Exercises

1. Choosing the units such that $c = 1$ but $\hbar \neq 1$, work out the dimensionalities of fields and couplings in the Lagrangian.

2. Given the Lagrangian
$$\mathcal{L} = -\frac{1}{2}\alpha\partial_\mu A_\nu \partial^\mu A^\nu - \frac{1}{2}\beta\partial_\mu A_\nu \partial^\nu A^\mu + \frac{m^2}{2}A_\mu A^\mu + J_\mu A^\mu \tag{1}$$
Calculate the equation of motion of A_μ. Take the divergence of it and show that $\alpha = -\beta$ eliminates the scalar kinetic term of $\partial_\mu A^\mu$, leaving only a massive spin-1 field propagating. Next, from the propagator of this massive spin-1 field:
$$\frac{\eta_{\mu\nu} - \frac{q_\mu q_\nu}{m^2}}{q^2 - m^2}\,, \tag{2}$$
show that in the limit $m \to 0$, A_μ can only be consistently coupled to a conserved current ($\partial_\mu J^\mu = 0$).

3. Show that Parity and Charge Conjugation are accidental symmetries of QED.

4. Consider the Weinberg operator
$$\frac{1}{\Lambda}\bar{L}_L^{c\,i} H_i\, H_j L_L^j \tag{3}$$
where H is the Higgs field, L_L is the left-handed lepton, and i, j are $SU(2)_L$ indices. Derive the bound on the scale of new physics Λ to ensure that the neutrino mass is below 0.1 eV.

5. Consider the following dimension-six operator
$$\frac{1}{\Lambda^2}\epsilon^{\alpha\beta\gamma}[\bar{Q}_{L\alpha}^{c\,i}\gamma^\mu u_{R\beta}][\bar{d}_{R\gamma}^c \gamma_\mu L_{L\,i}]\,. \tag{4}$$
where α, β, γ are color indices. Using the constraint on the proton lifetime, $\tau_p \gtrsim 10^{34}$ years, derive a bound on the new physics scale Λ.

6. Consider the operator
$$\frac{Y_e}{\Lambda} H \bar{L}_L \sigma^{\mu\nu} e_R B^{\mu\nu}\,. \tag{5}$$
Using the experimental bound on the electric dipole moment of the electron, $d_e \lesssim 10^{-28}\, e \cdot cm$, obtain the constraint on the scale Λ.

7. Consider an $SU(N_c)$ gauge theory with N_F Dirac fermions, q_L^i and q_R^i ($i = 1, 2, ..., N_F$), in the fundamental representation of the gauge group. Show that the accidental symmetry of the model is $SU(N_F)_L \times SU(N_F)_R$, where q_L^i and q_R^i transform as a $(\mathbf{N_F}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{N_F})$. Also show that a nonzero condensate $\langle \bar{q}_L^i q_R^j \rangle = \delta^{ij}$ leads to the symmetry breaking pattern $SU(N_F)_L \times SU(N_F)_R \to SU(N_F)_V$, resulting in $N_F^2 - 1$ Goldstone bosons.

8. Show that a free theory of a Weyl spinor ψ^α and a complex scalar ϕ are invariant under supersymmetry.