1. Show for the 3+1 mass scheme, the neutrino probabilities, for short-baseline experiments are

$$\begin{aligned} P(\nu_\mu \to \nu_e) &= \sin^2(2\theta_{\mu e})\sin^2\left(\frac{\Delta m_{41}^2 L}{4E}\right) \\ P(\nu_\mu \to \nu_\mu) &= 1 - \sin^2(2\theta_{\mu\mu})\sin^2\left(\frac{\Delta m_{41}^2 L}{4E}\right) \\ P(\nu_e \to \nu_e) &= 1 - \sin^2(2\theta_{ee})\sin^2\left(\frac{\Delta m_{41}^2 L}{4E}\right) \end{aligned}$$

where

$$\begin{aligned} \sin^2(2\theta_{\mu\mu}) &= 4|U_{\mu 4}|^2(1 - |U_{\mu 4}|^2) \\ \sin^2(2\theta_{ee}) &= 4|U_{e4}|^2(1 - |U_{e4}|^2) \\ \sin^2(2\theta_{\mu e}) &= 4|U_{\mu 4}|^2|U_{e4}|^2 \end{aligned}$$

where $U_{\mu 4}(U_{e4})$ is the projection of the $\nu_\mu(\nu_e)$ into the ν_4, Δm_{41}^2 is the squared mass difference, $\Delta m_{41}^2 \equiv m_4^2 - m_1^2 \sim m_4^2 - m_2^2 \sim m_4^2 - m_3^2$. Here we have oscillations of all four neutrinos.

(a) Show that in the limit of $|U_{\mu 4}|^2 << 1$ then

$$\sin^2(2\theta_{\mu e}) \sim \frac{\sin^2(2\theta_{\mu\mu})\sin^2(2\theta_{ee})}{4} \tag{1}$$

(b) Assume that it have more one neutrino, a 3+2 model. What did you think it happen with the constraint given in Eq. (1).

2. In atmospheric neutrinos there are two neutrino flavors, ν_μ and ν_e. Assume that there is no difference between neutrinos and anti-neutrinos.

(a) Assume that always the flux of non-oscillated atmospheric neutrino flux obeys $\phi_{\nu_\mu} = 2\phi_{\nu_e}$. Assume that $\theta_{23} = 45$ degrees and that $\theta_{13} = 0$. Here we have oscillations of all three neutrinos. Write down the oscillated flux of electron neutrinos. Is there the flux of oscillated electron neutrinos bigger, smaller or equal to the non-oscillated flux?