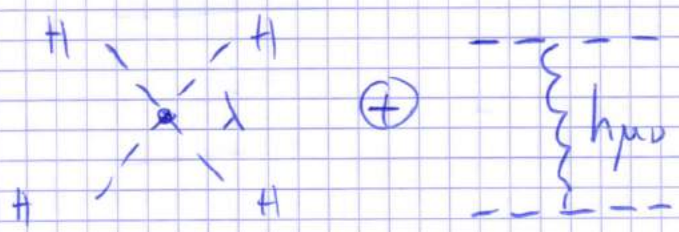
Consistency of the SM

Keeping couplings below 4π:

- All SM couplings become weaker at high energies except g' and gravity
- Gravity:



$$\underset{E\ \text{large}}{\sim} \lambda + G_N E^2 \lesssim 4\pi$$

$$\Downarrow$$

$$E\Big|_{max} \sim 4\pi/\sqrt{G_N} = 4\pi M_P \sim \boxed{4\pi\, 10^{19}\, \text{GeV}}$$

$$\Lambda \sim 10^{20}\ \text{GeV} \quad \text{Scale of new physics}$$

UV-completion:

String theory

NATURALNESS IN THE SM

Parameters:

Mass dimension : Λ_{cosmo} , m_H^2 , $M_P \approx \Lambda$

Dimension less : $g_{1,2,3}$, λ , Y_e , Y_d , Y_u

θ_0

the only coupling that does not selfrenormalize
$\Delta\lambda \neq \lambda$

THE STRONG CP PROBLEM

$$\frac{\theta_0}{32\pi^2} G^a \tilde{G}^a \qquad \theta = \theta_0 + \mathrm{Arg\,Det}\, M_q$$

$$\boxed{\theta = \theta_0 + \sum_i \mathrm{Arg}\, \underbrace{M_{q_i}}_{\textstyle\sum_i \alpha_i}}$$

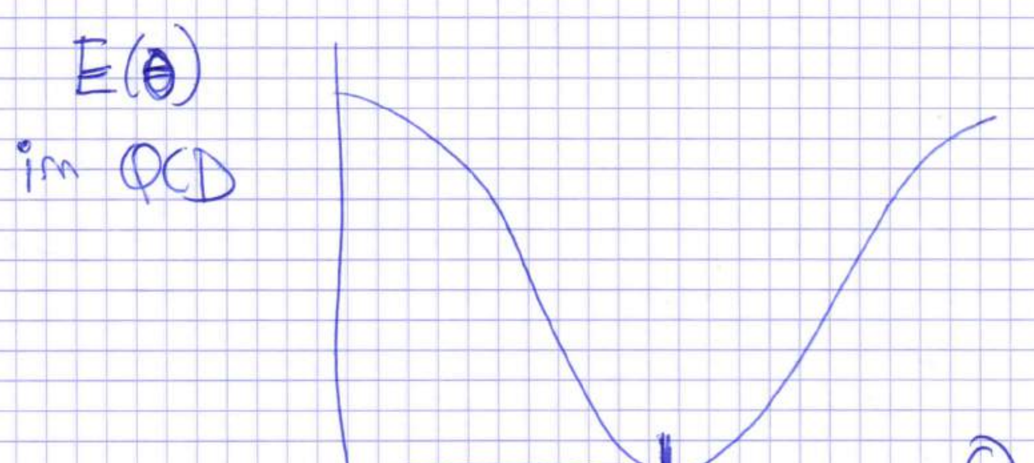
$$+\ \bar{q}^i_{f_L} q^i_{f_R} \tilde{M}_{q_i} e^{i\alpha_i} \qquad 0 \leq \theta < \pi$$

VIOLATE CP: $d_n \sim 2.\,|\theta| \cdot 10^{-16}\ e \cdot cm$

$$d_n\Big|_{exp} < 2.9 \cdot 10^{-26}\ e\cdot cm$$

$$\Downarrow$$

$$\boxed{\theta < 10^{-10}}$$



If θ is promoted to a field, it could dynamically go to zero ≡ Axi

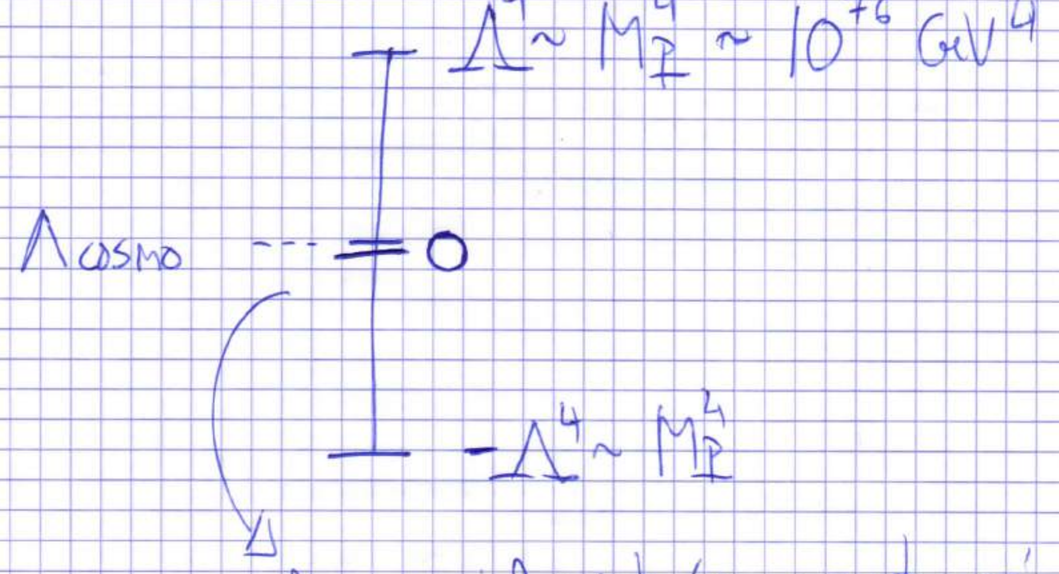
$19 \times 4 = 76$

Cosmological Constant

$$[M]^4 = [\Lambda_{cosm}]^4 = \left(\frac{\hbar}{L^4}\right) \frac{1}{[g^2]}$$

$$\sqrt{G_N}\, h_{\mu\nu} T^{\mu\nu} \qquad T^{\mu\nu} = h^{\mu\nu} \Lambda^4_{cosmo}$$

$$\Lambda_{cosmo}\Big|_{EXP} = 10^{-47}\, GeV^4 \sim 10^{-121} M_P^4$$

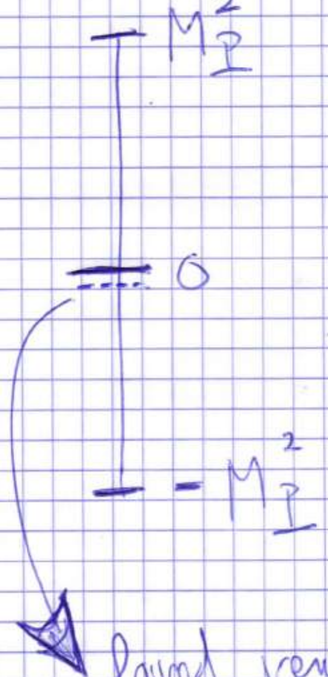


No a special point (no symmetry gain)

$$\Delta\Lambda_{cosmo} \neq \Lambda_{cosmo}$$

HIERARCHY PROBLEM

Expected range for μ^2:



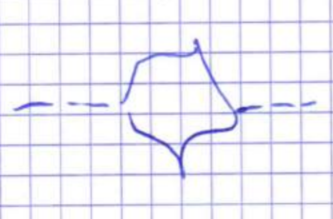
found very close to zero. Why?

$\approx 100^2\ GeV^2 \approx 10^{-32} \cdot M_P^2$

Point of enhanced symmetry?

No, in the SM

Furthermore:

 $\propto \dfrac{\Lambda^2}{16\pi^2}$

3 POSSIBILITIES

(I) → μ^2 not a fondamental parameter an Λ_{QCD}, derived from $\exp\left[-\frac{4\pi}{\beta\alpha}\right]$ ← fondamental param of $\mathcal{O}(1)$

Higgs = Composite state as pions in Φ(?)

or state from a warped extra dim

(II) $\mu^2 = 0$ special point ≡ theory gains a symmetry

$\boxed{\text{supersymmetry}}$

(III) M_P^2 not a fondamental scale

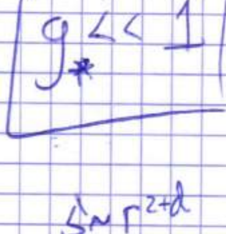
$$G_N \equiv M_P^{-2} \equiv \frac{g_*^2}{\Lambda^2}$$

$\boxed{g_* \ll 1}$ ⟹ due to the dilution of gravity in "d" extra dimension

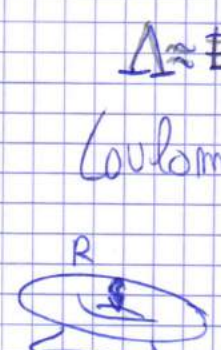
$\Lambda \approx$ EW scale

$S \sim r^{2+d}$

Coulomb law:



$$F \sim \frac{m_1 m_2}{r^{2+d}}$$



$\int d\Phi \propto m_1$

$$\frac{1}{\Lambda^2}\left(\frac{1}{\Lambda R}\right)^d$$

Must be compactified: $r < R$: $V \sim \frac{1}{\Lambda^{2+d}} \frac{1}{r^{1+d}}$ at $r > R$: $V \sim G_N \frac{1}{r}$ ⟹ $r = R$ $\boxed{G_N = \frac{1}{\Lambda^{2+d}} \frac{1}{R^d}}$

COMPOSITE HIGGS (HIGGSLESS MODEL)

Idea from QCD. Take QCD with two-flavor in the massless limit ($m_q = 0$):

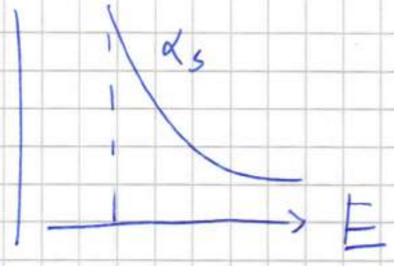
$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

$\boxed{Y = 2T_3^R + B}$

Accidental global symmetry: also $U(1)_B$

$$\left.\begin{array}{l} q_L \to U_L\, q_L \\ q_R \to U_R\, q_R \end{array}\right\} SU(2)_L \otimes SU(2)_R$$

At low-energy this symmetry is spontaneously broken



$\langle \bar q q \rangle \sim \Lambda_{QCD}$

Let's see it explicitly:

$$H^{ij} \equiv \bar q_L^i\, q_R^j \longrightarrow U_R\, H\, U_L^\dagger$$

$$\begin{pmatrix} \bar u_L u_R & \bar d_L u_R \\ \bar u_L d_R & \bar d_L d_R \end{pmatrix} \equiv H$$

Spontaneous breaking:

$$\langle \Sigma \rangle = \mathbb{1} \cdot f_\pi$$

↳ can be always written like this by an U_L & U_R rotation

$$\boxed{SU(2)_L \otimes SU(2)_R \to \underline{SU(2)_V}} \equiv (U_L \equiv U_R)$$

$$\langle \Sigma \rangle \to U \langle \Sigma \rangle U^\dagger = \langle \Sigma \rangle$$

▷ Goldstones: one per broken generator

3+3 generator ▷ 3 generator

⇓

3 Goldstones $= \pi^+, \pi^-, \pi^0$

polar coordinates

$$\Sigma = e^{i \sigma^a \pi^a / f_\pi} \cdot \mathbb{1} \left(1 + \frac{\sigma}{f_\pi} \right)$$

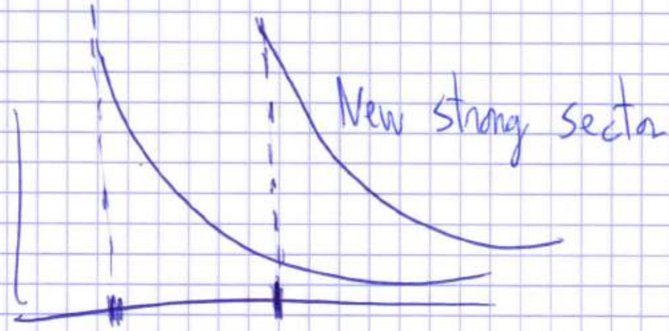
σ^a = generator broken

Exercise: $SU(2)_L \otimes SU(2)_R / SU(2)_V$ coset

$V(\Sigma^\dagger \Sigma)$ only contain σ ⟹ $\underline{\pi^a \text{ are massless!}}$

Composite PGB Higgs

μ^2 not fondamental:



Λ_{QCD} $\Downarrow$ $\pi, \rho, \ldots$

$\Lambda \sim \text{TeV}$ $\Downarrow$ Composite Higgs, ρ, ...

$\simeq \pi$ <u>to be lighter than the other!</u>

- In QCD with 2 flavors:

$$\psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \in 2_L \qquad \psi_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \in 2_R$$

$$\text{of } SU(2)_L \otimes SU(2)_R \xrightarrow[\langle \bar\psi\psi \rangle \neq 0]{} SU(2)_V$$

$$\Downarrow$$

$$\pi^0, \pi^\pm \equiv 3\,\text{PGB}$$

described by chiral lag.

- For the Higgs, we need at least

$$\underset{10}{SO(5)} \xrightarrow[\langle \bar\psi\psi \rangle \neq 0]{} \underset{6}{SO(4)} \approx SU(2)_L \otimes SU(2)_R$$

$$\Downarrow$$

$$5\,\text{PGB} = 2_{1} \text{ of } SU(2)_L \otimes U(1)_Y \qquad \boxed{Y = T_3^R}$$

We could have a replica at TeV but no Higgs in the spectrum

$\Downarrow$

$\boxed{\text{MCHM = Minimal Composite Higgs Model}}$

- Assume a strong sector (an QCD) at TeV
- Assume the global symmetry pattern:

Minimal Case: $SO(5) \to SO(4) \approx SU(2)_L \otimes SU(2)_R$

rotations in 5-space — generators $\equiv 10$

rotations in 4-space — generators $= 6$

$\Downarrow$

4 goldstones $=$ π^a, h !

$\boxed{\vec{H} = \text{5-vector}} = \begin{pmatrix} x \\ x \\ x \\ x \\ x \end{pmatrix}$ $\quad \vec{H} \to R_{5\times 5} \vec{H}$

$\langle \vec{H} \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ breaking to $SO(4)$: $\langle \vec{H} \rangle \to \begin{pmatrix} U_{4\times 4} & \\ 0 & 1 \end{pmatrix} \langle \vec{H} \rangle = \langle \vec{H} \rangle$

$$\vec{H} \equiv e^{-i\sqrt{2}\, T^a \cdot h^a / f} \langle \vec{H} \rangle \left(1 + \frac{\sigma}{f}\right)$$

(PROJECTING OUT σ: $\vec{H} \cdot \vec{H} = 1$)

$$T^a h^a = \left(\begin{array}{cccc|c} 0 & & & & h_1 \\ & 0 & & & h_2 \\ & & 0 & & h_3 \\ & & & 0 & h_4 \\ \hline -h_1 & -h_2 & -h_3 & -h_4 & 0 \end{array}\right)$$

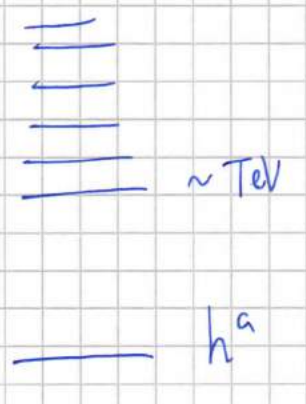
$\hookrightarrow$ generator broken : $\boxed{SO(5)/SO(4) \text{ Coset}}$

$$T^a_{ij} = -\frac{i}{\sqrt{2}}\left(\delta^a_i \delta^5_j - \delta^a_j \delta^5_i\right)$$

$$a = 1,2,3,4$$
$$i,j = 1,\dots,5$$

$$\vec{\Sigma} = \frac{\sin h/f}{h}\begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h \cot h/f \end{pmatrix} \qquad h \equiv \sqrt{\sum_a h^a h^a}$$

SPECTRUM of THE MODEL:



$$SO(4) \approx SU(2)_L \otimes SU(2)_R$$

Including $SU(2)_L \otimes U(1)_Y$ break the global $SO(5) \Longrightarrow V(h)$ generated (*)

Below heavy resonances ($E \lesssim \text{TeV}$), we have the effective theory of Goldstones:

$$\mathcal{L} = \frac{f^2}{2}(D_\mu \vec{H})^T (D^\mu \vec{H}) + \dots$$

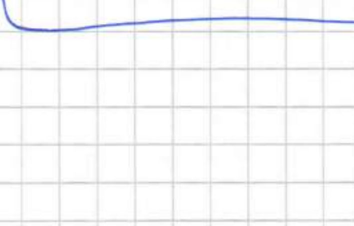
$\mathcal{L}$ in the unitary gauge: (G of SM $=0$)

$$\vec{H} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \sin h/f \\ \cos h/f \end{pmatrix}$$

$$\boxed{\frac{1}{2}(\partial_\mu h)^2 + \frac{1}{4} g^2 f^2 \sin^2\frac{h}{f}\left[W_\mu W^\mu + \frac{1}{2\cos^2\theta_W} Z^\mu Z_\mu\right] + \dots}$$

Taylor expand around $\langle h \rangle$

$$\underbrace{f^2\left[\sin^2\frac{\langle h\rangle}{f}\right.}_{v^2} \left. + 2\sin\frac{\langle h\rangle}{f}\cdot\cos\frac{\langle h\rangle}{f}\,\frac{h}{f} + \dots\right]$$

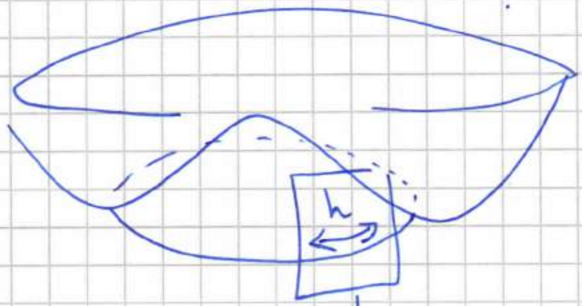


$$= g\, m_W \cos\frac{\langle h\rangle}{f} = \boxed{g^{SM}_{hWW}\cdot a}$$

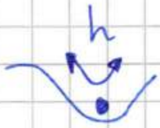
$$\cos\frac{\langle h\rangle}{f} = \sqrt{1-s^2} = \sqrt{1-v^2/f^2} \equiv a$$

Different from the SM where $a=1$!

$\circledast$



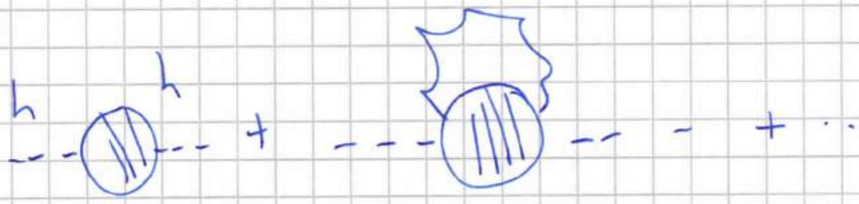
DUE TO THE STRONG TeV SECTOR



DUE TO SM-LOOPS

$$\boxed{\text{Higgs as a Pseudo-Goldstone Boson}}$$

$\langle h \rangle \approx f$



$= 0$

$$\approx \frac{g^2}{16\pi^2} \, TeV^2 \gg (100\ GeV)^2$$

Light Higgs

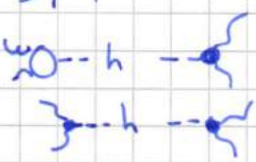
$$V \sim \frac{g^2}{16\pi^2} f(\sin h/f)$$

it is a phase

$\langle h \rangle \sim f$ will need to be smaller

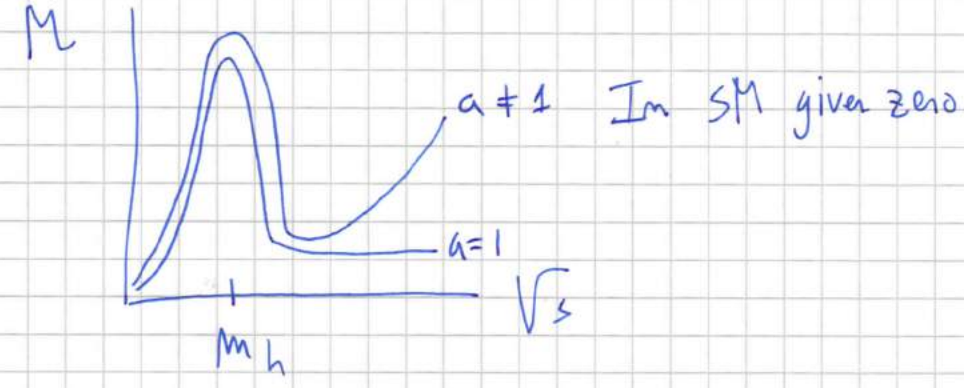
Implication of $a \neq 1$

(1) hWW coupling different from the SM:



(2) $W_L W_L \to W_L W_L$ grow with s

for $s \gg v$: $\sim \dfrac{s}{v^2} \quad - a^2 \dfrac{s^2}{s-m_h^2}\dfrac{1}{v^2} \xrightarrow[s \gg m_h]{} \boxed{\dfrac{s}{v^2}\left(1-a^2\right)} + \cdots$



↳ we must see that $W_L W_L$ - scattering grow with E

(regularize by other resonances)

HIGGS COUPLING TO FERMIONS:

We assume linear mixing of an SM fermion with an operator of the strong sector:

$$\boxed{\mathcal{L}_{int} = \lambda \cdot \bar{\Psi}_{SM} \cdot \mathcal{O}_{\Psi} + h.c.}$$

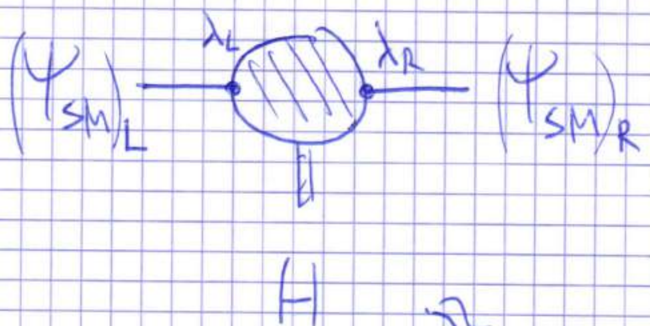
$\hookrightarrow$ comes in complete SO(5) multiplet

$\hookrightarrow$ do not fit in a complete SO(5)

$\Downarrow$

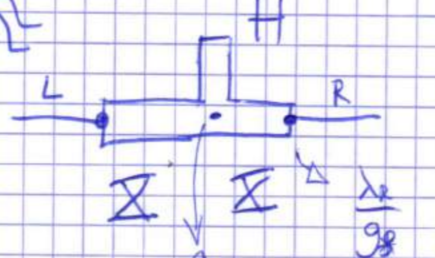
Breaking of the SO(5) symmetry

YUKAWA COUPLINGS:



$$\boxed{\sim \lambda_L \lambda_R \frac{1}{g_*}}$$

$\simeq$



SUPERSYMMETRY

- Consistent theory of a photon: $A_\mu J^\mu$ $\quad \partial_\mu J^\mu = 0$
- Consistent theory of a graviton: $h_{\mu\nu} T^{\mu\nu}$ $\quad \partial_\mu T^{\mu\nu} = 0$
- Consistent theory of a gravitino ($s = 3/2$): $\psi^\alpha_\mu \cdot J^\mu_\alpha$ $\quad \partial_\mu J^\mu_\alpha = 0$

α = weyl spin

ψ^α_ϵ

$\hookrightarrow$ Not known ~~of the~~ higher spin

Fermionic current

by Noether theorem: $\equiv$

super symmetry

Relates fermion to bosons:

ϕ complex $\longleftrightarrow$ ψ^α spinor

$\eta_{\mu\nu} = (-1, 1, 1, 1)$

$$\boxed{\mathcal{L} = -\partial_\mu \phi^* \partial^\mu \phi + i\, \psi^+ \bar{\sigma}^\mu \partial_\mu \psi}$$

$\bar{\sigma}^\mu = \{\mathbb{1}, -\sigma_i\}$

invariant under $\begin{cases} \phi \to \phi + \delta\phi & \delta\phi = \epsilon^\alpha \cdot \psi_\alpha \\ \delta\psi_\alpha \to \psi_\alpha + \delta\psi_\alpha & \end{cases}$

$\delta\psi_\alpha = -i (\sigma^\mu \epsilon^+)_\alpha \partial_\mu \phi$

$\hookrightarrow$ 2-component weyl fermion parametrizing the inf. transformation (anticommuting)

$$J^\alpha_\mu = \frac{\delta \mathcal{L}}{\delta \epsilon_\alpha(x)}$$

$$\boxed{\text{HIGGS - TOP SECTOR}}$$

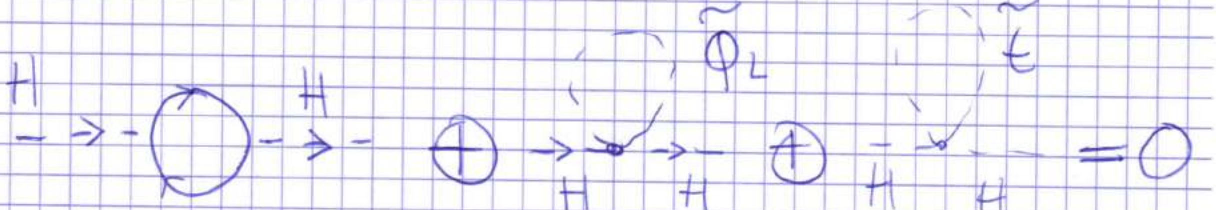
$H \longleftrightarrow \tilde{H}$ Higgsino

$t_R \longleftrightarrow \tilde{t}_R$ stop_R

$\varphi_L^3 \longleftrightarrow \tilde{\varphi}_L^3$ stop_L

$$\begin{cases} \tilde{H} : \text{natural to have mass zero} \\ H : \text{" " " " "} \end{cases}$$

$$\mathcal{L}_{int} = Y_t H \bar{\varphi}_L t_R + Y_t^2 |H \tilde{\varphi}_L^+|^2 + Y_t^2 |H \tilde{t}_R|^2 + Y_t^2 |\tilde{\varphi}_L^+ \tilde{t}_R|^2$$



boson loops cancel by fermion loops

Must be broken since not seen particle stops:

$$\boxed{\underbrace{\mu^2}_{(100\ \text{GeV})^2} \approx \frac{-3Y_t^2}{8\pi^2} \left(m^2_{\tilde{\varphi}_L} + m^2_{\tilde{t}_R} \right) \cdot \ln \frac{\Lambda}{m_{\tilde{t}}}}$$

Λ: scale where $m_{\tilde{t}}$ is generated

$$m_{\tilde{\varphi}} \approx m_{\tilde{t}_R} \approx 400\ \text{GeV}$$

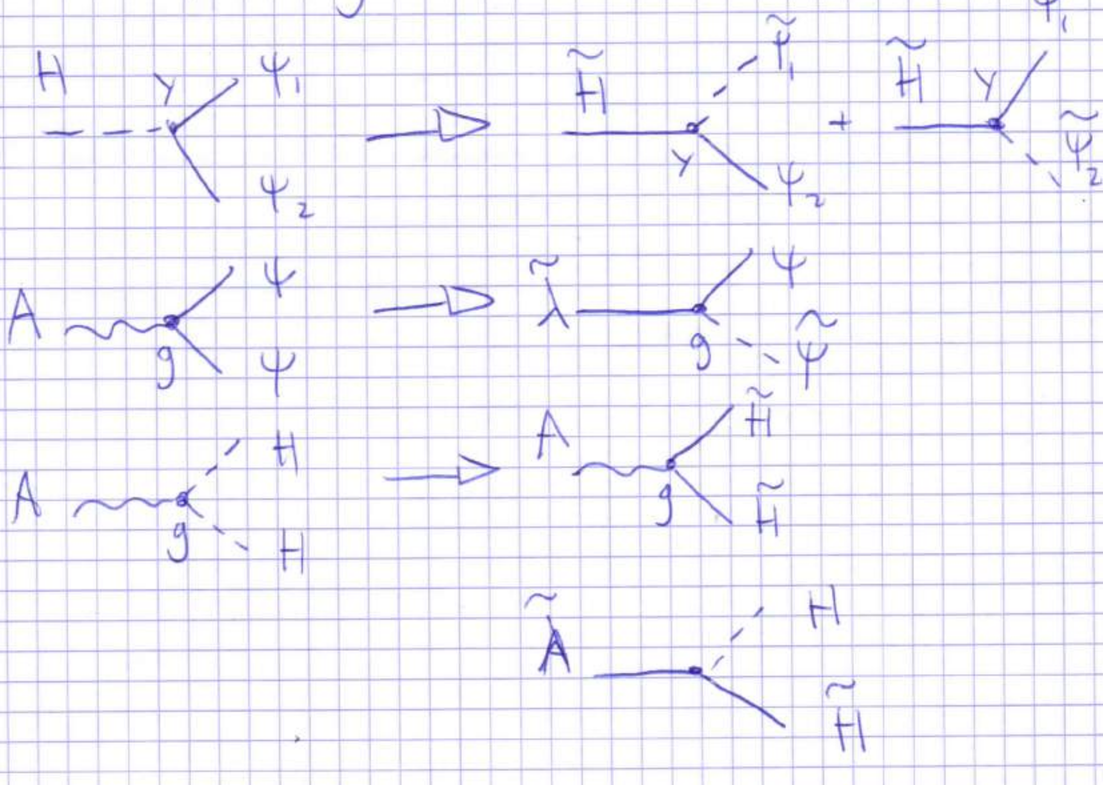
To give mass to $\tilde{H}$, avoid anomalies, and have Yukawa couplings to down-quark/lepton sector extra (Higgs, Higgsino needed):

$$(H_1, \tilde{H}_1) \quad Y = -1$$
$$(H_2, \tilde{H}_2) \quad Y = 1$$

Other particles:

$$A_\mu^a \longleftrightarrow \tilde{\lambda}_a \quad \text{gauginos}$$
$$\psi \longleftrightarrow \tilde{\psi} \quad \text{s-leptons, s-quarks}$$

For trilinears, supersymmetrization works:



SUSY PREDICTIONS

① Yukawa structure:

$$H_2 \to \text{coupling to } \bar{Q}_L u_R$$

$$H_1 \to \text{coupling to } \bar{Q}_L d_R \, , \, \bar{L}_L e_R$$

② Higgs quartics:

$$V = \frac{1}{8}(g^2+g'^2)\left(|H_1|^2-|H_2|^2\right)^2+\frac{1}{2}g^2\left|H_1^+H_2^{0*}+H_1^0H_d^{-*}\right|^2$$

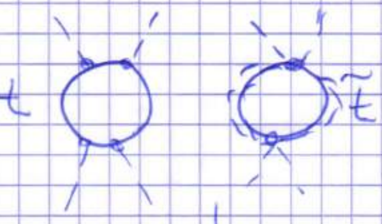
$$\Downarrow$$

$$\boxed{m_h^2 \leq m_Z^2}$$

Loops:

$$\boxed{m_h^2 \leq m_Z^2 + \frac{3m_t^4}{4\pi^2 v^2}\ln\frac{m_{\tilde{Q}}^2}{m_t^2}+\cdots}$$

$v \approx 246\,\text{GeV}$



Mass of the squark

$$m_{\tilde{Q}} \approx \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \gtrsim 4\,\text{TeV}$$

to get $m_h \sim 125\,\text{GeV}$

Derive it from the SM alone:

H_1 — $m_{\tilde{Q}}$: $\lambda_h = \lambda_{MSSM}^{MAX} = \frac{1}{8}(g^2+g'^2)$

RGE running:

$H_2 = H_{SM}$ — m_t: $\frac{d\lambda}{dt} = \frac{-6y_t^4}{32\pi^2}+\cdots$

$\boxed{V = \lambda |H|^4}$

$$\Delta\lambda = +\frac{3y_t^4}{16\pi^2}\ln m_{\tilde{Q}}/m_t$$

$$m_h^2(\text{MAX}) = 2\lambda v^2 + 2\Delta\lambda v^2$$

with: $m_t = \frac{y_t}{\sqrt{2}}v$