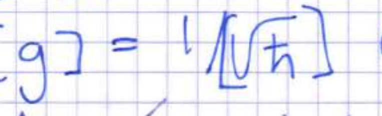
NATURAL UNITS OR NOT SO

Keeping $\hbar \neq 1$ units but $c=1$

$$[S] = [\hbar] = E \cdot L$$

$$[\mathcal{L}] = [\hbar]/L^4$$

$$\begin{cases} [D_\mu] = 1/L & \\ [\phi] = [\sqrt{\hbar}]/L & \text{fields} \\ [\psi] = [\sqrt{\hbar}]/L^{3/2} & \\ [M] = 1/L & \text{Masses} \\ [g] = 1/[\sqrt{\hbar}] & \text{Couplings} \end{cases}$$



Dimensionless:

$$\frac{g\phi}{M}, \quad \frac{D_\mu}{M}, \quad \frac{g\psi}{M^{3/2}}, \quad \frac{gF_{\mu\nu}}{M^2}$$

BUILDING THEORIES

Basics:

$$QFT \oplus \text{Lorentz Invariance}$$

then, given the dynamical degrees of freedom, we can build up a Lagrangian:

$$S = \int d^4x\, \mathcal{L} \quad : \quad \boxed{\mathcal{L} = \frac{\Lambda^4}{g_*^2}\hat{\mathcal{L}}\left(\frac{D_\mu}{\Lambda}, \frac{g_*\Phi}{\Lambda}, \frac{y\cdot\Psi}{\Lambda}, \frac{gF_{\mu\nu}}{\Lambda^2}\right)}$$

see behind

Given a complicated $\hat{\mathcal{L}}$, we can always go to low-energy (distance larger than $1/\Lambda$), weak fields ($g_*\langle\Phi\rangle/\Lambda$; $g_*\frac{\langle D\Phi\rangle}{\Lambda^2} \ll 1$) to be able to perform a Taylor expansion:

EFT = Effective Field Theory

$$\mathcal{L} = \mathcal{L}_{\leq 4} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \dots$$

$\mathcal{L}_d$ = contains terms (operators) made with fields and derivatives with dimension d

e.g.
$(\partial_\mu\phi)^2 \to d=4$
$\phi^4 \to d=4$
$\phi^5 \to d=5$
⋮

$\Lambda \equiv$ cut-off scale of the ~~EFT~~ (not valid for larger energies $E \gtrsim \Lambda$)

NO PREDICTIVITY

(*) An alternative approach could be to go
for consistent theories of S-matrices
(e.g. imposing CFT, ...)

I

$\boxed{\text{QED = theory of a massless spin-1 + fermions}}$

Usually, built using the principle of gauge inv.

$\Downarrow$

Not a symmetry
A trick to obtain a theory of massless spin-1 objects

After field redefinitions:

$$\boxed{\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + i\bar{\Psi}\not{D}\Psi - m\bar{\Psi}\Psi} + \text{higher dimensional operators}$$

$$D_\mu \equiv \partial_\mu - ieA_\mu$$

$\boxed{e > 0}$ $\boxed{m > 0}$

Accidental symmetries: Symmetries of $\mathcal{L}_{\leq 4}$:

e.g. PARITY: $\vec{X} \to -\vec{X}$

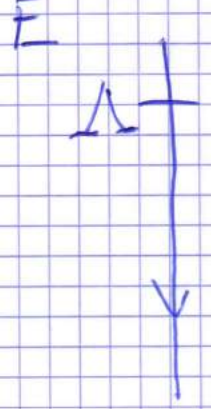
$\Psi \to \gamma_0 \Psi = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \Psi$

CHARGE CONJUGATION:

$\Psi \to C\bar{\Psi}^T \qquad C \equiv i\gamma^0\gamma^2$

RANGE OF VALIDITY OF THE THEORY:

(loss of predictivity or non-sense predictions)



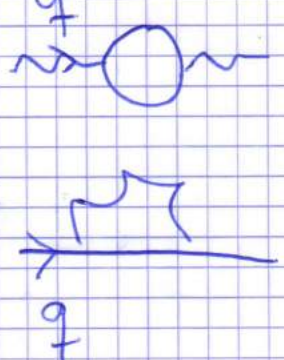
CUT-OFF SCALE: Higher dim operators equally important
Loss of predictivity

Important to know Λ:

At $E > \Lambda$ discovery guaranted (No lose theorem for discovery)

But, can we have $\Lambda \to \infty$?

In QED we can make sensible predictions as far as e^2 is small:

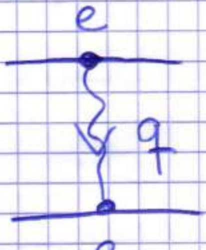


$$\approx e^2 \int \frac{d^4p}{(2\pi)^4} f(p,q) \overset{(*)}{=} \underbrace{\frac{e^2}{16\pi^2}}_{\text{Expansion parameter}} \overbrace{\int \underset{2p\,dp}{\underset{\shortparallel}{dp^2}}\, p^2 f(p,q)}^{\ln p^2 \text{ for dimensionless quantities}}$$

$$\boxed{\frac{e^2}{16\pi^2} \ll 1}$$

(*) $$\int d^n p = \int S_n \, dp$$

$$S_n = n \frac{\pi^{n/2}}{\Gamma(1+\frac{n}{2})} \begin{cases} S_4 = 2\pi^2 \\ S_5 = \frac{8}{3}\pi^2 \end{cases}$$



$\sim \dfrac{e^2}{q^2} \xrightarrow[\text{main quantum correction}]{} \dfrac{e^2(q)}{q^2}$

$$\boxed{e(q_1)=\frac{e(q_2)}{1-\frac{e^2(q_2)}{12\pi^2}\ln\frac{q_1}{q_2}}}$$

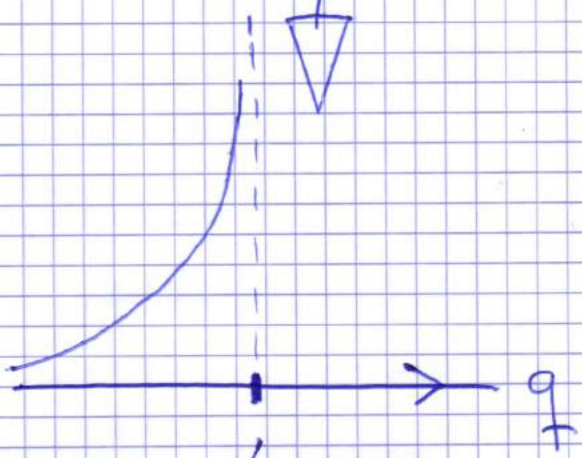
RGE: $\dfrac{de(q)}{d\ln q}=\dfrac{e^3}{12\pi^2}+\dots$

"Landau pole":

$e\to\infty$

at some scale:

$$\boxed{\Lambda}\approx e^{\frac{3\pi}{2\alpha}}\cdot M_w\approx \boxed{10^{264}\ \text{GeV}}$$



Predictivity is lost:

Not even in the Lattice it is found a consistent theory in the continuum limit unless $\alpha\to 0$ (triviality)

Possibility of New Physics: New d.o.f. could enter at $E\gg$

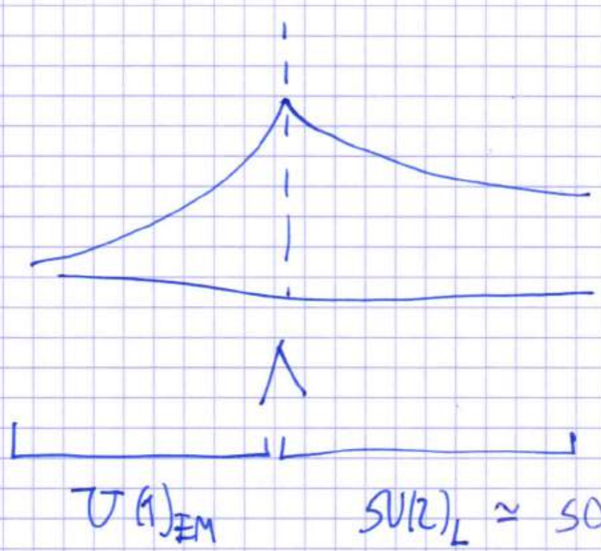
(Different from QCD (non-abelian theories) where no new physics can enter in the IR:

)

UV-completions of QED

$\equiv$ Finding a new theory that make QED consistent (beyond Λ)

e.g. $U(1)$ is embedded in a $SU(2)$ at high energy



$\hookrightarrow$ theory of $\gamma \oplus W^\pm$

$$m_W \sim \Lambda \sim g\langle \vec{H} \rangle$$

$$\vec{H} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Lambda$$

or $\langle H \rangle = \Lambda T_3$

Fermion (electrons)

$\vec{\Psi} \in 3$ of $SU(2)$:

$$U(1)_{EM} = e^{iT_3}$$

$$\begin{pmatrix} e_R^c \\ \nu_L \\ e_L^- \end{pmatrix} \Longleftarrow \quad \varphi \equiv T_3 = \begin{pmatrix} 1 & & \\ & 0 & \\ & & -1 \end{pmatrix}$$

New neutral state

New heavy state!!

<u>Mass to ν</u>: $\underset{\begin{pmatrix} e_R^c \\ e_L \\ \nu_L \end{pmatrix}}{\vec{\Psi}} \cdot \underset{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}{\vec{H}} \, \nu_R$

NATURALNESS

Are the parameters of the lagrangian taking

unexpected values? Unnatural?

Range of e: 0 to 4π

$e(0)\big|_{\text{exper.}} \approx 1/3$

Range of m: 0 to Λ

$$\boxed{m_e\big|_{\text{exper.}} \sim 1/2 \text{ MeV} \ll \Lambda}$$

very close to zero (boundary!)

Expectation from quantum effects:

$\cancel{\dfrac{1}{\not p - m_e}}$ $\cancel{\dfrac{1}{\not p - m_e + \Sigma(p)}}$

Correction:

we do not trust the theory beyond Λ

$$\sim \frac{e^2}{16\pi^2} m_e \underbrace{\int_0^\Lambda dp \frac{1}{p}}_{\ln \Lambda}$$

proportional to m_e: If m_e is sm

the corrections are small

"technically natural"

(*) Also if e would have been very small it would have been technically natural:

$$\Delta e \propto \frac{e^2}{16\pi^2} e$$

$\boxed{\text{why } \Delta m_e \propto m_e \text{ ?}}$

Extra symmetry in the limit $m_e \to 0$:

$$\left.\begin{array}{l}\psi_L \to \psi_L \\ \psi_R \to e^{i\theta}\,\psi_R\end{array}\right\} \text{ chiral symmetry}$$

$\boxed{m_e = 0 \text{ is a special point}}$

spurion technique:

Promote the parameter m_e to a complex field that transforms

$$\left.\begin{array}{l} m_e \to m_e\, e^{-i\theta} \\ \psi_L \to \psi_L \\ \psi_R \to e^{i\theta}\,\psi_R \end{array}\right\} \quad m_e \bar{\psi}_L \psi_R + h.c.$$

Invariant!

this symmetry tells you how loop correction depend on m_e

$\propto m_e \bar{\psi}_L \psi_R$

to be invariant

$\propto m_e^2 \left(\bar{\psi}_L \psi_R\right)^2$

BEHIND THE SM

the SM as an EFT:

Degrees of freedom:

spin-0 : H

spin-1/2 : Ψ_i = quarks and leptons

spin-1 : γ, W, Z, G

spin-2 : $h_{\mu\nu}$

$$\boxed{\mathcal{L}_{\leq 4}} = m_H^2 |H|^2 - \frac{1}{4g^2} F_{\mu\nu}^{a\,2} + i\,\overline{\Psi}_i \underbrace{\not{D}}_{\partial_\mu - i g_a T^a A_\mu^a} \Psi + |D_\mu H|^2$$

$$+ Y_{ij} H \overline{\Psi}_i \Psi_j + h.c. + \lambda |H|^4 + \theta\, \tilde{G}^a \tilde{G}^a$$

$$\underbrace{(\partial_\mu h_{\mu\nu})^2}_{\text{representative}} + G_N h_{\mu\nu} T^{\mu\nu}_{SM} \qquad \left(\underbrace{T^{\mu\nu}_{SM}}_{\text{Energy-momentum tensor}} = \eta^{\mu\nu} \Lambda_{cosmo} + |\partial_\mu H|^2 + \dots \right)$$

	SU(3)	SU(2)	U(1)
Q_L	3	2	1/3
u_R	3	1	4/3
d_R	3	1	-2/3
L_L	1	2	-1
e_R	1	1	-2
H	1	2	1

Clever basis:

(*)
$$\boxed{\begin{aligned} Y_e &= Y_e^{diag} \\ Y_d &= V_{CKM}\, Y_d^{diag} \\ Y_u &= Y_u^{diag} \end{aligned}}$$

V_{CKM} = 3 angles and a phase

Accidental symmetries: = only in the operators of dim' < 4

(I) Lepton number: $e_{L,R} \to e^{i\theta_e} e_{L,R}$

$\mu_{L,R} \to e^{i\theta_\mu} \mu_{L,R}$

$\tau_{L,R} \to e^{i\theta_\tau} \tau_{L,R}$

(II) Baryon number: $q_{L,R} \to e^{i\theta_B} q_{L,R}$

If masses are zero: $U(3)_Q \otimes U(3)_u \otimes U(3)_d \otimes U(3)_L \otimes U(3)_e$

↳ some of them broken by quantum effects

(*) $$\bar{Q}\, Y_u\, u_R = \bar{Q}\, U_L^{u\dagger} \underbrace{U_L^u Y_u U_R^{u\dagger}}_{Y_u^{diag}} U_R^u u = \bar{Q}'\, Y_u^{diag}\, u'$$

$$\bar{Q}\, Y_d\, d = \bar{Q}\, U_L^{d\dagger} U_L^d\, Y_d\, U_R^{u\dagger} U_R^u d =$$

$$= \bar{Q}\, U_L^{\dagger d}\, Y_d^{diag}\, d' = \bar{Q}'\, \underbrace{U_L^u \cdot U_L^{d\dagger}}_{V_{CKM}}\, Y_d^{diag}\, d'$$

Interesting physics from higher-dimensional operators:

Lepton number and baryon number violated:

In dim ≤ 4 operators, there is an accidental symmetry:

① $L_L^i \to e^{iL_i} L_L^i$ 3 indep. rotations since Y_e is diagonal

② $Q_L \to e^{iB} Q_L$

$q_R \to e^{+iB} q_R$

Baryon = 3 quarks "accidentally" stable because of symmetry $B_{baryons} = 3B$

Broken by:

dim = 5 operator:

$\psi^c = G\bar{\psi}^T$

$G = \begin{pmatrix} \varepsilon & \\ & \varepsilon \end{pmatrix}$

$$\frac{1}{\Lambda}\left(\bar{L}_L^c \cdot H\right)\left(H \cdot L_L\right)$$ the only one

$\varepsilon^{ab} H^a L_L^b$ (*)

When $\langle H\rangle \neq 0$, give masses to ν_L:

$$\frac{v^2}{\Lambda}\bar{\nu}_L^c \nu_L$$

Majorana mass

$m_\nu = \frac{v^2}{\Lambda}$ can be very small if $\Lambda \gg v$

For $m_\nu \lesssim 0.1\, eV \to \Lambda \sim 10$

(*) SU(2) invariance:

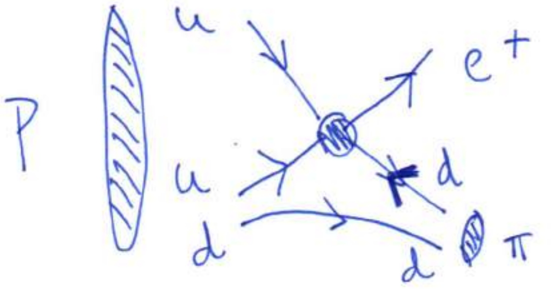
$$\varepsilon_{rs}\,\psi^r\psi^s = \varepsilon_{rs}\,U^r{}_{r'}\psi^{r'}\,U^s{}_{s'}\psi^{s'}$$

$$= \underbrace{\varepsilon_{rs}\,U^r{}_{r'}\,U^s{}_{s'}}_{\operatorname{Det} U \cdot \varepsilon_{r's'}}\,\psi^{r'}\psi^{s'}$$

$$\operatorname{Det} U = 1$$

dim-6 operator leading to B violation (proton decay)

$$\frac{1}{\Lambda^2}\,\varepsilon_{\alpha\beta\gamma}\,\varepsilon_{rs}\left[\bar{q}_L^{c\;r\alpha}\,\gamma^\mu\,u_R^\beta\right]\left[\bar{d}_R^{c\;\gamma}\,\gamma_\mu\,L_L^s\right]$$



$p \to \pi^0 e^+$

For $\Lambda \gtrsim 10^{15}$ GeV $\quad \tau_p \gtrsim 10^{34}$ years

Experimental bound!

Another important observables (CP-odd):

$\boxed{\text{Electric Dipole Moment (EDM)}}$

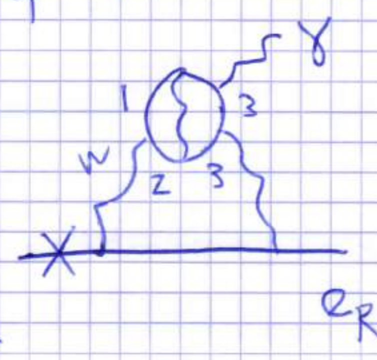
$H = -\vec{d}\cdot\vec{E}$

Let's concentrate in the one of the electron:

Since it is CP-odd, we need CP violation

$\equiv$ 3 quarks in the loop

e.g.



surprisingly, in the SM cancel (Pospelov et al)

$d_e \lesssim 10^{-38}\ e\cdot cm$

$d_e|_{exp} \lesssim 10^{-28}\ e\cdot cm$

BSM operator (dim 6):

$$\frac{c\, Y_e}{\Lambda^2} H \sigma^a \bar{L}_L \sigma^{\mu\nu} e_R W^a_{\mu\nu}$$

$\Longrightarrow \Lambda \gtrsim 500\,\text{TeV}$

Im $c \sim \mathcal{O}(1)$

CHALENGES FOR BSM

Strong bounds on dim-6 operator induced:

Λ (GeV)

- 10^{15} — proton decay, neutrino masses
- $4 \cdot 10^{7}$ — ϵ_K
- 10^{6} — $\Delta m_K / m_K$
- 10^{5} — EDM of e
- 10^{3} — EWPT

LHC accesible

Accidental symmetries happen because of the simplicity of the theory (e.g. as soon as we add more particles to QED we have C, P and CP breakings)

but BSM are not simple (only hope: Emerge from a simple theory)

$\Downarrow$

strong dynamics, strings

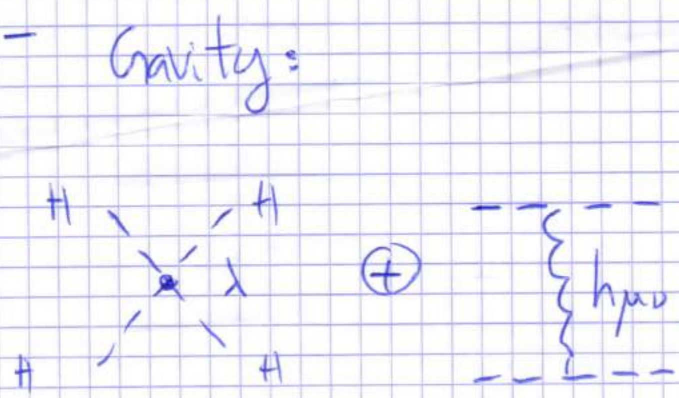
as QCD

CONSISTENCY OF THE SM

Keeping couplings below 4π:

- All SM couplings become weaker at high energie
  except g' and gravity

- Gravity:



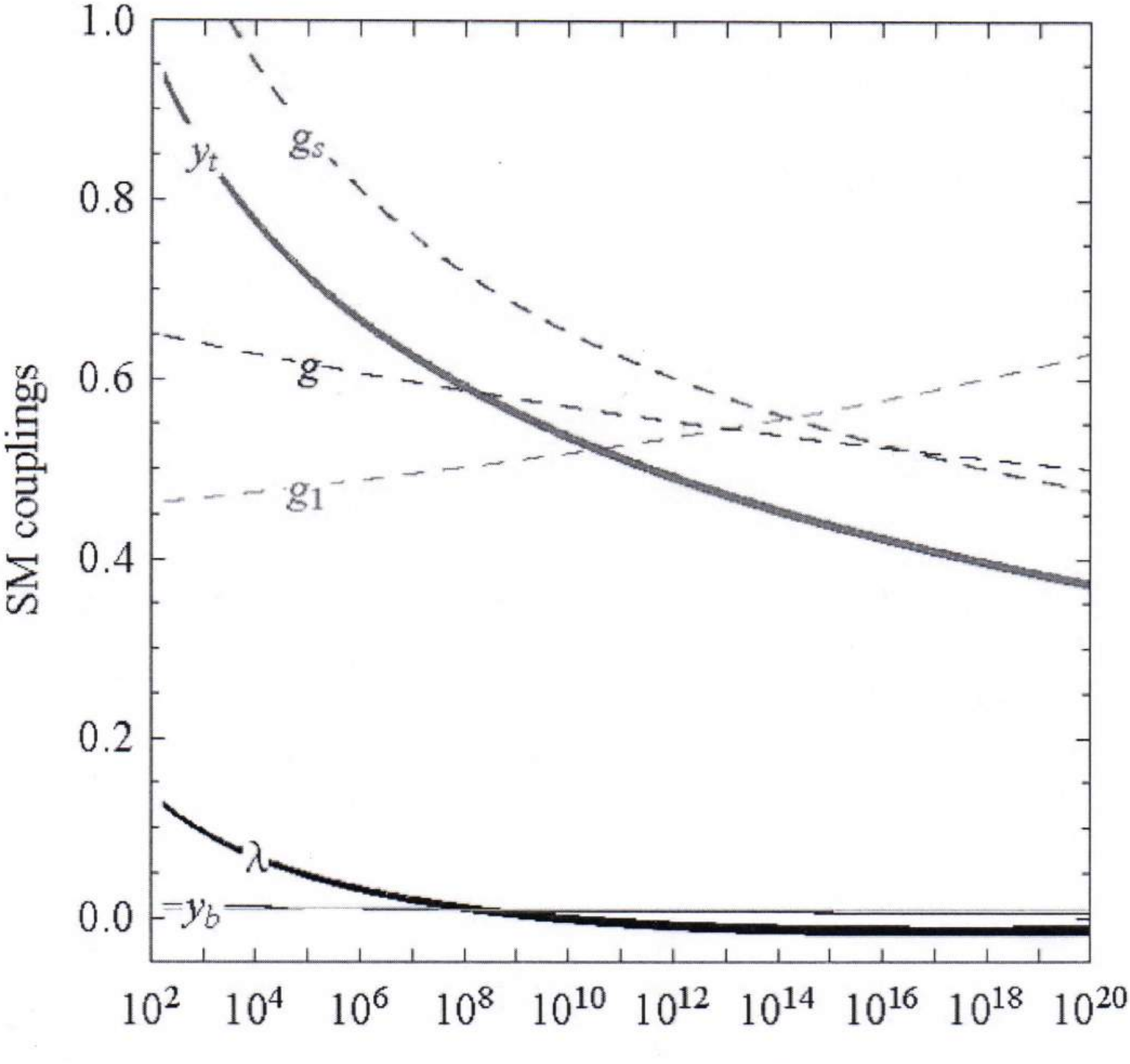
$$\underset{E\ \text{large}}{\sim} \lambda + G_N E^2 \lesssim 4\pi$$

$$E\Big|_{max} \sim 4\pi/\sqrt{G_N} = 4\pi M_P \sim \boxed{4\pi\, 10^{19}\,\text{GeV}}$$

$$\Lambda \sim 10^{20}\ \text{GeV} \quad \text{Scale of new physics}$$

UV- COMPLETION:

String theory



1.0
0.8
0.6
0.4
0.2
0.0
SM couplings
y_t
g_s
g
g_1
λ
y_b
10^2 10^4 10^6 10^8 10^{10} 10^{12} 10^{14} 10^{16} 10^{18} 10^{20}
RGE scale μ in GeV