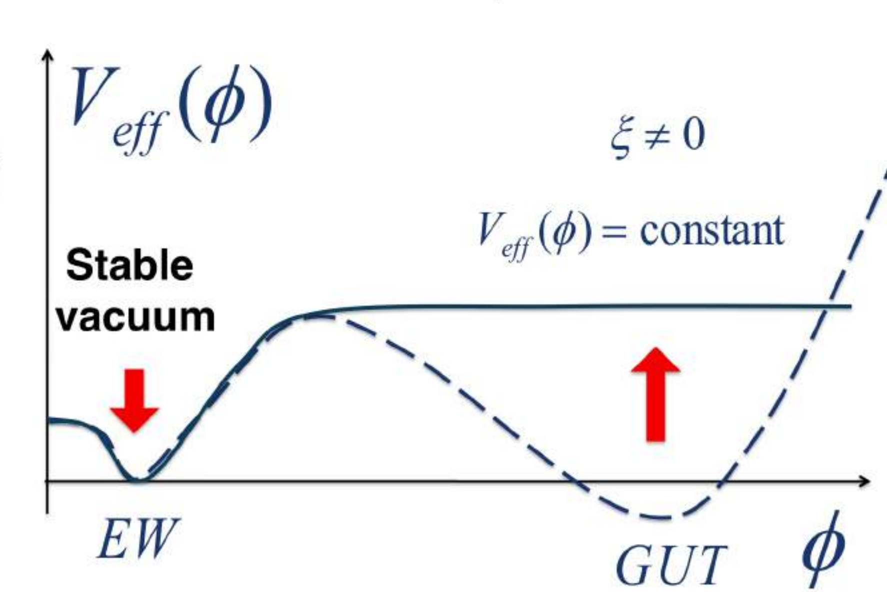


Higgs Potential at M_Pl?









References

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- Physics Report on “Particle Physics models of inflation and the cosmological density perturbations” by David H. Lyth and Antonio Riotto, arXiv:hep-ph/9807278
- Planck results on cosmological parameters/inflation: arXiv:1502.01589[astro-ph.CO] arXiv:1502.02114[astro-ph.CO]