Dark Matter

II. Properties and framework

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The slides of these classes have been put together by looting the excellent ones created by some of the teachers of the "School on Dark Matter", held at ICTP-SAIFR in São Paulo in 2016. (And some additional material.)

Reorganized and trimmed for a more compact purpose, for this class I have used mostly material from classes of P.D. Serpico's, as well as E. Cypriano's classes on lensing.

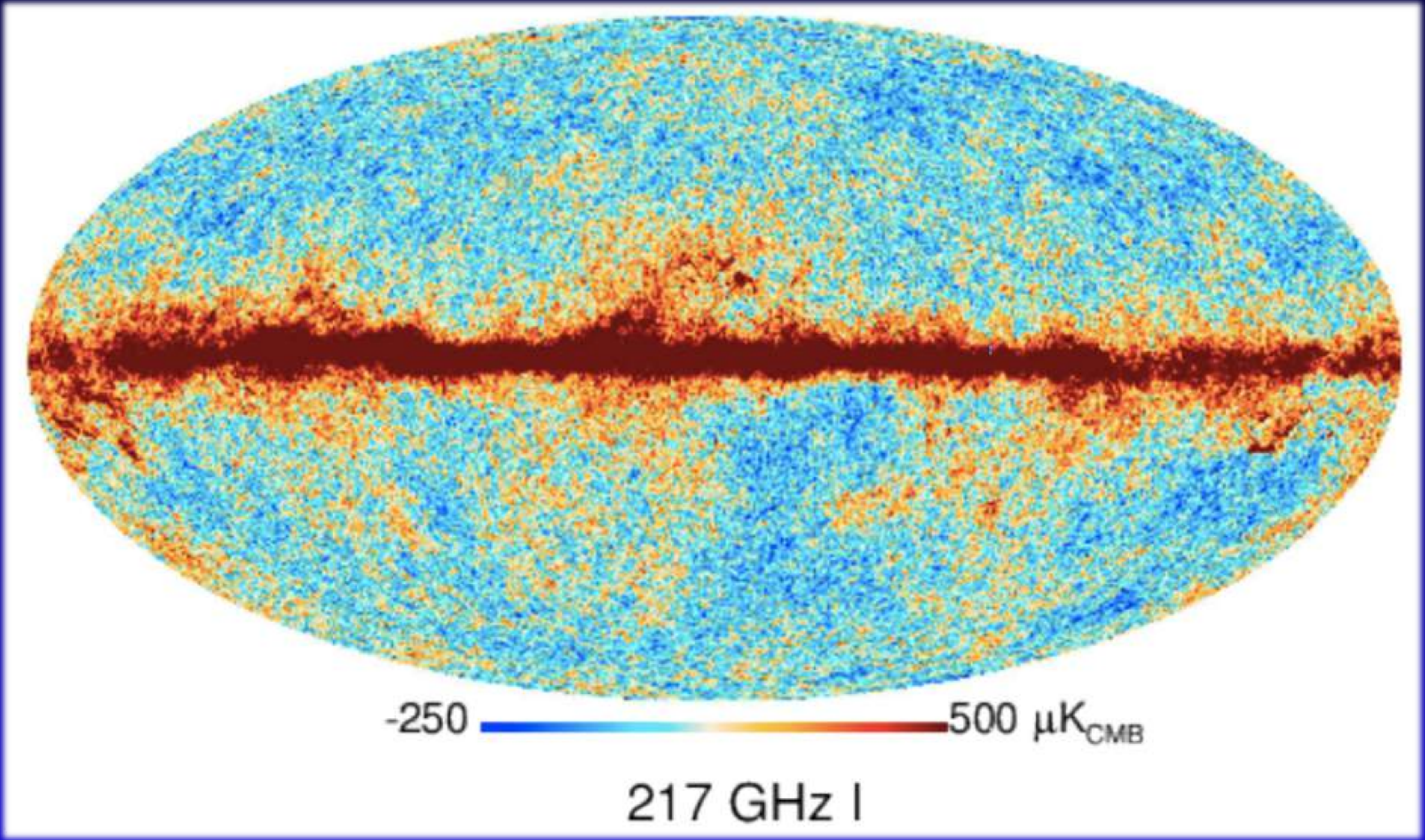
The complete material can be found at this address

http://www.ictp-saifr.org/school-on-dark-matter-2/

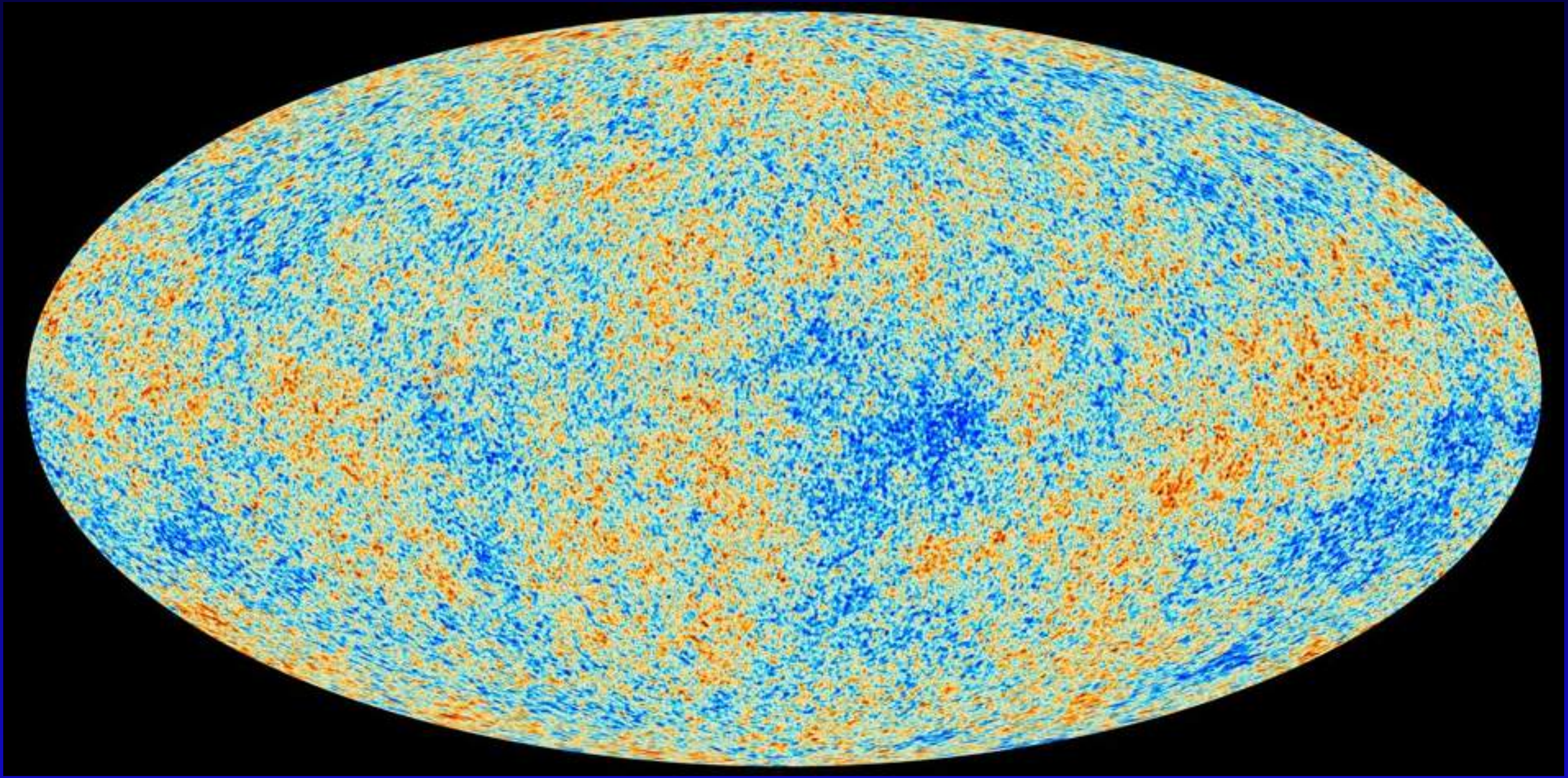
I strongly encourage you to download and study them to have a broader view on the subject. Excellent exercises are suggested, and references available.

Of course, do not hesitate to contact me for any question you may have.

CMB, a dark matter probe

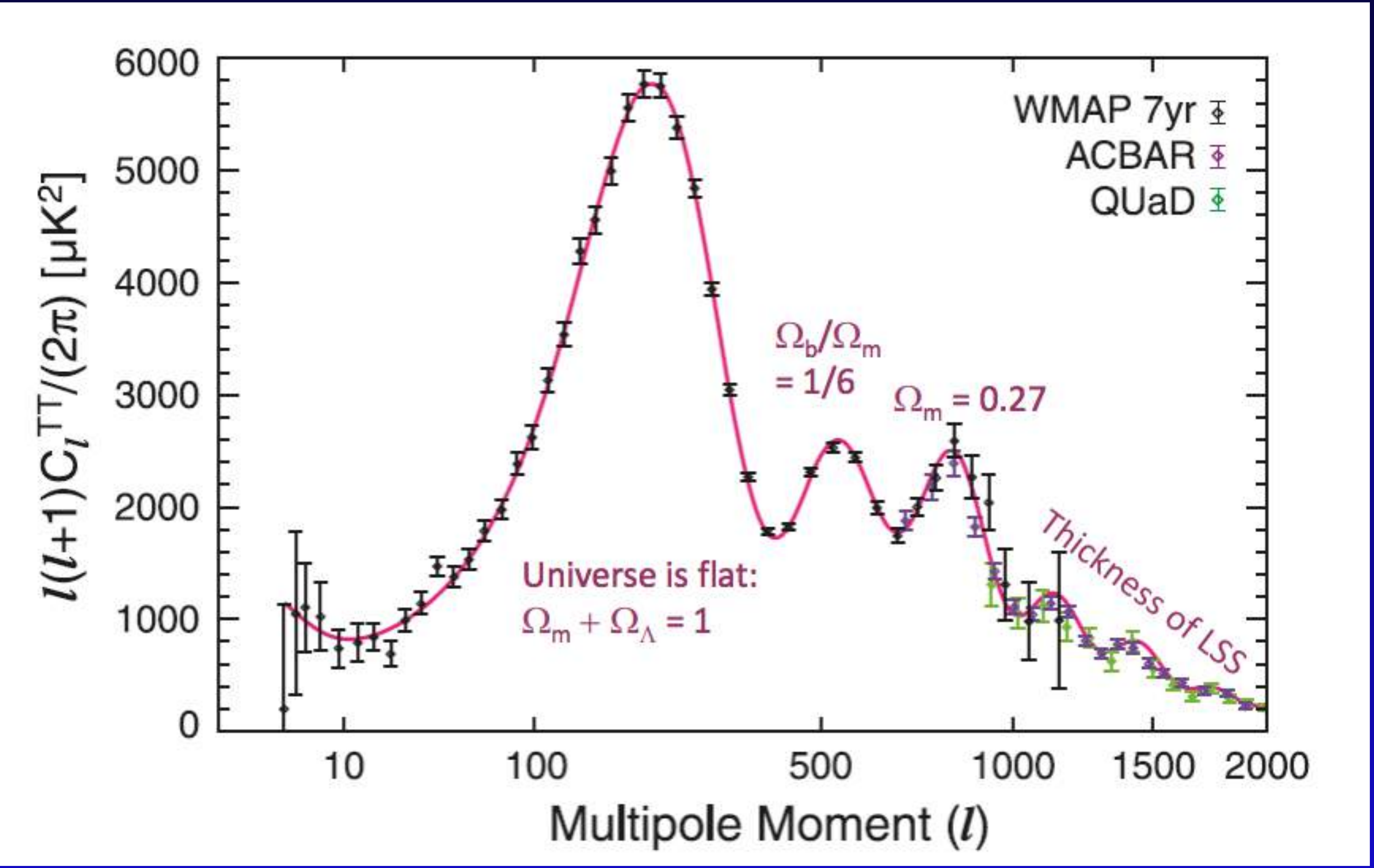


CMB, a dark matter probe

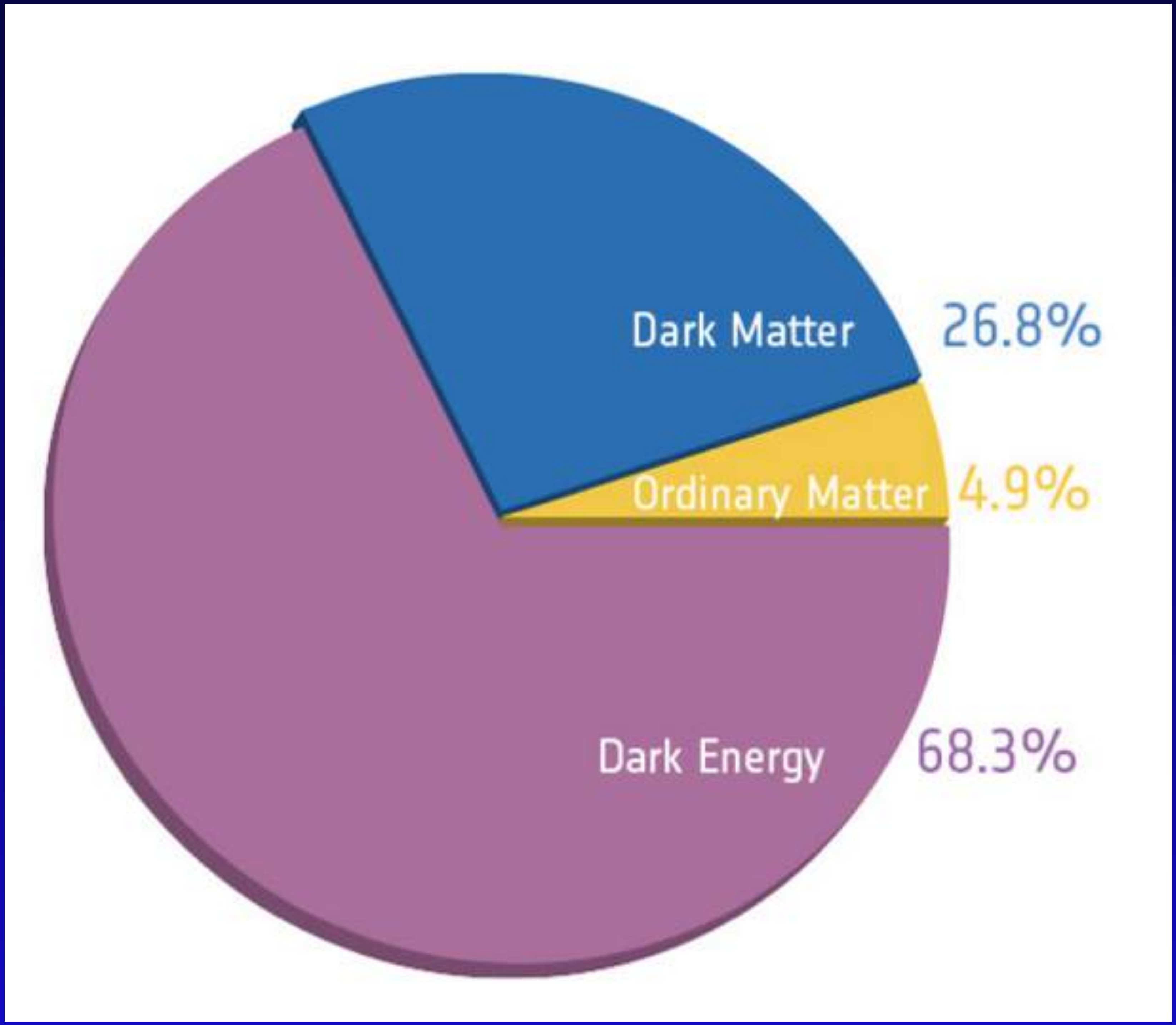


[Planck coll.]

CMB, a dark matter probe

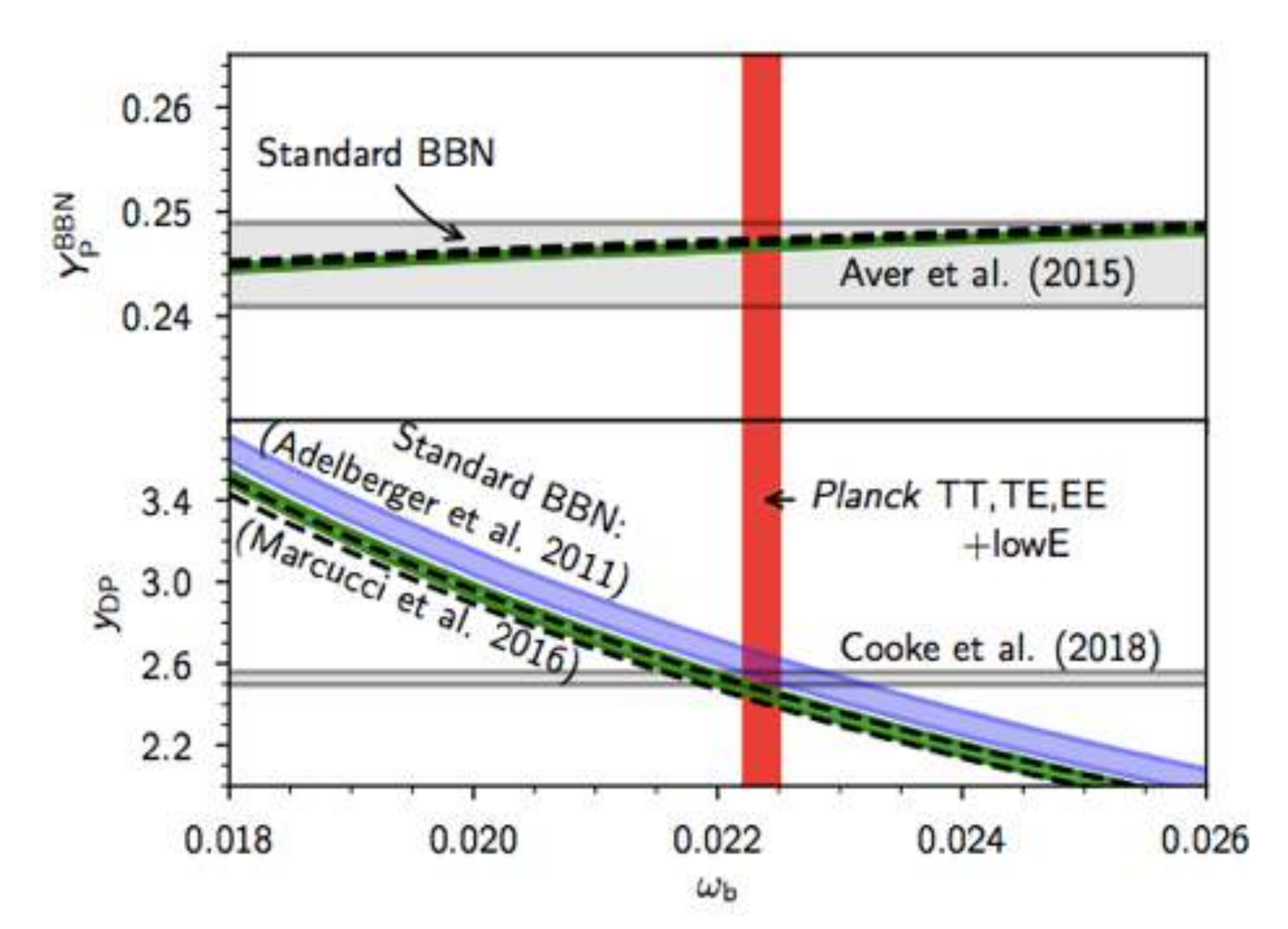


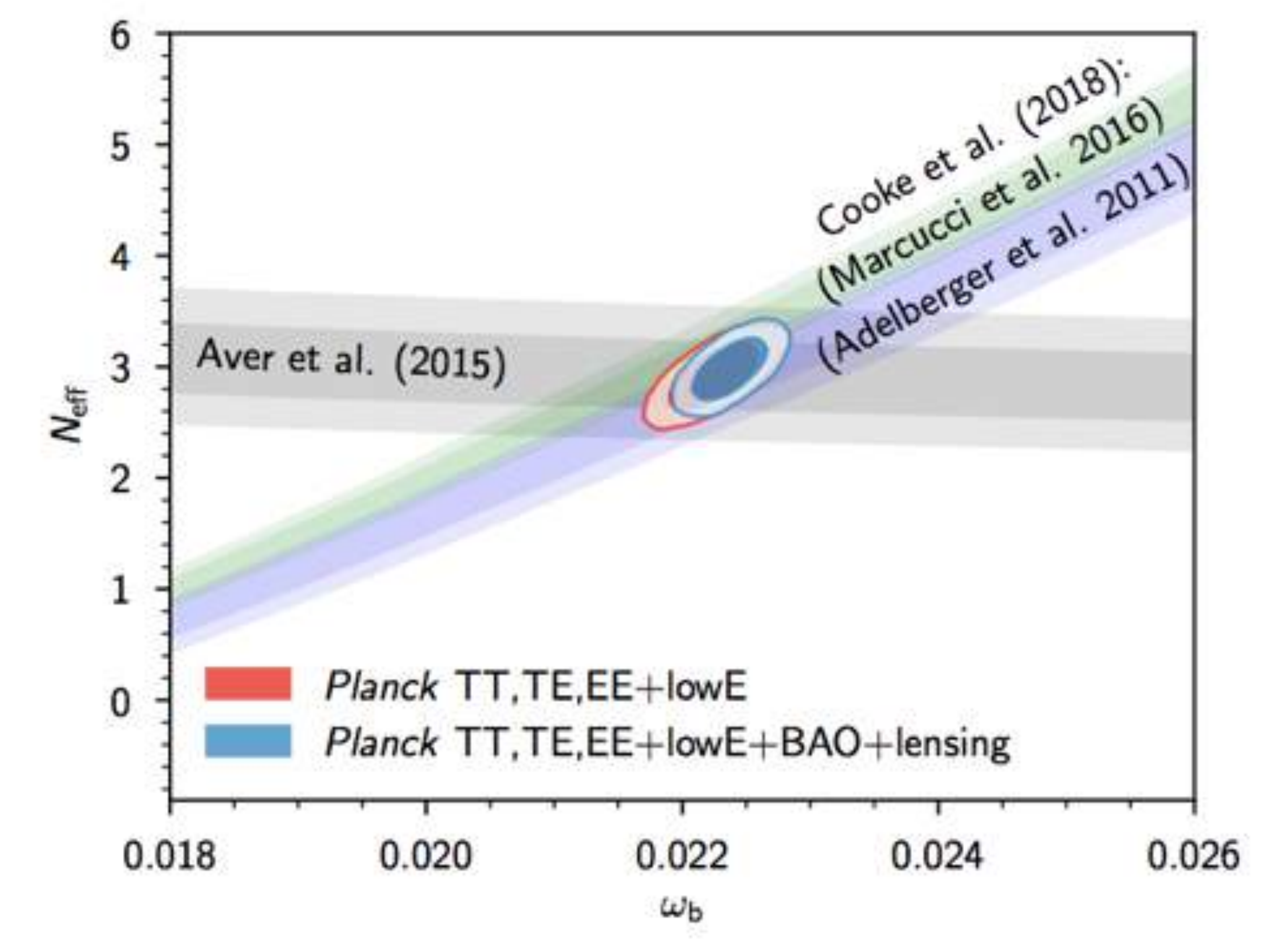
Common ground to start with



[e.g. Planck coll.]

Reality check, latest CMB results

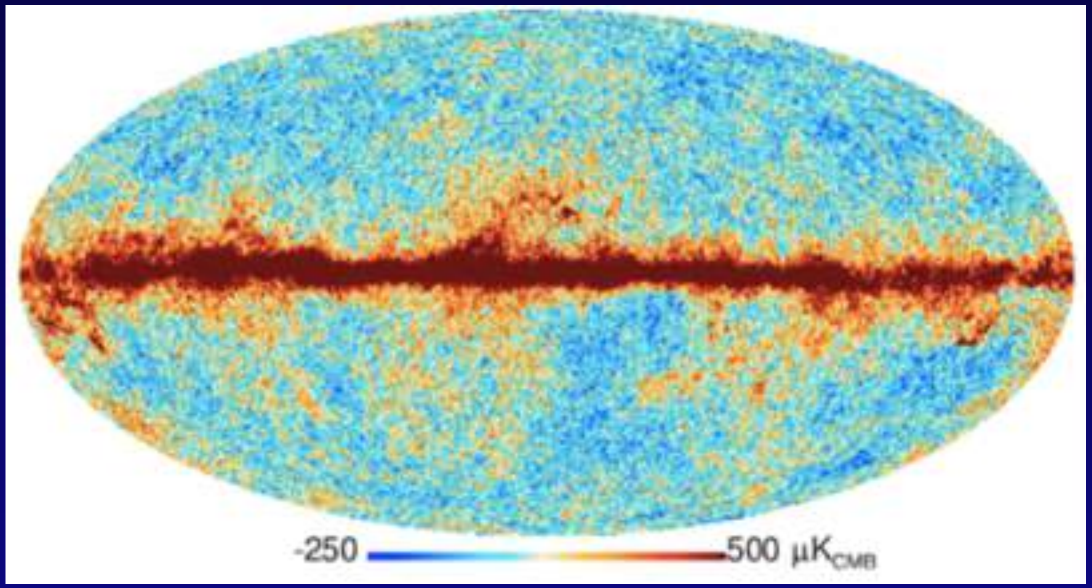


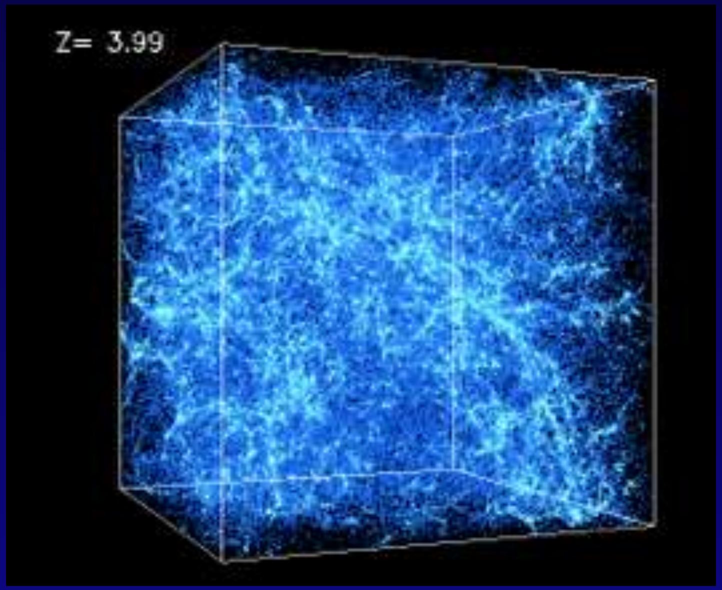


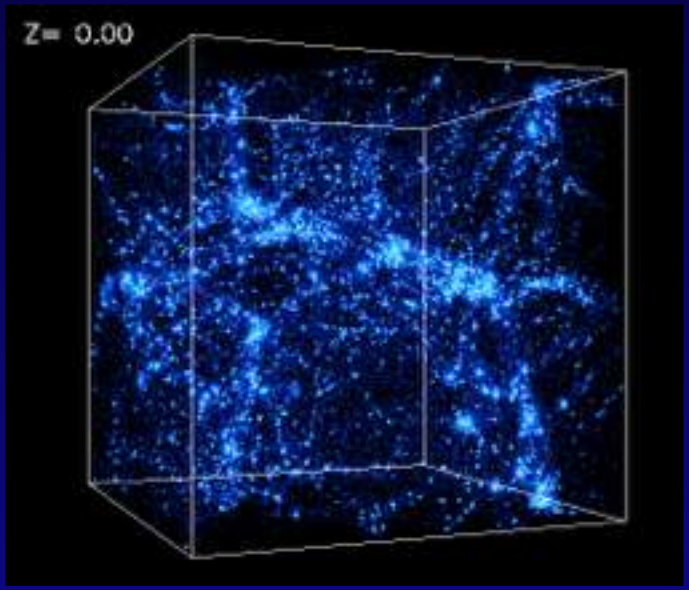
[Planck coll., 2018]

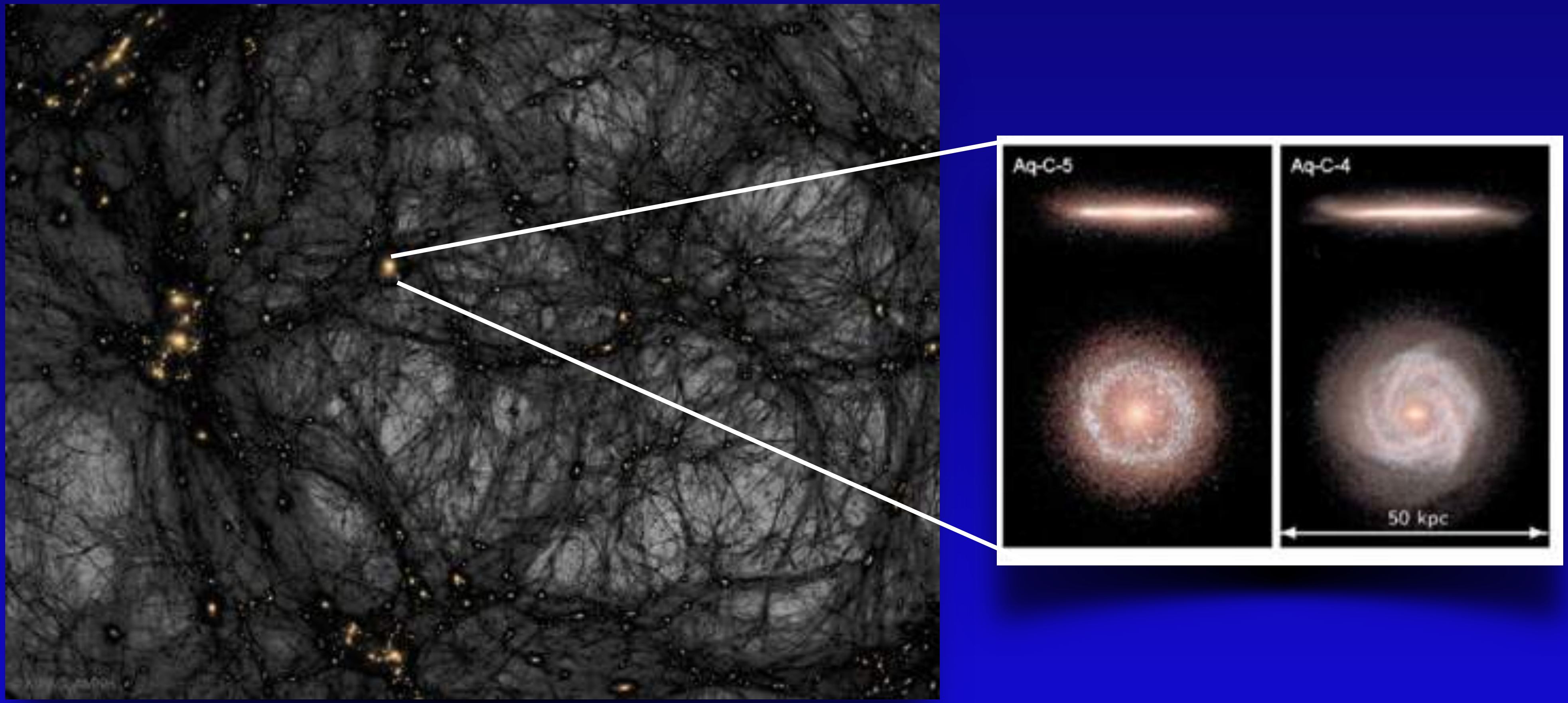
A story of LCDM
I: structure formation

age of Universe



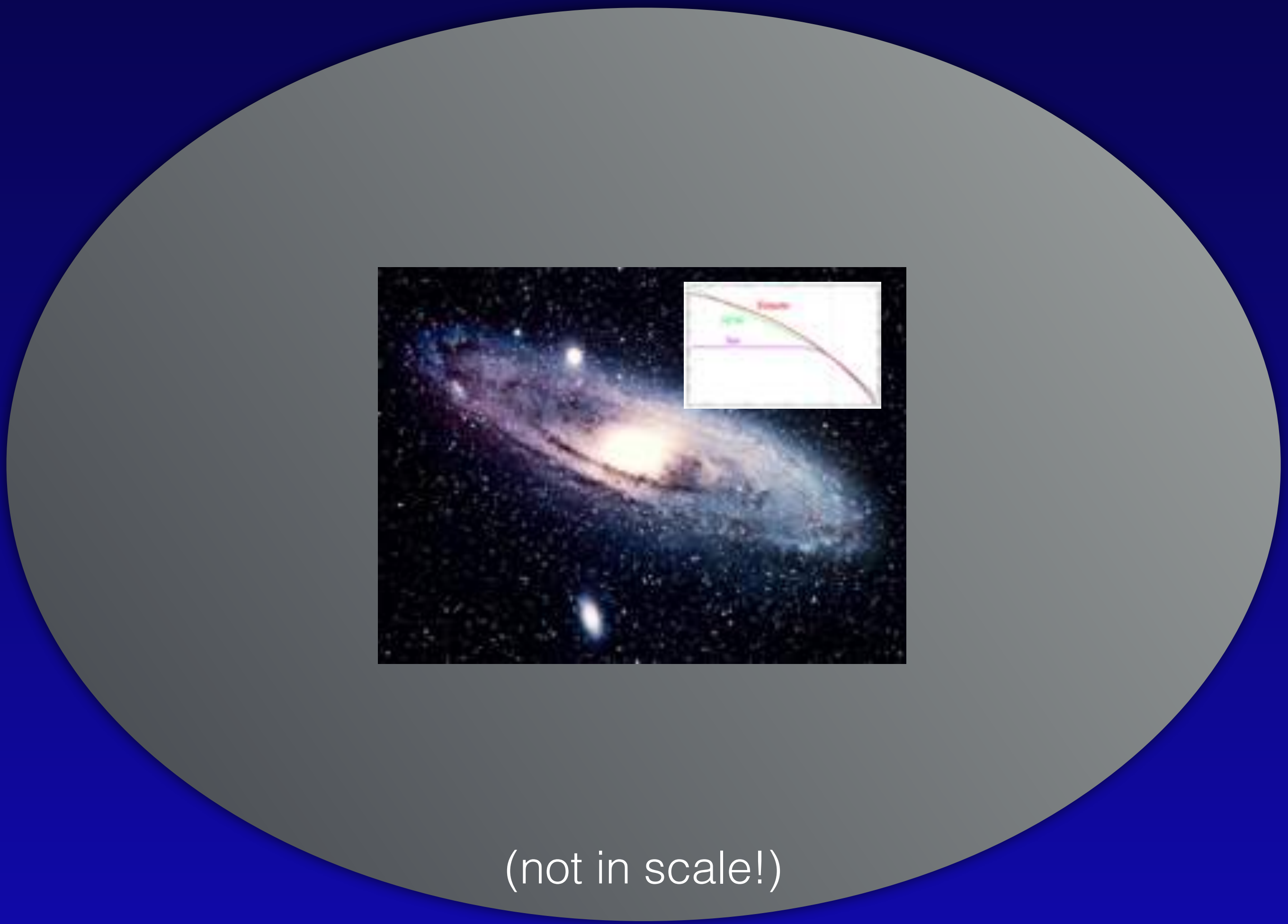






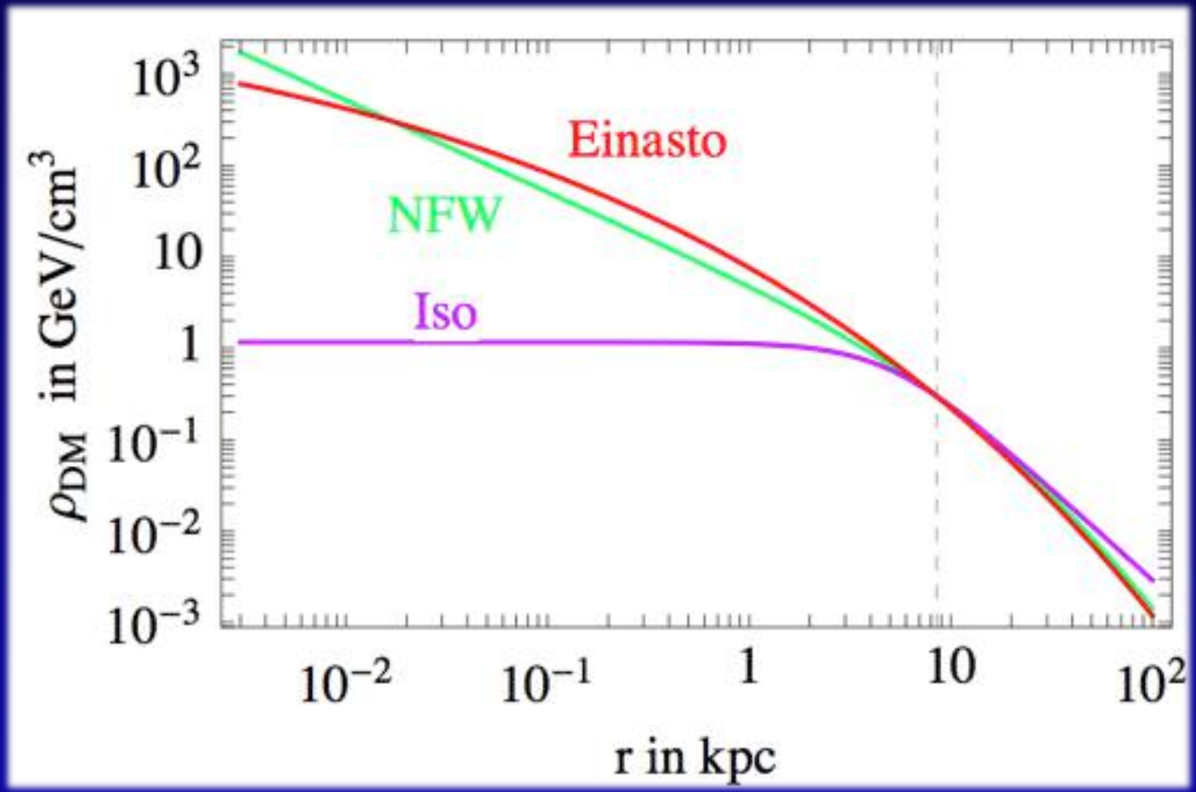
physical size

A story of LCDM
II: the single halo



(not in scale!)

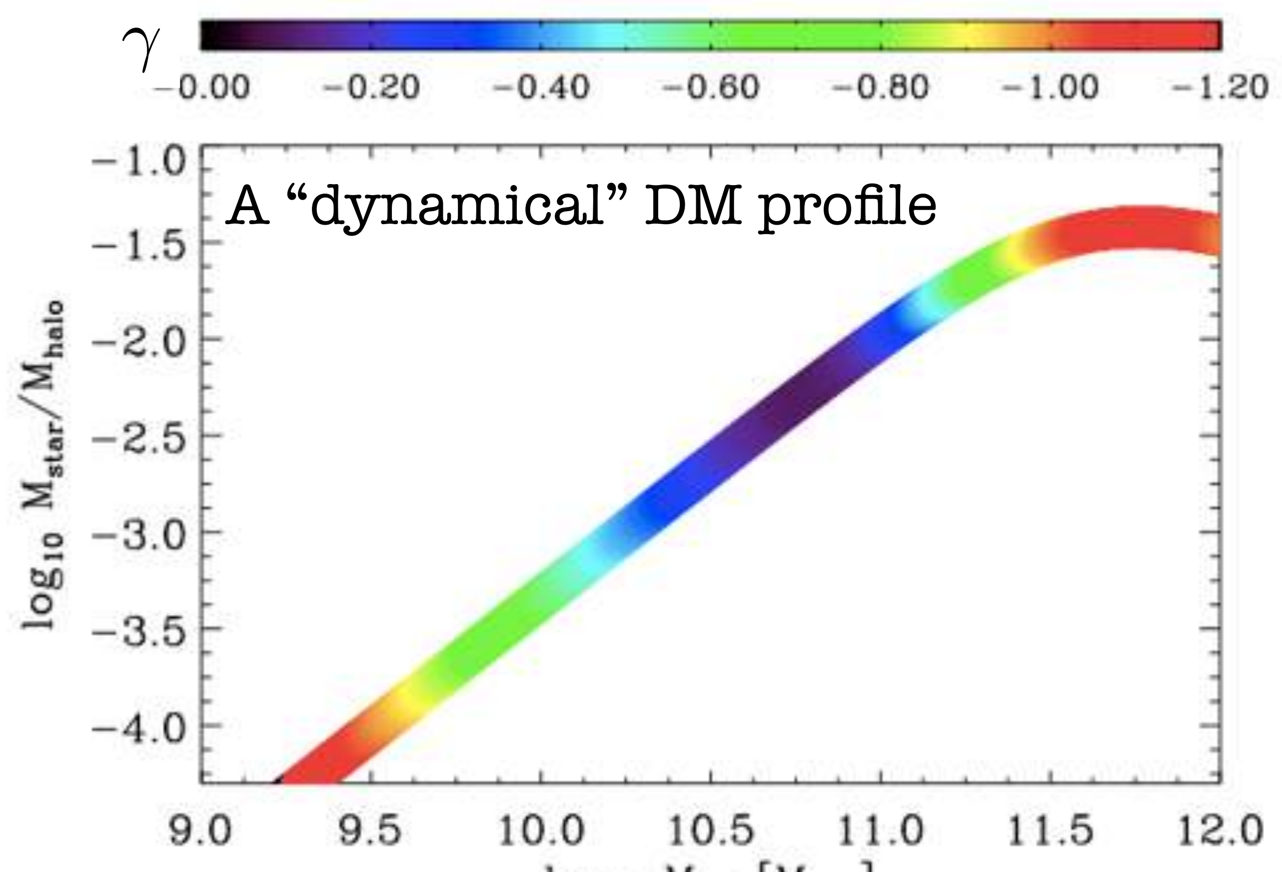
A “universal” DM profile?



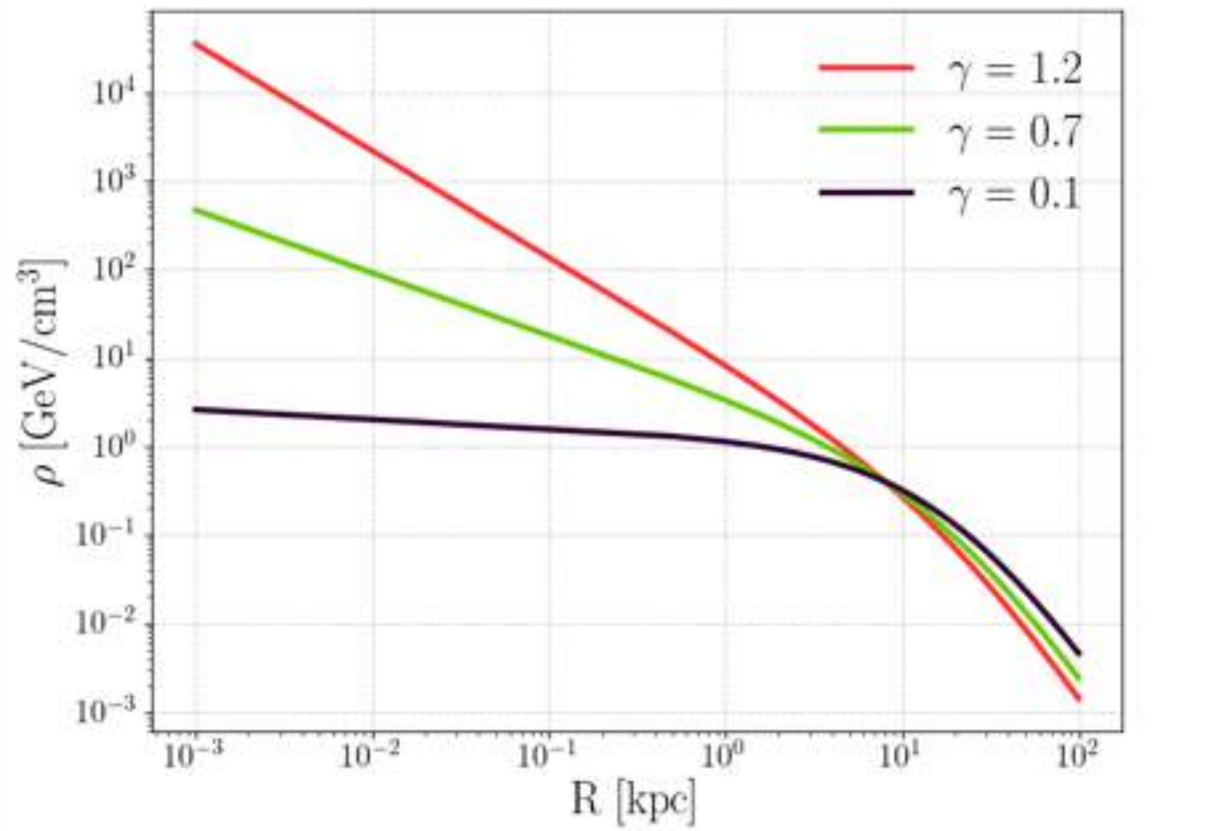
NAVARRO-FRENK-WHITE

$$\rho(R) \propto \frac{R_s}{R}\left(1+\frac{R}{R_s}\right)^{-2}$$

A story of LCDM
III: the dark matter distribution



[Di Cintio et al., 2013]

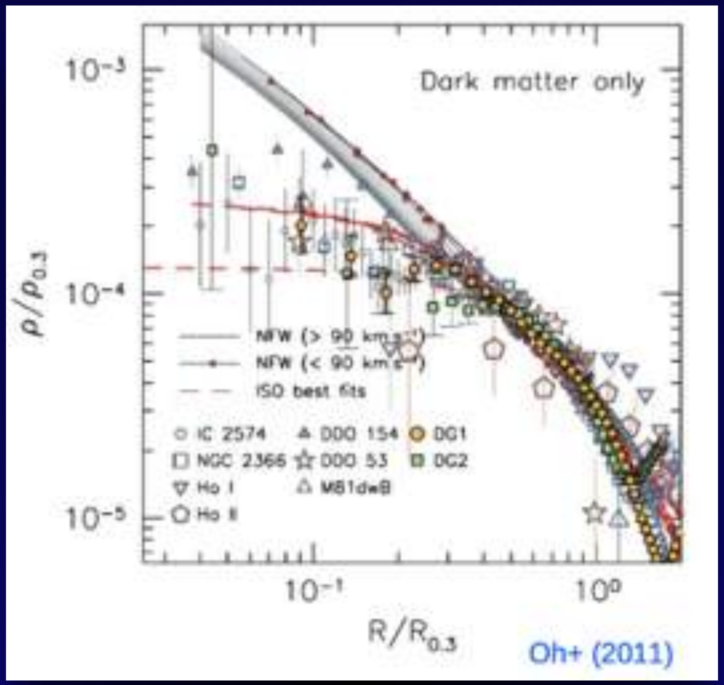


generalized NFW

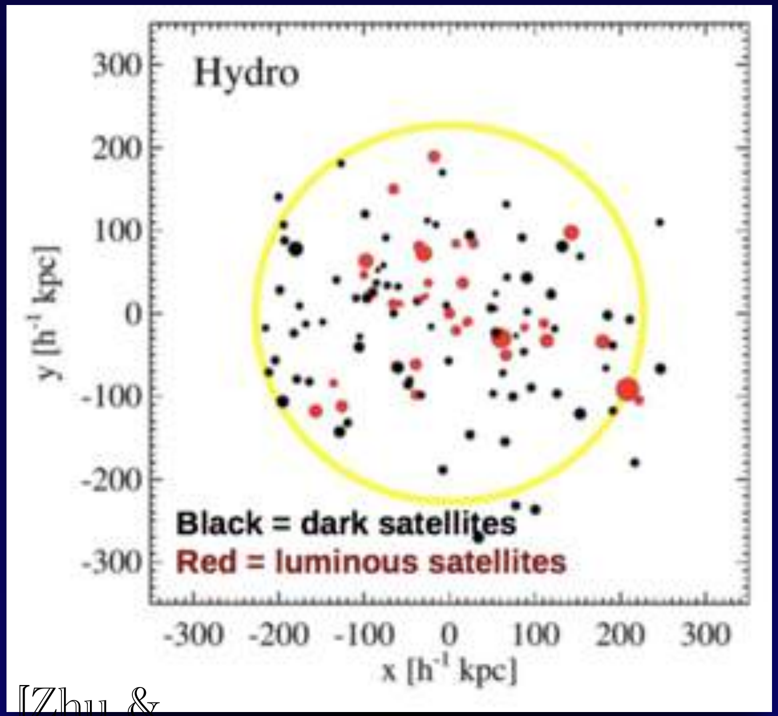
$$\rho_{DM}(R) \propto \rho_0 \left(\frac{R}{R_s}\right)^{-\gamma} \left(1 + \frac{R}{R_s}\right)^{-3+\gamma}$$

A story of LCDM
IV: the small scale problems

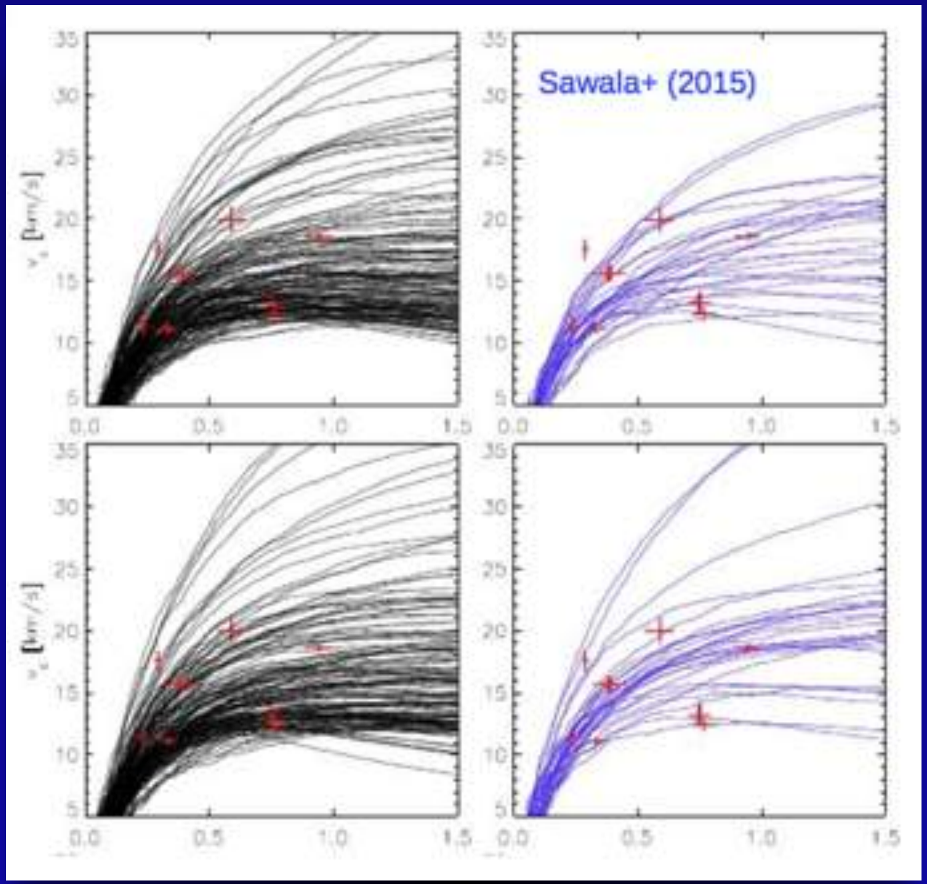
Cusp vs core



Missing satellite



Too big to fail



Ask me later, if interested

Spiral galaxies
disk dynamical structure

$$F_{grav} = F_{cent}$$

$$G_N \frac{M}{R^2} = \frac{v_c^2}{R}$$

$$v_c \propto \sqrt{\frac{M}{R}}$$

Disk is Rotation supported:
observable velocity traces enclosed mass

Spiral galaxies disk dynamical structure

$$M = M(R)$$

$$v_c = v_c(R)$$

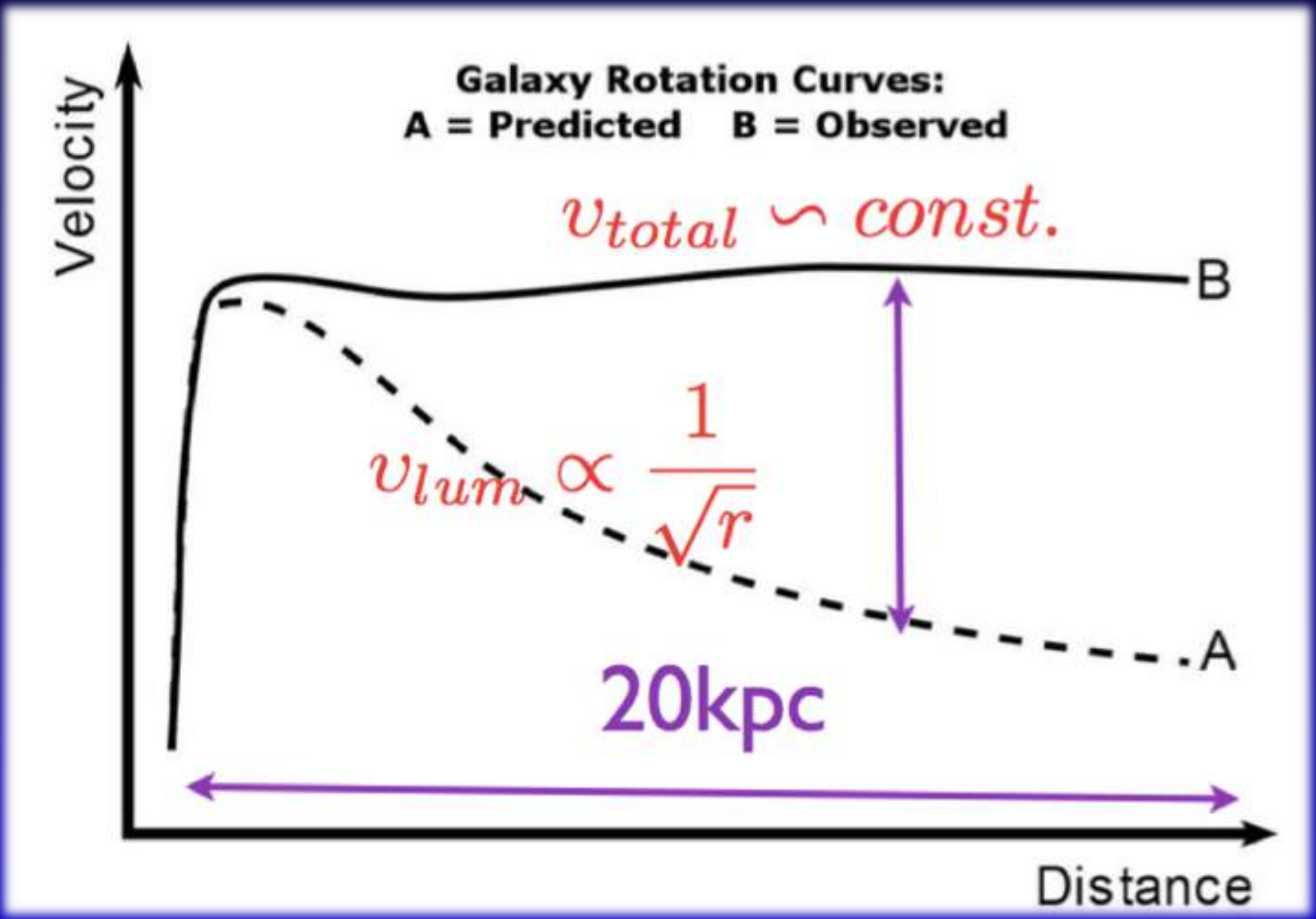
$$v_c(R) \propto \Phi(R)$$

$$v_c(R) \propto \sqrt{\frac{M(R)}{R}}$$

Using observed circular velocities to infer the potential (total enclosed mass)

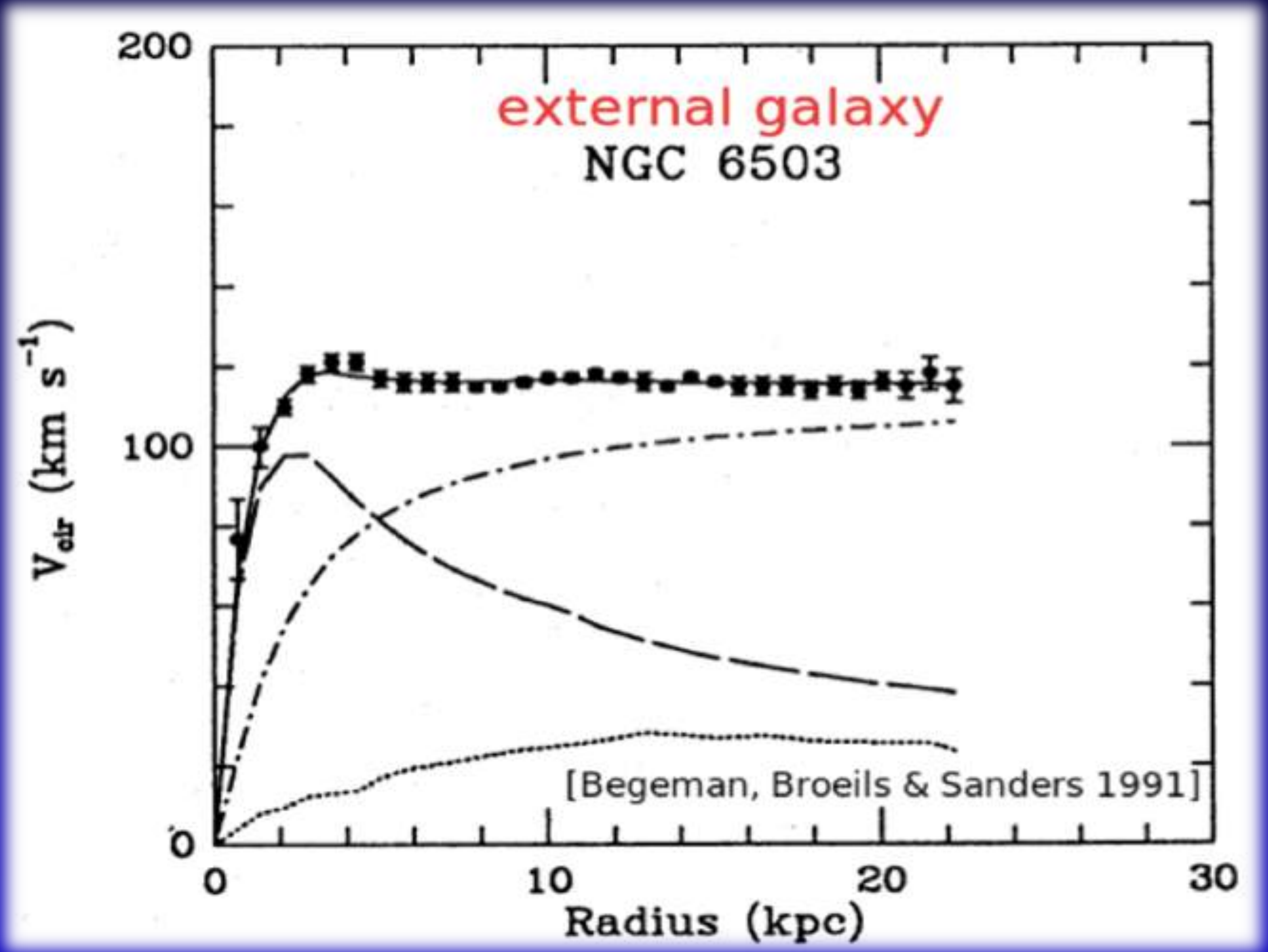
Rotation Curves in local galaxies: an evergreen classic

(with interesting twists)



discrepancy between observed and predicted (from visible matter only)

Rotation Curves in local galaxies: an evergreen classic



discrepancy between observed and predicted (from visible matter only)

Not only the disk: Jeans analysis

Fluid continuity equation

$$\frac{\partial f}{\partial t} + \sum_{\alpha=1}^{6} \frac{\partial}{\partial w_\alpha}(f \dot{w}_\alpha) = 0$$

Collisionless Boltzmann Equation

$$\frac{\partial f}{\partial t} + \vec{\mathbf{v}} \cdot \vec{\nabla} f - \vec{\nabla}\Phi \cdot \frac{\partial f}{\partial \vec{\mathbf{v}}} = 0$$

(CBE)

Jeans analysis

$$\frac{\partial f}{\partial t} + \vec{\mathbf{v}} \cdot \vec{\nabla} f - \vec{\nabla}\Phi \cdot \frac{\partial f}{\partial \vec{\mathbf{v}}} = 0$$

Collisionless Boltzmann Equation

$$\nu(\vec{\mathbf{x}}) \equiv \int f(\vec{\mathbf{x}}, \vec{\mathbf{v}}) d^3\vec{v}$$

$$\bar{v}_i(\vec{\mathbf{x}}) \equiv \frac{1}{\nu(\vec{\mathbf{x}})} \int v_i f(\vec{\mathbf{x}}, \vec{\mathbf{v}}) d^3\vec{v}$$

$$\overline{v_i v_j}(\vec{\mathbf{x}}) \equiv \frac{1}{\nu(\vec{\mathbf{x}})} \int v_i v_j f(\vec{\mathbf{x}}, \vec{\mathbf{v}}) d^3\vec{v} = \sigma_{ij}^2 + \bar{v}_i \bar{v}_j$$

$$\nu \frac{\partial \bar{v}_j}{\partial t} \quad + \quad \bar{v}_i \nu \frac{\partial \bar{v}_j}{\partial x_i} \quad = \quad -\nu \frac{\partial \Phi}{\partial x_j} \; - \; \frac{\partial}{\partial x_i}(\nu \sigma_{ij}^2) \quad (j = 1, 2, 3)$$

$$\text{acceleration} + \begin{array}{c}\text{kinematic}\\ \text{viscosity}\end{array} \quad = \quad \text{gravity} \; + \; \text{pressure}$$

Jeans analysis: a practical example in a specific case cylindrical symmetry

$$\frac{1}{\nu}\frac{\partial}{\partial z}(\nu\sigma_{zz}^2) = -\frac{\partial\Phi}{\partial z} = -2\pi G\Sigma(z)$$

$$\Sigma(z) = \int_{-z}^{z}\rho(z')dz'$$

$$\Sigma(z) = -\frac{\sigma_{zz}^2}{2\pi G}\left(\frac{d}{dz}\ln\nu\right)$$

$$h^{-1} \equiv \frac{d}{dz}\ln\nu$$

From observations

$$\sigma_{zz}^2 = (20\ \mathrm{km/s})^2$$

$$h = 300pc$$

$$\downarrow$$

$$\Sigma \approx 50\ M_{\odot}\mathrm{pc}^{-2}$$

Some words about Gravitational lensing

Gravitational lensing geometry

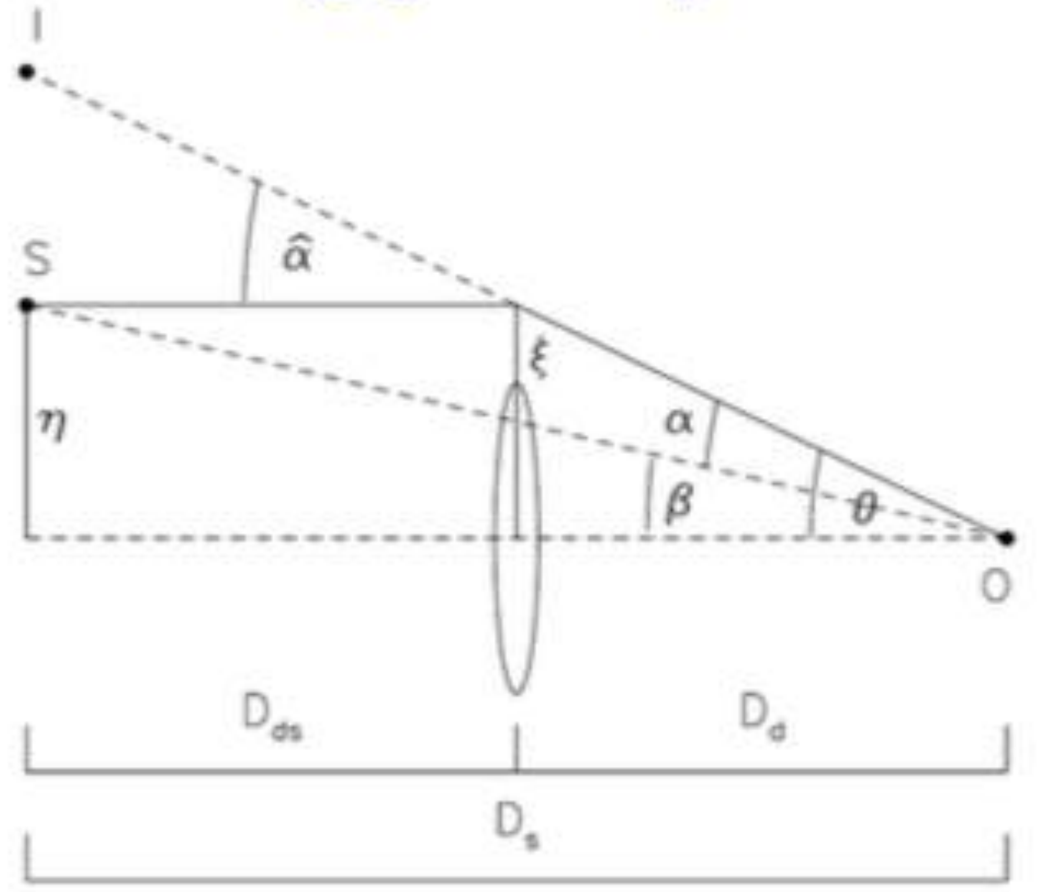
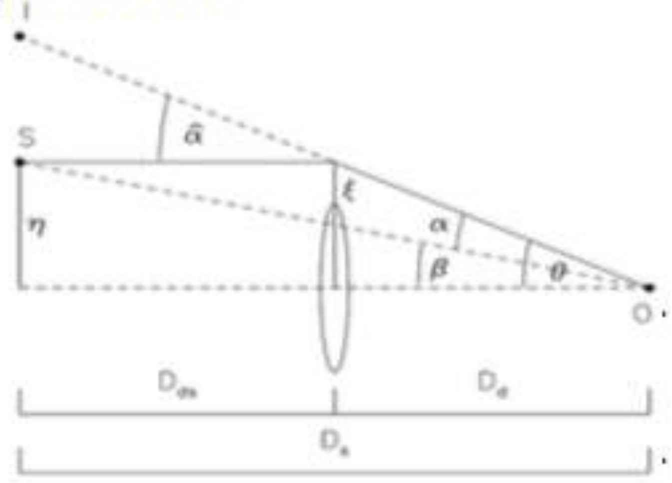


Figure from Narayan & Bartelmann (1996; arXiv:astro-ph/9606001)

- Parameters:
  - Angular diameter distances: D_s, D_d and D_{ds}
  - Source position: $\beta = \eta / D_s$ - *Impact parameter*
  - Image position: $\theta = \xi / D_d$
  - Deflexion angle: $\alpha = \hat{\alpha}\ D_{ds} / D_s$

Some words about Gravitational lensing

The Lensing Equation



- $$\eta = \frac{D_s}{D_d}\xi - D_{ds}\hat{\alpha}(\xi)$$
- Using the angular quantities instead of the physical ones we get:
$$\beta = \theta - \frac{D_{ds}}{D_s}\hat{\alpha}(D_d\theta)$$
- Then by using the *physical* deflexion angle we arrive to the **Lens Equation**:
$$\beta = \theta - \alpha(\theta)$$

Some words about Gravitational lensing

The deflection angle

- From GR one can estimate the deflection angle due to a mass point:

$$\hat{\alpha} = \frac{4\ G\ M}{c^2 \xi}$$

- The projected angle is then:

$$\alpha = |\boldsymbol{\alpha}| = \frac{D_{ds}}{D_s} \frac{4\ G\ M}{c^2 D_d |\boldsymbol{\theta}|}$$

- Considering now a direction, as all the angles lies in the image-lens direction, we have:

$$\boldsymbol{\alpha} = \frac{D_{ds}}{D_s} \frac{4\ G\ M}{c^2 D_d |\boldsymbol{\theta}|} \frac{\boldsymbol{\theta}}{|\boldsymbol{\theta}|}$$

The Einstein angle

- The lens equation can be easily solved in this case.
- Lets first define a 'natural scale', the *Einstein angle* for this lens:

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{ds}}{D_s D_d}}$$

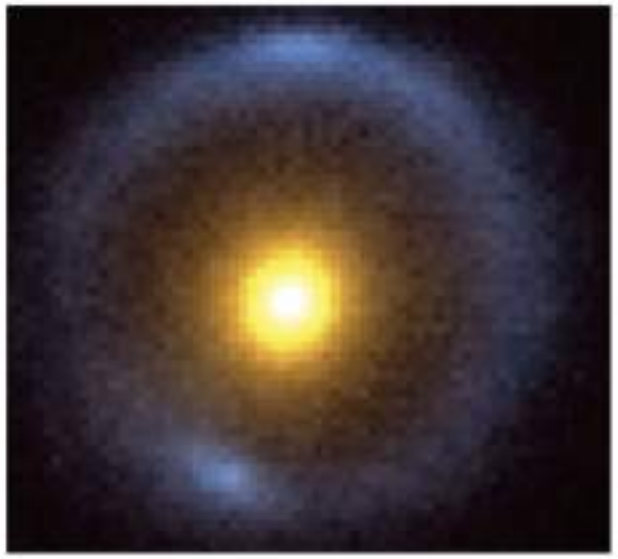
- The lens equation then becomes:

$$\beta = \theta - \frac{\theta_E^2}{\theta}$$

- It has two solutions:

$$\theta_\pm = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2}\right)$$

The Einstein angle

- What is 'natural' about the *Einstein angle or radius* ?
- If $\beta = 0 \rightarrow \theta = \theta_E$ 
- The distance between two images (solutions) is $2\theta_E$
- The density inside the the Einstein radius is the critical lensing density

$$\Sigma_{cr} = \frac{c^2}{4\pi G} \frac{Ds}{D_d \ D_{ds}}$$

above which multiple imaging occur.

Lensing regimes

The lensing effect

- Gravitational lensing causes several effects on the images of the sources:
  1. Radial displacement
  2. Multiple imaging (given certain conditions)
  3. Magnification of the angular size $\xrightarrow{\textit{surface brightness conservation}}$ and flux
  4. Distortion
  5. Time Delay
- The prevalence/interest of one or more of those effects over the others have to do with the *lensing regime*.

Lensing regimes

Lensing regimes

- Different lensing regimes differentiate themselves by:
  1. The lens mass distribution
  2. The distances involved
  3. The impact parameter
- The main lensing regimes are:
  1. Micro lensing $\theta \leq \theta_E \quad dist. \sim kpc$
  2. Strong lensing $\theta \leq \theta_E \quad dist. \sim Gpc$
  3. Weak lensing $\theta \geq \theta_E \quad dist. \sim Gpc$

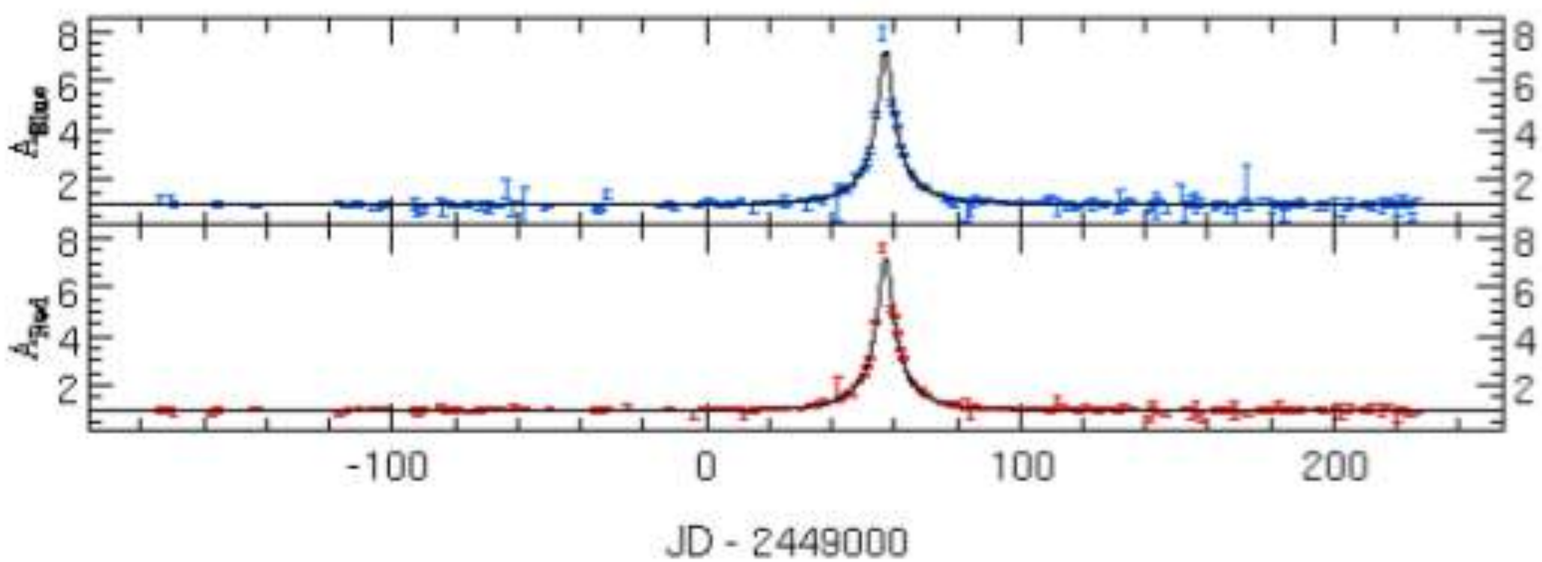
Micro Lenses

Micro lenses

- Typical *Einstein radius*

$$\theta_E = 0.902mas \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D_d}{10kpc}\right)^{-1/2} \left(1 - \frac{D_d}{D_s}\right)^{-1/2}$$

- We cannot observationally resolve the multiple imaging, but we can measure the variation of the flux due to the magnification



- Note that microlensing should produce symmetric and achromatic light curves as above.

Strong Lenses