Commuting Transfer Matrix Methods

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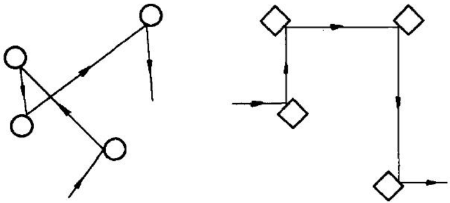
Motion in Random Enviroment

Particles move independent of each other through fixed obstacles.



- Fraction of lattice sites is occupied with given scatter.
- The scattering rules are deterministic.
- The obstacles are randomly distributed on the lattice.
- Partition sum of classical model of statistical mechanics.

Scaling Behaviour of the Paths

Let S be the size of the open trajectory:

- Perimeter of the percolation cluster

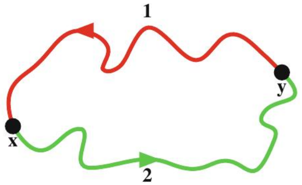
$$\mathrm{R}(\mathrm{S}) \sim \mathrm{S}^{1/d_f}$$

- Probability to find the percolation cluster

$$\mathrm{P}(\mathrm{S}) \sim \mathrm{S}^{2-\tau_f}$$

Correlations among trajectories $G_2(x, y)$:

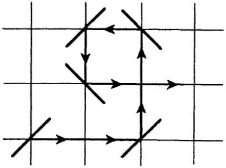
- Probability of two links lie on the same loop

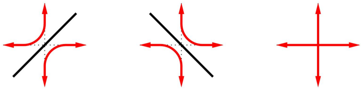


The Mirror Model

Gunn,Ortuño (1985) ; Ruijgrok,Cohen (1988)

- Obstacles are right and left mirrors and vacancies





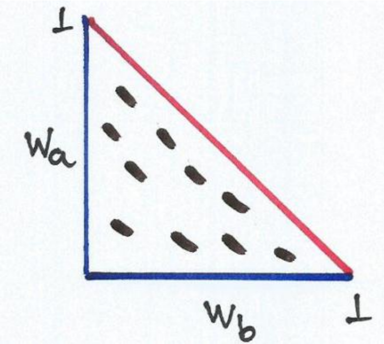
w_a w_b w_c

- Weighting every closed loop in a given state by a fugacity Q

$$\mathrm{Z} = \sum_{\textit{scatter configurations}} w_a^{n_a} w_b^{n_b} w_c^{n_c} Q^{\# paths}$$

Simulation Results

- Phase diagram



- Probability to find an open orbit after time t

$$P_o(t) \sim \begin{cases} t^{-1/7} & \text{for} \quad w_a + w_b = 1 \\ \frac{A_1}{log(t)+A_2} & \text{for} \quad w_a + w_b < 1 \end{cases}$$

Integrability Results

- Yang-Baxter

$$\begin{aligned}
cb'a'' &= bb'c'' + bc'a'' \\
ac'a'' &= aa'c'' + ca'a'' \\
ab'c'' &= ac'b'' + cb'b'' \\
ab'a'' &= Qba'b'' + aa'b'' + ca'b'' + bb'b'' + bc'b'' + ba'a'' + ba'c''
\end{aligned}$$

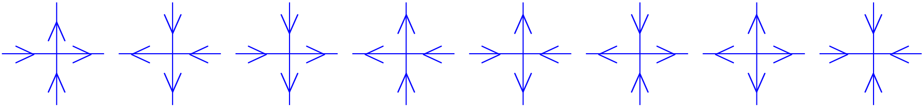
- Largest Eigenvalue

$$\begin{aligned}
\Lambda_0(\lambda) &= \left[\frac{4\Delta^2}{|\Delta|-\lambda}\right] \frac{\Gamma\left(1+\frac{\lambda}{2|\Delta|}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2|\Delta|}+\frac{\lambda}{2|\Delta|}\right)\Gamma\left(\frac{3}{2}-\frac{\lambda}{2|\Delta|}\right)}{\Gamma\left(\frac{1}{2}+\frac{\lambda}{2|\Delta|}\right)\Gamma\left(\frac{1}{2-n+2m}+\frac{\lambda}{2|\Delta|}\right)\Gamma\left(1-\frac{\lambda}{2|\Delta|}\right)} \\
&\times \frac{\Gamma\left(1+\frac{1}{2|\Delta|}-\frac{\lambda}{2|\Delta|}\right)}{\Gamma\left(\frac{1}{2}+\frac{1}{2|\Delta|}-\frac{\lambda}{2|\Delta|}\right)}
\end{aligned}$$

- Loop Density $< \rho_{\text{loops}} >= \frac{d}{dQ} \log \Lambda_0(\lambda), \quad Q \to 1$

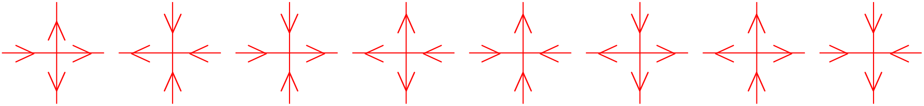
Even and Odd Eight-Vertex

- Even Eight-Vertex (Baxter 1971)



$a \quad a \quad b \quad b \quad c \quad c \quad d \quad d$

- Odd Eight-Vertex (Wu and Kunz 2004)



$\bar{a} \quad \bar{a} \quad \bar{b} \quad \bar{b} \quad \bar{c} \quad \bar{c} \quad \bar{d} \quad \bar{d}$

$$W_{ev} = \left[\begin{array}{cc|cc} a & 0 & 0 & d \\ 0 & b & c & 0 \\ \hline 0 & c & b & 0 \\ d & 0 & 0 & a \end{array}\right] \qquad W_{od} = \left[\begin{array}{cc|cc} 0 & \bar{a} & \bar{d} & 0 \\ \bar{b} & 0 & 0 & \bar{c} \\ \hline \bar{c} & 0 & 0 & \bar{b} \\ 0 & \bar{d} & \bar{a} & 0 \end{array}\right]$$