LECTURE 1

The Standard Model of Electroweak Interactions

Gauged symmetry group $SU(2)\times U(1)$ ← called hypercharge (denote hypercharges by "Y")

SSB: $SU(2)\times U(1) \to U(1)_{em}$

$SU(2)$ generators T^a; on fundamental $\to \frac{1}{2}\sigma^a$ (σ^a = Pauli); 2-dimensional, "doublet"

$U(1)$ generator acting on doublet, $Y\mathbb{1}_2$ ← 2×2 identity; Y ← hypercharge of particular doublet.

$U(1)_{em}$: generator is linear combination of T^a and Y that is not broken (leaves vacuum invariant).

Higgs field: $H(x)$ is a doublet with $Y=\frac{1}{2}$ (write $2_{1/2}$). VEV breaks symmetry. By gauge transformation free to choose $\langle H\rangle = \begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}$

Now $\left(\frac{1}{2}\tau^3 + Y\mathbb{1}_2\right)\langle H\rangle = \left(\frac{1}{2}\tau^3+\frac{1}{2}\mathbb{1}\right)\langle H\rangle = \begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix} = 0$

So the generator $Q = T^3+Y$ is unbroken; it is the "charge" of fields under $U(1)_{em}$. (electromagnetic charge).

Let's see how this works.

Lepton fields: $l_L(x) : 2_{-1/2}$ $\quad e_R = 1_{-1}$ $\quad$ (here $\psi_{L,R} = \left(\frac{1\mp\gamma_5}{2}\right)\psi_{L,R}$ $\quad \frac{(1\pm\gamma_5)}{2}\psi_{L,R} = 0$).

Note different reps for L vs R fields: a "chiral" gauge theory.

Now Q acting on l_L is $Q = \frac{1}{2}\sigma^3 - \frac{1}{2}\mathbb{1} = \begin{pmatrix}0&0\\0&-1\end{pmatrix}$, or $l_L = \begin{pmatrix}\nu_L \leftarrow \text{charge } 0\\ e_L \leftarrow \text{charge } -1\end{pmatrix}$

" " e_R " $Q = 0-1=-1$ $\quad$ $e_R \leftarrow$ charge -1

e_L & e_R form e, electron 4-component spinor field.

Quarks: $q_L: 2_{1/6}$ $\quad u_R: 1_{2/3}$ $\quad d_R: 1_{-1/3}$

$Q q_L = \left(\frac{1}{2}\sigma^3 + \frac{1}{6}\mathbb{1}\right) q_L = \begin{pmatrix}2/3&0\\0&-1/3\end{pmatrix} q_L$ $\quad q_L = \begin{pmatrix}u_L\\ d_L\end{pmatrix}$ with charge $\begin{matrix}2/3\\-1/3\end{matrix}$ corresponding to u_R and d_R:

u_L & u_R form u, "up-quark", d_L & d_R form down-quark, 4 component spinors

Note: no partner of ν_L → remains 2 component (will result in massless field).

SUMMARY:

$H: 2_{1/2}$	$q_L: 2_{1/6}$	$Q = T^3+Y$ $\quad$ (for $\langle H\rangle = \begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}$)
$l_L: 2_{-1/2}$	$u_R: 1_{2/3}$	
$e_R: 1_{-1}$	$d_R: 1_{-1/3}$	

☆ Exercise: re-do with choice $\langle H\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$. What changes?

Mass generation I: quarks and leptons

$\mathcal{L} = \sum_{\psi} \bar{\psi} i \not{D} \psi - V$ Want to construct gauge invariant terms in V.

Note $\bar{\psi}_L \chi_L = 0$ or $\bar{\psi}_R \chi_R = 0$, for any two fields ψ & χ.

Gauge invariant mass (quadratic) terms? → None!
Need $\bar{\psi}_L \chi_R$ but all ψ_L are doublets and χ_R are singlets!

(Pedantic: the charge conjugate fields have opposite chirality, so we could have also, eg, $q_R^c: 2_{-1/6}$ $u_L^c: 1_{-2/3}$ $d_L^c: 1_{1/3}$. Even then can't find quadratic invariants).

Cubic invariants: Yukawa terms, ie, terms of the form $\phi \bar{\psi}_L \chi_R$ ← scalar
Just quantum numbers: $2\times 2 = 1+3$ (or $\frac{1}{2}\times\frac{1}{2} = 0+1$, from spin in your QM course).
and Y is additive. So $V = -\mathcal{L}_{Yuk}$

$H (l_L)^* e_R$ has $2\times 2\times 1$ ✓ and $\frac{1}{2} - (-\frac{1}{2}) + (-1) = 0$ ✓ $\longrightarrow y^E \bar{l}_L H e_R$ + h.c.

$H (q_L)^* d_R$ ✓✓ ✓ $\frac{1}{2} - (\frac{1}{6}) + (-\frac{1}{3}) = 0$ ✓ $y^D \bar{q}_L H u_R$ + h.c.

$H^* (q_L)^* u_R$ ✓✓ ✓ $(-\frac{1}{2}) + (-\frac{1}{6}) + \frac{2}{3} = 0$ ✓ ?

If $H \to UH$ and $q_L \to U q_L$ under $SU(2)$ $\Rightarrow$ $\bar{q}_L H \to \bar{q}_L U^\dagger U H = \bar{q}_L H$

but H^*? $H^* \to U^* H^*$ But there is a matrix S ∋ $S U^* S^{-1} = U$. Then if $\tilde{H} = S H^*$ we have $\tilde{H} \to S U^* H^* = S U^* S^{-1} S H^* = U \tilde{H}$, and $Y(\tilde{H}) = -Y(H)$.
In fact, we can take $S = i\sigma^2$, since $i\sigma^2 [\exp(i\omega^a \sigma^a)]^* (i\sigma^2)^{-1} = \exp(i\omega^a \sigma^a)$ and $\tilde{H} = i\sigma^2 H^*$.

Then above: for u_R term $y^U \bar{q}_L \tilde{H} u_R$ + h.c.

Notation: often exchange order, eg, $y^E H \bar{l}_L e_R$ etc, with correct index contraction understood.

When $H = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} + \cdots$ we obtain (assuming y's are real)

$$-\mathcal{L}_{Yuk} = \frac{y^E v}{\sqrt{2}} \bar{e} e + \frac{y^D v}{\sqrt{2}} \bar{d} d + \frac{y^U v}{\sqrt{2}} \bar{u} u \quad \Rightarrow \quad m_e = \frac{v}{\sqrt{2}} y^E \quad m_d = \frac{v}{\sqrt{2}} y^D \quad m_u = \frac{v}{\sqrt{2}} y^U$$

Vector bosons: denote $D_\mu = \partial_\mu + i g_1 Y B_\mu + i g_2 T^a W_\mu$

Let
$$\sigma^\pm = \frac{\sigma^1 \pm i\sigma^2}{\sqrt{2}} \qquad W^\pm = \frac{W^1 \mp i W^2}{\sqrt{2}}$$

Note: $W_\mu = W^a \sigma^a$ transforms as $W_\mu \to U W_\mu U^\dagger$. If $U = e^{i\omega Q} = e^{i\omega(T^3+Y)} \to e^{i\omega\frac{1}{2}\sigma^3}$

then $e^{i\omega\frac{1}{2}\sigma^3}\sigma^\pm e^{-i\omega\frac{1}{2}\sigma^3} = \sigma^\pm + \frac{i\omega}{2}[\sigma^3, \sigma^\pm] + \cdots$

And $[\sigma^3, \sigma^\pm] = \frac{1}{\sqrt{2}}[\sigma^3, \sigma^1] \pm \frac{i}{\sqrt{2}}[\sigma^3, \sigma^2] = \frac{1}{\sqrt{2}}\left(2i\sigma^2 \pm i(-2i\sigma^1)\right) = \pm 2\sigma^\pm$

so each additional $\frac{\omega}{2}[\sigma^3, -]$ becomes a $\pm\omega$

$$\Rightarrow \quad U\sigma^\pm U^\dagger = e^{\pm i\omega}\sigma^\pm$$

Now $\sigma^+ W^+ + \sigma^- W^- = \frac{1}{2}(\sigma^1 + i\sigma^2)(W^1 - iW^2) + \frac{1}{2}(\sigma^1 - i\sigma^2)(W^1 + iW^2) = \sigma^1 W^1 + \sigma^2 W^2$

so with this definition $W^\pm$ are fields of charge $Q = \pm 1$.

So we write $D_\mu = \partial_\mu + i g_1 Y B_\mu + i g_2 (T^3 W^3_\mu + T^+ W^-_\mu + T^- W^+_\mu)$.

Some combination of B_μ and W^3_μ comes with generator Q. This will correspond to the photon A_μ. The orthogonal combination is a massive vector called Z_μ:

$$\begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} \Rightarrow \begin{aligned} &g_1 Y B_\mu + g_2 T^3 W^3_\mu \\ &= (g_1\cos\theta\, Y + g_2 \sin\theta\, T^3) A_\mu + (g_1 \sin\theta\, Y - g_2\cos\theta\, T^3) Z_\mu \end{aligned}$$

1^{st} term $= eQA_\mu = e(T^3 + Y)A_\mu \Rightarrow g_1\cos\theta = g_2\sin\theta = e \left(\text{so: } \sin\theta = \frac{g_1}{\sqrt{g_1^2+g_2^2}}\right)$

Then 2^{nd} term
$$\sqrt{g_1^2+g_2^2}\,(\sin^2\theta\, Y - \cos^2\theta\, T^3) Z_\mu = \sqrt{g_1^2+g_2^2}\,(\sin^2\theta\, Q - T^3) Z_\mu$$

eg, for components of a doublet $\sqrt{g_1^2+g_2^2}\,(\sin^2\theta\, Q - \frac{1}{2}\sigma^3) Z_\mu$, for singlet $\sqrt{g_1^2+g_2^2}\,\sin^2\theta\, Q$

Exercise: write explicitly the gauge interaction terms in $\sum \bar\Psi i \not{D} \psi$ where the sum runs over $\psi = l_L, e_R, q_L, u_R, d_R$.

Mass generation II: vector bosons

Know they arise from $|D_\mu H|^2$:

$$\mathcal{L}_{M^2} = (D_\mu\langle H\rangle)^\dagger D_\mu\langle H\rangle = \left|\left(i g_1 \tfrac{1}{2} B_\mu + i g_2\left(\tfrac{\sigma^3}{2}W^3_\mu + \tfrac{1}{2}(\sigma^+ W^- + \sigma^- W^+)\right)\right)\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}\right|^2$$

Use $\sigma^+ = \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}$ $\sigma^- = \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix}$ $\sigma^-\sigma^+ = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$ $\sigma^+\sigma^- = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$ etc

$$\Rightarrow \mathcal{L}_{M^2} = \frac{1}{2}v^2\left(\frac{g_2}{2}\right)^2 2 W^-_\mu W^+_\mu + \frac{1}{2}v^2\left(\frac{1}{2}\right)^2 (g_1 B_\mu - g_2 W^3_\mu)^2 = \frac{1}{4}g_2^2 v^2 W^+_\mu W^-_\mu + \frac{1}{8}v^2(g_1^2+g_2^2) Z_\mu Z_\mu$$

where we have used $\begin{pmatrix} A \\ Z \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} B \\ W^3 \end{pmatrix}$ $\quad M_W^2 = \frac{1}{4}g_2^2 v^2 \quad M_Z^2 = \frac{1}{4}(g_1^2+g_2^2)v^2 \Rightarrow M_W = M_Z\cos\theta$

The higgs: in unitary gauge $H(x) = e^{i\alpha^a(x)T^a + i\alpha^0(x)Y}\begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}}\end{pmatrix}$

(note that the combination with $Q = T^3 + Y$ does not enter) then h is a massive, real field.

Take $\mathcal{L} = (D_\mu H)^\dagger (D_\mu H) - \frac{1}{4}\lambda\left(H^\dagger H - \frac{1}{2}v^2\right)^2$

then in unitary gauge this is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{4}g_2^2(v+h)^2 W^+W^- + \frac{1}{8}(g_1^2+g_2^2)(v+h)^2 Z^2 - \frac{1}{16}\lambda\left(h^2 + 2hv\right)^2$$

We used our previous result for W/Z masses; we now see, in addition, $\mathcal{L} = \dots - \frac{1}{4}\lambda v^2 h^2 \Rightarrow m_h^2 = \frac{1}{2}\lambda v^2$

Parameters:

$$\lambda, v, g_1, g_2 \quad \Longleftrightarrow \quad M_W, M_Z, e \quad m_h^2 \Leftarrow \text{physical}$$

To see $\Leftarrow$, w/ from $\cos\theta = M_W/M_Z$ and $g_1\cos\theta = e$ get g_1 and g_2

then get v from, e.g. $\frac{1}{2}g_2 v = M_W$ and λ from m_h.

There are infinitely many choices of 4 independent measurable parameters that can be traded for the unphysical parameters λ, v, g_1, g_2.

PROJECT

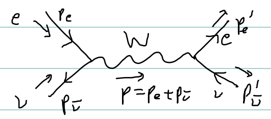
Since we have not looked at generations, assume all we have is one, that is one each of q_L, u_R, d_R, l_L & e_R (no muons, strange quarks, etc). Compute the total width of the Z, Γ_Z (this is the inverse of the life-time). Do this at tree level only. Then compute the Branching fractions into up quarks, down-quarks, electrons and neutrinos.

Low energy interactions, ρ parameter, custodial symmetry

Fermi-theory: obtained at low momentum exchange of vectors

Charged currents: exchange of W's

$$\bar{l}_L i\not{\partial} l_L \to \bar{l}_L i\left(i\tfrac{g_2}{2}(\sigma^+ \not{W}^- + \sigma^- \not{W}^+)\right) l_L = -\frac{g_2}{\sqrt{2}}\left(\bar{\nu}_L \not{W}^- e_L + \bar{e}_L \not{W}^+ \nu_L\right)$$



$$= \left(-\frac{i g_2}{\sqrt{2}}\right)^2 \bar{v}(p_{\bar\nu})\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)u(p_e)\; -i\,\frac{\eta_{\mu\nu} - \cancel{p_\mu p_\nu/M_W^2}}{\cancel{p^2} - M_W^2}\; \bar{u}(p_e')\gamma_\nu\left(\frac{1-\gamma_5}{2}\right)v(p_{\bar\nu}')$$

$$= -i\,\frac{\tfrac{1}{2} g_2^2}{M_W^2}\, J_\mu J_\mu'^{\dagger}$$

For future reference

$$\tfrac{1}{2}\, g_2^2/M_W^2 = \tfrac{1}{2}\, g_2^2/\tfrac{1}{4}g_2^2 v^2 = 2/v^2 \quad \text{, independent of } g_2 .$$

Fermi constant G_F: Fermi wrote $\mathcal{H}' = 4G_F/\sqrt{2}\; J_\mu J^{\dagger\mu} = \frac{G_F}{\sqrt{2}}\bar{\nu}\gamma^\mu(1-\gamma_5)e\;\bar{e}\gamma_\mu(1-\gamma_5)\nu$

\+ Marshak/Sudarshan

\+ Feynman/Gell-Mann

$$= \frac{G_F}{\sqrt{2}}\text{ “}V-A \otimes V-A\text{”}$$

Comparing: $\frac{4G_F}{\sqrt{2}} = \frac{2}{v^2} \Rightarrow \boxed{v^2 = \frac{1}{\sqrt{2}G_F}}$

$G_F = 1.169\times 10^{-5}\ \text{GeV}^{-2} \Longrightarrow v = 246\ \text{GeV}$.

☆ Exercise: Muon lifetime (spell out in exercises handout)

Neutral currents:

$$\bar{l}_L i\left(i\sqrt{g_1^2+g_2^2}\right)\left(\sin^2\theta\, Q - \tfrac{1}{2}\sigma^3\right)\not{Z} l_L = -\sqrt{g_1^2+g_2^2}\left(-\tfrac{1}{2}\bar{\nu}_L\gamma^\mu\nu_L + \left(\tfrac{1}{2}-\sin^2\theta\right)\bar{e}_L\gamma^\mu e_L\right) Z_\mu$$

$$\bar{e}_R i\left(i\sqrt{g_1^2+g_2^2}\right)\left(\sin^2\theta\, Q - 0\right)\not{Z} e_R = -\sqrt{g_1^2+g_2^2}\left(-\sin^2\theta\right)\bar{e}_R\gamma^\mu e_R\, Z_\mu$$

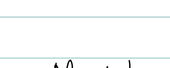
$$\text{sum} = -\tfrac{1}{2}\sqrt{g_1^2+g_2^2}\left[-\bar{\nu}_L\gamma^\mu\nu_L + \bar{e}\left[\gamma^\mu\left(\tfrac{1-\gamma_5}{2}\right) - 2\sin^2\theta\,\gamma^\mu\right]e\right]Z_\mu$$

Sometimes $\propto \bar{e}\,(c_V\gamma^\mu + c_A\gamma^\mu\gamma_5)\,e$ with $c_V = 1-4\sin^2\theta \quad c_A = -1$

so $\mathcal{L} = -\frac{1}{4}\sqrt{g_1^2+g_2^2}\left[-\bar{\nu}\gamma^\mu(1-\gamma_5)\nu + \bar{e}\,(c_V\gamma^\mu + c_A\gamma^\mu\gamma_5)e\right]Z_\mu$

More generally $(2\sin^2\theta\, Q - \sigma^3)\gamma^\mu(1-\gamma_5) + 2\sin^2\theta\, Q\,\gamma^\mu(1+\gamma_5) \Rightarrow c_V = 4\sin^2\theta\, Q - \sigma^3 \quad c_A = \sigma^3$

where the σ^3 refers to the left handed field, of course.



$$\left(-\tfrac{i}{4}\sqrt{g_1^2+g_2^2}\right)^2 \frac{(-i)\,\eta_{\mu\nu}}{-M_Z^2}\;\bar{v}(p_\nu')\gamma^\mu(1-\gamma_5)u(p_\nu)\;\bar{u}(p_q)\left[c_V\gamma^\mu + c_A\gamma^\mu\gamma_5\right]v(p_q)$$

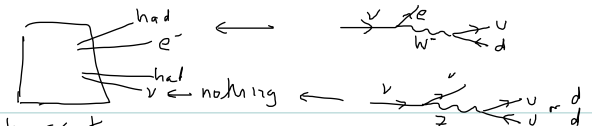
$$= -\frac{i}{8}\,\frac{(g_1^2+g_2^2)}{\tfrac{1}{4}(g_1^2+g_2^2)v^2}\; J^\mu_{n,\nu} J^\mu_{n,q} = -i\,\frac{2}{v^2}\left(\frac{1}{4}J_n^\mu J_n^\mu\right)$$

(the $\frac{1}{4}$ is to compare with charged currents which have $(1-\gamma_5)/2$ factors).

(the 2 from $(J_\nu + J_n)^2$ cross-term).

Modulo $c_{V,A} \neq 1$ this is just like W exchange!

NC/CC : $\overrightarrow{\nu \text{ beam}}$



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or

$$\rho = \frac{NC}{CC} = \frac{\left(\sqrt{g_1^2+g_2^2}\right)^2 \frac{1}{M_Z^2}}{g_2^2 \frac{1}{M_W^2}} = \frac{M_W^2}{\cos^2\theta M_Z^2} = 1$$

To see why this is interesting, take a generalization:

$$\mathcal{L}_{M^2} = M_W^2 W^+ W^- + \frac{1}{2}\left(M_3^2 W_3^2 + 2M_{3B}^2 W_3 B + M_B^2 B^2\right)$$

Note that in order to have a massless photon (zero eigenvalue of $\begin{pmatrix} M_3^2 & M_{3B}^2 \\ M_{3B}^2 & M_B^2 \end{pmatrix}$) need $M_{3B}^2 = M_3 M_B$

Second eigenvalue is $M_Z^2 = M_3^2 + M_B^2$. As before

$$\begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} \qquad \cos\varphi = \frac{M_3}{M_Z} \qquad \sin\varphi = \frac{M_B}{M_Z}$$

Now $g_2 W_3 J_3 + g_1 B J_B$. In order to have A_μ couple to $Q = T^3 + Y$ need

$g_2 \sin\varphi = g_1 \cos\varphi$ as before. So $g_2 W_3 J_3 + g_1 B J_B = \left(g_2 \cos\varphi J_3 - g_1 \sin\varphi J_B\right) Z$

$$= g_2\left(\cos\varphi J_3 - \frac{\sin^2\varphi}{\cos\varphi} J_B\right) Z$$

$$= \frac{g_2}{\cos\varphi}\left(\cos^2\varphi T^3 - \sin^2\varphi (Q - T^3)\right) = \frac{g_2}{\cos\varphi}\left(T^3 - \sin^2\varphi Q\right)$$

so again $\rho = \frac{NC}{CC} = \frac{\left(\frac{g_2}{\cos\varphi}\right)^2 \frac{1}{M_Z^2}}{g_2^2 \frac{1}{M_W^2}} = \frac{M_W^2}{M_3^2} \equiv 1 - \Delta\rho \qquad \Delta\rho = \frac{M_W^2 - M_3^2}{M_3^2} \qquad \frac{\Delta\rho}{\rho} = \frac{M_W^2 - M_3^2}{M_W^2}$

Experimentally $\Delta\rho < 1\%$. How can $\Delta\rho$ be non-zero?

Suppose we use a representation other than $2_{1/2}$ for H.

Take case H is a spin-1 (ie, dim 3) rep. with $\langle H \rangle = \begin{pmatrix} 0 \\ 0 \\ v/\sqrt{2} \end{pmatrix}$

Start from $T^1 = T^{23} = i\begin{pmatrix} 0&0&0 \\ 0&0&1 \\ 0&-1&0 \end{pmatrix}$, $T^2 = T^{31} = i\begin{pmatrix} 0&0&-1 \\ 0&0&0 \\ 1&0&0 \end{pmatrix}$ $\quad T^3 = T^{12} = -i\begin{pmatrix} 0&1&0 \\ -1&0&0 \\ 0&0&0 \end{pmatrix}$ $\quad$ (This is just Adj: $(T^a)^{bc} = -i\epsilon^{abc}$)

diagonalize T^3: $-i\begin{pmatrix} 0&1&0 \\ -1&0&0 \\ 0&0&0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = (\pm 1, 0)\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow$ i) $\left.\begin{array}{l} -iy = \pm x \\ ix = \pm y \end{array}\right\}$ same $\quad y = \mp i x$, $z = 0$ $\quad$ ii) $x = y = 0 \quad z = 1$

$$\Rightarrow S^{-1} = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -i/\sqrt{2} & 0 & i/\sqrt{2} \\ 0 & 1 & 0 \end{pmatrix} \Rightarrow S\frac{T^3}{-i}S^{-1} = \begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & -i/\sqrt{2} & 0 \end{pmatrix}\begin{pmatrix} 0&1&0 \\ -1&0&0 \\ 0&0&0 \end{pmatrix} S^{-1} = \begin{pmatrix} -i/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0&0&0 \\ i/\sqrt{2} & 1/\sqrt{2} & 0 \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -i/\sqrt{2} & 0 & i/\sqrt{2} \\ 0&1&0 \end{pmatrix} = \begin{pmatrix} -i & 0 & 0 \\ 0&0&0 \\ 0&0&i \end{pmatrix}$$

or $ST^3S^{-1} = \text{diag}(1, 0, -1)$

$$\text{Then } ST^1S^{-1} = \begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} & 0 \\ 0&0&1 \\ 1/\sqrt{2} & -i/\sqrt{2} & 0 \end{pmatrix}(-i)\begin{pmatrix} 0&0&0 \\ 0&0&1 \\ 0&-1&0 \end{pmatrix} S^{-1} = -i\begin{pmatrix} 0&0&i/\sqrt{2} \\ 0&-1&0 \\ 0&0&-i/\sqrt{2} \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -i/\sqrt{2} & 0 & i/\sqrt{2} \\ 0&1&0 \end{pmatrix} = -i\begin{pmatrix} 0 & i/\sqrt{2} & 0 \\ i/\sqrt{2} & 0 & -i/\sqrt{2} \\ 0 & -i/\sqrt{2} & 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0&1&0 \\ 1&0&-1 \\ 0&-1&0 \end{pmatrix}$$

$$ST^2S^{-1} = \begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} & 0 \\ 0&0&1 \\ 1/\sqrt{2} & -i/\sqrt{2} & 0 \end{pmatrix}(-i)\begin{pmatrix} 0&0&-1 \\ 0&0&0 \\ 1&0&0 \end{pmatrix} S^{-1} = -i\begin{pmatrix} 0&0&-1/\sqrt{2} \\ 1&0&0 \\ 0&0&-1/\sqrt{2} \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -i/\sqrt{2} & 0 & i/\sqrt{2} \\ 0&1&0 \end{pmatrix} = -i\begin{pmatrix} 0 & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0&i&0 \\ -i&0&-i \\ 0&i&0 \end{pmatrix}$$

$T^\pm = \frac{T^1 \pm iT^2}{\sqrt{2}} = \begin{pmatrix} 0&0&0 \\ 1&0&0 \\ 0&-1&0 \end{pmatrix}, \begin{pmatrix} 0&1&0 \\ 0&0&-1 \\ 0&0&0 \end{pmatrix}$. Now

$\langle H\rangle^\dagger (T^+T^- + T^-T^+)\langle H\rangle = \langle H\rangle^\dagger T^-T^+\langle H\rangle = \frac{v^2}{2} \Rightarrow M_W^2 = g_2^2\frac{v^2}{2}$, $\langle H^\dagger\rangle T^3T^3\langle H\rangle = \frac{v^2}{2} \Rightarrow \frac{1}{2}M_3^2 = g_2^2\frac{v^2}{2} \Rightarrow \frac{\Delta\rho}{\rho} = -1$

If instead we have $\langle H \rangle = \frac{v}{\sqrt{2}}\begin{pmatrix}0\\1\\0\end{pmatrix}$

Then $\langle H \rangle^{\dagger}(T^+T^- + T^-T^+)\langle H \rangle = 2\frac{v^2}{2}$ and

$$\langle H \rangle^{\dagger} T^3 T^3 \langle H \rangle = 0$$

So now $\frac{\Delta\rho}{\rho} = 2$.

Suppose we have both, say $\langle H_1 \rangle = \frac{v_1}{\sqrt{2}}\begin{pmatrix}0\\0\\1\end{pmatrix}$ and $\langle H_1 \rangle = \frac{v_2}{\sqrt{2}}\begin{pmatrix}0\\1\\0\end{pmatrix}$

Then $M_W^2 = g_2^2\left(\frac{v_1^2}{2} + v_2^2\right)$ $\quad M_3^2 = g_2^2(v_1^2 + 0)$ $\quad \frac{\Delta\rho}{\rho} = \frac{v_2^2 - \frac{1}{2}v_1^2}{\frac{v_1^2}{2} + v_2^2}$

and we can get $\Delta\rho = 0$ if we set $v_2^2 = \frac{1}{2}v_1^2$.

The Georgi-Machacek model has a doublet ($\Delta\rho \sim 0$) a complex triplet χ with $Y=1$ with $\langle \chi \rangle = v_\chi\begin{pmatrix}0\\0\\1\end{pmatrix}$ and a real triplet ξ ($Y=0$) with $\langle \xi \rangle = v_\xi\begin{pmatrix}0\\1\\0\end{pmatrix}$, so $\Delta\rho = 0$ for $v_\chi = v_\xi$.

ASIDE: GENERAL CASE

$\langle H\rangle^+ T^a T^b \langle H\rangle W^a W^b + \langle H^+\rangle \langle H\rangle Y^2 B^2$

$Q = T^3 + Y \qquad Q\langle H\rangle = 0 \Rightarrow$

$$T^3\langle H\rangle = -Y\langle H\rangle \qquad\qquad T^3\psi_m = m\psi_m$$

$$[T^3, T^1 \pm i T^2] = i T^2 \pm i(-iT^1) = \pm T^1 + i T^2 = \pm(T^1 \pm i T^2) \quad \Rightarrow \quad T^3 T^\pm \psi_m = (m \pm 1) T^\pm \psi_m$$

$$\left[\frac{T^1 + iT^2}{\sqrt{2}}, \frac{T^1 - iT^2}{\sqrt{2}}\right] = -i[T^1, T^2] = T^3$$

$\psi_j \quad T^3\psi_j = j\psi_j \quad T^+\psi_j = 0$

$$T^+\psi_m = A_m \psi_{m+1} \qquad T^-\psi_m = B_m\psi_{m-1}$$

$$\|T^+\psi_m\|^2 = \psi_m^+ T^- T^+ \psi_m = B_{m+1} A_m = |A_m|^2 \Rightarrow A_m^* = B_{m+1}$$

$$\|T^-\psi_m\|^2 = \psi_m^+ T^+ T^- \psi_m = B_m A_{m-1} = |B_m|^2 \Rightarrow A_{m-1} = B_m^* \text{ same}$$

$$\psi_m^+[T^+, T^-]\psi_m = |B_m|^2 - |A_m|^2 = |A_{m-1}|^2 - |A_m|^2 = m$$

and $T^+\psi_j = 0 \Rightarrow A_j = 0$

$$|A_{j-1}|^2 - |A_j|^2 = j \qquad \Rightarrow |A_{j-1}|^2 = j$$

$$|A_{j-2}|^2 - |A_{j-1}|^2 = j-1 \qquad |A_{j-2}|^2 = 2j-1$$

$\vdots$

$$|A_{m-1}|^2 - |A_m|^2 = m \Rightarrow |A_{m-1}|^2 = m + (m+1) + \cdots + [m + (j-m)]$$

$$= m(j-m+1) + \underbrace{(1 + 2 + \cdots + (j-m))}_{\frac{1}{2}(j-m)(j-m+1)}$$

$$|A_{m-1}|^2 = \tfrac{1}{2}(j-m+1)(j+m)$$

$$|A_m|^2 = \tfrac{1}{2}(j-m)(j+m+1)$$

check: $|A_j|^2 = 0$ ✓ $\quad |A_{j-1}|^2 = \frac{1}{2}(j-j+1)(j+j-1+1) = j$ ✓ $\quad |A_{j-2}|^2 = \frac{1}{2}\,2\,(2j-1)$ ✓

For us $T^3\langle H\rangle =$ "m"$\langle H\rangle$ is $-Y\langle H\rangle$ since $Q = T^3 + Y$ has $Q\langle H\rangle = 0$

So $\langle H\rangle^+(T^+T^- + T^-T^+)\langle H\rangle = |A_m|^2 + |A_{m-1}|^2 = \frac{1}{2}(j-m+1)(j+m) + \frac{1}{2}(j-m)(j+m+1)$

$$= j^2 - m^2 + \tfrac{1}{2}(j+m) + \tfrac{1}{2}(j-m) = j^2 - m^2 + j$$

$\langle H\rangle^+ T^3 T^3 \langle H\rangle = Y^2 \langle H^+ H\rangle$

$$M_W^2 = g_2^2 |\langle H\rangle|^2 (j^2 - Y^2 + j) \qquad M_3^2 = g_2^2 |\langle H\rangle|^2\, 2\,Y^2$$

$$\frac{\Delta\rho}{\rho} = \frac{M_W^2 - M_3^2}{M_W^2} = \frac{j(j+1) - 3Y^2}{j(j+1) - Y^2}$$

More interesting: H is in $2_{1/2}$, ϕ in $(2j+1)_Y$, with $\langle\phi\rangle \ll \langle H\rangle$

$$\Rightarrow \frac{\Delta\rho}{\rho} = \frac{g_2^2|\langle\phi\rangle|^2}{M_W^2}(j(j+1) - 3Y^2) \qquad \text{or even} \quad \frac{\Delta\rho}{\rho} = \frac{g_2^2}{M_W^2}\sum_i [j_i(j_i+1) - 3Y_i^2]\langle\phi_i\rangle^2$$

Custodial symmetry

Suppose the Lagrangian is such that it is invariant under an $SU(2)$ transformation with (W^1, W^2, W^3) a 3-plet, and that the vacuum is invariant as well. Then $\mathcal{L} = M_W^2 W^+ W^- + M_3^2 W_3^2 = M^2 (W^{1\,2} + W^{2\,2} + W^{3\,2}) \Rightarrow M_W = M_3 \Rightarrow \Delta\rho = 0$

The SM has this property in a certain limit. To see this, introduce a 2x2 matrix made of columns of H and $\tilde{H}$:

$$\phi = \left((\tilde{H})\ (H) \right)$$

Under $SU(2)_L$ ('L' = left, the one we are gauging) $\phi \to L\phi$ $\quad L \in SU(2)_L$

Let's introduce a second group, $SU(2)_R$ and define $\phi \to \phi R^\dagger$ $\quad R \in SU(2)_R$

Now $\mathrm{Tr}(D_\mu \phi^\dagger D_\mu \phi) = 2 D_\mu H^\dagger D_\mu H$ is our $\mathcal{L}$ and it is invariant!?

We take the W's to transform under $SU(2)_L \otimes SU(2)_R$ by $W_\mu \to L W_\mu L^\dagger$ (as before)

Now $\langle \phi \rangle = \left(\begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \right) = \frac{v}{\sqrt{2}} \mathbb{1}$

so it is invariant under $\langle \phi \rangle \to L \langle \phi \rangle R^\dagger$ for $R = L$ (the "diagonal" $SU(2)$ subgroup of $SU(2)_L \times SU(2)_R$). This is a custodial symmetry for the standard model.

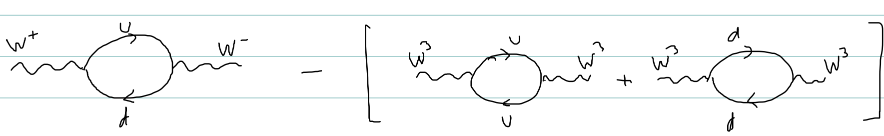
It is approximate because

$D_\mu \phi = \cdots ig_1 B_\mu \phi Y$ where $Y = \begin{pmatrix} -1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$ so $\phi \to \phi R^\dagger$ is a problem (so I lied above)

Also because $\bar{q}_L \tilde{H} u_R + \bar{q}_L H d_R = \bar{q}_L \phi \begin{pmatrix} u_R \\ d_R \end{pmatrix}$ is symmetric with $\begin{pmatrix} u_R \\ d_R \end{pmatrix} \to R \begin{pmatrix} u_R \\ d_R \end{pmatrix}$

But different yukawa couplings break the symmetry.

These enter, in the SM, at 1-loop order. For example corrections to the $W^\pm$ & W^3 masses arise from



⊛Exercise (hard!). Compute the $g_{\mu\nu}$, 0-momentum part of the self-energy (using any regularization — I recommend dim-reg): $\Pi_{\mu\nu}(0) = g_{\mu\nu} \Pi(0)$. Use this to compute

$$\frac{\Delta\rho}{\rho} = \frac{\Pi^\pm(0) - \left(\Pi^{3(u)}(0) + \Pi^{3(d)}(0)\right)}{M_W^2} = \frac{3}{64\pi^2} \frac{g_2^2}{M_W^2} \left[m_u^2 + m_d^2 - \frac{2 m_u^2 m_d^2}{m_u^2 - m_d^2} \ln \frac{m_u^2}{m_d^2} \right]$$

Ex (cont'd)
Show $\Delta\rho = 0$ for $m_d = m_u$, else $\Delta\rho > 0$
(It turns out this is also true if scalars run in loop instead of quarks).

Custodial Symmetry is a useful component of "new physics" (NP) models, particularly of those that give alternative mechanisms for SSB.

$\Delta\rho/\rho$ gives strong constraints on NP. In addition there are other constraints from electroweak physics that constrain NP models.

Project? Operator Analysis a la Grinstein & Wise PLB 265 (1991) 326

Project? Anomalies: show the SM is gauge and gravitational/gauge anomaly free
Do this for arbitrary Y's and find solutions.

Project: SU(5)?

Flavor Physics

What / Why / How

- Flavor physics: study the different types of quarks, "flavors", their spectrum and transitions among them (interactions).

  More generally: leptons

  Transitions: strengths, symmetries (eg CP/P/T, continuous?).

- Why? Richness (much to do & understand)

  * Stringent test of theory
  * Closely tied to all observed CPV(iolation)

- Methods involved are many/diverse: main challenge is strong interactions (uncover flavor physics)

  EFTs: electro-weak

  χ-$\mathcal{L}$ag

  HQET

  SCET ...

  Symmetries

  Non-perturbative (lattice)

Flavor in the SM

To account for 3 generations of leptons: e, μ, τ, (and 3 neutrinos) extend field content to include

$$l_{1L} = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \quad l_{2L} = \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \quad l_{3L} = \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$$

ie, 3 spin-1/2 L-fields in $(1,2)_{-1/2}$ under $(SU(3), SU(2))_{U(1)}$

and

$$e_{1R} = e_R, \quad e_{2R} = \mu_R, \quad e_{3R} = \tau_R$$

ie, 3 spin-1/2 R in $(1,1)_{-1}$

The relevant part of the SM Lagrangian is now

$$\mathcal{L} = \sum_{j=1}^{3} \bar{l}_{jL} i\cancel{\partial} l_{jL} + \sum_{j=1}^{3} \bar{e}_{jR} i\cancel{\partial} e_{jR} - \left(\sum_{j=1}^{3} y^E_i \bar{l}_{iL} H e_{iR} + \text{h.c.} \right)$$

In unitary gauge

$$\mathcal{L} = \sum_{j=1}^{3} \bar{e}_j \left(i\cancel{\partial} - \tfrac{1}{\sqrt{2}} y^E_i (v+h) \right) e_i + \sum_{j=1}^{3} \bar{\nu}_{jL} i\cancel{\partial} \nu_{jL} + \sum_{j=1}^{3} W \,\&\, Z \text{ interactions as before}$$

so that $m_e = \frac{1}{\sqrt{2}} v y^E_1$, $m_\mu = \frac{1}{\sqrt{2}} v y^E_2$, etc and the coupling of the higgs to e's is proportional to mass $\frac{m_e}{v} h \bar{e} e + \frac{m_\mu}{v} h \bar{\mu}\mu + \frac{m_\tau}{v} h \bar{\tau}\tau$

We have been unnecesarily restrictive, but nothing changes in the end: Let's write down the most general set of terms that are gauge (and Poincare) invariant, and of dimension 4 or less (for renormalizability).

$$\mathcal{L} = Z^l_{ij} \bar{l}_{iL} i\cancel{\partial} l_{jL} + Z^e_{ij} \bar{e}_{iR} i\cancel{\partial} e_{jR} - \left(y^E_{ij} \bar{l}_{iL} H e_{jR} + \text{h.c.} \right)$$

sum on i,j is implicit

The matrices Z^l and Z^e are hermitian (so that $\mathcal{L}$ is hermitian too).
Redefining (changing variables) by $l_L = U l_L$ (ie, $l_{iL} = U_{ij} l_{jL}$)
has the effect $Z^l \to U^\dagger Z^l U$. Choose U to diagonalize Z^l, $Z^l = \text{diag}(z^l_1, z^l_2, z^l_3)$
Now $z^l_i = z^{l*}_i > 0$, because the kinetic term is positive for consistency.
Redefine $l_{iL} \to \frac{1}{\sqrt{z_{il}}} l_{iL}$ (no sum), then the kinetic term is canonical.

$$Z^l_{ij} \bar{l}_{iL} i\cancel{\partial} l_{jL} \longrightarrow \bar{l}_{jL} i\cancel{\partial} l_{jL} \quad \text{as before}$$

Do the same for e_R, $e_R \to \frac{1}{\sqrt{Z^e}} V e_R$. We have canonical kinetic terms, and the Yukawas are as before, $y^E \to \frac{1}{\sqrt{Z_l}} U^\dagger y^E V \frac{1}{\sqrt{Z_e}}$ $\to$ redefine as new y^E:

$$\mathcal{L} = \bar{l}_L i\cancel{\partial} l_L + \bar{e}_R i\cancel{\partial} e_R - \left(y^E_{ij} \bar{l}_{iL} H e_{jR} + \text{h.c.} \right)$$

Finally, an additional $l_L \to U_l l_L$, $e_R \to U_e e_R$, diagonalizes $y^E \to U_l^\dagger y^E V_e = \text{diag}$

Now we turn to quarks: using the compact notation $q_L = \begin{pmatrix} q_{1L} \\ q_{2L} \\ q_{3L} \end{pmatrix}$, etc.
we can make redefinitions to bring the Lagrangian to the form

$$\mathcal{L} = \bar{q}_L i\not{D} q_L + \bar{u}_R i\not{D} u_R + \bar{d} i\not{D} d_R - y^u \bar{q}_L \tilde{H} u_R + y^D \bar{q}_L H d_R + h.c.)$$

where it is understood that $y^u \bar{q}_L \tilde{H} u_R = y^u_{ij} \bar{q}_{iL} \tilde{H} u_{jR}$, etc.

As opposed to the lepton case, now we cannot diagonalize <u>both</u> Yukawas by unitary redefinitions of the fields

$$q_L \to U_{q_L} q_L, \quad u_R \to U_{u_R} u_R \quad \text{and} \quad d_R \to U_{d_R} d_R$$

because we get $U^\dagger_{q_L} y^u U_{u_R}$ and $U^\dagger_{q_L} y^D U_{d_R}$, so we can choose U_{q_L} to diagonalize y^u or y^D (but not both).

But we want the mass terms

$$\mathcal{L}_m = \frac{v}{\sqrt{2}} y^u \bar{u}_L u_R + \frac{v}{\sqrt{2}} y^D \bar{d}_L d_R + h.c.$$

to be diagonal, so we can identify the physical states (often called "mass eigenstates").

Since the kinetic terms are

$$\mathcal{L}_K = \bar{q}_L i\not{D} q_L + \bar{u}_R i\not{D} u_R + \bar{d}_R i\not{D} d_R$$

$$= \bar{u}_L i\not{\partial} u_L + \bar{d}_L i\not{\partial} d_L + \bar{u}_R i\not{\partial} u_R + \bar{d}_R i\not{\partial} d_R + \text{interactions}$$

we can do separate unitary redefinitions of u_L, u_R, d_L and d_R that will retain the canonical $\bar{\psi} i\not{\partial}\psi$ form, at the expense of non-diagonal interactions:

$$u_L \to U_{u_L} u_L, \quad d_L \to U_{d_L} d_L, \quad u_R \to U_{u_R} u_R \quad \& \quad d_R \to U_R d_R$$

and choose these so that

$$\frac{v}{\sqrt{2}} U^\dagger_{u_L} y^u U_{u_R} = \mathrm{diag}(m_u, m_c, m_t) \qquad \frac{v}{\sqrt{2}} U^\dagger_{d_L} y^D U_{d_R} = \mathrm{diag}(m_d, m_s, m_b)$$

Then, in unitary gauge, the higgs interaction is still diagonal:

$$\mathcal{L}_h = \frac{m_u}{v} h \bar{u} u + \frac{m_c}{v} h \bar{c} c + \cdots$$

Now look at gauge interactions:

$$\mathcal{L}_{\bar\psi\psi A} = -e\bar q_L \not{A} Q q_L - e\bar u_R \not{A} Q u_R - e\bar d_R \not{A} Q d_R$$

$$= -\tfrac{2}{3}e\,\bar u_L \not{A} u_L + \tfrac{1}{3}e\,\bar d_L \not{A} d_L - \tfrac{2}{3}e\,\bar u_R \not{A} u_R + \tfrac{1}{3}e\,\bar d_R \not{A} d_R$$

$$\to -\tfrac{2}{3}e\,\bar u_L U_{u_L}^\dagger \not{A} U_{u_L} u_L + \tfrac{1}{3}e\,\bar d_L U_{d_L}^\dagger \not{A} U_{d_L} d_L - \tfrac{2}{3}e\,\bar u_R U_{u_R}^\dagger \not{A} U_{u_R} u_R + \tfrac{1}{3}e\,\bar d_R U_{d_R}^\dagger \not{A} U_{d_R} d_R$$

$$= -\tfrac{2}{3}e\,\bar u \not{A} u + \tfrac{1}{3}e\,\bar d \not{A} d$$

still diagonal.

A similar computation gives that the Z coupling are still diagonal:

This is a very important result/property of the SM:
there are no Flavor Changing Neutral Currents (FCNC) in $\mathcal{L}_{SM}$
($Z^\mu J_\mu$ with $J^\mu \sim \bar d\Gamma^\mu d + \bar s\Gamma^\mu s$ but no $\bar d\Gamma^\mu s + \bar s\Gamma^\mu d$)

* Exercise: write the Z couplings explicitly.

The $W^\pm$ interactions are different:

$$\mathcal{L}_{\bar\psi\psi W} = -g_2\,\bar q_L\left(\frac{\sigma^+}{2}\not{W}^+ + \frac{\sigma^-}{2}\not{W}^-\right) q_L$$

$$= -\frac{g_2}{\sqrt2}\,\bar u_L \not{W}^+ d_L + h.c.$$

$$\longrightarrow -\frac{g_2}{\sqrt2}\,\bar u_L U_{u_L}^\dagger \not{W} U_{d_L} d_L + h.c.$$

$$= -\frac{g_2}{\sqrt2}\,\bar u_L V \not{W} d_L + h.c$$

where $V = U_{u_L}^\dagger U_{d_L}$ is the unitary ($V^\dagger V = 1$) Kobayashi-Maskawa matrix (Also called Cabibbo-K-M matrix, or CKM).

CKM matrix simplify: Freedom (leaving $m_{u,d}$ diag, positive) $u \to \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})\,u$
$d \to (e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})\,d$. $V^\dagger V = 1 \Rightarrow 3+6$ conditions $\Rightarrow 18-9 = 9$ parameters $(3+6)$, -5 phase diffs $(\alpha_i - \beta_j) \Rightarrow 4$ parameters
3 ANGLES + 1 PHASE

$D_\mu = \partial_\mu + i g_3 A^a_\mu T^a + i g_2 W^a_\mu \frac{\sigma^a}{2} + i g_1 B_\mu Y$

Flavor "symmetry".

For $y^{U,D,E} = 0$ $\mathcal{L}$ has $U(3)^5$ symmetry

(a U(1) is anomalous, we only need the SU(3) factors).

with $q^i_L \to U_q{}^i{}_j\, q^j_L$, $u^i_R \to U_u{}^i{}_j\, u^j_R$, etc.

Symmetry is broken explicitly by Yukawa interactions.

Can keep track of pattern of symmetry bkg by treating Yukawa couplings as "spurions" (constant fields):

$\tilde{H} y^u_{ij} \bar{q}_{iL} u_{jR} \to \tilde{H} U^\dagger_{q\,ki}\, y^u_{ij} U_{u\,jm} \bar{q}_{kL} u_{mR}$ so, we take $y^u_{ij} \to U_{q\,ik}\, y^u{}_{km} U^\dagger_{u\,mj}$

and then the term is invariant.

Better notation: matrix

$$q_L \to U_q q_L, \quad u_R \to U_u u_R, \quad y^u \to U_q y^u U_u^\dagger$$

$$\Rightarrow \bar{q}_L y^u u_R \to \bar{q}_L U_q^\dagger (U_q y^u U_u^\dagger) U_u u_R = \bar{q}_L y^u u_R$$

and so on.

As we will see, new interactions that break this "symmetry" tend to produce rates of flavor tranformations inconsistent with observation (absent tuning or large parametric supression).

(Hence the usefulness of the symmetry).

We will be mostly concerned with hadronic (quark) physics (only $U(3)^3$).

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

1. One irremovable phase: CP is violated in $\bar{u}_L V \slashed{W}^+ d_L + \bar{d}_L V^\dagger \slashed{W}^- u_L$

Under CP $\bar{u}_L \gamma^\mu d_L \to \bar{d}_L \gamma_\mu u_L$ and $W^{+\mu} \to W^-_\mu$; so CP $\Rightarrow V^\dagger = V$.

Exercise: In QED $C: \bar{e}\gamma^\mu e \to -\bar{e}\gamma^\mu e$ and $A^\mu \to -A^\mu$, so $\bar{e}\slashed{A}e \to \bar{e}\slashed{A}e$

In QCD ? $C: \bar{q}T^a\gamma^\mu q \to \bar{q}(-T^a)^T\gamma^\mu q$, but $(-T^a)^T = (-T^a)^* \neq T^a$

What does C symmetry mean in QCD? How does A^a_μ transform?

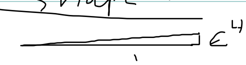
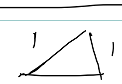
Exercise: If in m_u (or m_d) two entries are equal, show V can be brought into a real matrix (ie, in $O(3)$).

2. Precise knowledge of the elements of V is necessary to constrain new physics (or to test the validity of the SM/CKM thy). Will describe below how well we know it and how. But for now a sketch

$$V \sim \begin{pmatrix} \epsilon^0 & \epsilon^1 & \epsilon^3 \\ -\epsilon^1 & \epsilon^0 & \epsilon^2 \\ -\epsilon^3 & -\epsilon^2 & \epsilon^0 \end{pmatrix} \quad \text{with } \epsilon \sim 10^{-1}$$

3. $V^\dagger V = VV^\dagger = 1 \Rightarrow$ rows & columns of V are ortho-normal vectors.

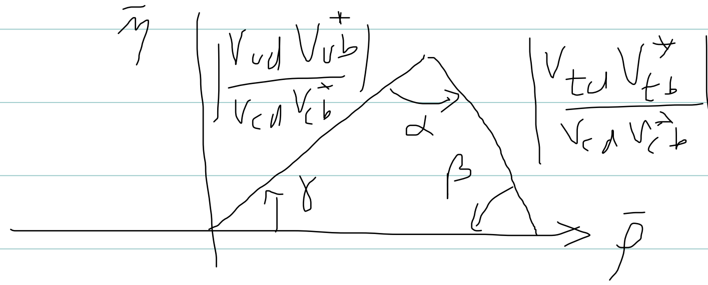
$\sum V_{ij} V^*_{kj} = 0$ for $i \neq k$: sum of 3 complex = 0 is a triangle in z-plane

ik	$\Sigma = 0$	$\sim \epsilon^n$	shape (normalized so base = 1).
12	$V_{ud}V^*_{cd} + V_{us}V^*_{cs} + V_{ub}V^*_{cb} = 0$	$\epsilon + \epsilon + \epsilon^5 = 0$	
23	$V_{cd}V^*_{td} + V_{cs}V^*_{ts} + V_{cb}V^*_{tb} = 0$	$\epsilon^4 + \epsilon^2 + \epsilon^2 = 0$	
13	$V_{ud}V^*_{td} + V_{us}V^*_{ts} + V_{ub}V^*_{tb} = 0$	$\epsilon^3 + \epsilon^3 + \epsilon^3$	

These are "unitarity triangles". The most commonly discussed is in the 1-3 columns:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \quad \Rightarrow$$

Dividing by middle $\quad \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$



Other convention:
$\beta = \phi_1$
$\alpha = \phi_2$
$\gamma = \phi_3$

Exercise: (i) Show that

$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \quad \alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \quad \gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

(ii) These are invariant under phase transformations of quarks.

(ie, under remaining arbitrariness) $\Rightarrow$ physical

(iii) Area $= -\frac{1}{2}\,\mathrm{Im}\,\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} = -\frac{1}{2}\,\frac{1}{|V_{cd}V_{cb}^*|^2}\,\underbrace{\mathrm{Im}(V_{ud}V_{cd}^*V_{cb}V_{ub}^*)}_{J\,=\,\text{Jarlskog invariant}}$

(under phase transf).

Note that $\quad \mathrm{Im}\,V_{ij}V_{kl}V_{il}^*V_{kj}^* = J\,(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj})$

is the common area of all triangles.

The normalized triangles have area $J/\text{largest}\,|V_{ij}V_{kl}^*|$

As we'll see, the area of the normalized triangle dictates the CP-asymmetries.

(small, order $\epsilon^?$ or ϵ^4 for squashed triangles, order 1 for fat triangles).

Show bounds on $\bar{\rho}, \bar{\eta}$ plane

4. Parametrizations of V.

Standard:

$$V = CBA \,, \quad A = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; B = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}, \; C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

$c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ with θ_{ij} in 1st quadrant.

Wolfenstein:

$$s_{12} = \lambda \,, \quad s_{23} = A\lambda^2 \qquad s_{13}e^{i\delta} = A\lambda^3(\rho + i\eta) = \frac{A\lambda^3(\bar\rho + i\bar\eta)\sqrt{1-A^2\lambda^4}}{\sqrt{1-\lambda^2}\,[1 - A^2\lambda^4(\bar\rho + i\bar\eta)]}$$

Exercise (i) Show that

$$\bar\rho + i\bar\eta = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$, hence phase invariant.

(ii) Expand in $\lambda \ll 1$ to show

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

Back to flavor symmetry: $U(3)^3$

Suppose we extend the SM by adding terms (local, Lorentz & gauge inv) dim>4 operators that are invariant under $U(3)^3$ – including spurions λ_u, λ_d. For example

Recall
$u_R \to U_u u_R$
$q_L \to U_q q_L$
$d_R \to U_d d_R$
$y^U \to U_q^\dagger y^U U_u$
$y^D \to U_q^\dagger y^D U_d$

$$\Delta\mathcal{L} = \sum_i c_i \mathcal{O}_i$$

with
$$\mathcal{O}_1 = G^a_{\mu\nu} \tilde{H} \bar{u}_R T^a \sigma^{\mu\nu} y^{U\dagger} q_L$$
$$\mathcal{O}_2 = \bar{q}_L \gamma^\mu y^U y^{D\dagger} q_L \, \bar{d}_R \gamma_\mu y^{D\dagger} y^D d_R$$

Now, go to basis with diagonal mass matrices:

$$\mathcal{O}_1 \to G_{\mu\nu} \tilde{H} \bar{u}_R T^a \sigma^{\mu\nu} U^\dagger_{u_R} y^{U\dagger} \begin{pmatrix} U_{u_L} u_L \\ U_{d_L} d_L \end{pmatrix}$$
$$= G_{\mu\nu} \tilde{H} \bar{u}_R T^a \sigma^{\mu\nu} \underbrace{(U^\dagger_{u_R} y^{U\dagger} U_{u_L})}_{y^{U\prime} = y^{U,\text{diag}}} \begin{pmatrix} u_L \\ \underbrace{U^\dagger_{u_L} U_{d_L}}_{V} d_L \end{pmatrix}$$

The only off-diagonal interaction (in flavor) is determined by $y^{U,\text{diag}} V$

Similarly, $\mathcal{O}_2 \to \bar{q}'_L \gamma^\mu y^{U\prime} y^{U\prime\dagger} q'_L \, \bar{d}_R \gamma_\mu y^{D\prime\dagger} y^{D\prime} d_R$, where $q'_L = \begin{pmatrix} u_L \\ V d_L \end{pmatrix}$

Of course $y^{U,D\prime\dagger} = y^{U,D\prime}$ so $y^{U,D\prime} y^{U,D\prime\dagger} = (y^{U,D\prime})^2$

Exercise: show this is generally true, that is, flavor-change is determined by V, or more specifically $y^{U\prime} V$ or $V y^{D\prime}$ (or $V^\dagger y^{U\prime}$ or $y^{D\prime} V^\dagger$).

This is the principle of Minimal Flavor Violation (MFV).

Extensions of the SM in which the only breaking of $U(3)^3$ is by y^U & y^D automatically satisfy MFV. As we will see they are least constrained by flavor changing and CPV observables.

Two Higgs Doublet Model(s) (2HDM)

Why?

* Minimal SM extension, use to test SM correctness (are the predictions of the SM unique? differentiate between models?)
  (Note: extending N of generations is also very minimal but does not probe SSB).
* Additional symmetries (eg, Peccei-Quinn → axions, not for these lec's)
* Must in Supersymmetric version's of SM (MSSM and all variants)

Use SM gauge group and fermion fields, and instead of one scalar doublet (H) introduce two, H_1 and H_2, both in $(1,2)_{1/2}$

Assume $\langle H_1\rangle = \frac{v_1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}$ and $\langle H_2\rangle = \frac{v_2}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}$

Note: this is not obvious. Suppose both $\langle H_1\rangle \neq 0$ and $\langle H_2\rangle \neq 0$ (and constant, ie, $\vec{x}$ & t independent). By a gauge transformation $H_{1,2} \to U H_{1,2}$ we can arrange to have $\langle H_1\rangle = \begin{pmatrix}0\\ *\end{pmatrix}$, but $\langle H_2\rangle = \begin{pmatrix}*\\ *\end{pmatrix}$ generally.

If the upper component is non-zero, then $Q = \begin{pmatrix}1&0\\0&0\end{pmatrix}$ will be broken, $Q\langle H_2\rangle \neq 0$
$\Rightarrow$ photon get a mass, "Coulomb" will turn into $\frac{1}{r}e^{-\mu r}$, ...

That $\langle H_2\rangle = \begin{pmatrix}0\\ *\end{pmatrix}$ when $\langle H_1\rangle = \begin{pmatrix}0\\ *\end{pmatrix}$ puts some constraints on the potential $V(H_1, H_2)$

*Exercise: write explicitly the most general potential (up to quartic terms). Which terms care about alignment of $\langle H_2\rangle$ vs $\langle H_1\rangle$? How would you arrange the coefficients so that $\langle H_1\rangle$ & $\langle H_2\rangle$ are aligned?

SSB: Z/W - masses

$$\mathcal{L}_{K,H} = |D_\mu H_1|^2 + |D_\mu H_2|^2$$

$$\frac{1}{4} g_2^2 v_1^2 W_\mu^+ W_\mu^- + \frac{1}{8}(g_1^2+g_2^2) v_1^2 Z^2 \qquad \frac{1}{4} g_2^2 v_2^2 W_\mu^+ W_\mu^- + \frac{1}{8}(g_1^2+g_2^2) v_2^2 Z^2$$

$$M_W^2 = \frac{1}{4} g_2^2 v^2 \quad M_Z^2 = \frac{1}{4}(g_1^2+g_2^2) v^2 \qquad \text{where } v^2 = v_1^2 + v_2^2 \approx 246\,\text{GeV}$$

Useful $\cos\beta \equiv \frac{v_1}{\sqrt{v_1^2+v_2^2}} = \frac{v_1}{v}$ $\qquad \sin\beta \equiv \frac{v_2}{\sqrt{v_1^2+v_2^2}} = \frac{v_2}{v}$

Note: this "β" has nothing to do with the unitarity triangle "β"
The notation is standard in the literature → live with it!

Scalar spectrum:

Counting: 2 complex doublets: $2\times2\times2 = 8$ real fields
-3 eaten $(W^\pm, Z)$
$= 5$

1 charged scalar, $h^\pm = 2$
2 neutral scalars, $h, H = 2$
1 neutral pseudoscalar, $A = 1$

Quick and dirty: look for eaten 1st, cross term $\partial_\mu \phi \times W_\mu v$

$$H_m = \begin{pmatrix} \phi_m^+ \\ \frac{1}{\sqrt{2}}(v_m + \rho_m + i\eta_m) \end{pmatrix}$$

(1) Charged: $D_\mu H_m = \begin{pmatrix} \partial_\mu \phi_m^+ \\ 0 \end{pmatrix} + i\frac{g_2}{\sqrt{2}} \begin{pmatrix} W_\mu^+ v_m/\sqrt{2} \\ W_\mu^- \phi_m^+ \end{pmatrix}$

$$\rightarrow |D_\mu H_m|^2 = \frac{i g_2}{\sqrt{2}}\left(\partial_\mu \phi_m^- W_\mu^+ \frac{v_m}{\sqrt{2}} - \partial_\mu \phi_m^+ W_\mu^- \frac{v_m}{\sqrt{2}}\right)$$

$$\Rightarrow \phi^{+\,\text{eaten}} = \cos\beta\, \phi_1^+ + \sin\beta\, \phi_2^+ \Rightarrow h^\pm = -\sin\beta\, \phi_1^+ + \cos\beta\, \phi_2^+$$

Note on CP:

Complex field $\phi(\vec{x},t) \xrightarrow[CP]{} \phi^*(-\vec{x},t)$

☆Exercise: Why not $\phi \to e^{i\omega}\phi^*$? (Hint: redefine field).

$$\rho + i\eta \xrightarrow[CP]{} \rho + i(-\eta)$$

(ii) Scalars: not eaten. In general $V(H_1, H_2)$ will contain a mass matrix with ρ_1^2, $\rho_1\rho_2$ and ρ_2^2 terms. Mass eigenstates: h, H

$$\begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

Note: again "α" notation std, not the same as UT "α"

(iii) $\eta^{eaten} = \cos\beta\,\eta_1 + \sin\beta\,\eta_2 \quad A = -\sin\beta\,\eta_1 + \cos\beta\,\eta_2$

☆Exercise: Show the cubic couplings of h and H to gauge bosons are given by

$$2\frac{M_W^2}{v}\sin(\alpha+\beta)\, h W_\mu^+ W_\mu^- + 2\frac{M_Z^2}{v}\sin(\alpha+\beta)\, h Z_\mu Z_\mu$$

$$+ \left(h \to H \text{ and } \sin(\alpha+\beta) \to \cos(\alpha+\beta)\right)$$

☆Exercise: Show that the cubic couplings $h^\pm VV$ and AVV (with V = vector field) vanish.

☆Exercise: Compute the HAZ coupling.

Since $h^+ W^-_\mu Z_\mu$ coupling vanishes, $h^+ \to W^+ Z$ at LHC would indicate New Physics. But what?

☆ Exercise: show that in more than 2 doublet models, still no coupling.

☆ Exercise: How do 5 scalars transform under custodial $SU(2)$?

Georgi–Machacek model

Like the 2HDM, a simple extension of the SM with some new features.

Field Content

$$\left.\begin{array}{l} H \;:\; 2_{1/2} \\ \chi \;:\; 3_1 \end{array}\right\} \text{necessarily complex}$$

$$\xi \;:\; 3_0 \quad \text{real}$$

$Q = T^3 + Y$; for triplet, $T^3 = \mathrm{diag}(1,0,-1)$ (just spin-1, with $m=1,0,-1$).

$$\chi = \begin{pmatrix} \chi^{++} \\ \chi^{+} \\ \chi^{0} \end{pmatrix} \qquad \xi = \begin{pmatrix} \xi^{+} \\ 0 \\ \xi^{-} \end{pmatrix} \quad (\xi^- = (\xi^+)^*).$$

$$\text{SSB} : \langle H \rangle = \frac{v_H}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle \chi \rangle = v_\chi \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \langle \xi \rangle = v_\chi \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Same VEV for χ^0 and ξ^0 to ensure $\frac{\Delta\rho}{\rho} = 0$ (at tree level) → we saw this earlier!

☆ Exercise: $M_W^2 = \frac{1}{4} g_2^2 v^2$ where $v^2 = v_H^2 + 8 v_\chi^2$.

Define, for later use: $\cos\theta_H = \frac{v_H}{v}$ $\left(\sin\theta_H = \frac{\sqrt{8}\, v_\chi}{v}\right)$.

That $\Delta\rho = 0$ at $v_\chi = v_\xi$ actually follows from custodial symmetry. As before

$\Phi = \begin{pmatrix} h^{0*} & h^+ \\ h^- & h^0 \end{pmatrix}$ has $\langle \Phi \rangle = \frac{v_H}{\sqrt{2}} \mathbb{1}$ and $\Phi \to L \Phi R^\dagger$ under $SU(2)_L \times SU(2)_R$ has $\langle \Phi \rangle \to \langle \Phi \rangle$ under $SU(2)_c \subset SU(2)_L \times SU(2)_R$ (diagonal, ie, $R = L$).

We can do something similar for the triplets

$$X = \begin{pmatrix} \chi^{0*} & \xi^{+} & \chi^{++} \\ \chi^{-} & \xi^{0} & \chi^{+} \\ \chi^{--} & \xi^{-} & \chi^{0} \end{pmatrix} \quad \text{has} \quad \langle X \rangle = v_\chi \mathbb{1} \quad (\text{here } \mathbb{1} \text{ is } 3\times3 \text{ identity})$$

Again $X \to L X R^\dagger$, where now L & R are 3-dim irreducible representations of $SU(2)_L$ & $SU(2)_R$, resp.

And again, $\langle X \rangle \to \langle X \rangle$ under $SU(2)_c$ ($L=R$).

☆ Project: This model has 13 real scalars − 3 eaten = 10 physical scalars. Work it out: find the physical fields in terms of components of Φ and X. Find their couplings to two vector bosons. Can a singly charged Higgs decay to WZ? Can a doubly charged Higgs decay to WW? If so compute the rates. Can a doubly charged higgs decay to W^+ plus a singly charged higgs, and what's the rate? Neglecting decay of doubly charged higgs to two higgses (or 3-bodies) compute the branching fractions for decays of doubly and singly charged higgses (as function of masses). (You can see we can keep on investigating this: push the envelope as far as you want). (Note: $H^{++} \to W^+ H^+$ only if $m_{H^{++}} > m_W + m_{H^+}$, of course). (Rates in terms of $\sin\theta_H$)

<u>Bottom line</u> $h^+ W^- Z$ coupling not zero in some extensions of SM ⇒ look!

Add here: new LHC bounds in $H^+ \to W^+ Z$ decays. ⇐

☆☆ Exercise: Show the 10 states form multiplets of $SU(2)_c$: $5 \oplus 3 \oplus 1 \oplus 1$ (and the eaten fields —would-be-goldstone-bosons— also form a 3).

Flavor in 2HDM

Now: Yukawa couplings in 2HDM

Most general: matrix notation in flavor space

$$-\mathcal{L}_{Yuk} = \tilde{H}_1 \bar{q}_L y_1^U u_R + \tilde{H}_2 \bar{q}_L y_2^U u_R + H_1 \bar{q}_L y_1^D d_R + H_2 \bar{q}_L y_2^D d_R$$
$$+ H_1 \bar{l}_L y_1^E e_R + H_2 \bar{l}_L y_2^E e_R \quad + h.c.$$

The mass terms are

$$-\sqrt{2}\,\mathcal{L}_{\psi,mass} = \bar{u}_L (v_1 y_1^U + v_2 y_2^U) u_R + \bar{d}_L (v_1 y_1^D + v_2 y_2^D) d_R + \bar{e}_L (v_1 y_1^E + v_2 y_2^E) e_R \quad + h.c.$$

As before, $u_{L,R} \to U_{u_{L,R}} u_{L,R}$, $d_{L,R} \to U_{d_{L,R}} d_{L,R}$ $e_{L,R} \to U_{e_{L,R}} e_{L,R}$ with $U_x^\dagger U_x = \mathbb{1}$

so that $\bar{u}_L i\not{\partial} u_L \to \bar{u}_L i\not{\partial} u_L$, etc. and

$$U_{u_L}^\dagger (v_1 y_1^U + v_2 y_2^U) U_{u_R} = \sqrt{2}\, m^U = \sqrt{2}\,\mathrm{diag}(m_u, m_c, m_t) \quad ,\text{etc.}$$

Gauge couplings of quarks and leptons as in SM, with $V = U_{u_L}^\dagger U_{d_L} = \text{CKM}$ as before.

Charged Higgs couplings to quarks and leptons:

Recall: $\phi^{+\,\text{eaten}} = \cos\beta\, \phi_1^+ + \sin\beta\, \phi_2^+ \Rightarrow h^+ = -\sin\beta\, \phi_1^+ + \cos\beta\, \phi_2^+$

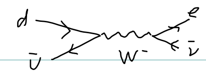
Invert: $\phi_1^+ = \cos\beta\, \phi^{+\,\text{eaten}} - \sin\beta\, h^+ \qquad \phi_2^+ = \sin\beta\, \phi^{+\,\text{eaten}} + \cos\beta\, h^+$

Collecting $h^\pm$ couplings:

$$-\mathcal{L}_{h^\pm \bar{\psi}\psi} = -h^- \bar{d}_L U_{d_L}^\dagger (-\sin\beta\, y_1^U + \cos\beta\, y_2^U) U_{u_R} u_R + h^+ \bar{u}_L U_{u_L}^\dagger (-\sin\beta\, y_1^D + \cos\beta\, y_2^D) U_{d_R} d_R$$
$$+ h^+ \bar{\nu}_L U_{e_L}^\dagger (-\sin\beta\, y_1^E + \cos\beta\, y_2^E) U_{e_R} e_R \quad + h.c.$$

In general these couplings change flavor differently than in W^+ couplings.

This can be a problem. For example, in SM the "beta-decays" $b \to u l \nu$ ($l = e, \mu, \tau$) are suppressed relative to $b \to c l \nu$ by $|V_{ub}|^2/|V_{cb}|^2 \sim \epsilon^2 \sim 10^{-2}$. Here the quark coupling is unsuppressed in general. The problem is even worse for $B^+ \to l^+ \nu$. B^+ is a pseudo-scalar meson with $\bar{b}u$ quarks (like π^+ has $\bar{d}u$ and K^+ has $\bar{s}u$).

$\pi^- \to e^- \nu$: in SM, 

$$= V_{ud}^* \, \bar d_L \gamma^\mu u_L \left(-\frac{g_2}{\sqrt2}\right)^2 \left[-i\, \frac{\eta_{\mu\nu} - p_\mu p_\nu/M_W^2}{p^2 - M_W^2}\right] \bar e_L \gamma^\nu \nu_L \approx i\,\frac{g_2^2}{8M_W^2}\, \underbrace{\bar d \gamma^\mu(1-\gamma_5) u}_{\text{“}V^\mu - A^\mu\text{”}}\, \bar e \gamma_\mu (1-\gamma_5)\nu$$

or $\mathcal{L}_{eff} = V_{ud}^* \frac{G_F}{\sqrt2}\, \bar d\gamma^\mu(1-\gamma_5)u\, \bar e\gamma_\mu(1-\gamma_5)\nu$ where we used $\frac{1}{2v^2} = \frac{G_F}{\sqrt2}$

The decay amplitude is

$$a(\pi^-\to e^-\bar\nu) = \langle e^-\bar\nu|\,\mathcal{L}_{eff}\,|\pi^-\rangle = V_{ud}^*\frac{G_F}{\sqrt2}\langle 0|\,\bar d\gamma^\mu(1-\gamma_5)u\,|\pi^-\rangle\,\langle e^-\bar\nu|\,\bar e\gamma_\mu(1-\gamma_5)\,|0\rangle$$

Now $\langle 0|V^\mu|\pi^-\rangle = 0$ by CP invariance of strong interactions

★ Exercise: show this!

and $\langle 0|A^\mu(0)|\pi^-(\vec p)\rangle = f_\pi p^\mu$: has to be proportional to only 4-vector that the right hand side (RHS) depends on, and f_π is a dynamically determined constant of proportionality called "the pion decay constant". It can be determined using lattice simulations of QCD.

★ Exercise: show that $\langle 0|A^\mu(x)|\pi^-(\vec p)\rangle = e^{-ip\cdot x} f_\pi p^\mu$

On the other hand $\langle e^-\bar\nu|\,\bar e\gamma_\mu(1-\gamma_5)|0\rangle = \bar u(p_e)\gamma_\mu(1-\gamma_5)v(p_{\bar\nu})$ where u,v are Dirac spinors.

So $$a(\pi^-\to e^-\bar\nu) = V_{ud}^*\frac{G_F}{\sqrt2} f_\pi\, p^\mu\, \bar u(p_e)\gamma_\mu(1-\gamma_5)v(p_{\bar\nu}) \qquad (p = p_e + p_{\bar\nu})$$
$$= V_{ud}^*\frac{G_F}{\sqrt2} f_\pi \boxed{m_e}\, \bar u(1-\gamma_5)v$$

$$\sum_{spins}|\bar u(1-\gamma_5)v|^2 = \mathrm{Tr}\left[(1-\gamma_5)\not p_\nu(1+\gamma_5)(\not p_e + m_e)\right] = 8\,p_e\cdot p_{\bar\nu} = 4\left[(p_e+p_{\bar\nu})^2 - p_e^2 - p_{\bar\nu}^2\right] = 4(m_\pi^2 - m_e^2)$$

So $$\Gamma(\pi^-\to e^-\nu) = \frac{1}{8\pi}\frac{|\vec p_e|}{m_\pi^2}\sum_{spins}|a|^2 = \frac{1}{8\pi}\frac{(m_\pi^2-m_e^2)}{2m_\pi^3}\frac{G_F^2}{2}|V_{ud}|^2 f_\pi^2 m_e^2\cdot 4(m_\pi^2-m_e^2) = \boxed{\frac{1}{8\pi}G_F^2|V_{ud}|^2\left(1-\frac{m_e^2}{m_\pi^2}\right)^2 f_\pi^2 m_e^2 m_\pi}$$

Note that $\frac{\Gamma(\pi^-\to e^-\nu)}{\Gamma(\pi^-\to\mu^-\nu)} = \left(\frac{m_\pi^2-m_e^2}{m_\pi^2-m_\mu^2}\right)^2\left(\frac{m_e^2}{m_\mu^2}\right) = 1.28\times10^{-4}$ (compare with PDG 1.23×10^{-4} w. small errors)

★ Exercise: Show that $\langle 0|\,\bar d\,\gamma_5 u\,|\pi^-(p)\rangle = \frac{f_\pi m_\pi^2}{m_u+m_d}$.

★ Add to the SM a charged scalar h^+, of mass M, with $\mathcal{L}_{Yuk} = \kappa_q h^- \bar d\gamma_5 u + \kappa_\ell h^- \bar e(1-\gamma_5)\nu$ + h.c.

Compute the contribution $\Delta\Gamma$, to the partial width for $\pi^-\to e^-\bar\nu$. Compare with SM. If $\kappa_q\sim\kappa_\ell\sim 1$, give a lower bound on M so that $\Delta\Gamma$ is not in conflict with $\Gamma^{exp}(\pi^-\to e^-\bar\nu)$.

Next, neutral higgses h, H, A:

Recall $\quad H_m = \begin{pmatrix} \phi_m^+ \\ \frac{v_m + \rho_m + i\eta_m}{\sqrt{2}} \end{pmatrix} \quad m = 1,2 \qquad \eta^{\text{eaten}} = \cos\beta\,\eta_1 + \sin\beta\,\eta_2 \qquad A = -\sin\beta\,\eta_1 + \cos\beta\,\eta_2$

Mass eigenstates $\quad \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix} \quad \Bigg\| \quad \begin{matrix} \eta_1 = \cos\beta\,\eta^{\text{eaten}} - \sin\beta\,A \\ \eta_2 = \sin\beta\,\eta^{\text{eaten}} + \cos\beta\,A \end{matrix}$

Then look at quark/lepton couplings:

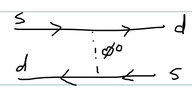
$$-\sqrt{2}\,\mathcal{L}_{\phi\psi\psi} = \bar{u}_L U_{u_L}^\dagger \left[y_1^U (\rho_1 - i\sin\beta A) + y_2^U (\rho_2 + i\cos\beta A) \right] U_{u_R} u_R$$
$$+ \bar{d}_L U_{d_L}^\dagger \left[y_1^D (\rho_1 - i\sin\beta A) + y_2^D (\rho_2 + i\cos\beta A) \right] U_{D_R} d_R$$

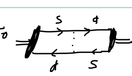
+ idem for leptons.

This means that generically we will have "Flavor Changing Neutral Currents" effects.

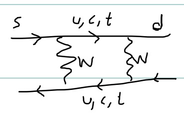
We will study these later, but for now: they're "bad" ☺.

For example:



$\Rightarrow$ $\bar{K}^0$  K^0 $\Rightarrow$ $K^0\bar{K}^0$ oscillations

We will see later that in the SM this comes from



suppressed by
- G_F
- 1 loop $\frac{1}{16\pi^2}$
- CKM + GIM: $\left(V_{cd}V_{cs}^* \frac{m_c^2}{M_W^2}\right)^2 \sim \epsilon^2 \left(\frac{m_c}{M_W}\right)^4$ or $(V_{td}V_{ts}^*)^2 \sim \epsilon^6$

so very roughly we want $\frac{1}{M_{\phi^0}^2} \ll G_F \frac{1}{16\pi^2} \cdot 10^{-6}$ or $\frac{M_\phi}{v} \gg 10^4\, G_F^{-1/2} \sim 10^3$ TeV !!

But M_ϕ is supposed to be of order $v \sim 10^{-1}$ TeV. We need tiny coupling constants, ugh.

Solution:

(i) 2HDM type III: tune coefficients to be tiny

(ii) (better) Find symmetries to make coefficients vanish $\rightarrow$ Type I, II, lepton specific, flipped

For example (Glashow–Weinberg)

Type I: $H_1 \to H_1$, $H_2 \to -H_2$, $\psi_{L,R} \to \psi_{L,R}$. Then, eg, $\tilde{H}_2 \bar{q}_L u_R$, $H_2 \bar{q}_L d_R$ and $H_2 \bar{l}_L e_R$ are forbidden by symmetry. Set $y_2^{U,D,E} = 0$ in above formulae. Mass is from $y_1^{U,D,E}$ only $\Rightarrow$ just like SM

Similarly (still Glashow-Weinberg) : Type II

$$-\mathcal{L}_{Yuk} = \tilde{H}_2 \bar{q}_L y^U u_R + H_1 \bar{q}_L y^D d_R + H_1 \bar{l}_L y^E e_R \quad + h.c.$$

This can be achieved by two "$\mathbb{Z}_2$" symmetries

a) $H_2 \to -H_2$, $u_R \to -u_R$, all other neutral

b) $H_1 \to -H_1$, $d_R \to -d_R$, $e_R \to -e_R$, all other neutral

Looking back, setting $y_1^U = 0$, $y_2^D = y_2^E = 0$ we have

$$-\mathcal{L}_{\text{dophy}} = (\cos\alpha H + \sin\alpha h)\, \bar{u} \frac{m^U}{v_2} u - \sin\beta A\, \bar{u} \frac{m^U}{v_2} i\gamma_5 u$$

$$+(-\sin\alpha H + \cos\alpha h)\left[\bar{d}\frac{m^D}{v_1} d + \bar{e}\frac{m^E}{v_1} e\right] + \cos\beta A \left[\bar{d}\frac{m^D}{v_1} i\gamma_5 d + \bar{e}\frac{m^E}{v_1} i\gamma_5 e\right]$$

where $m^U = \text{diag}(m_u, m_c, m_t)$

Note $\frac{m^U}{v_2} = \frac{v}{v_2}\frac{m^U}{v} = \frac{1}{\sin\beta}\frac{m^U}{v}$, $\sqrt{2}\frac{m^U}{v}$ = value of Yukawa coupling in SM.

Similarly: $\frac{m^D}{v_1} = \frac{1}{\cos\beta}\frac{m^D}{v}$

$\Rightarrow$ we can have similar Yukawa couplings for m_b and m_t if $\tan\beta \sim \frac{m_t}{m_b} \sim \frac{173}{5} \sim 35$

That's a crude estimate since Yukawa couplings depend on renormalization scale, and

$y_b(m_b) \sim 2\, y_b(m_t)$, so for $y_b(m_t) \sim y_t(m_t)$ need $\tan\beta \sim 70$.

So the hierarchy $m_t \gg m_b$ is explained by $v_2 \gg v_1$. Then $m_c \gg m_s$, but not as pronounced $\left(\frac{m_c}{m_s} \sim 15\right)$ is a combination of $v_2 \gg v_1$ and corrections from large $y_{t,b}$, and $m_u \lesssim m_d$ is all dominated by corrections.

☆Exercise: Compute charged higgs couplings to quarks. Express your answer in terms of V (CKM), diagonal mass matrices, v (SM vev) and β.

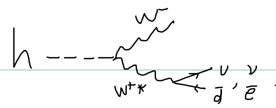
So what do we know from data?

A word about higgs data fits.

(i) Measure: $pp \to h + X$

$\qquad\qquad \hookrightarrow \gamma\gamma,\ b\bar{b},\ WW^*,\ ZZ^*,\ \tau\bar{\tau}$

W^* means "off shell": $2M_W > m_H$. So, really $h \to WW^*$ means

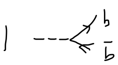


etc.

And, of course, $W^- \to e\bar{\nu}$ or $d\bar{u}$. The difference between $W \to f f'$ and $W^* \to f f'$ is that for $W \to ff'$ the "invariant mass" of the pair $m_{ff'}^2 = (p_f + p_{f'})^2 = p_W^2 = M_W^2$.

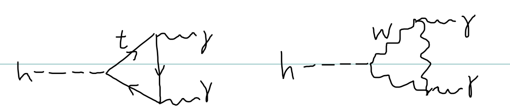
Since W is unstable, really have $m_{ff} = m_W \pm \Gamma_W$

Similarly for Z^*.

(ii) $h \to b\bar{b}, WW^*, \tau\bar{\tau}, \ldots$ is tree level  etc

But $h \to \gamma\gamma$ is from 1-loop. In SM dominated by t & W

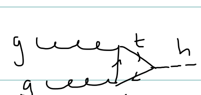
$\boxed{\text{Want formulas? See 0503172}}$



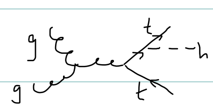
$\hookrightarrow \sim \frac{y_t}{m_t} h F_{\mu\nu} F^{\mu\nu}$, $\frac{y_t}{m_t} \sim \frac{1}{v}$ (vs, eg Type II, $\frac{\sin\alpha}{\sin\beta}\frac{1}{v}$).

The loop computations give W of opposite sign as t, and W about 5 times larger than t.

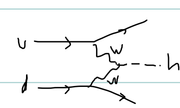
(iii) The production mechanisms:



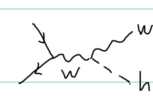
"gluon fusion"
↑
dominant
$\sigma \sim 4 \times 10^4$ fb



"$t\bar{t}h$"
smallest
$\sigma \sim 7 \times 10^2$ fb



"Vector boson fusion"
2nd largest (x10 smaller)
$\sigma \sim 4 \times 10^3$ fb



"vector boson associated production"
3rd (another x few smaller)
$\sigma \sim 2 \times 10^3$ fb

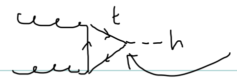
Applying different experimental (kinematic) cuts, gives some sensitivity to other production mechanisms.

(iv) Experiments report "signal strength" $\mu = \frac{\sigma}{\sigma_{SM}} \cdot \frac{Br}{Br_{SM}}$

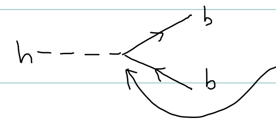
where σ = h production cross section, Br = particular decay's branching fraction.

When fitting to Beyond the Standard Model (BSM) models, both σ and Br may be modified.

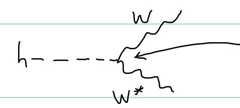
For example, in 2HDM-type II



has $\frac{\cos\alpha}{\sin\beta}$ and



has $\frac{\sin\alpha}{\cos\beta}$



$\sin(\alpha+\beta)$

TABLE/PLOT OF μ_{LHC}

Since $\mu^{exp} \simeq 1 \pm 20\%$ish we want $|\sin(\alpha+\beta)| \approx 1$

For $\alpha+\beta = \pm\frac{\pi}{2}$ we also have $\cos\alpha = \cos(\pm\frac{\pi}{2}-\beta) = \pm\sin\beta$

$\sin\alpha = \sin(\pm\frac{\pi}{2}-\beta) = \pm\cos\beta$

(see next page) FIGURE

In this limit all of the above 2HDM-II correction factors are either 1 or all -1 $\Rightarrow$

$\Rightarrow$ cannot distinguish from the SM

This is called the "decoupling" limit.

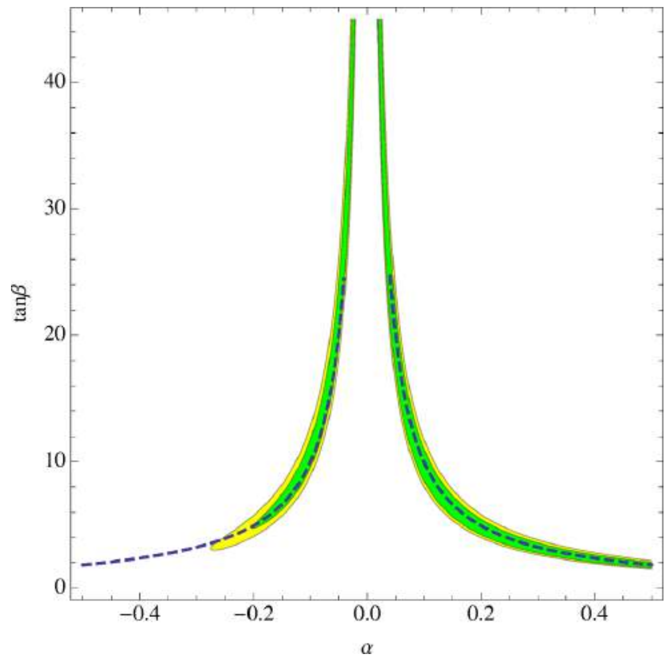


Figure 1. The region of parameter space within 1- and 2-σ of the best fit values. The dashed line is the decoupling limit, $\alpha + \beta = \pm\pi/2$, where the couplings are SM-like (up to a possible sign flip for the down Yukawa couplings).

We also impose the following perturbativity constraint on the couplings

$$\frac{y_i^2}{4\pi} \leq 1\,, \qquad \frac{\lambda_i}{4\pi} \leq 1\,. \tag{4.2}$$

We insist on these constraints up to the cutoff scale for all the Yukawa and scalar couplings. We list the beta-functions used in evolving the coupling constants in appendix B.

4.2 Experimental bounds

A wealth of experimental data, particularly from precision measurements, places strong constraints on the spectrum of the 2HDM-II. A newly published result on a direct search for the charged Higgs at LEP yields the 95% C.L. lower bound $M_{H^\pm} \geq 80\,\mathrm{GeV}$ [38]. At present there is no lower bound on the charged Higgs mass from the Tevatron or LHC data. A much tighter constraint on the charged Higgs mass can be deduced from rare decay processes. By analyzing the branching ratio $\mathrm{Br}(\bar{B} \to X_s\gamma)$, ref. [39] obtained the bound $M_{H^\pm} \geq 380\,\mathrm{GeV}$ at 95% confidence level. A direct search at LEP places a 95% limit $M_A \gtrsim 93\,\mathrm{GeV}$ for the MSSM CP-odd Higgs, A [40]. However this limit doesn't apply to the 2HDM case studied here. Nevertheless, we employed this bound in in the rest of the paper. The reader should keep in mind that $M_A \lesssim 93\,\mathrm{GeV}$ is not experimentally excluded.

More examples of MFV models:

1. SUSY-SM. In the absence of ~~SUSY~~ this is MFV:

$$\mathcal{L} = \int d^4\theta \left[\bar{Q} e^V Q + \bar{U} e^V U + \bar{D} e^V D \right] + \text{gauge kinetic terms} \left(\int d^2\theta\, W_\alpha W^\alpha \right) + \text{higgses} \times 2 + \text{leptons}$$

$$+ \int d^2\theta\, W + \text{h.c.}$$

with $W = H_1 Q y^U U + H_2 Q y^D D$ + non-quark terms

Here Q, U, D are superfields with

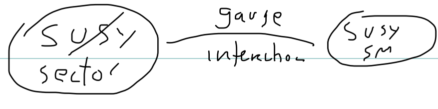
$$Q \sim (3,2)_{1/6} \qquad H_1 \sim (1,2)_{1/2}$$
$$U \sim (\bar{3},1)_{-2/3} \qquad H_2 \sim (1,2)_{-1/2}$$
$$D \sim (\bar{3},1)_{1/3}$$

Add soft-susy breaking

$$\Delta\mathcal{L}_{\cancel{susy}} = \phi_q^* m_q^2 \phi_q + \phi_U^* m_U^2 \phi_U + \phi_d^* m_d^2 \phi_d$$
$$+ \left(\phi_{h_1} \phi_q g^U \phi_U + \phi_{h_2} \phi_q g^D \phi_d + \text{h.c.} \right)$$

Unless $m^2_{q,U,d} \propto \mathbb{1}$ and $g^{U,D} \propto y^{U,D}$ new flavor changing interactions are present and large (the effects can be made small if the masses of scalars (from diagonalizing $\sim m_q^2 + m_U^2 + v^2 y^U y^{U\dagger} \ldots$) have large eigenvalues.

This is the motivation for gauge-mediated $S\cancel{USY}$.

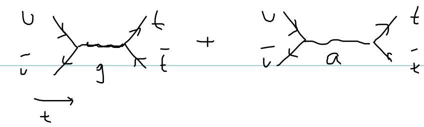


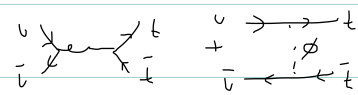
Gauge interactions in $\bar{Q} e^V Q + \cdots$ are flavor $\sim \mathbb{1}$

(In SUGRA mediated the problem is severe).

Note that for $U(3)^3$ to be a symmetry the squarks must transform too, just like quarks: $q_L \to U_q q_L$ $\quad \tilde{\phi}_q \to U_q \tilde{\phi}_q$, etc.

2. "MFV fields" Recently $t\bar{t}$ FB asymmetry, possibly explained by

(i) s-channel, eg, axigluon 

(ii) t-channel, eg, scalar 

I won't explain why axi-gluon introduces $U(3)^3$ breaking (roughly, one needs opposite sign couplings of $a^\mu \bar{u}\gamma_\mu\gamma_5 u$ and $a^\mu \bar{t}\gamma_\mu\gamma_5 t$).

Concentrate on t-channel models. Clearly $\phi \bar{t} u$ breaks $U(3)^3$.

Unless one has extreme fine tuning one will also have $\bar{c}u$ & $\bar{c}t$ couplings, and if L-handed quarks are involved, also $\bar{b}s$, $\bar{b}d$ & $\bar{s}d$ couplings.

One can have a $U(3)^3$-symmetric model by including a scalar multiplet that transforms under $U(3)^3$. For example, one can have

$$\bar{q}_L \phi u_R \quad \text{with} \quad \phi \to U_{q_L} \phi U^\dagger_{u_R} \quad (\text{and a } 2_{-1/2} \text{ under } SU(2)_W \times U(1)_Y).$$

This actually works (see:

Exercise: classify all possible dim-4 interactions $\sim \phi \bar{\psi} \psi'$ and corresponding transformation laws for ϕ (under $U(3)^3 \times$ SM-gauge grp)
(i) to order $(y^{U,D})^0$, (ii) with up to $(y^{U,D})^1$

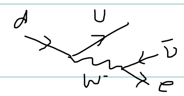
FCNC's

stands for Flavor Changing Neutral Currents

but is used more generally to mean FCN-transitions.

FC transitions in SM (review from previous lecture):

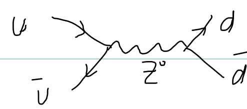
1. Tree level. Only $W^\pm$:

$n \to p e \bar{\nu}$: 

Exercise: If you have never computed μ-lifetime, check that

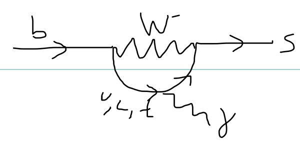
$$\Gamma(\mu \to e \nu_\mu \bar{\nu}_e) = \frac{G_F^2 m_\mu^5}{192\pi^3} \quad \text{for } m_e = 0$$

But Z^0 & h^0 interactions are diagonal in flavor

e.g. 

(or any $q\bar{q}$ or $\ell\bar{\ell}$).

2. 1-loop: Can we have FCNC's at 1-loop? Say $b \to s\gamma$?

Yes! 

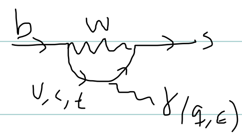
$\Rightarrow$ FCNC's are supressed in SM relative to tree level by $\sim \frac{g_2^2}{16\pi^2} \sim \frac{\alpha}{4\pi s_W^2}$

GIM-mechanism: more supression of FCNC in SM!

(i) "Old" Let's imagine a world with $m_u < m_c < m_t << M_W$

(Real world $m_u << m_c << M_W \approx \frac{1}{2} m_t$).

Without explicit computation of integrals, we see that



see below

$$= e q_\mu \epsilon_\nu \bar{u}(p_s) \sigma^{\mu\nu} \left(\frac{1+\gamma_5}{2}\right) u(p_b) \frac{\boxed{m_b}}{M_W^2} \frac{g_2^2}{16\pi^2} \times I$$

Dirac spinors

where $I = \sum_{i=u,c,t} V_{ib} V_{is}^* F\left(\frac{m_i^2}{M_W^2}\right)$

Now, expand in Taylor series $F(x) = F(0) + x F'(0) + \dots$

and use $\sum_{i=u,c,t} V_{ib} V_{is}^* = 0$

$$I = \underbrace{\left(\sum V_{ib} V_{is}^*\right)}_{0} F(0) + \left(\sum_i V_{ib} V_{is}^* \frac{m_i^2}{M_W^2}\right) F'(0) + \dots$$

Moreover $\sum_{i=u,c} V_{ib} V_{is}^* = - V_{tb} V_{ts}^*$

so $I = F'(0) \sum_{j=u,c} V_{ib} V_{is}^* \frac{m_i^2 - m_t^2}{M_W^2}$

$\Rightarrow$ The FCNC is supressed, in addition to the loop, by

$$\sim V_{ub} V_{us}^* \frac{m_u^2 - m_t^2}{M_W^2} + V_{cb} V_{cs}^* \frac{m_c^2 - m_t^2}{M_W^2} \sim \epsilon^4 \frac{m_t^2}{M_W^2} + \epsilon^2 \frac{m_t^2}{M_W^2}$$

that is, both by $\frac{m_q^2}{M_W^2}$ AND by ϵ^2.

Note: This "old" GIM is a good description in the lepton sector, since all masses $<< M_W$

Note #2: The $m_b \left(\frac{1+\gamma_5}{2}\right)$: the W couples to left handed fields $\Rightarrow (1-\gamma_5)$. But $\bar{\psi}_L \sigma^{\mu\nu} \psi_L = 0$. Need chirality flip $\psi_L \to \psi_R$: Computation gives $\bar{\psi}_L \sigma^{\mu\nu} i \partial b_L = m_b \bar{\psi}_L \sigma^{\mu\nu} b_R$

(ii) "Modern" GIM

Of course, $m_t \ll M_W$ is not a good approximation. But the supression by CKM's ϵ^2 is still there

$$I = \sum_i V_{ib} V_{is}^* F\left(\frac{m_i^2}{M_W^2}\right) = -\sum_{i=u,c} V_{ib} V_{is}^* \left(F\left(\frac{m_t^2}{M_W^2}\right) - \frac{m_i^2}{M_W^2}\right) \sim \epsilon^2 \left(F\left(\frac{m_t^2}{M_W^2}\right) - \frac{m_c^2}{M_W^2}\right)$$

It turns out that $F(x)$ is an increasing function with $F(1) \simeq \mathcal{O}(1) \gg \frac{m_c^2}{M_W^2}$ can be neglected.

$\Rightarrow$ the virtual t-quark exchange dominates this amplitude.

Exercise: show that for $s \to d\gamma$ it is no longer true that virtual t-quark dominates, that in fact c & t contributions are numerically (roughly) the same magnitude.

Bounds on NP, rough. Use $b\to s\gamma$ of previous discussion as example.

No MFV: extend SM by $\Delta\mathcal{L} = \frac{eF_{\mu\nu}}{\Lambda^2} H^\dagger \bar{b}_R \sigma^{\mu\nu} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \to \frac{ev}{\sqrt{2}\Lambda^2} \bar{b}_R \sigma^{\mu\nu} s_L F_{\mu\nu}$

So, roughly, $\frac{a_{NP}}{a_{SM}} \sim \frac{\frac{v}{\sqrt{2}\Lambda^2}}{|V_{tb}V_{ts}^*| \frac{\alpha}{4\pi s_w^2} \frac{m_b}{M_W^2}}$ and requiring $\frac{a_{NP}}{a_{SM}} \lesssim 10\%$

(since the SM prediction agrees with experiment with ~10% errors)

$$\Lambda^2 \gtrsim \frac{vM_W^2}{\sqrt{2}m_b} \frac{s_w^2}{|V_{tb}V_{ts}| \frac{\alpha}{4\pi}} \cdot \frac{1}{0.1} \to \Lambda > 70 \text{ TeV} \quad !$$

With MFV: $\Delta\mathcal{L} = \frac{eF_{\mu\nu}}{\Lambda^2} H^\dagger \bar{d}_R^i \lambda_d^{ij} \sigma^{\mu\nu} q_L^j \to 0$ (diagonal in flavor)

So same but $\lambda_d \to \lambda_d \lambda_u^\dagger \lambda_u \to \frac{ev}{\sqrt{2}\Lambda^2} \lambda_b \lambda_t^2 V_{tb} V_{ts}^* \bar{b}_R \sigma^{\mu\nu} s_L F_{\mu\nu}$

Now $\frac{a_{NP}}{a_{SM}} \sim \frac{\frac{\lambda_t^2}{\sqrt{2}\Lambda^2}}{\frac{\alpha}{4\pi s_w^2} \frac{1}{M_W^2}} \Rightarrow \Lambda^2 > \frac{1}{\sqrt{2}} M_W^2 \lambda_t^2 s_w^2 \left(\frac{4\pi}{\alpha}\right) \cdot \frac{1}{0.1} \Rightarrow \Lambda > 4 \text{ TeV}$

Determination of CKM / U.T.

Magnitudes: (i) V_{ud} (nuclei: superallowed $0^+ \to 0^+$ transitions)

(ii) V_{us}, V_{cd}, V_{cs}, V_{ub}, V_{cb} $\quad M \to M' \ell \nu$ (eg, $K^+ \to \pi^0 e^+ \nu$)

$$\langle p' | V^\mu | p \rangle = f_+(q^2)(p+p')^\mu + f_-(q^2)\, q^\mu \qquad q^\mu \equiv (p-p')^\mu$$

$V^\mu = \bar\psi' \gamma^\mu \psi$ (no A^μ by P in QCD)

$m_\ell \simeq 0 \to q^\mu \bar e \gamma^\mu (1-\gamma_5)\nu = 0 \to$ no f_- in rate
$\hookrightarrow$ (not quite for $\ell = \tau$).

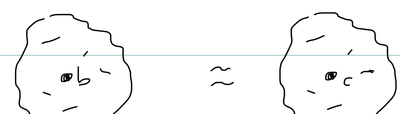
Central problem: f_+ ?

$\longrightarrow$ Symmetry: for same state (eg $\pi \to \pi$) $\partial_\mu V^\mu = 0 \Rightarrow f_- = 0$ and $f_+(0) = 1$

For $K \to \pi$ SU(3) symmetry. $f_+(0) = 1 + \cancel{\mathcal{O}(m_s)} + \mathcal{O}(m_s^2)$

Ademollo-Gatto Thm.

$\bar B \to \bar D \ell$ ($b \to c \ell \nu$): HQ symmetry (souped up with HQET)!



The "brown muck" $\leftarrow$! is in both cases bound by an ∞-heavy 3plet source of color

In some sense $f_+(0) = 1$ here too.

Long story short: get rid of m so as to take $m \to \infty$ limit, eg, use $v^\mu = \frac{p^\mu}{m}$ and normalize states to 1 (as in NR physics).

Then $\langle D(v') | V^\mu | B(v) \rangle = \xi(v \cdot v')(v+v')^\mu \qquad \xi(1) = 1$ is the Isgur-Wise function

(iii) $X^0 - \bar X^0$ mixing: we'll see $B^0 \to V_{tb} V_{td}^*$, $B^s \to V_{ts} V_{td}^*$

(iv) α, β, γ angles directly from CPV asymmetries

(we'll) study $\sin(2\beta)$ - the poster boy of those