Second day:

1. The QCD scale parameter Λ is defined by

$$\log \frac{Q^2}{\Lambda^2} = -\int_{\alpha_S(Q)}^{\infty} \frac{dx}{\beta(x)}\,, \tag{7}$$

where the β-function is

$$\beta(\alpha_S) = \mu^2 \frac{\partial \alpha_S}{\partial \mu^2} = -b\alpha_S^2 \left[1 + b'\alpha_S + b''\alpha_S^2 + \mathcal{O}(\alpha_S^3)\right]. \tag{8}$$

   (a) Consider the two renormalisation schemes A and B, where the couplings are related by

$$\alpha_S^B = \alpha_S^A \left[1 + c_1\alpha_S^A + c_2(\alpha_S^A)^2 + \mathcal{O}((\alpha_S^A)^3)\right]. \tag{9}$$

   Show that the first two β-function coefficients b and b' are scheme-independent, whereas the third is related in the two schemes by

$$b''_B = b''_A + c_2 - b'c_1 - c_1^2\,. \tag{10}$$

   (b) Show the scale parameters of the two schemes are related by

$$\Lambda_B = \Lambda_A \exp\left(\frac{c_1}{2b}\right). \tag{11}$$

   Hint: Combine the two formulas for $\Lambda_{A,B}$ and calculate

$$\log \frac{\Lambda_B}{\Lambda_A} = \frac{1}{2}\int_{\alpha_A(Q)}^{\alpha_B(Q)} \tag{12}$$

   and then take the limit $Q \to \infty$, $\alpha_{A,B} \to 0$.

2. Explain why in the NLO calculation of $\sigma(e^+e^- \to \text{hadrons})$ there is no need to renormalise the strong coupling (i.e., the UV divergences cancel).

3. List the properties that a function of the four momenta has to enjoy to be an "infrared-safe" quantity. Think of examples of infrared safe and unsafe quantities.

4. Compute the leading order $e^+e^- \to \gamma \to q\bar{q}$ cross section at LEP. How does the cross-section change when a Z boson is also included? Plot as a function of the centre-of-mass energy.

5. Can QCD be a fundamental theory? What about QED?

6. Show that the QCD lagrangian is isospin invariant of $m_d = m_u$ (and hence in particular if $m_d = m_u = 0$).