NEUTRINOS - HOMEWORK PROBLEMS (1)

1. The charged pion decays almost 100% of the time into a muon and a (muon-type) neutrino ($\pi^+ \to \mu^+\nu_\mu$). In the reference frame where the parent pion is at rest, compute the muon energy as a function of the muon-mass (m_μ), the charged pion mass (m_π), and the neutrino mass (m_ν). What is the absolute value of the muon momentum (tri)vector? Numerically, what is the relative change of the muon momentum between $m_\nu = 0$ and $m_\nu = 0.1$ MeV? It is remarkable that the muon momentum from pion decay at rest has been measured at the 3.4×10^{-6} level (Phys. Rev. D**53**, 6065 (1996)). This provides the most stringent current constraint on the "muon-neutrino mass."

2. At small enough energies ($\sqrt{s} < O(100)$ GeV), the neutrino cross section is approximately given by $\sigma_\nu \sim G_F^2 s/\pi$, where G_F is the Fermi constant and $s = (p+P)^2$ is the square of the center-of-mass energy of the neutrino (with four-momentum p_μ) plus target (with four-momentum P_μ) system. (a) Estimate the cross section, in cm^2 for neutrino–electron scattering and neutrino–neutron scattering when e and n are at rest and the neutrino energy $E_\nu = 10$ MeV. (b) Estimate the mean free path of a 10 MeV neutrino through lead, in AU (one Astronomical Unit – is the average Earth–Sun distance, equal to 1.5×10^{11} m (or 500 light-seconds). Assume that, at these energies, you can treat the nucleus as a collection of free nucleons and electrons (this is not really correct...), and that the scattering cross section on neutrons is larger than that on protons.

3. In February 1987, neutrinos from a Supernova that exploded in the Large Magellanic Cloud, located 50 kpc away from the Earth, reached the Kamiokande and the IMB experiments. This neutrino burst was reported in K. Hirata *et al.*, Phys. Rev. Lett. **58**, 1490 (1987) and R.M. Bionda *et al.*, Phys. Rev. Lett. **58**, 1494 (1987). Read the two papers and address the following questions: (a) Both experiments detect electron antineutrinos via $\bar{\nu}_e p \to e^+ n$. Calculate the incoming antineutrino energy, in the reference frame where the target protons are at rest, as a function of the recoil angle and energy of the positron, and the neutron and proton masses. You may set the positron mass to zero. Using the table of events in the Kamiokande paper, compute the energies of the antineutrinos that were deteced by the Kamiokande experiment. What is the highest (lowest) observed antineutrino energy? (b) Use the result from (a) to obtain an upper bound on the "electron antineutrino mass" (don't worry about mixing). The reasoning is the following: if neutrinos have mass, neutrinos with different energies propagate with slightly different velocities. Hence, the higher energy neutrinos should have arrived at the detectors before the lower energy ones. Compute the relative arrival time of two antineutrinos with two different energies (assume that the neutrino mass is a lot smaller than any neutrino energy in the problem). The time distribution of the events observed by Kamiokande is consistent with the expected antineutrino energy spread (a few seconds). From this fact, place an upper bound on the neutrino mass. [The last three events arrives more than nine seconds after the first event, and there is still a debate regarding whether these are really from the Supernova. Discard them, and consider only the first nine events.]

   Useful information: 1 parsec is about 3×10^{16} m.

4. To understand the effect of neutrino oscillations (consider two flavor $\nu_\mu \leftrightarrow \nu_\tau$ transitions) on the atmospheric muon-neutrino data, numerically calculate and draw histograms of the average muon neutrino survival probability in ten equal-size bins of $\cos\theta_z$, where θ_z is the angle between the neutrino direction and the vertical-axis at the detector's location ($\theta_z = 0$ for neutrinos coming straight from above, and $\theta_z = \pi$ for neutrinos coming from below). Make one histogram for $E_\nu = 0.2$ GeV, 2 GeV, and 20 GeV plus $\Delta m^2 = 2.5 \times 10^{-4}$ eV^2, 2.5×10^{-3} eV^2, and 2.5×10^{-2} eV^2, for a grand total of nine plots. Assume throughout that the mixing is maximal, i.e., $\sin^2 2\theta = 1$, and that neutrinos are produced 20 km above the surface of the Earth.