Focus on WIMPs

Historical aside - so far all DM measurements gravitational in nature

2 previous examples - 1, Neptune discovered through wobbles in orbit of Uranus. Original DM! Found in 1846 in 30 min of searching within 1° of prediction of Urbain Le Verrier.

2, Perihelion advance of Mercury. Newtonian gravity only approximation of GR. New physics! Although originally thought to be planet Vulcan.

Freeze out : why we like WIMPs. (See Cline for more details)

(For gory details see Bernstein - Kinetic Theory in expanding universe)

If DM has more than gravitational interactions there is a nice story.

Suppose $\chi\chi \to f\bar{f}$ ← SM fermion.

After big bang DM produced along with SM particles now in thermal soup

Phase space density of DM $f(|\vec{p}|, t)$ (NB $f \geq 0$ and funct of $|\vec{p}|$ not $\vec{p}$ ← FRW is homogeneous + isotropic)

Obeys Liouville Boltzmann eqn: $\dfrac{\partial f}{\partial t} - \underbrace{\left(\dfrac{\dot{a}}{a}\right)}_{=H} |p| \dfrac{\partial f}{\partial p} = C[f]$ ← collision term.

Often don't need to worry about full phase space distribution, assume α f thermal (Method of pseudo-chemical potential)

FRW : $ds^2 = +dt^2 - a^2(t)\,(dr^2 + r^2 d\Omega^2)$

Assumed $k = 0$.

$$H \equiv \frac{\dot{a}}{a}.$$

[NB — Technical aside: in general there are no equilibrium solutions to the non-interacting Liouville eqn $L(f) = 0 = \frac{\partial f}{\partial t} - \frac{\dot{a}}{a} p \frac{\partial f}{\partial p}$

<u>But</u> $m = 0$, $m \to \infty$ has solutions. So for very high and very low temperatures we can solve.

Assume $f_{eq} = e^{\mu(t) - \beta(t) E}$; μ chemical pot.
β $1/T$.

$E^2 = p^2 + m^2$

Then $L(f_{eq}) = 0 \Rightarrow \dot{\mu} - E\dot{\beta} + \frac{\dot{a}}{a} p \beta \frac{\partial E}{\partial p} = 0$ $\left[\frac{\partial E}{\partial p} = p/E \right]$

$$\frac{\dot{\mu}}{\dot{\beta}} = E - \frac{\dot{a}}{a} \frac{\beta}{\dot{\beta}} \frac{p^2}{E}.$$

<u>$m \to 0$</u>: $E = p$ ∴ $\frac{\dot{\mu}}{\dot{\beta}} = p\left(1 - \frac{\dot{a}}{a} \frac{\beta}{\dot{\beta}}\right)$ ⇒ soln is $\mu = 0$ $\beta = \text{const} \times a$.
So massless particles stay in equilibrium and $T \sim 1/a$.

$m \to \infty$ $E = m + p^2/2m + \dots$ $\frac{\dot{\mu}}{\dot{\beta}} - m = \frac{p^2}{2m}\left(1 - 2\frac{\dot{a}}{a} \frac{\beta}{\dot{\beta}}\right)$ ⇒ soln is $\mu = m\beta + \text{const}$.
$\beta \dot{a} = \frac{1}{2} a \dot{\beta}$ i.e. $\beta \sim a^2$.
So NR massive particles stay in equilibrium with $T \sim 1/a^2$.]

Since we don't usually care about full phase space we can take moments

$n = g \int \frac{d^3p}{(2\pi)^3} f(|p|, t)$, (Integrate by parts)   (g: d.o.f.)

Recall radiation $n \sim T^3$
matter $n \sim (mT)^{3/2} e^{\dots}$

$$\dot{n} + 3Hn = g \int \frac{d^3p}{(2\pi)^3} C[f].$$

$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\Omega^2)$
FRW.

Gondolo + Gelmini ... Improved Analysis $\qquad \pm$ Boson / Fermion $\quad$ Bose enhancement / Pauli blocking $\quad$ easier / harder

$$g_1\int\frac{d^3p_1}{(2\pi)^3}C[f_1] = -\sum_{\text{spins}}\int\Big(f_1f_2(1\pm f_3)(1\pm f_4)|\mathcal{M}_{12\to34}|^2 - \cancel{f_3f_4}(1\pm f_1)(1\pm f_2)f_3f_4 \times|\mathcal{M}_{34\to12}|^2\Big)$$

$(1,2\to3,4)$

$$\times(2\pi)^4\,\delta^{(4)}(p_1+p_2-p_3-p_4)$$

$$\times\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\cdots\frac{d^3p_4}{(2\pi)^3 2E_4}$$

Assume f_i are FD or BE ie thermal equilibrium.

Assume $T \ll E-\mu$ so that $f\sim$ MB ie $1\pm f\simeq 1$.

Assume 3,4 (SM states) are kept in equilibrium by other reactions within SM. So $f_3, f_4 = f_{3,4}^{eq}$.

$$\langle\sigma v_{mol}\rangle = \frac{\int \sigma v\, dn_1^{eq} dn_2^{eq}}{\int dn_1^{eq}\, dn_2^{eq}} \qquad e^{-E/T}d^3p_1$$

Then $\dot{n}+3Hn = \langle\sigma v\rangle\left(n_3^{eq}n_4^{eq} - n_1n_2\right)$

Detailed balance at equilibrium $\Rightarrow n_1^{eq}n_2^{eq} = n_3^{eq}n_4^{eq}$.

Finally assume $\overset{1=2}{n_1 = n_2}$ ie $\chi\chi\to\bar{f}f$

$$\boxed{\dot{n}+3Hn = \langle\sigma v\rangle\left(n_{eq}^2 - n^2\right)}$$

$\Leftarrow$ For Dirac or Maj $\sigma_M A^2 = \frac{\sigma_D A^2}{2}$ but remove 2 particles.

(typically solve numerically)

$n\sim a^{-3}$ (even w/o interactions)

HW

So useful to define $Y = n/s$ $\quad$ comoving density

s = entropy density.

Also $\cancel{s}$ $sa^3 = $ const. $\quad x = m/T$

$$\frac{dY}{dt} = \frac{\dot{n}}{s} - \frac{\dot{s}}{s^2}n = \frac{\dot{n}}{s}+\frac{3Hn}{s} \Rightarrow \frac{dY}{dt} = \langle\sigma v\rangle s\left(Y_{eq}^2 - Y^2\right)$$

$$\frac{dY}{dt} = \frac{dY}{dx}\cdot\frac{dx}{dt}. \quad \text{In RD} \quad H \simeq \overset{\text{really } 1.67\ldots}{\tfrac{5}{3}} g_*^{1/2} \frac{T^2}{M_P} \quad \text{and} \quad T \sim 1/a.$$

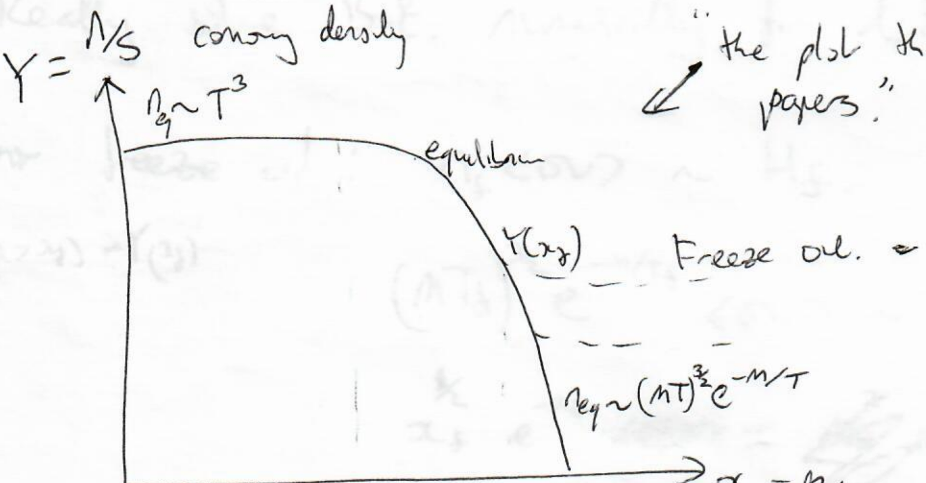
$M_P \leftarrow 1.2\times10^{19}$ GeV.

So $\quad \frac{\dot a}{a} \sim \frac{1}{a^2} \Rightarrow a \sim t^{1/2}$

$\Rightarrow \frac{dT}{dt} \sim -T^3 \Rightarrow \frac{dx}{dt}$

Putting in factors $\quad \frac{dY}{dt} = + \frac{5}{3} g_*^{1/2} \frac{m^2}{M_P} \frac{1}{x} \frac{dY}{dx}$

Finally, $$\boxed{\frac{dY}{dx} = \frac{x\, s \langle\sigma v\rangle}{\frac{5}{3} g_*^{1/2} \frac{m^2}{M_P}} \left(Y_{eq}^2 - Y^2\right)}$$ ← BE as usually presented. Want to solve for $Y(x\to\infty)$ $\left(s = s_0 x^{-3}\right)$



"the plot that launched a thousand papers." – "Christopher Marlowe" :)

gets into the eq of H
$$\frac{g^2}{\Lambda^2}(\bar\chi\chi)(\bar q q)$$
$$g \gtrsim 10^{-6}\left(\frac{\Lambda}{100\,\text{GeV}}\right)^2\left(\frac{\text{GeV}}{m_\chi}\right.$$

Competition between $n\langle\sigma v\rangle$ and H.

For simplicity assume $\langle\sigma v\rangle = \text{const}$.

Early $n\langle\sigma v\rangle \sim T^3$, $H \sim T^2$ so expansion not important & in equilibrium. $n \sim 1/a^3$, $Y = \text{const}$.

Once $T \sim m$ hard to do $f\bar f \to \chi\chi$, still do $\chi\chi \to f\bar f$ $\quad n \sim (mT)^{3/2} e^{-m/T}$, $H \sim T^2$

Expansion wins. Falls out of equilibrium. <u>Freeze Out</u>

Early on, T large, x small: $\underbrace{n\langle\sigma v\rangle \gg H}_{\text{ignore H term}} \Rightarrow \begin{matrix} n = n_{eq} & n \sim 1/a^3 \\ Y = Y_{eq} & Y \sim \text{const.}\end{matrix}$

$n \sim T^3$, $H \sim T^2$

RHS drives $Y \to Y_{eq}$.
$n \to n_{eq}$.

Once below mass, χ drops of eq. distribution at $Y \to Y_{eq}(x_f)$ (at $x = x_f$)

Late times $n\langle\sigma v\rangle \ll H$. ignore r.h.s $Y \to$ const $(Y_{eq}(x_f))$
$n \to 1/a^3$.

Cross over happens at $\underline{n\langle\sigma v\rangle \sim H}$ $\Leftarrow$ freeze out

[Really solve B.E. numerically for details]

Approx freeze out! $n_f\langle\sigma v\rangle \sim H_f$. $x = m/T$.

$Y(x>x_f) = Y(x_f)$

$$(mT_f)^{3/2} e^{-m/T_f} \langle\sigma v\rangle \sim T_f^2 / M_P.$$

$$x_f^{1/2} e^{-x_f} \cancel{\langle\sigma v\rangle} = \cancel{\frac{M_\chi^2}{M_P}} \propto \frac{1}{M_\chi M_P \langle\sigma v\rangle}.$$

Take logs: $\underline{x_f \sim \log(m_\chi M_P \langle\sigma v\rangle)}$ + log corrections depending on m_χ.

What of final density in DM? $n_f = \frac{H_f}{\langle\sigma v\rangle} \Rightarrow Y(x_f) = \frac{H_f}{s_f\langle\sigma v\rangle}$

So present $\Omega_\chi = \frac{Y(x_f) \times s_0 \times m_\chi}{\rho_{cr}}$ (present day entropy density: s_0) $\simeq \frac{s_0}{\rho_{cr}} \frac{T_f^2}{M_P} \frac{1}{T_f^3} \frac{m_\chi}{\langle\sigma v\rangle} = \frac{s_0}{\rho_{cr} M_P} \frac{x_f}{\langle\sigma v\rangle}$.

$\rho_{cr} = \frac{3H^2}{8\pi G_N} \simeq 8\times10^{-47} h^2\,\text{GeV}^{-4}$

Since x_f only log sensitive for large range of σv $x_f \sim 25 \Rightarrow \boxed{\Omega_\chi \sim \frac{1}{\langle\sigma v\rangle}}$

I have dropped lots of numerical factors, π's, g_* etc. (See Cline)

Putting them back in

0.1188 observed, Planck etc

$$\Omega_\chi h^2 \simeq 0.1 \left(\frac{x_f}{25}\right)\left(\frac{g_*}{80}\right)^{-1}\left(\frac{3\times10^{-26}\,\mathrm{cm}^3\mathrm{s}^{-1}}{\langle\sigma v\rangle}\right).$$

$$\sigma v \sim \frac{\alpha^2}{M_W^2} \simeq 3\times10^{-26}\,\mathrm{cm}^2\mathrm{s}^{-1}$$!! <u>WIMP</u>

but many other ways to $\frac{g_{hid}^4}{m_{hid}^2}$ "w

0803.4196 Feng + Kumar

<u>Lee-Weinberg</u> + <u>Unitarity</u> define mass range.

$\hookrightarrow \langle\sigma v\rangle \sim G_F^2 m_\chi^2 \Rightarrow \Omega h^2 \leq 0.12 \Rightarrow \sigma v \gtrsim$ something

$\Rightarrow m_\chi \gtrsim 2$ GeV.

$\langle\sigma v\rangle \sim \frac{4\pi\,(\text{or } 1)}{m_\chi^2} \Rightarrow m_\chi \lesssim 40-100$ TeV

Finite mass range for "WIMPs".

↓

Thermal relic, abundance set by annihilations in SM.

Note lower end of range set by SM couplings to W/Z.

If other new gauge forces that DM couples to $\sigma \sim \frac{m_\chi^2 g_D^4}{m_D^4}$

then DM can be lighter. <u>But</u> if thermal still need to avoid erasing structure i.e. no WDM $\rightarrow$ $m_\chi \gtrsim 1$ keV. + BBN $\gtrsim$ 1 MeV

Q: repeat freeze out for baryons?, Why are they not a thermal relic? Where do they come from? (Asymmetric DM)

[Other idea, WIMP (thermal) decays into totally inert DM (superWIMP) Feng...]

Recap

DM stable, WIMP thermal relic gives correct abundance, NO particle in SM $\Rightarrow$ new physics.

What type of new physics?

Why stable if $m \sim 100$ GeV and coupled to SM?

LPOPs

Proton stable since lightest baryon.

Electron " " " charged particle.

DM stable because lightest particle charged under new symmetry

Simplest example is Z_2 SM even

New odd.

LPOP stable. Can only couple in pairs to SM.

eg Z_3 Agashe + Servant 0403143

$Z_2 \rightarrow Z_n \rightarrow U(1) \longrightarrow SU(N) \longrightarrow$ gauged or global

Interestingly many BSM models have weak scale particles + parities for other reasons!!