Bose-Einstein Condensation in ultra-cold atomic gases

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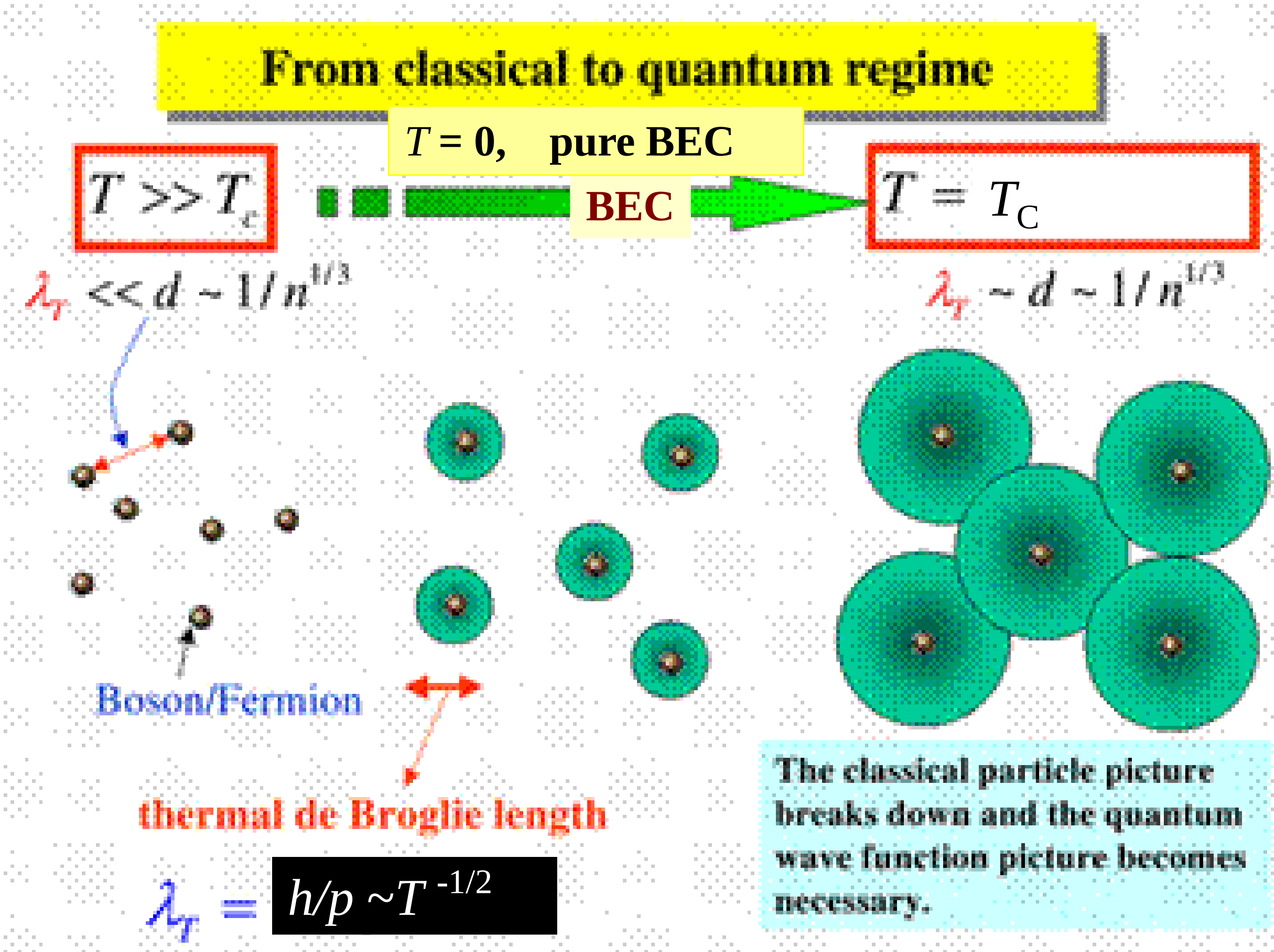
OUTLINE

- Bose-Einstein Condensate (BEC)
- Special Properties: Interference, vortex lattice, atom laser, quantum phase transition
- Mean-Field Equation with two-body interaction
- Variational approximation
- Three-body interaction
- **Current Research**: Soliton & self-bound systems in 3 dimensions
- Concluding remark

Boson versus fermion

- Boson: integral spin, example ^{1}H, ^{4}He, ^{7}Li atoms
- Fermion: half-integral spin, example ^{2}H, ^{3}He, ^{6}Li atoms

Bose-Einstein Condensate (BEC)



Bose-Einstein Condensation
Predicted 1924

S. N. Bose

A. Einstein

ose -
nstein
ondensation

- Quantum vs classical mechanics
- Wave vs particle behavior
- A phase-coherent giant molecule
- Lack of viscosity: leakage through microscopic hole and superfluidity
- Infinite thermal conductivity

Liquid He experiments

- Liquid He is a strongly interacting system, no controlable theory possible.
- A dilute weakly interacting BEC will allow a perturbative theory to be developed
- Experiment of BEC in ultra-cold ultra-dilute atomic gas ➔Controllable perturbative theory

Bose-Einstein Condensation in dilute trapped gas

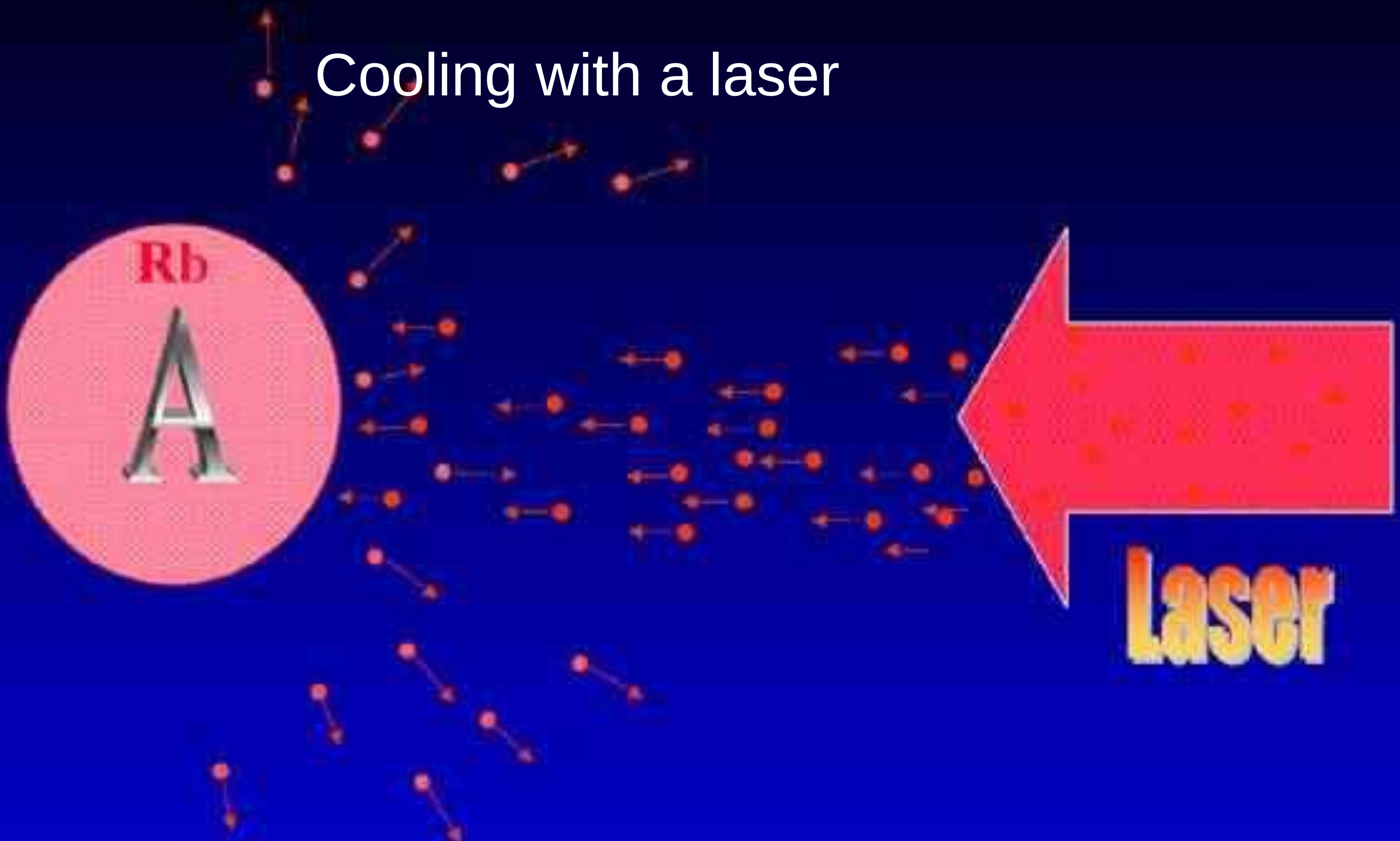
Detected 1995

BEC Physicists win the 2001 Nobel Prize!

Eric Cornell   Carl Wieman   Wolfgang Ketterle

(JILA)   (JILA)   (MIT)

Cooling with a laser



Pushing atoms with light

Acceleration 100 000 g's!!!!

de Broglie Wave Length λ (H atom)

- $\lambda = \frac{h}{mv} = \frac{39}{v \frac{\text{cm}}{\text{s}}}\ \mu\text{m}$

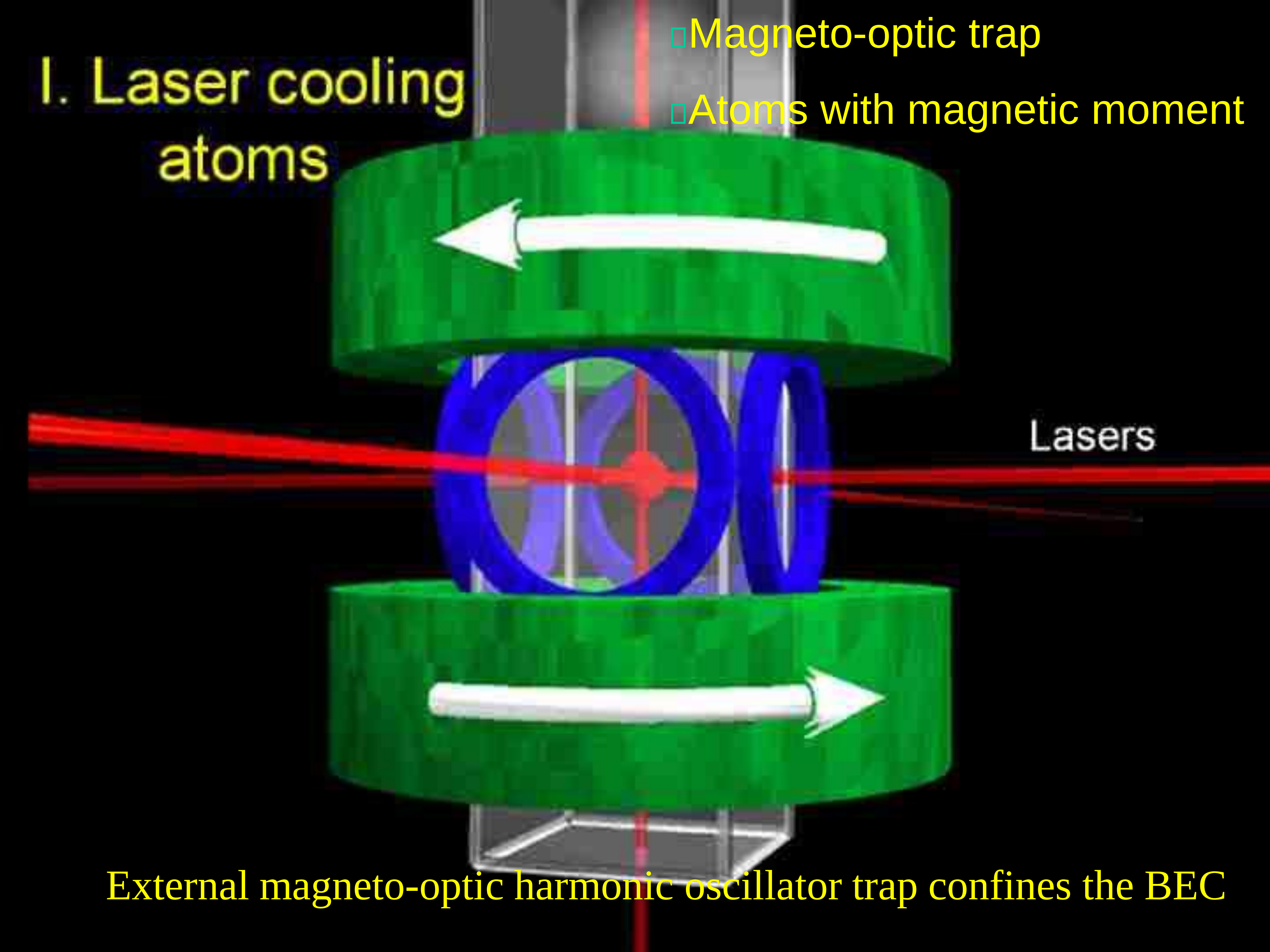
- $v = \sqrt{\frac{3kT}{m}} = \left[16 \times 10^3 \sqrt{T(K)}\right] \frac{\text{cm}}{\text{s}}$

- OVEN (400 K): $v = 3.2 \frac{\text{km}}{\text{s}}, \lambda = 0.1$ nm $= 2a_0$

- Laser Cool.(50 μK): $v = 1 \frac{\text{m}}{\text{s}}, \lambda = 0.4\ \mu\text{m}$

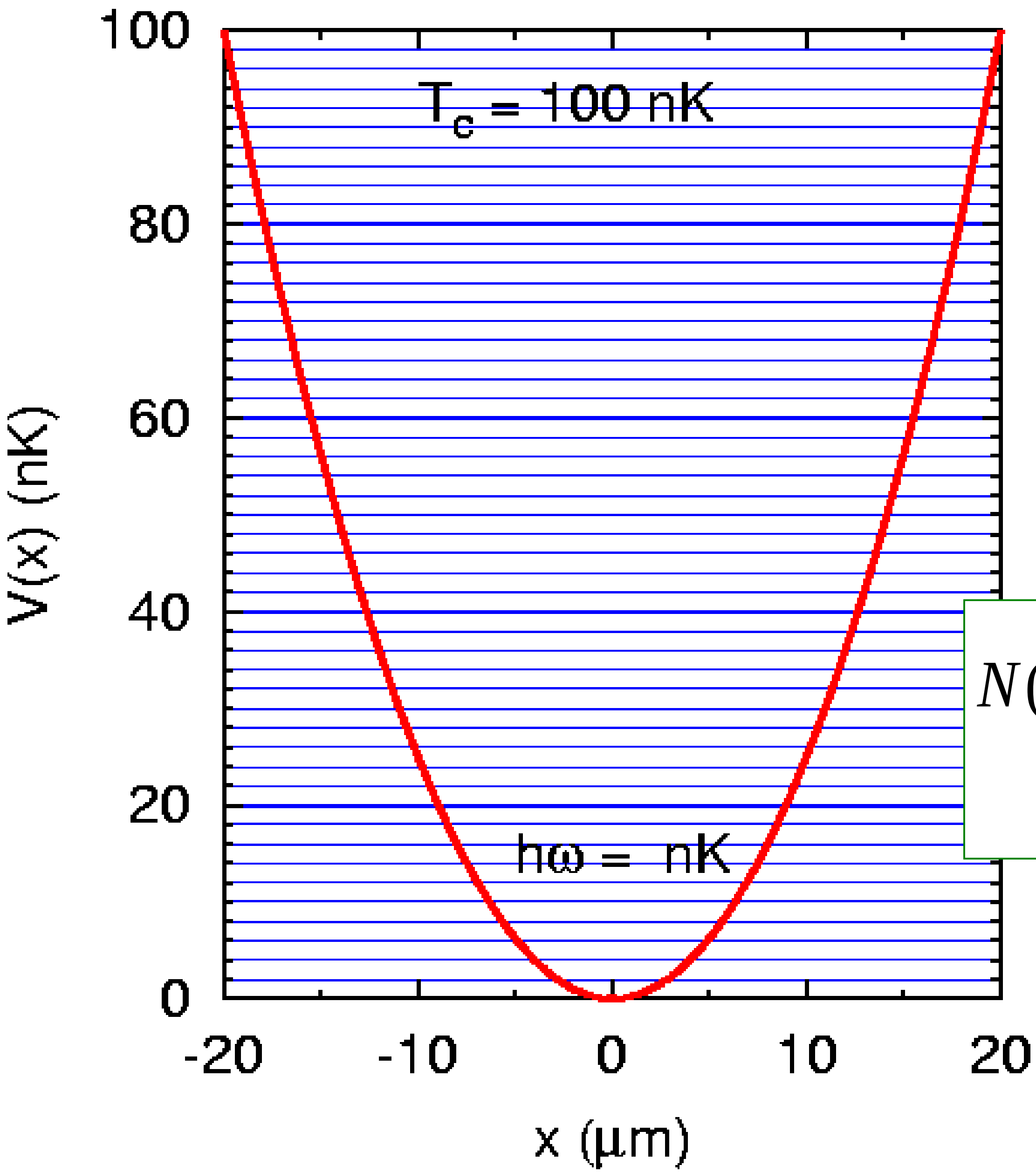
- BEC (50 nK): $v \approx 1 \frac{\text{cm}}{\text{s}}, \lambda = 40\ \mu\text{m}$

- Magneto-optic trap
- Atoms with magnetic moment



External magneto-optic harmonic oscillator trap confines the BEC

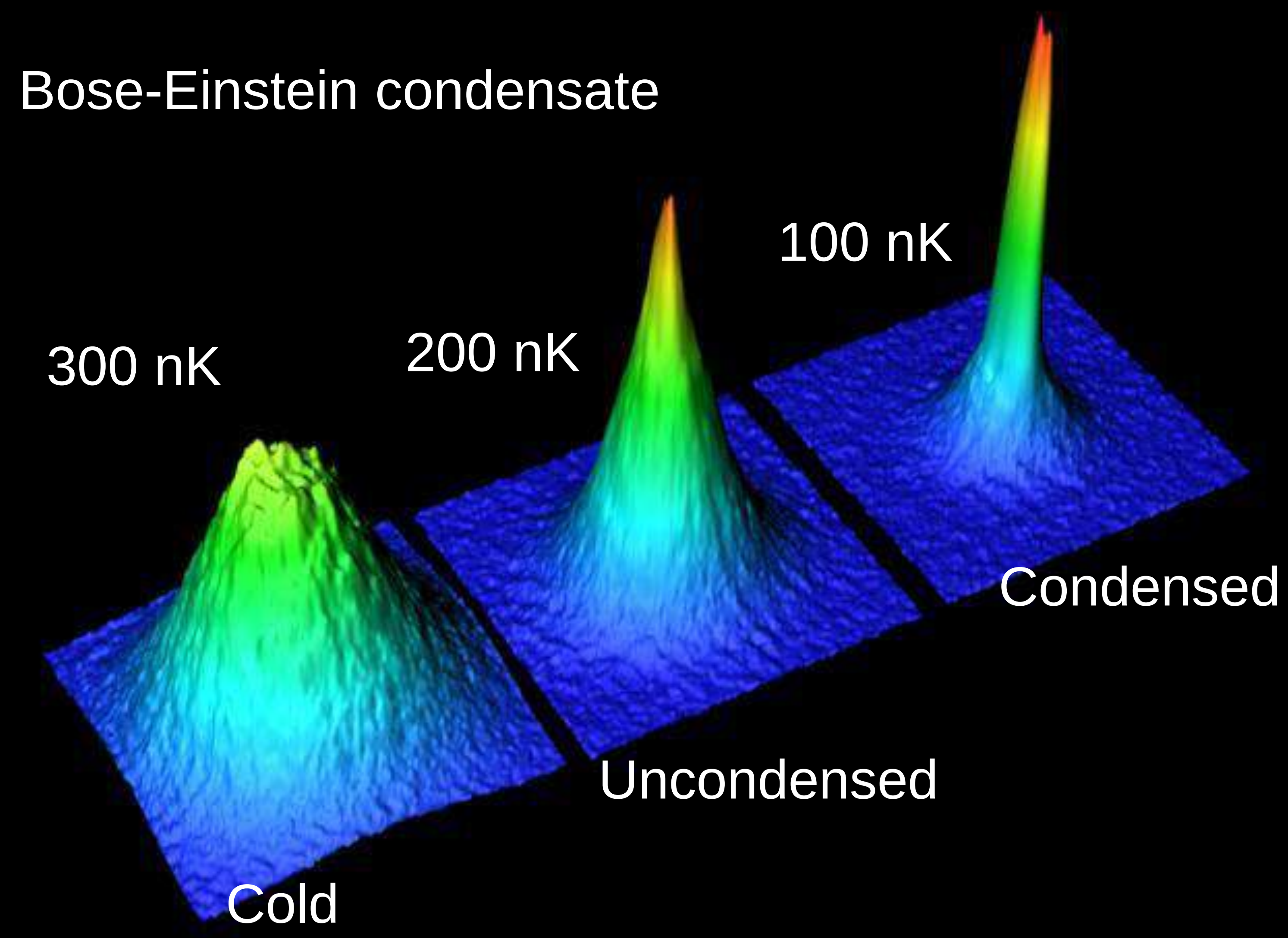
Harmonic trap and quantum statistics



$$N(E) = \frac{1}{\exp \frac{E - \mu}{kT} - 1}$$

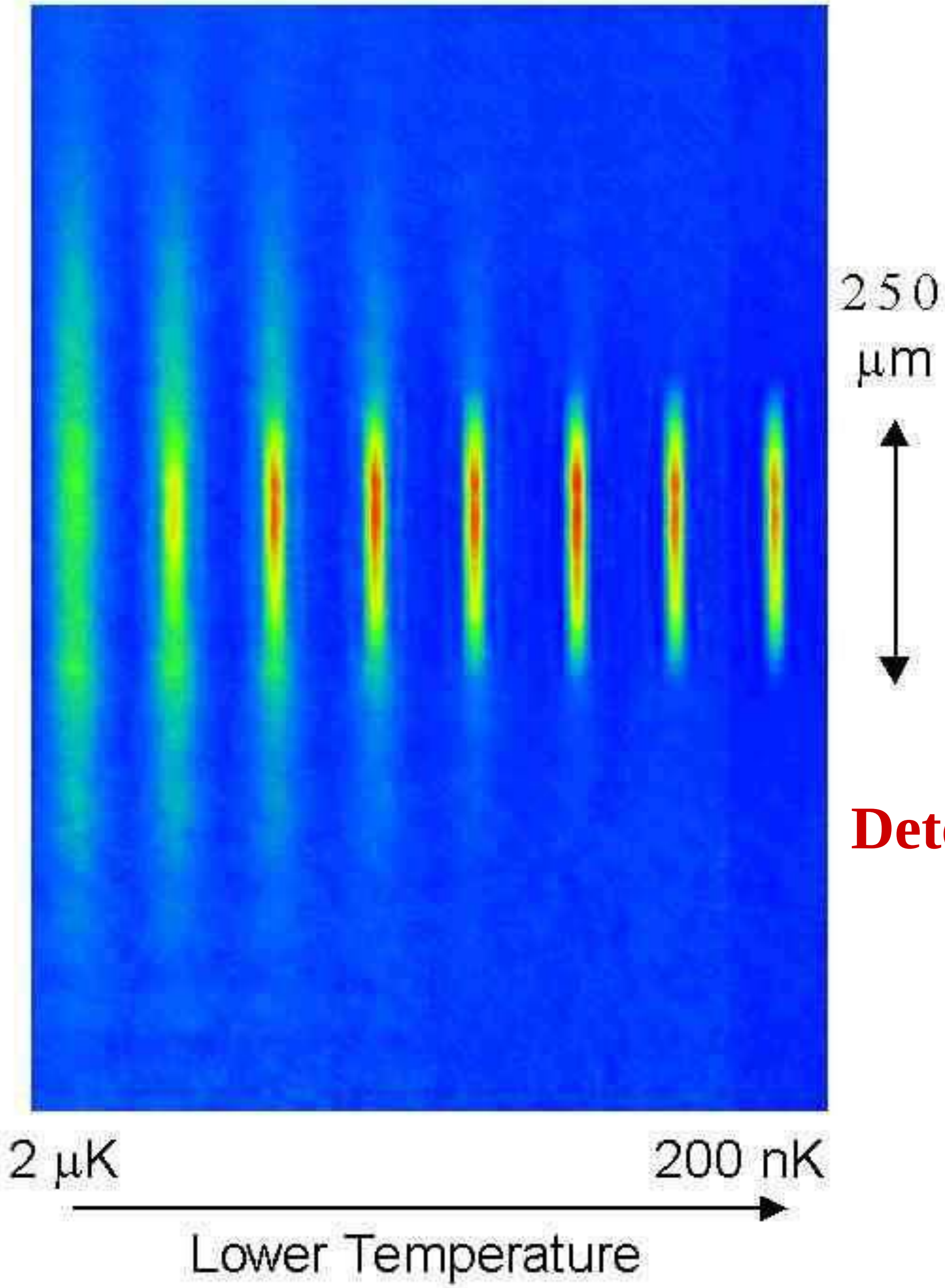
Evaporative cooling

- At the end of laser cooling binary collision makes atoms with a distribution of energy with mean temperature μK
- Magnetic trap is lowered so that the hottest atoms escape (90%)
- Remaining 10% atoms have much reduced **velocity (cm/sec) & temperature (n K) & pressure (10^{-10} atm)** & undergoes BEC of size (μm)


Bose-Einstein condensate
300 nK
200 nK
100 nK
Cold
Uncondensed
Condensed

History of BEC

- Rb: June 1995 JILA Cornell/Wieman
- Na: Sept 1995 MIT Ketterle
- Li: July 1995 Rice Hulet
- H: 1998 MIT Kleppner
- He*: Feb-Mar 2001 Orsay/Paris Aspect & Cohen Tannoudji
- K: Oct 2001 Firenze Inguscio
- Cs: Oct 2002 Innsbruck Grimm
- Cr: Mar 2005 Stuttgart Pfau
- Yb: Sept 2007 Kyoto Takahashi
- Sr: Sept 2009 Amsterdam Schreck
- Dy: Sept 2011 Stanford Lev

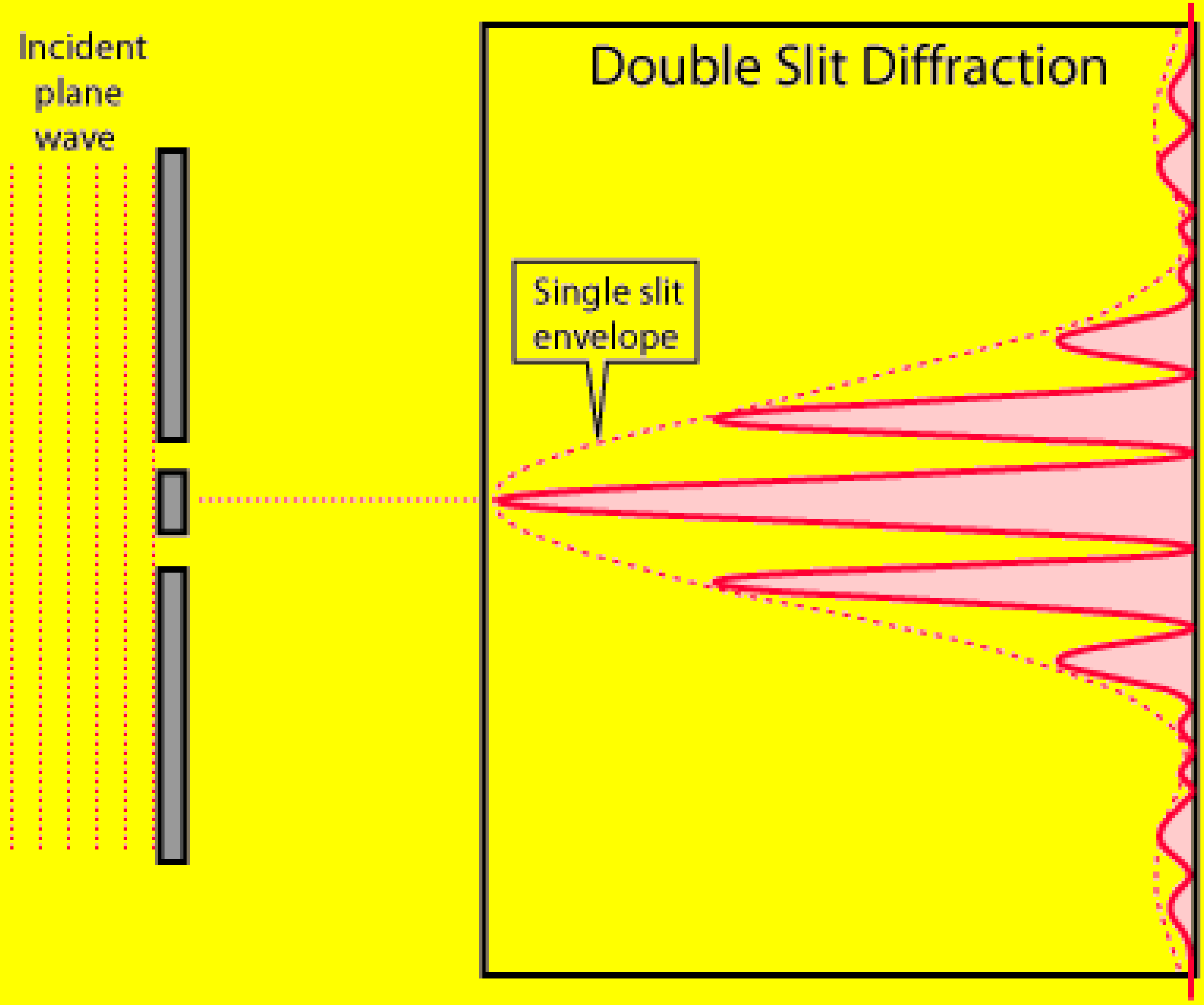


BEC =
“Droplet” formation
in a
quantum saturated
vapor

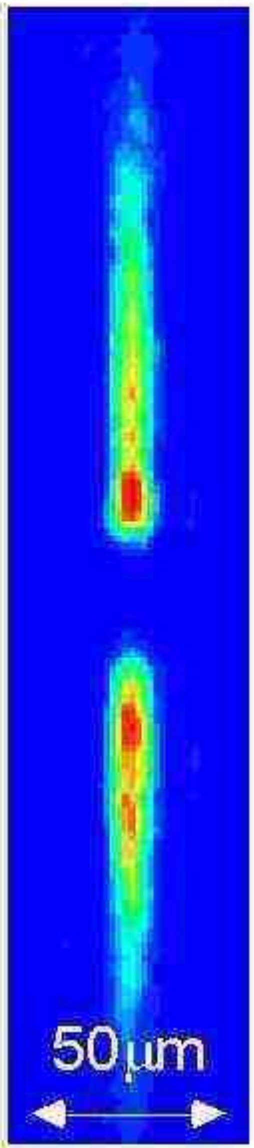
Detection in First Experiments 1995

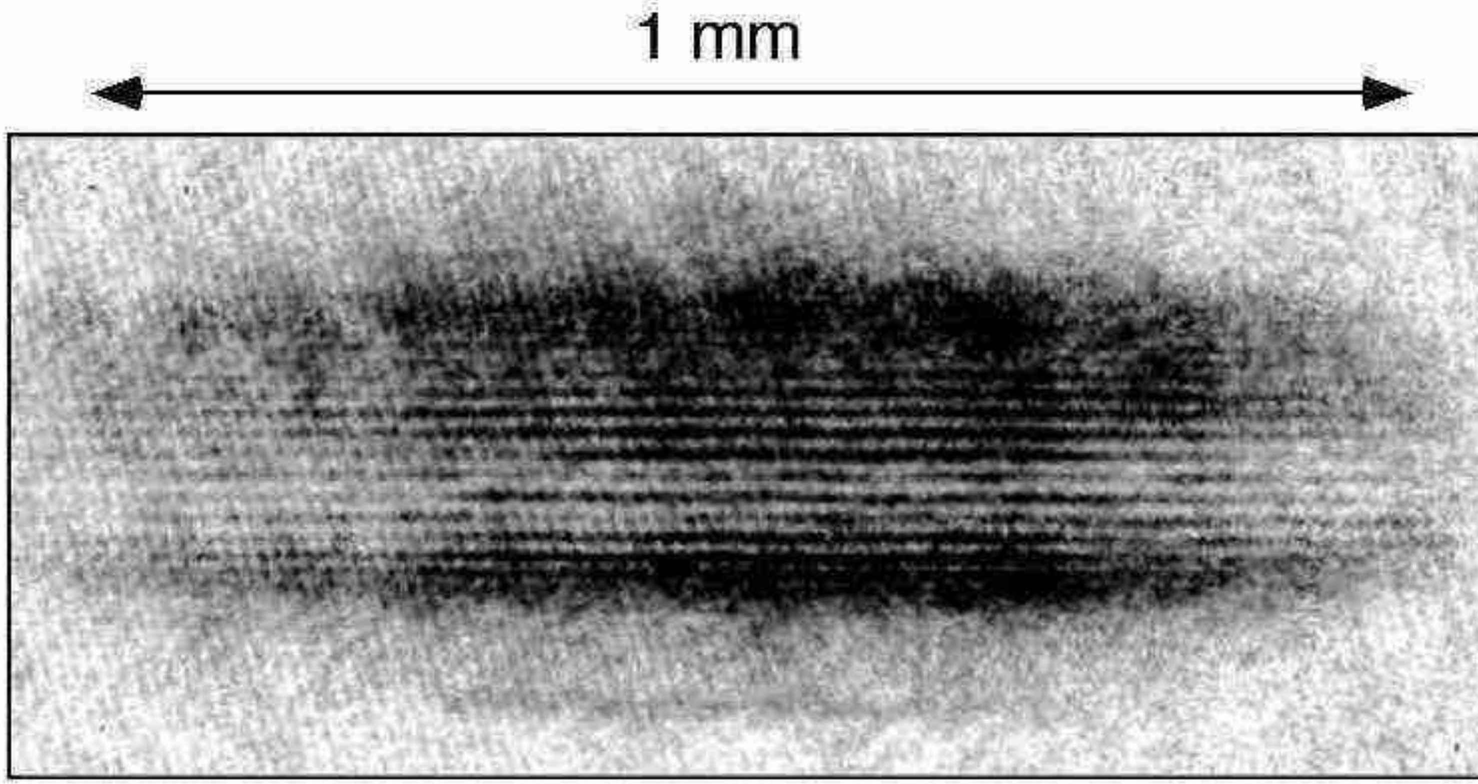
Trapped cloud

(Phase-contrast images)


Incident
plane
wave
Double Slit Diffraction
Single slit
envelope

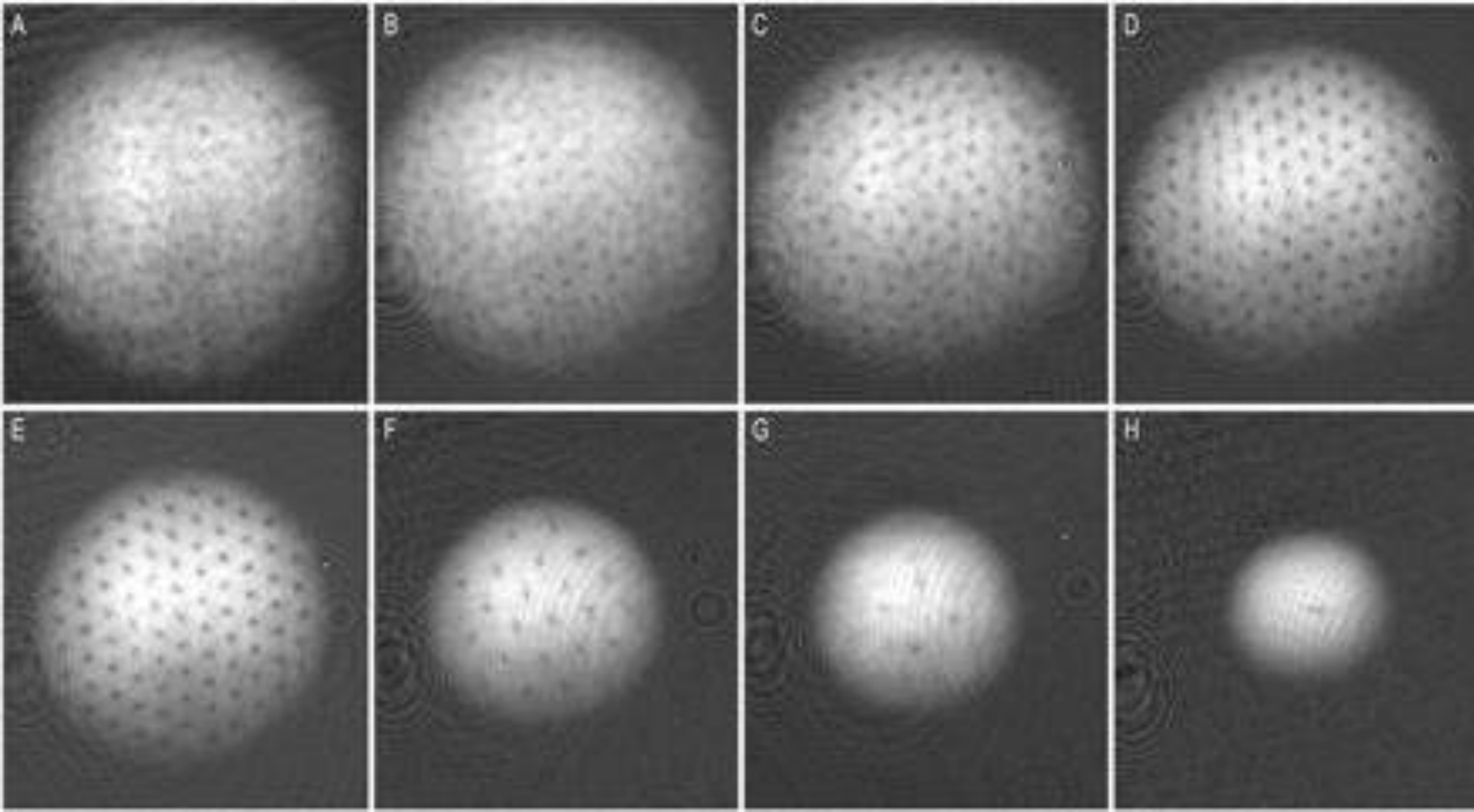
Two condensates ... interfere!





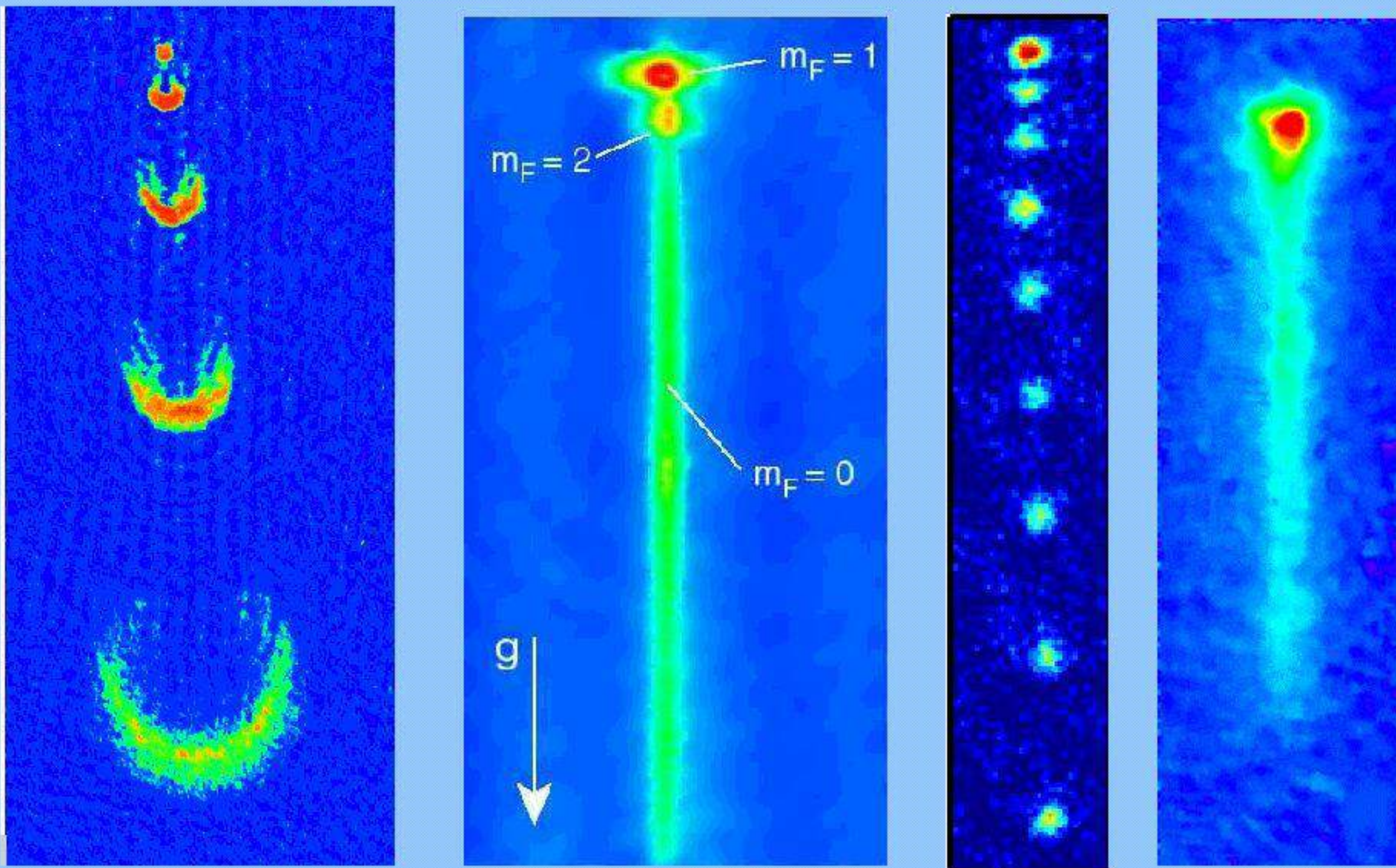
Andrews, Townsend, Miesner, Durfee, Kurn, Ketterle, Science **275**, 589 (1997)

Vortex-lattice formation: times 25, 100, 200, 500 ms, 1, 5, 10, 40 s on 1 mmX1.2mm view.
Ketterle Science 292, 476 (2001)



Atom laser gallery

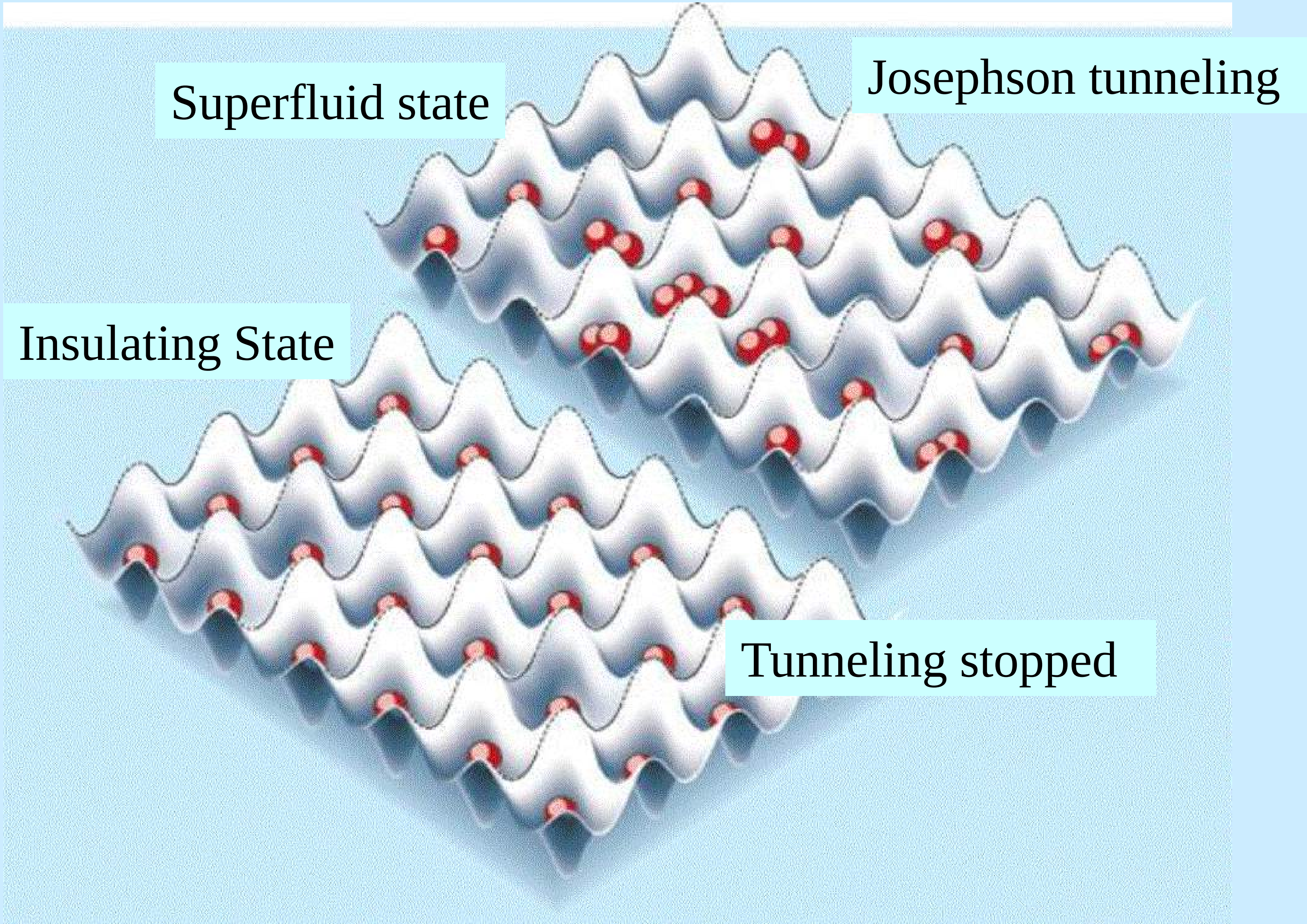
Height:
5, 2, 0.5, 1 mm



MIT '97 | Munich '99 | Yale '98 | NIST '99

QUANTUM PHASE TRANSITION

Periodic optical-lattice trap



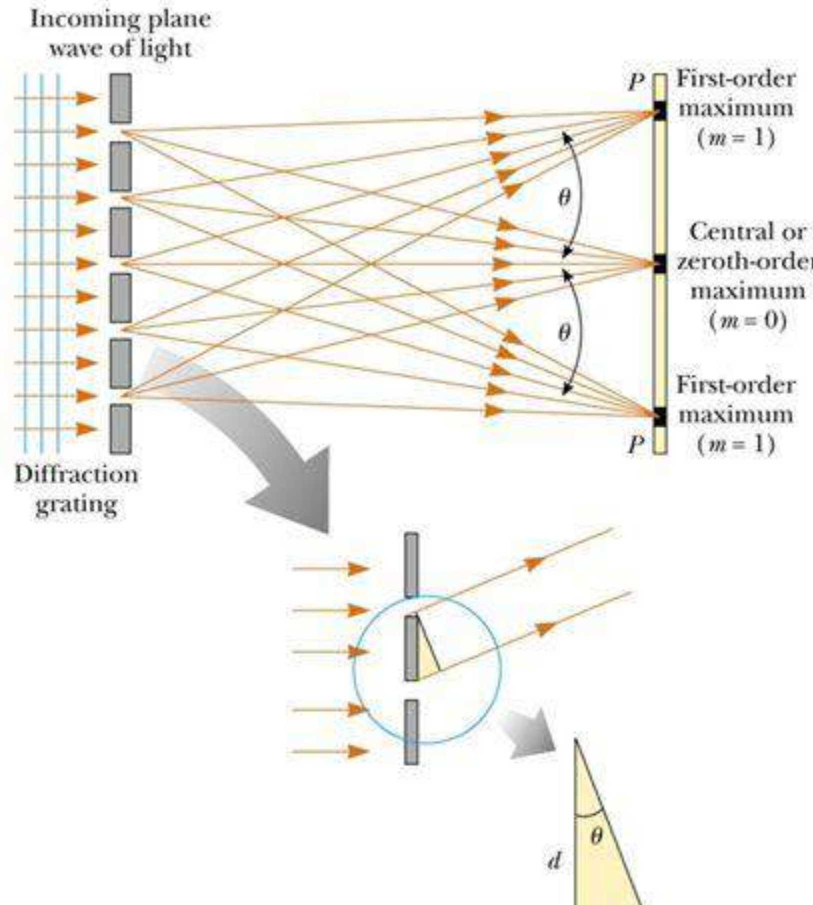
Quantum phase transition (QPT)

M. Greiner et al., Nature 415,39(2002).

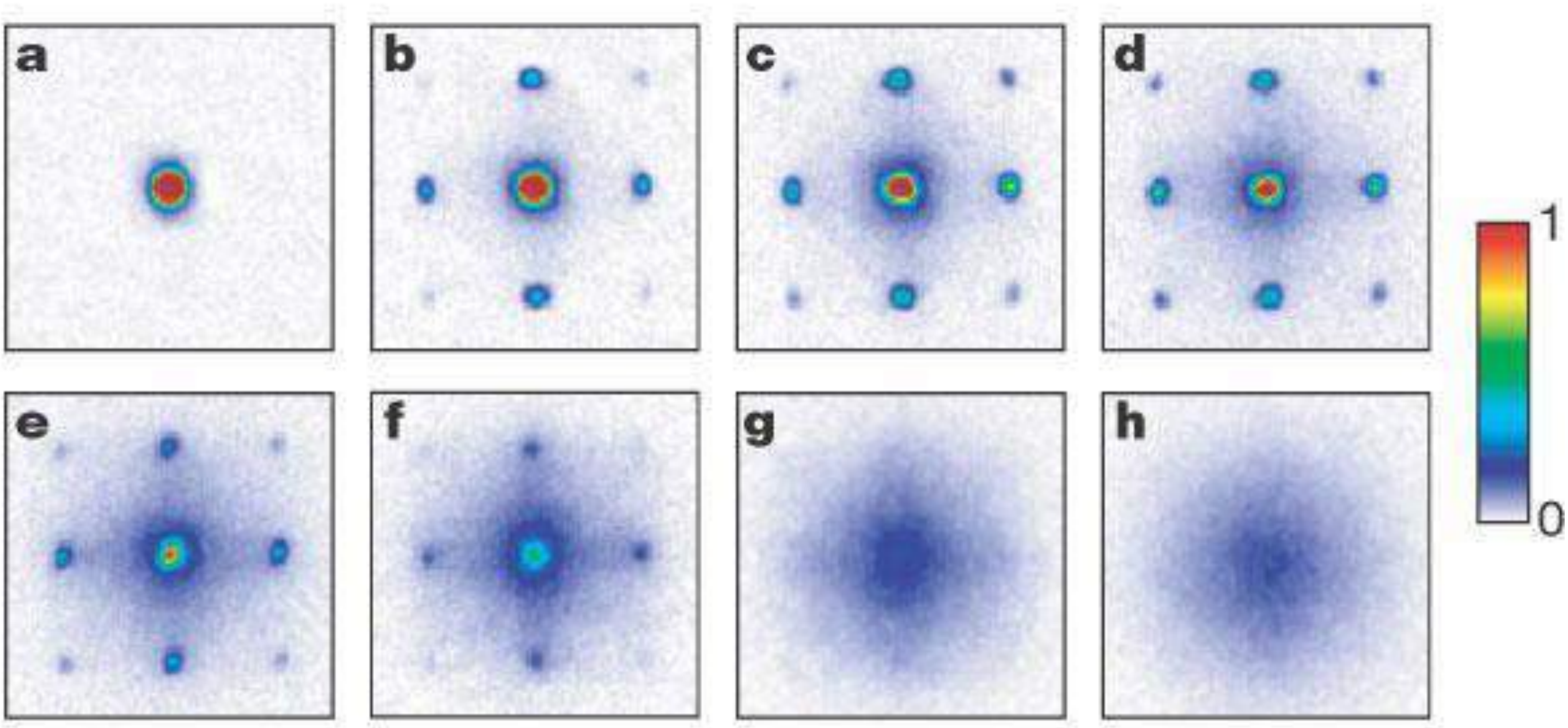
- Classical phase transition ➔ thermal fluctuation
- QPT ➔ quantum fluctuation at T = 0
- Superfluid phase: Atoms freely move from one site to other so number is not known and phase known
- Insulator phase: Number known and phase arbitrary
- Optical-lattice strength beyond a value blocks tunneling and destroys superfluidity and interference.

Diffraction Grating

- The condition for *maxima* is
  - $d \sin \theta_{bright} = m \lambda$
    - $m = 0, 1, 2, \ldots$
- The integer m is the *order number* of the diffraction pattern



Quantum Phase Transition



Degenerate Fermi gas

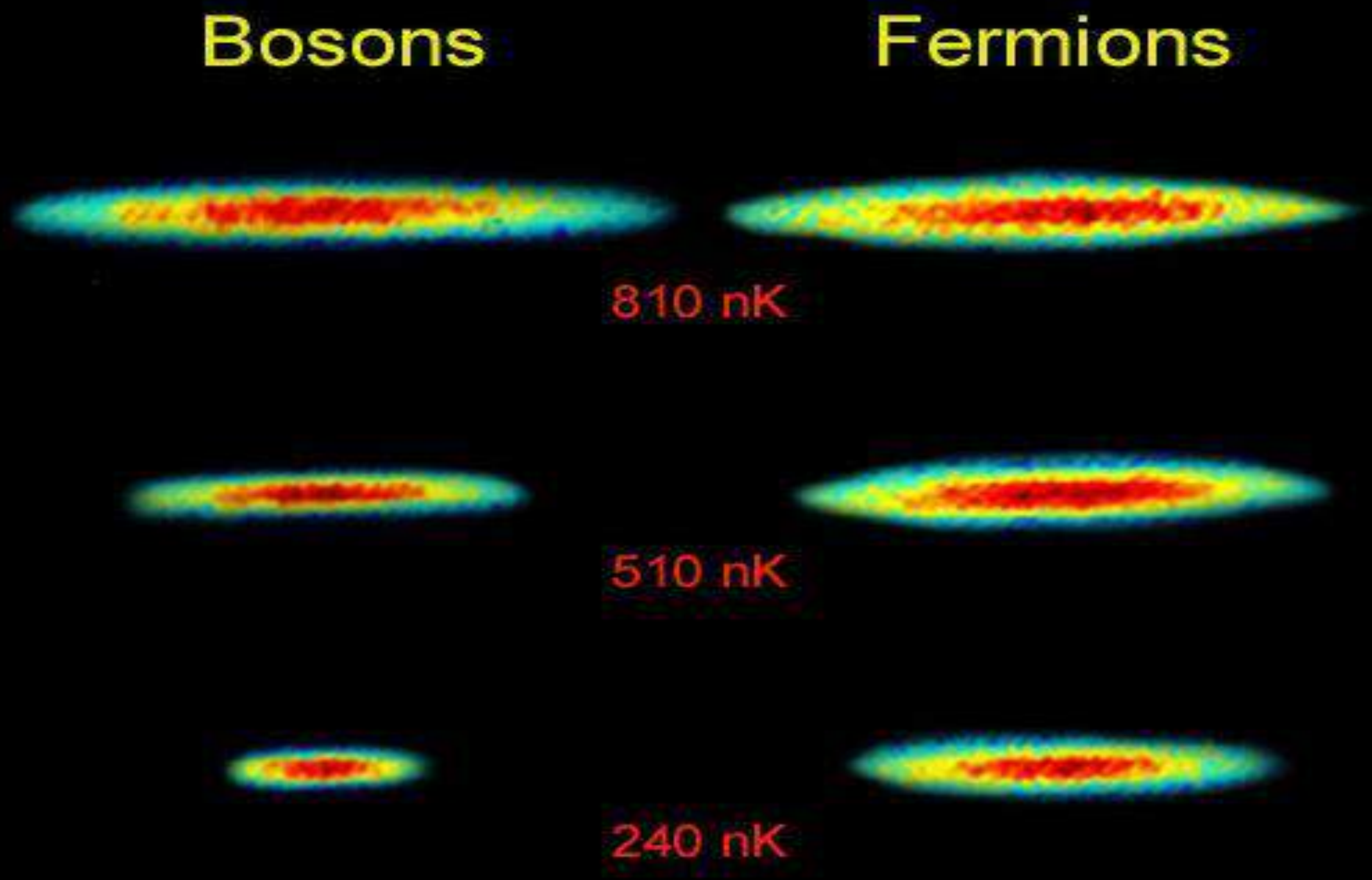
- Fermionic atoms are strongly repulsive at short distances due to Pauli repulsion and are impossible to condense by evaporative cooling

- Condensation possible in Bose-Fermi and Fermi-Fermi mixtures by sympathetic cooling

Bose-Fermi and Fermi-Fermi degenerate gas

- Bose-Fermi mixtures
- ^{40}K-^{87}Rb 2002 Firenze Inguscio
- ^{6}Li-^{23}Na 2002 MIT Ketterle
- ^{6}LI-^{7}Li 2001 Rice Hulet &
  2001 France Salomon

- Fermi-Fermi mixtures
- ^{40}K-^{40}K* 1999 JILA Jin
- ^{6}Li-^{6}Li* 2002 Duke Thomas



^{7}Li ^{6}Li

Bose-Einstein Condensate (BEC)

Uncertainty relation:

$$\Delta x \Delta p \approx h,$$

$$\Delta p = m \Delta \mathrm{v}$$

$$\Delta x \Delta \mathrm{v} \approx \frac{h}{m}$$

Mass (BEC) = $10^8 \sim 10^{10}$ X Mass (electron)

Makes the experimetal realization much easier

Theory

- BEC involves many-body dynamics
- Quantum field theory
- Many-body Schrödinger equation: Monte Carlo and other methods
- Simple mean-field model very useful but may lose some physics

Many-boson problem-> mean-field model:

Trap

$$H = \sum_i H_i + \frac{1}{2}\sum_{i\neq j} V_{ij} \equiv \sum_i \left(\frac{p_i^2}{2m} + U_i \right) + \frac{1}{2}\sum_{i\neq j} V_{ij}, \quad H_i = T_i + U_i$$

Hartree approximation

$$\psi = \prod_i \psi_i(\mathrm{r}_i)$$

Assumptions & loss of some quantum effects

$$<\psi | H | \psi> = \int \left(\prod_i \psi_i^* \right) \left(\sum_i H_i + \frac{1}{2}\sum_{i\neq j} V_{ij} \right) \left(\prod_i \psi_i \right) \left(\prod_i d\mathrm{r}_i \right)$$

$$= \sum_i <\psi_i | H_i | \psi_i> + \frac{1}{2}\sum_{i\neq j} <\psi_i \psi_j | V_{ij} | \psi_i \psi_j>$$

Extremize $<\psi | H | \psi>$ subject to $<\psi_i | \psi_i> = 1.$

Extremize a Lagrangian

Introduce the generalized Lagrangian functional

$$L=\sum_i <\psi_i|H_i|\psi_i>+\frac{1}{2}\sum_{i,j,i\neq j}<\psi_i\psi_j|V_{ij}|\psi_i\psi_j>$$

$$-\sum_i \mu_i\left(<\psi_i|\psi_i>-1\right)$$

Extremize L wrt $\psi_i{}^*$ and μ_i to get

$$\int d\mathrm{r}_i\left[H_i+\int\sum_{j,j\neq i}d\mathrm{r}_j\psi_j{}^*V_{ij}\psi_j-\mu_i\right]\psi_i=0,\qquad \int d\mathrm{r}_i\,|\psi_i|^2=1$$

$$\left[H_i+\sum_{j,j\neq i}<\psi_j|V_{ij}|\psi_j>\right]|\psi_i>=\mu_i|\psi_i>,\qquad <\psi_i|\psi_i>=1$$

We have approximated N-boson problem by N 1-boson problems determining the wave function of each boson.

The eqn looks like a time-independent Schrödinger eqn with a nonlinear potential. The time dependent form

$$\left[H_i + \sum_{j,j \neq i} < \psi_j | V_{ij} | \psi_j > \right] | \psi_i > = i\hbar \frac{\partial}{\partial t} | \psi_i >$$

A stationary state corresponds to $\psi_i(r_i, t) = \mathrm{Exp}\left(-\frac{i\mu_i t}{\hbar} \right) \psi_i(r_i),$

μ_i are chemical potential.

Energy and Chemical Potential

Chemical potential

$$\mu_i = < \psi_i \,|\, H_i \,|\, \psi_i > + \sum_{j,\, j \neq i} < \psi_i \psi_j \,|\, V_{ij} \,|\, \psi_i \psi_j >$$

$\sum_i \mu_i$ is not the total energy E.

Energy

$$E = \sum_i \left[< \psi_i \,|\, H_i \,|\, \psi_i > + \frac{1}{2} \sum_{j,\, j \neq i} < \psi_i \psi_j \,|\, V_{ij} \,|\, \psi_i \psi_j > \right]$$

Scattering length

In a BEC all the atoms occupy the same state $\psi_i = \psi$:

$$\left[H(\mathrm{r}) + (N-1)\int d\,\mathrm{r}'\psi^*(\mathrm{r}')V(\mathrm{r},\mathrm{r}')\,\psi(\mathrm{r}')\right]\psi(\mathrm{r},t) = i\hbar\frac{\partial}{\partial t}\psi(\mathrm{r},t)$$

Quantum dynamics involves multiple interaction and *t* matrix or scattering length is a measure of atomic interaction.

$$V(\mathrm{r}-\mathrm{r}') = \frac{4\pi\hbar^2 a}{m}\delta(r-r')$$

This potential in the Born approximation & gives a *t* matrix with scattering length *a*.

Gross-Pitaevskii (GP) Equation

Then the nonlinear Schrödinge r equation becomes the well - known mean - field Gross - Pitaevskii (GP) equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + U(r) + \frac{4\pi\hbar^2 a}{m}(N-1)\,|\psi(\mathrm{r})|^2\right]\psi(\mathrm{r},t) = i\hbar\frac{\partial}{\partial t}\psi(\mathrm{r},t)$$

positive a repulsion

negative a attraction

Lowest - order perturbati on theory proportioa l to a

Dimensionless equation

For large N and a harmonic trap the GP equation is

$$\left[-\frac{\hbar^2}{2m}\nabla^2+\frac{1}{2}m\omega^2(\gamma x^2+\kappa y^2+\lambda z^2)+\frac{4\pi\hbar^2 aN}{m}|\psi|^2\right]\psi(x,y,z,t)$$

$$=i\hbar\frac{\partial}{\partial t}\psi(x,y,z,t)$$

In terms of harmonic length $l=\sqrt{\hbar/m\omega}$, redefine lengths as $x'=x/l,\ t'=t\omega,\ \psi'=\psi l^{3/2}$, etc to get the dimensionl ess

$$\left[-\frac{1}{2}\nabla^2+\frac{1}{2}(\gamma x^2+\kappa y^2+\lambda z^2)+4\pi aN|\psi|^2\right]\psi(x,y,z,t)$$

$$=i\frac{\partial}{\partial t}\psi(x,y,z,t)$$

Rapidly rotating BEC

Rotating BEC, vortex - lattice formation.

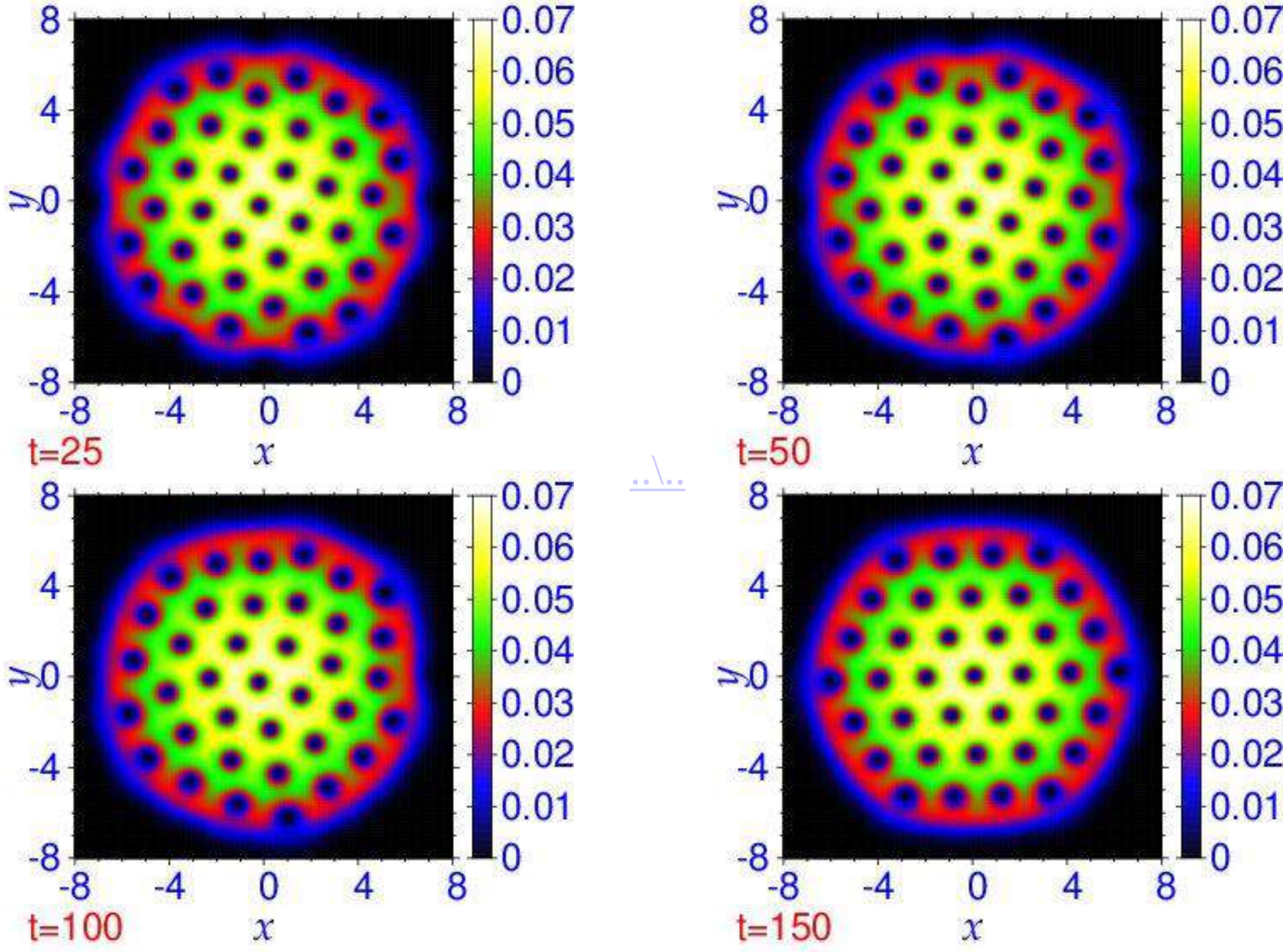
The trap rotates with angular frequency Ω around z axis.

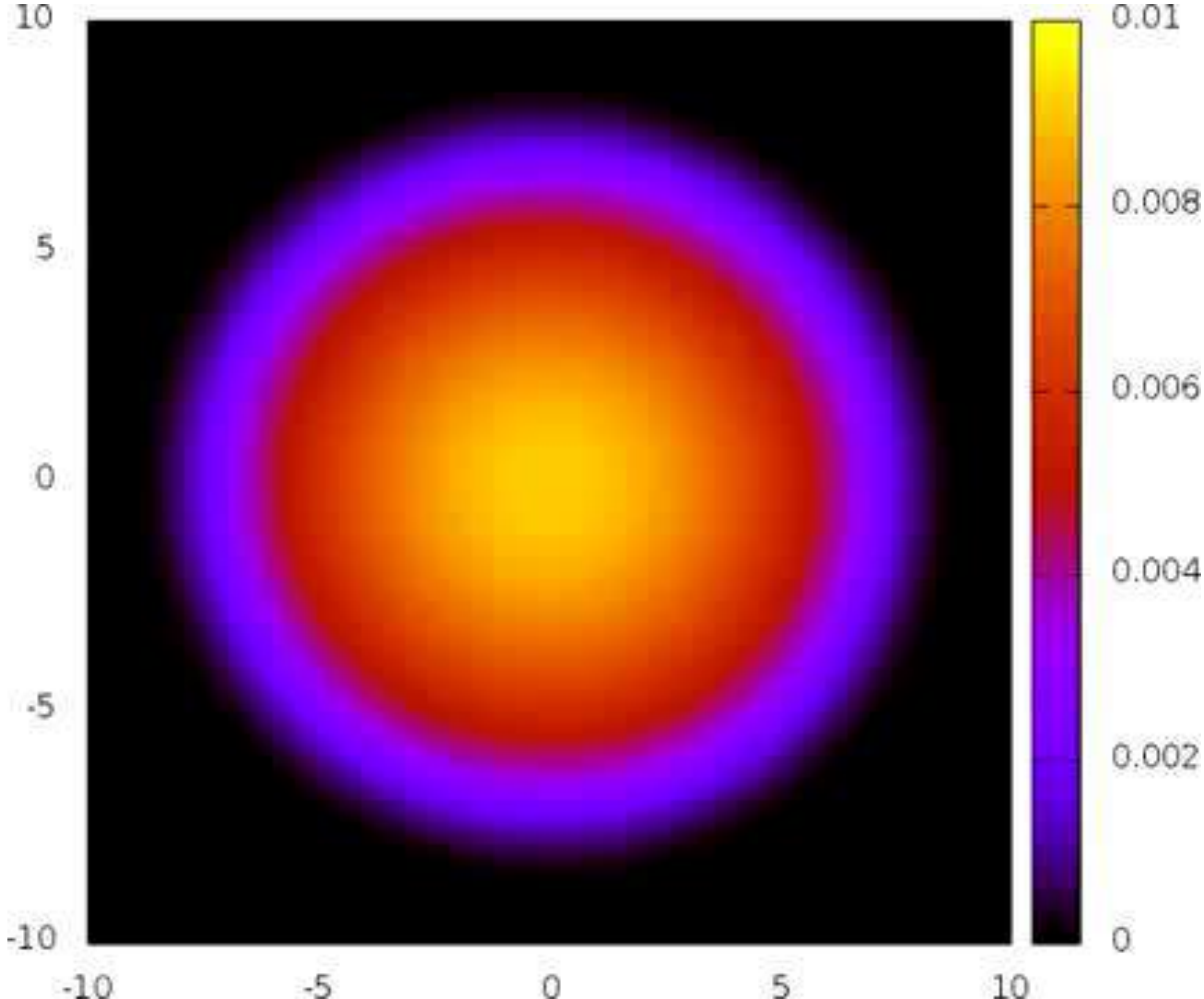
$$\left[-\frac{1}{2}\nabla_{\rho}^{2}-\frac{1}{2}\frac{\partial^{2}}{\partial z^{2}}+\frac{1}{2}\left(\gamma\rho^{2}+\lambda z^{2}\right)-\ell_{z}\Omega+4\pi aN\,|\,\psi\,|^{2}\right]\psi=i\frac{\partial\psi}{\partial t}$$

In the rotating frame the original Hamiltonia n changes to

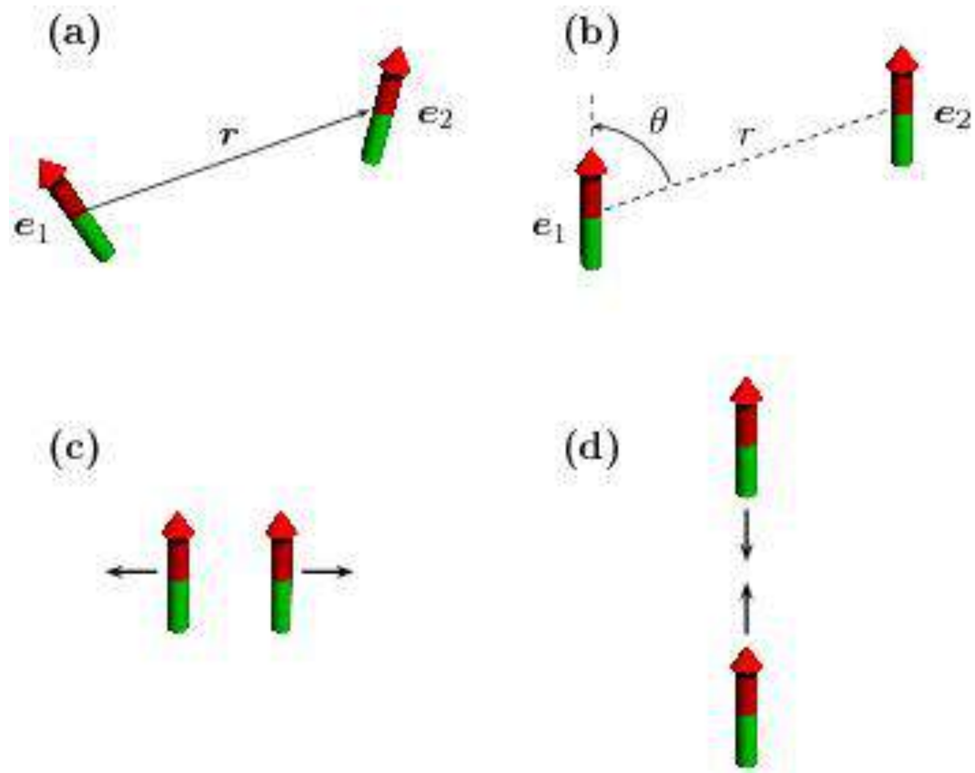
$H\,'=H-\ell_{z}\Omega,$ viz. Landau + Lifshitz, Mechanics

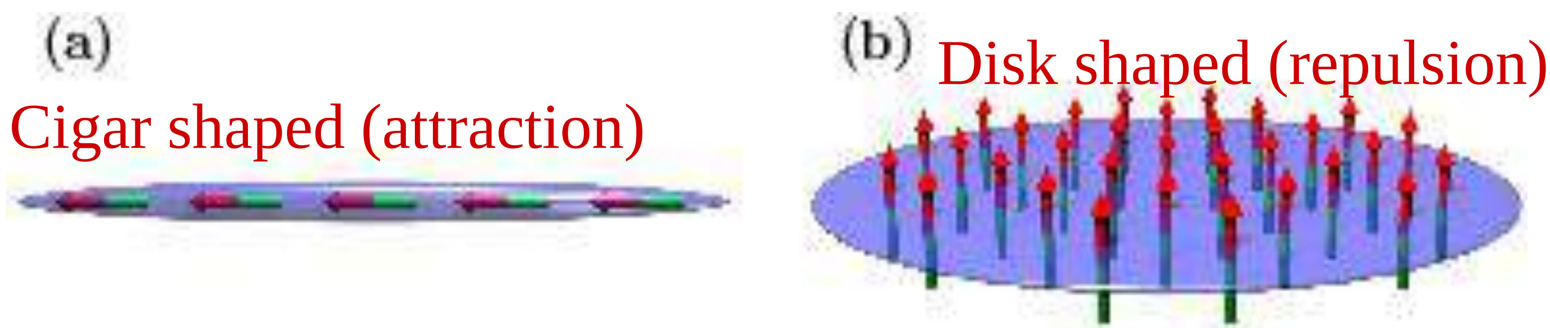
Angular momentum $\ell_{z}=-i\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial z}\right).$


t=25
t=50
t=100
t=150
x
y
-8
-4
0
4
8
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0

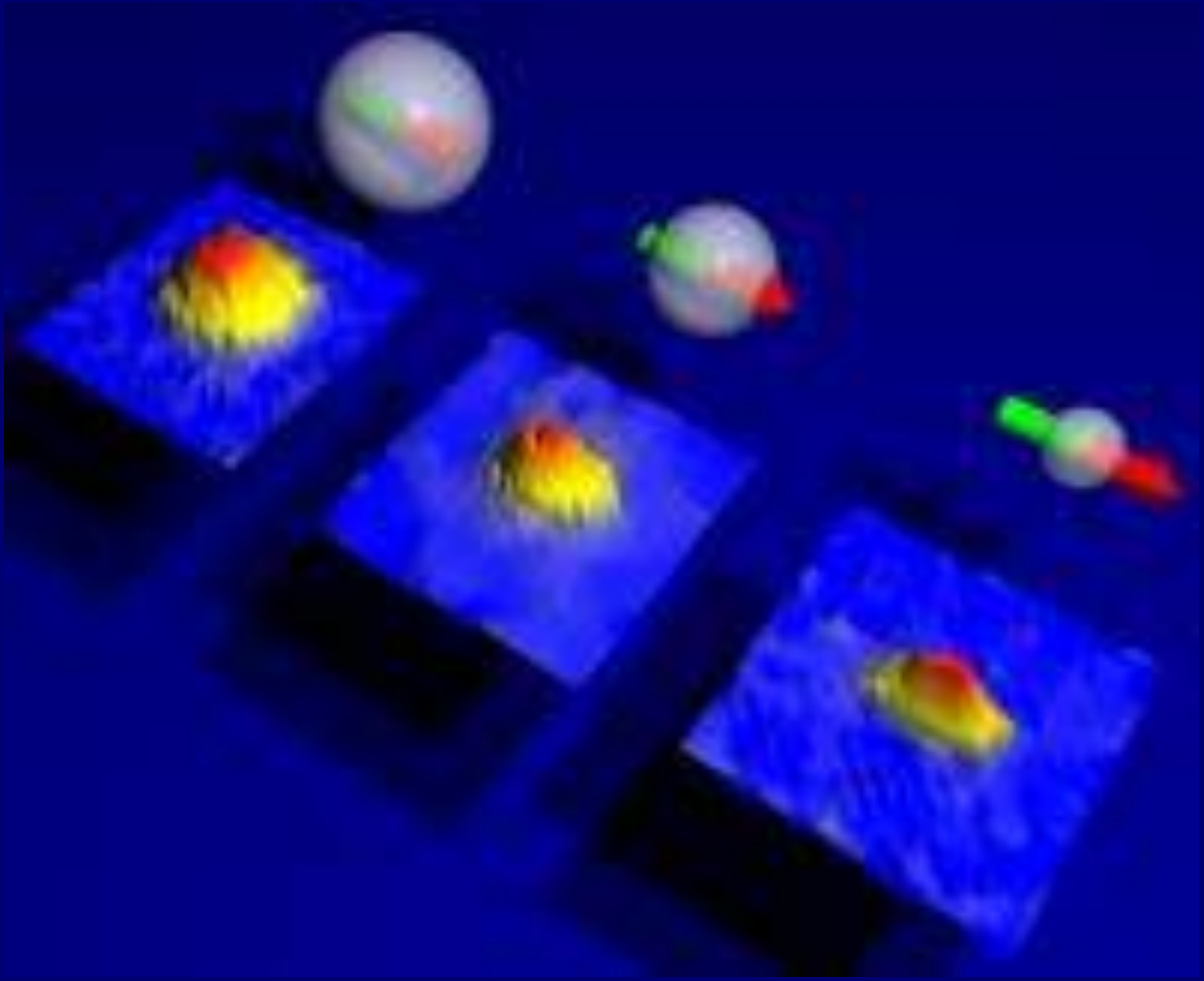

10
5
0
-5
-10
-10
-5
0
5
10
0.01
0.008
0.006
0.004
0.002
0

Atoms with large dipole moment

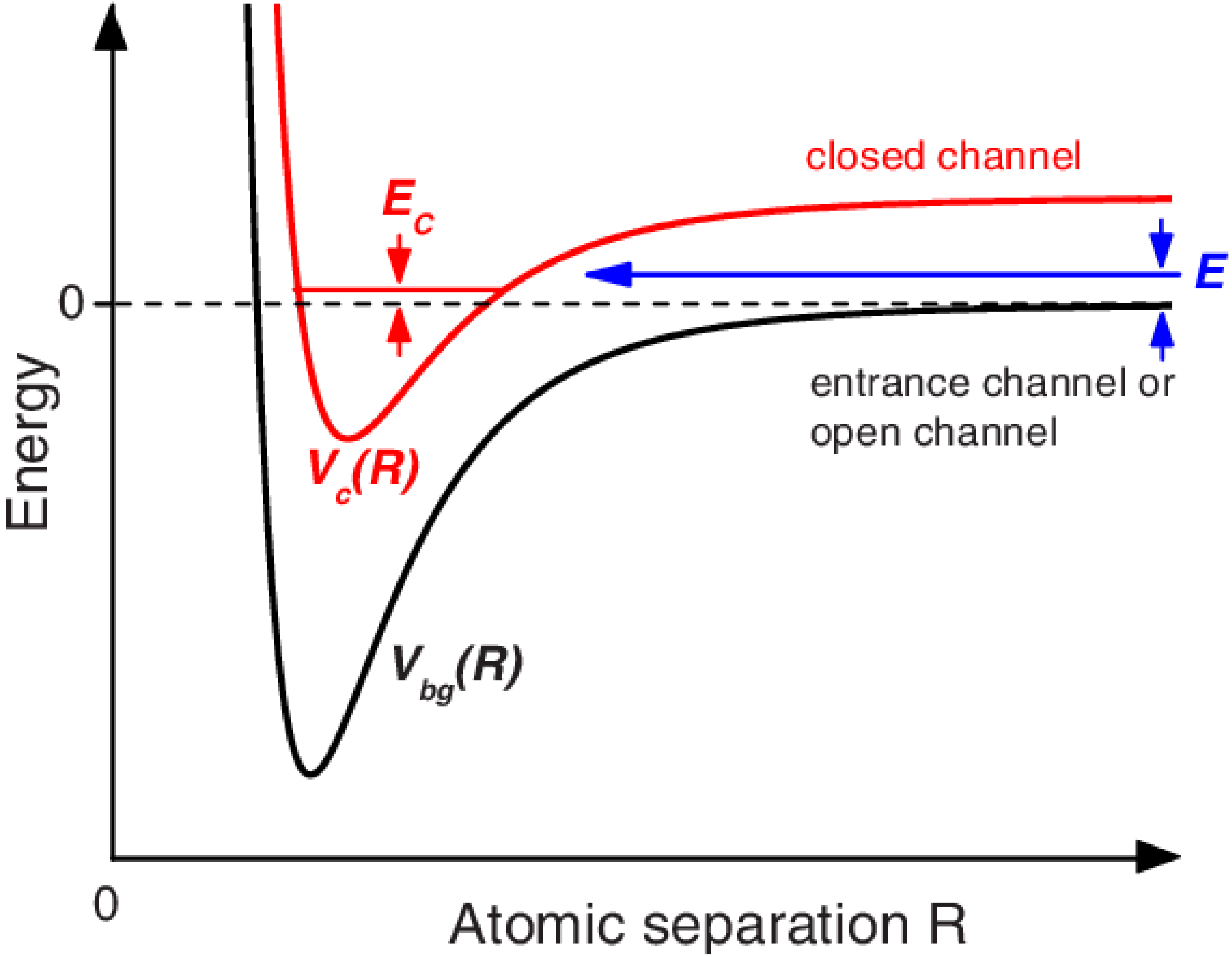




Change of shape of BEC as the atomic interaction is reduced in a dipolar BEC



Feshbach resonance



One dimensional equation: Cigar shape

Unlike the thermal gas, the BEC is a very dilute super-fluid and takes the shape of the container or confining trap. Hence the BEC can have the shape of a disk or cigar, rather than a sphere, in an appropriate trap.

Cigar Shape: $\nu = \gamma >> \lambda$,

$$\psi(\mathrm{r},t) = \psi_{1D}(z,t)\phi(\rho) = \frac{\psi_{1D}(z,t)}{\sqrt{\pi d_\rho{}^2}} \exp\left(-\frac{\rho^2}{2d_\rho{}^2}\right), \quad \lambda d_\rho{}^2 = 1, \quad \rho = \{x, y\}$$

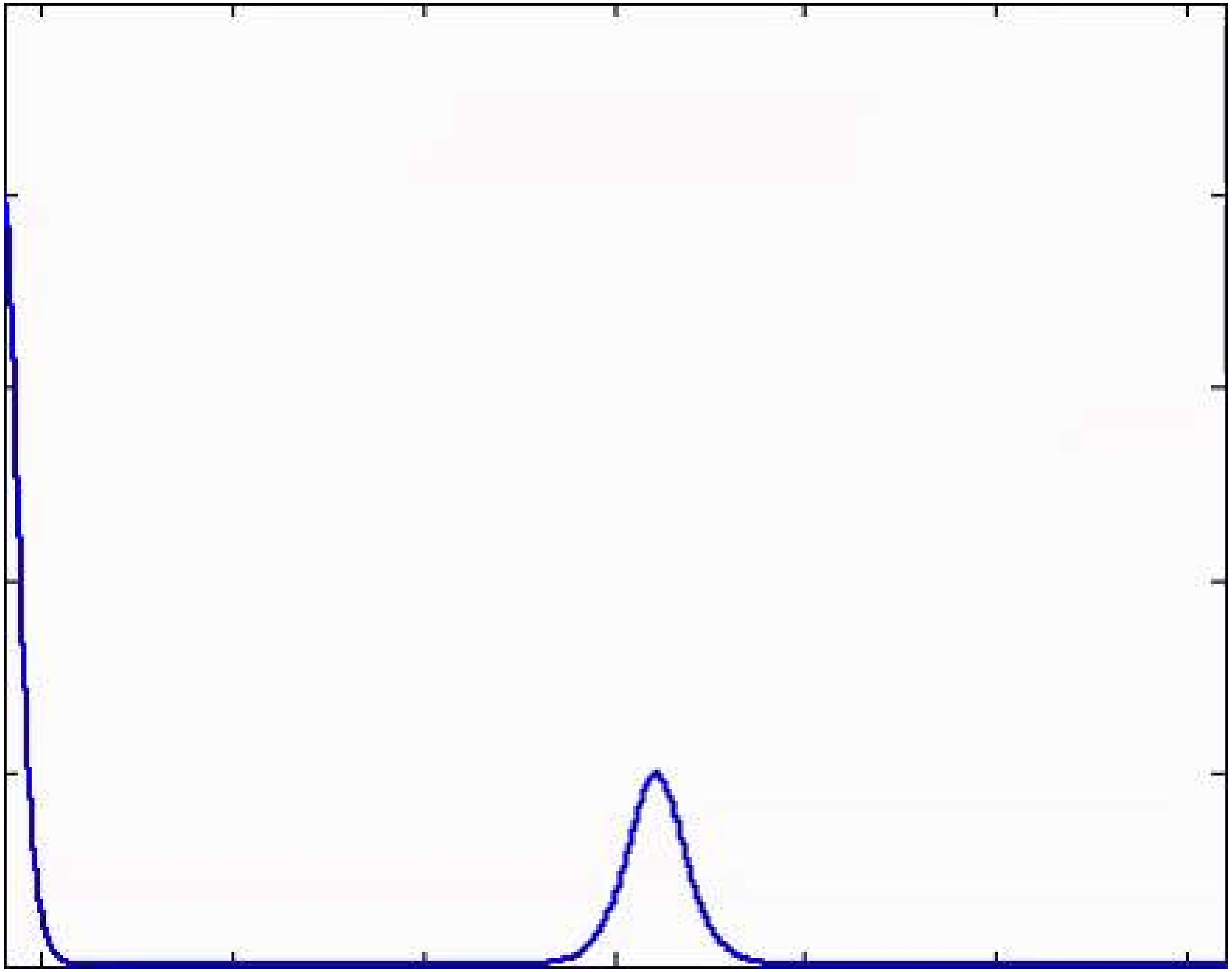
Substitute in the GP equation, multiply by the wave function $\phi(\rho)$ and integrate over ρ to obtain

$$i\frac{\partial \psi_{1D}(z,t)}{\partial t} = \left[-\frac{1}{2}\frac{\partial^2}{\partial z^2} + \frac{1}{2}\lambda^2 z^2 + \frac{2aN|\psi_{1D}|^2}{d_\rho{}^2}\right]\psi_{1D}(z,t)$$

Soliton in one dimension (1D)

- A 1D **soliton** is a solitary wave that maintains its shape while travelling.
- It is generated from a balance between repulsive kinetic energy and attractive nonlinear interaction.
- No collapse & energy-momentum conservation
- Elastic collision: Two 1D solitons can pass through each other in collision without a change in shape.

Soliton-soliton collision

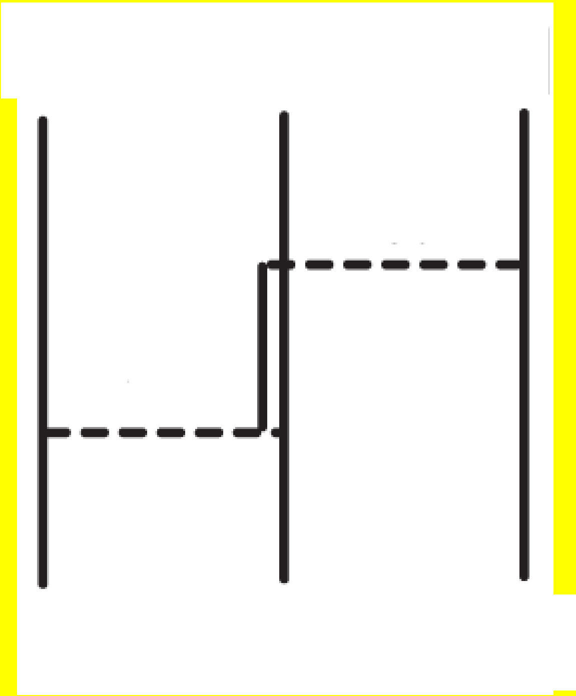


Generalized GP Equation with higher-order correction

$$i\frac{\partial\psi(\mathrm{r},t)}{\partial t}=\left[-\frac{1}{2}\nabla^2-4\pi\,|a|\,N|\psi|^2+\frac{N^2K_3}{2}|\psi|^4\right]\psi(\mathrm{r},t);$$

Two-body force

K_3 Three - body force



There are many different higher-order corrections each of which leads to a strong repulsion at center and stops collapse

Variational Approximation

Variational Gaussian Ansatz for wave function

$$\psi(\mathrm{r}) = \frac{\pi^{-3/4}}{w^{3/2}} \exp\left[-\frac{r^2}{2w^2}\right], \quad w \text{--> width of the QB}$$

$$\varepsilon(\mathrm{r}) = \frac{|\nabla \psi(\mathrm{r})|^2}{2} - 2\pi N |a| |\psi(\mathrm{r})|^4 + \frac{K_3 N^2}{6} |\psi(\mathrm{r})|^6,$$

$$E = \int \varepsilon(\mathrm{r}) d\mathrm{r} = \frac{3}{4w^2} - 2\pi N |a| \frac{\pi^{-3/2}}{2\sqrt{2} w^3} + \frac{K_3 N^2}{2} \frac{\pi^{-3}}{9\sqrt{3} w^6},$$

Energy minimum determines width w:

$$\frac{dE}{dw} = 0, \text{-->} \qquad \frac{1}{w^3} - \frac{4\pi N |a|}{(2\pi)^{3/2} w^4} + \frac{N^2 K_3}{2} \frac{4}{9\sqrt{3}\pi^3 w^7} = 0$$

Collapse versus a stable state
Formation of a trapless BEC

weak attraction -> leakage

strong attraction ->collapse

$$\text{Energy}\quad E=\frac{3}{4w^2}-2\pi N\,|\,a\,|\,\frac{\pi^{-3/2}}{2\sqrt{2}w^3}+\frac{K_3N^2}{2}\frac{\pi^{-3}}{9\sqrt{3}w^6},$$

If $K_3=0,$ the minimum of energy $E ==> w=0$ a collapsed state of 0 size. Any positive three-body force K_3 stops collapse.

Three-dimensional soliton (trapless BEC) Experiments on QB formation

- H. Kadau et al., Nature 530, 194 (2016)
  → **Observing the Rosensweig instability of a quantum ferrofluid** .

- C. R. Cabrera et al., Science 359, 301 (2018)
  → **Quantum liquid droplets in a mixture of Bose-Einstein condensates**.

- G. Semeghini et al., Phys. Rev. Lett. **120**, 235301 (2018)
  →Self-Bound Quantum Droplets of Atomic Mixtures in Free Space

Experiments on trapless BEC
Self-bound BEC

- BEC of dipolar atoms: long-range dipolar attraction & higher order-repulsion: ^{52}Cr atoms

- BEC of a mixture of atoms: intraspecies repulsion with higher-order repulsion and interspecies attraction: ^{39}K atoms

Experimental procedure

- Always BEC is made in a trap.
- When the parameters are appropriate for the formation of a 3D soliton, the characteristic size of the BEC usually will be very small and very different from trap size (oscillator length).

Suggestion for future experiments on trapless BEC

- An attractive BEC with three-body force

- A mixture of repulsive bosons and fermions with attractive boson-fermion interaction and three-boson force

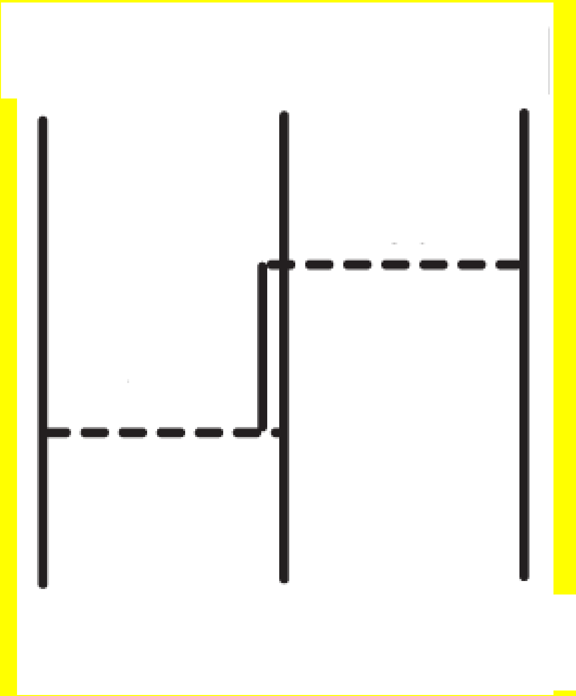
Trapless BEC of attractive ^{7}Li atoms with repulsive three-body and attractive two-body interactions (negative scattering length): A theoretical study

Generalized GP Equation with three-body force

$$i\frac{\partial\psi(\mathrm{r},t)}{\partial t}=\left[-\frac{1}{2}\nabla^2-4\pi\,|a|\,N|\psi|^2+\frac{N^2K_3}{2}|\psi|^4\right]\psi(\mathrm{r},t);$$

Two-body force

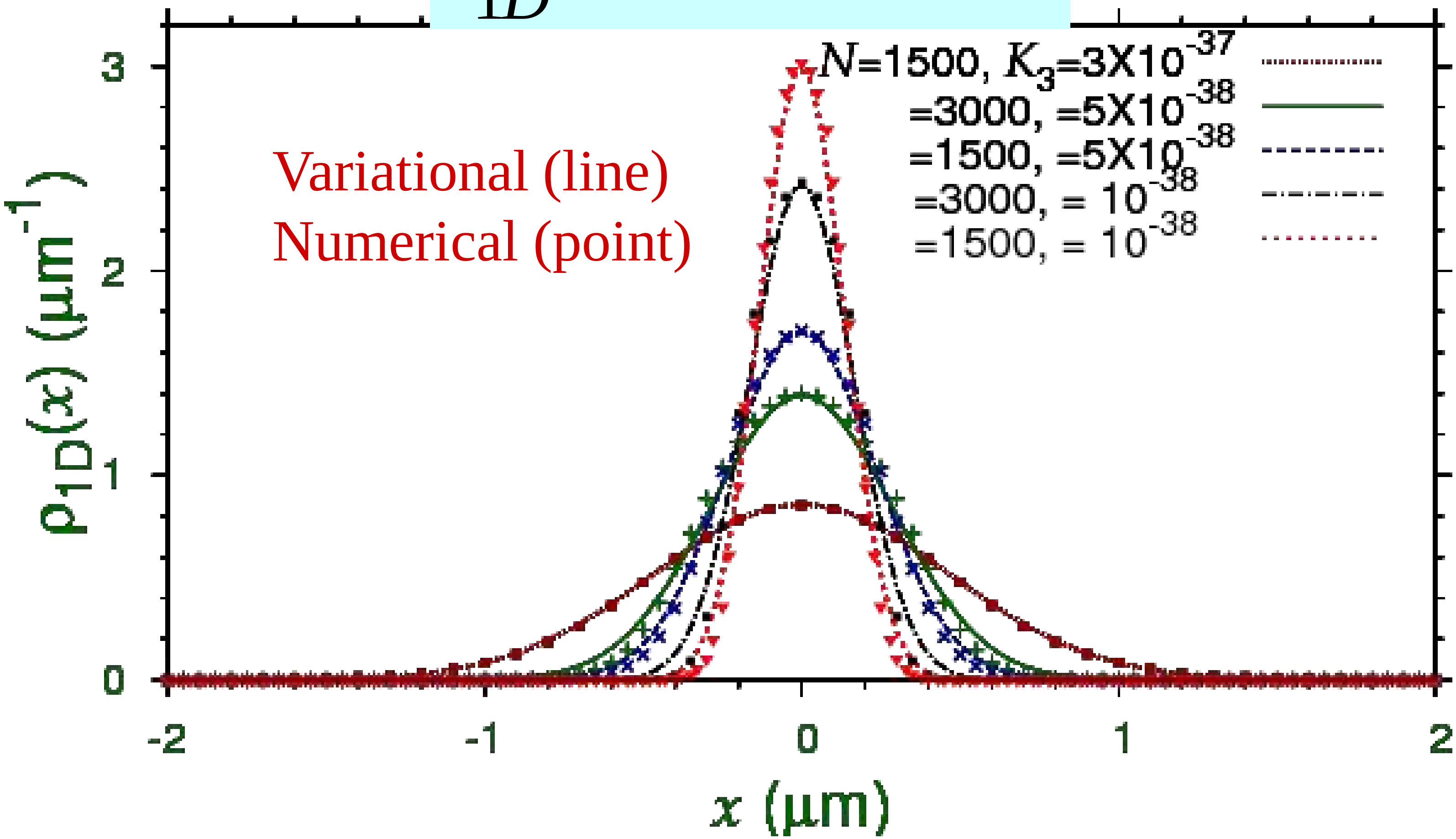
K_3 Three - body force



a = -27.4a_0

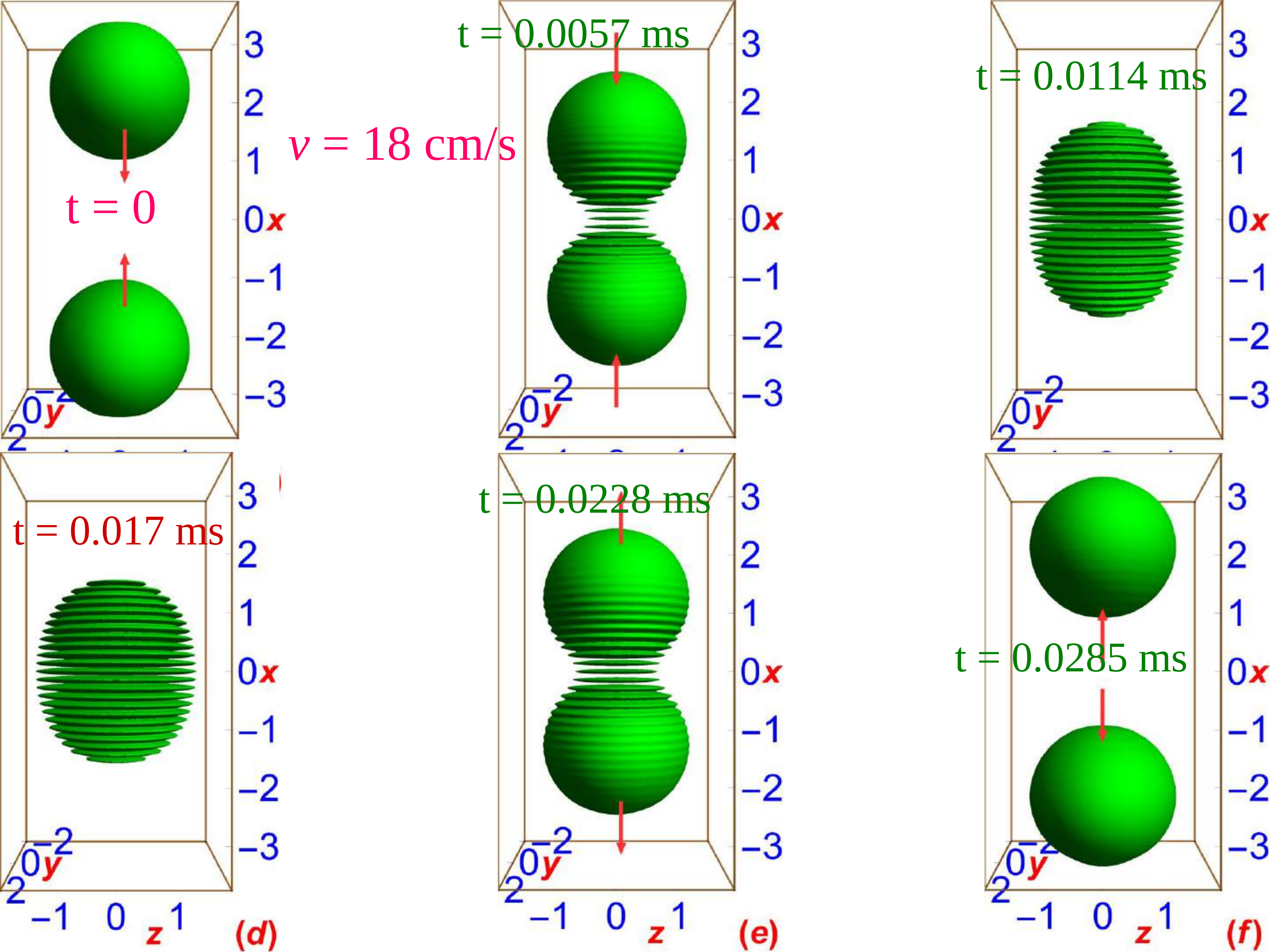
Variational (line) and numerical (point) one-dimensional (1D) density

$$\rho_{1D}(x) = \int |\psi(\mathrm{r})|^2 \, dydz$$

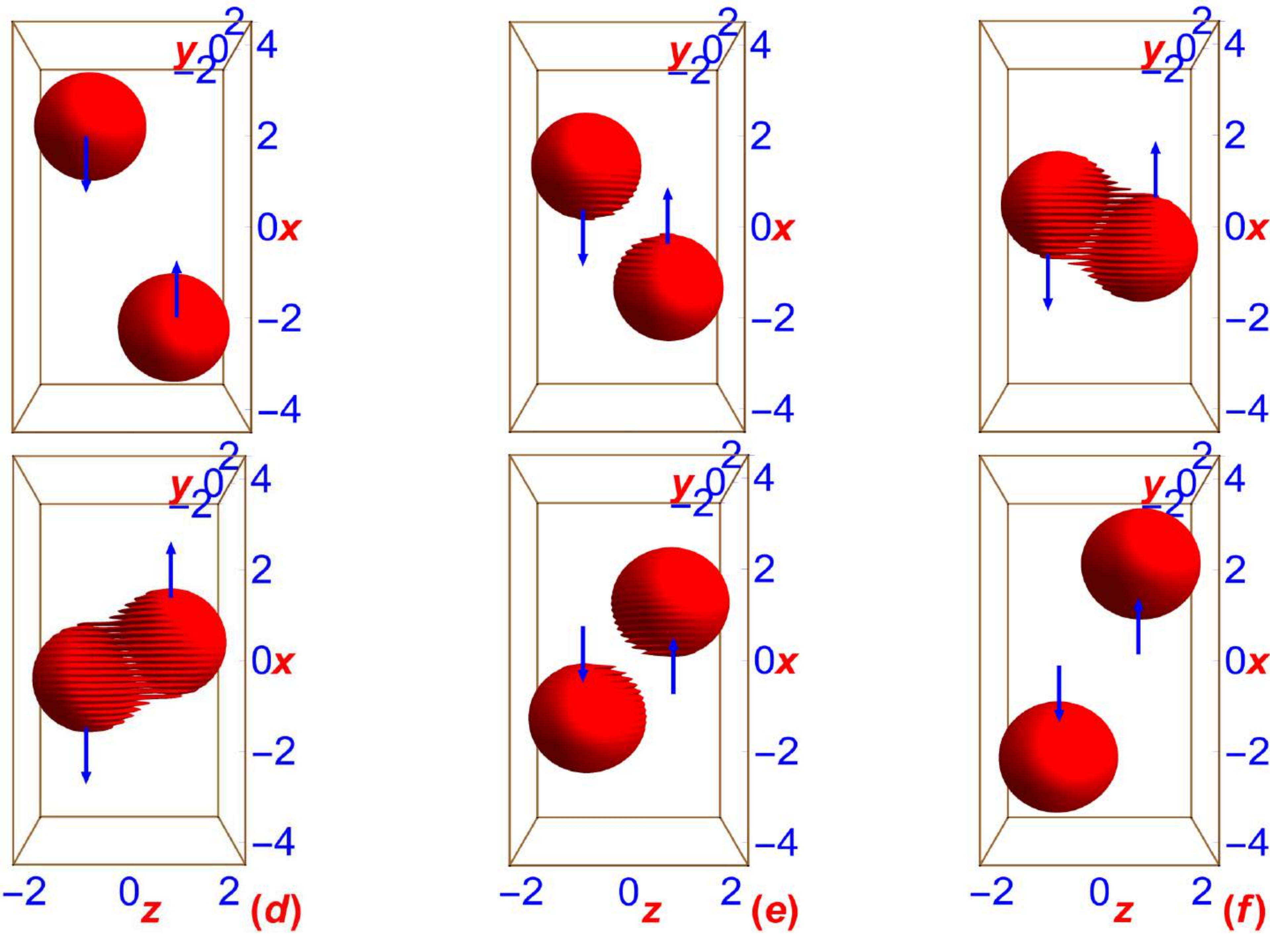


Collision of two ^{7}Li balls, with N = 1500,
$K_3 = 3X10^{-37}(1-i)\ m^6/s$

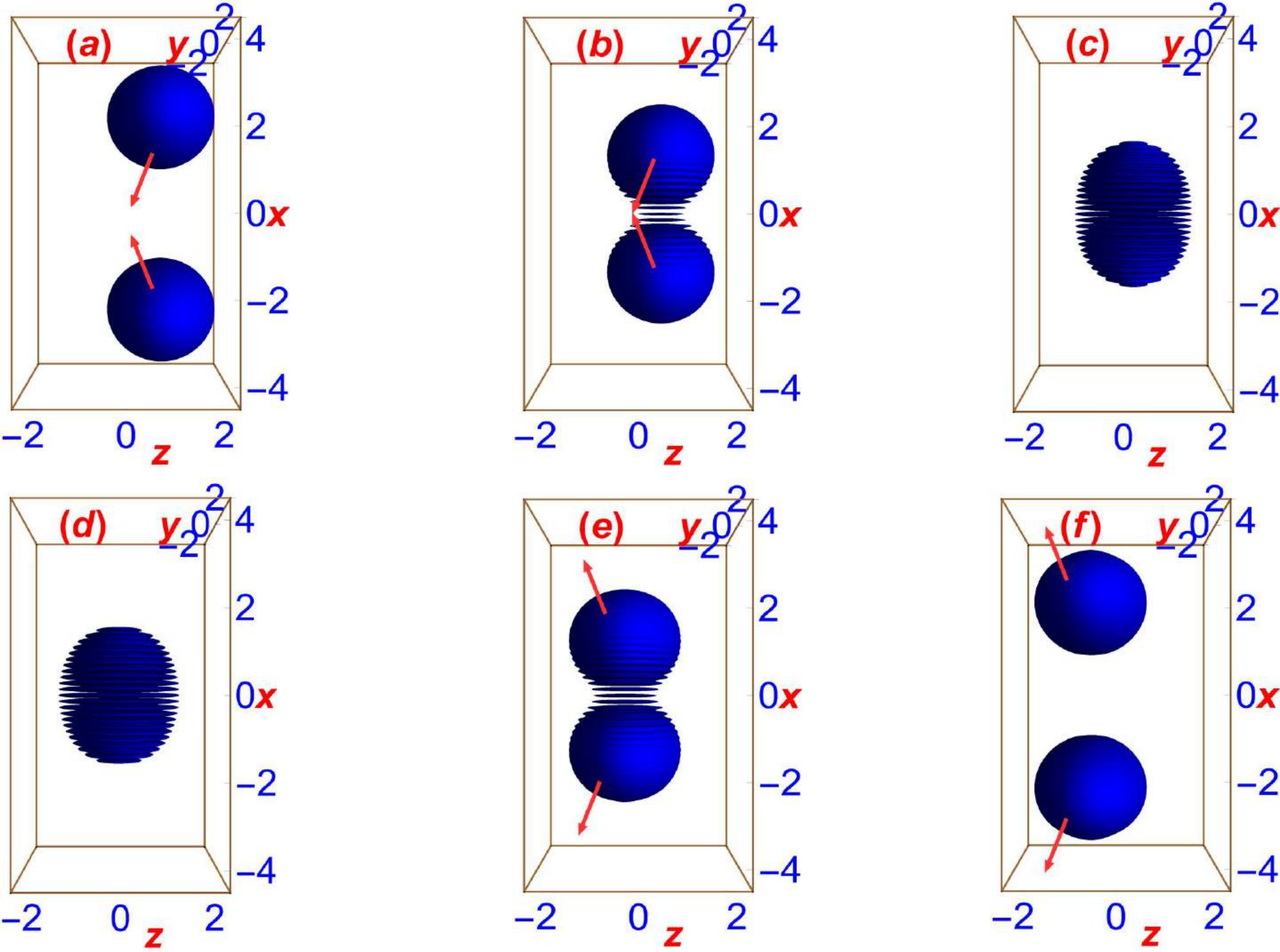
Moving in opposite directions along x axis with velocity 18 cm/s, at times (a) t = 0, (b) = 0.0057 ms, (c) = 0.0114 ms, (d) = 0.017 ms, (e) = 0.0228 ms, (f) = 0.0285 ms. The density on the contour is 10^{10} atoms/cm^3 and unit of length is 1 μm.



t = 0
v = 18 cm/s
t = 0.0057 ms
t = 0.0114 ms
t = 0.017 ms
t = 0.0228 ms
t = 0.0285 ms
x
y
z
(d)
(e)
(f)

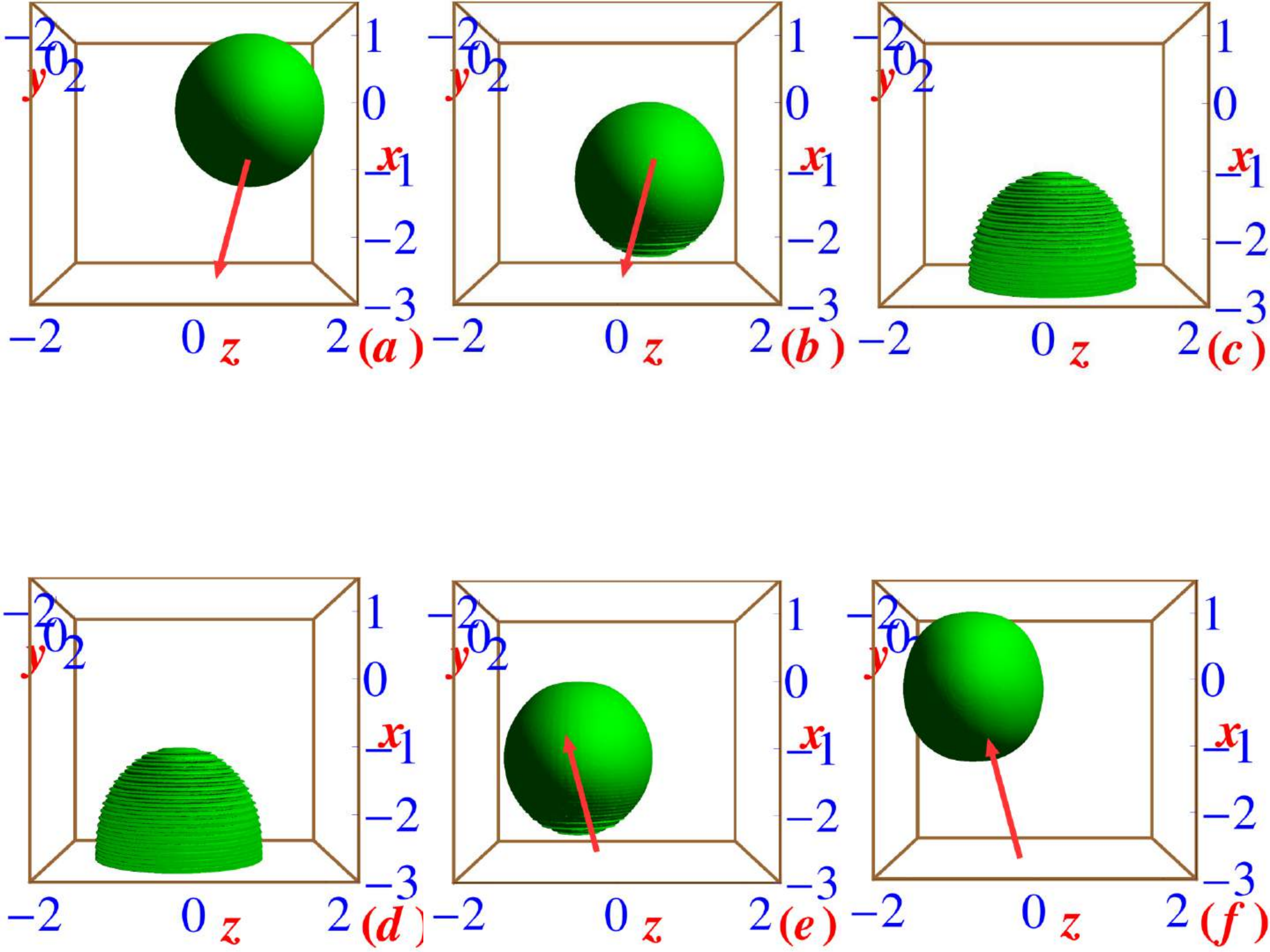


y
2
0
−2
4
2
0 x
−2
−4
−2
0 z
2
(d)
(e)
(f)



(a)
(b)
(c)
(d)
(e)
(f)
y
x
z

Bouncing of a ^{7}Li ball, with $N = 1500$, $K_3 = 3 \times 10^{-37}(1-i)\ m^6/s$, against a hard rigid noninteracting wall



−2
0
2
y
1
0
x
−1
−2
−3
−2
0
z
2
(a)
(b)
(c)
(d)
(e)
(f)

Trapless boson-fermion ^{7}Li- ^{6}Li mixture with repulsive three-boson and two-boson interactions and attractive boson-fermion interaction: A theoretical study

Boson-fermion mixture

Trapped boson - fermion mixture :

$$[-\frac{\hbar^2}{2m_1}\nabla^2 + U + \frac{4\pi\hbar^2 a_1 N_1}{m_1}|\psi_1|^2$$

$$+\frac{2\pi\hbar^2 a_{12} N_2}{m_R}|\psi_2|^2]\ \psi_1(\mathrm{r},t) = i\hbar\frac{\partial}{\partial t}\psi_1(\mathrm{r},t)$$

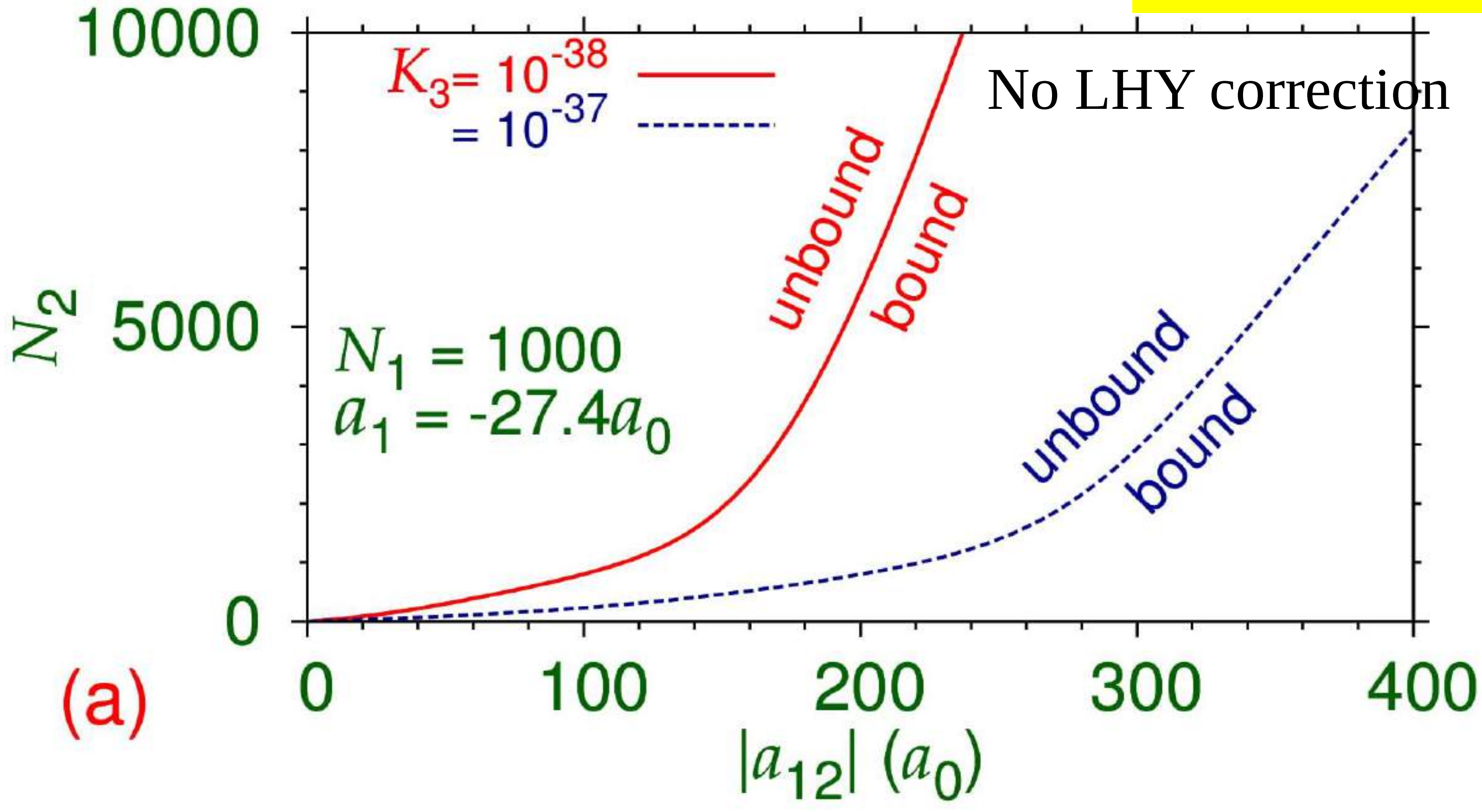
$$[-\frac{\hbar^2}{8m_2}\nabla^2 + U + \frac{\hbar^2(3\pi^2 N_2|\psi_2|^2)^{2/3}}{2m_2}$$

$$+\frac{2\pi\hbar^2 a_{12} N_1}{m_R}|\psi_1|^2]\ \psi_2(\mathrm{r},t) = i\hbar\frac{\partial}{\partial t}\psi_2(\mathrm{r},t)$$

SKA+Salasnich, Phys. Rev. A 78 (2008) 043616

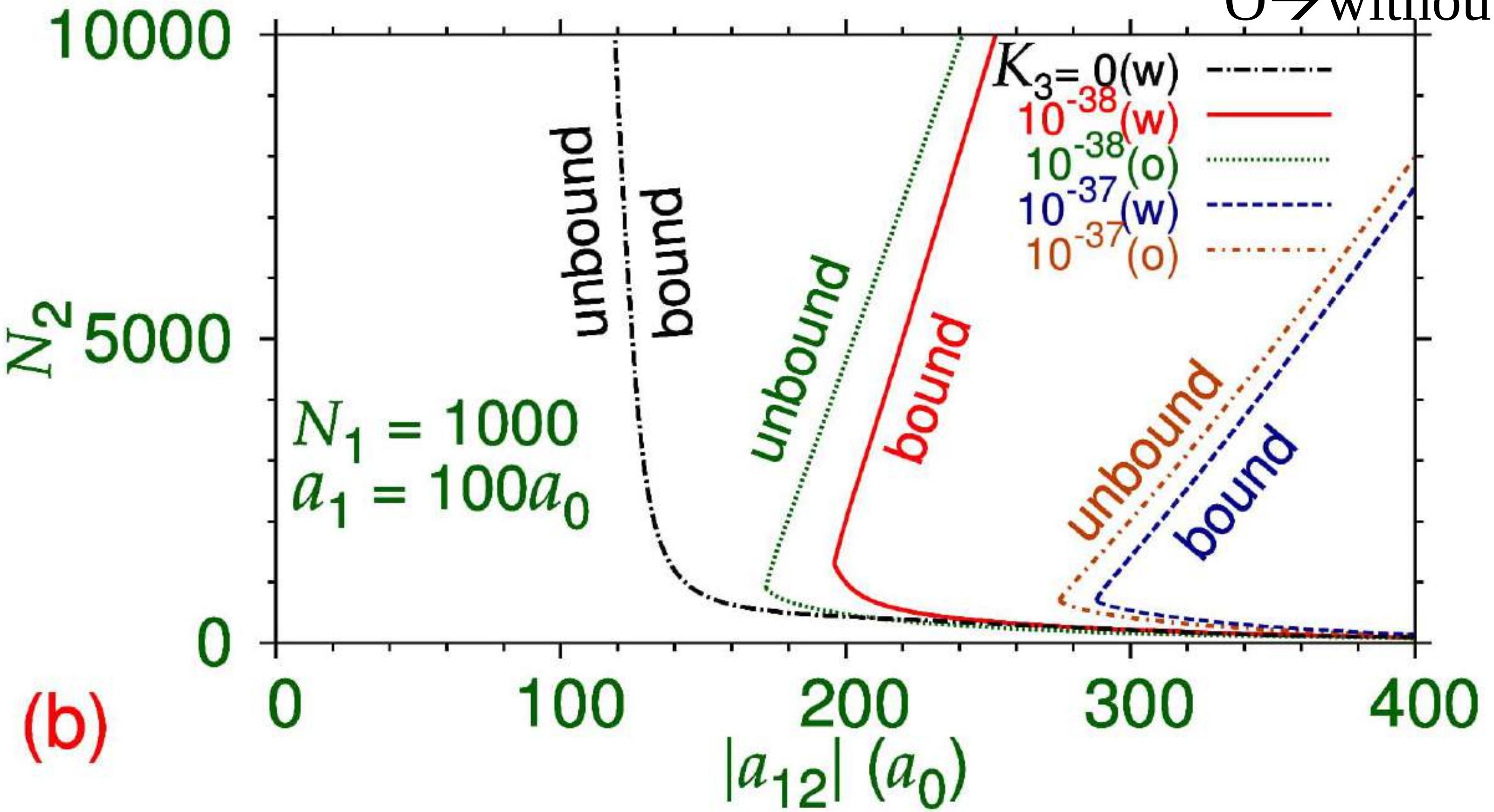
Boson-fermion quantum ball for attractive boson-boson interaction

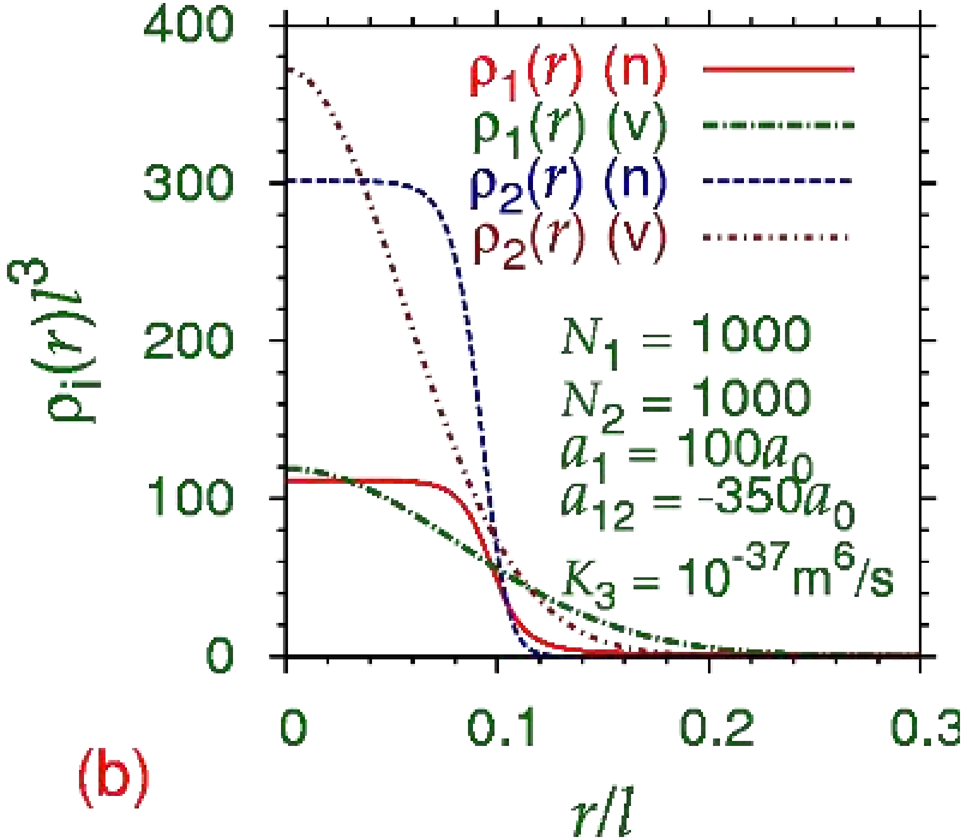
^{7}Li-^{6}Li mixure



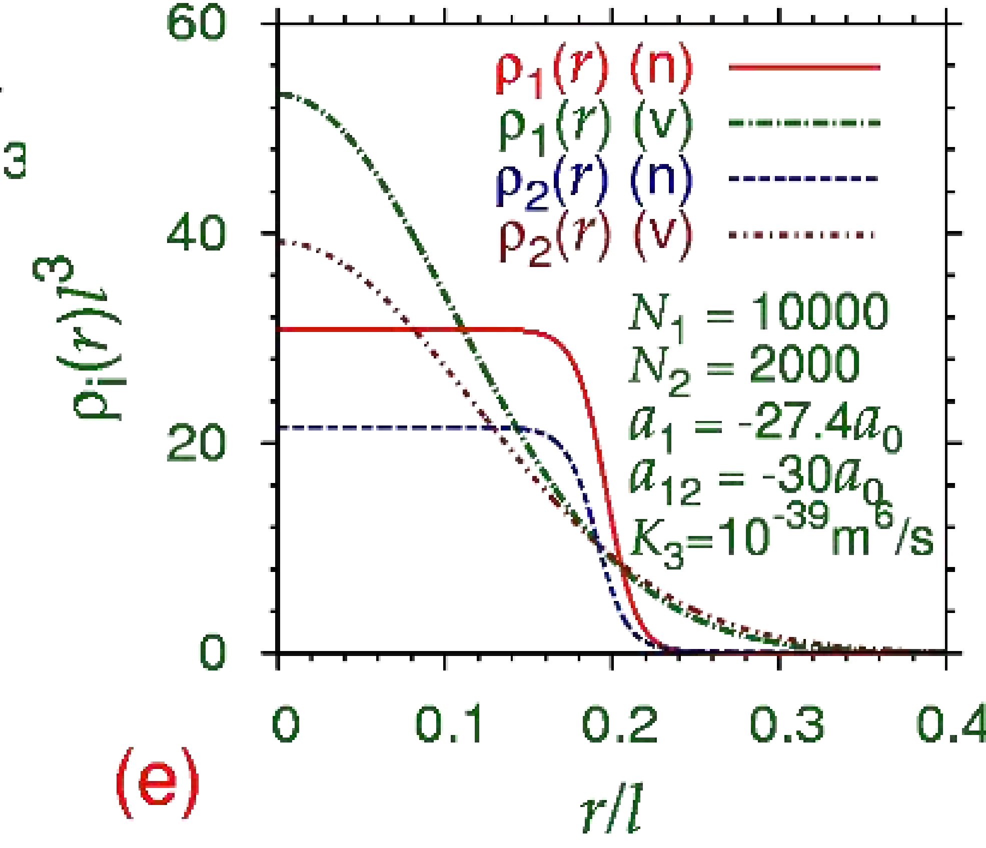
Boson-fermion quantum ball for repulsive boson-boson interaction

W→ with LHY
O→without




$\rho_1(r)$ (n)
$\rho_1(r)$ (v)
$\rho_2(r)$ (n)
$\rho_2(r)$ (v)
$N_1 = 1000$
$N_2 = 1000$
$a_1 = 100a_0$
$a_{12} = -350a_0$
$K_3 = 10^{-37}\mathrm{m}^6/\mathrm{s}$
$\rho_i(r)l^3$
r/l
0
100
200
300
400
0.1
0.2
0.3

(b)


$\rho_1(r)$ (n)
$\rho_1(r)$ (v)
$\rho_2(r)$ (n)
$\rho_2(r)$ (v)
$N_1 = 10000$
$N_2 = 2000$
$a_1 = -27.4a_0$
$a_{12} = -30a_0$
$K_3=10^{-39}\mathrm{m}^6/\mathrm{s}$
$\rho_i(r)l^3$
r/l
0
20
40
60
0.1
0.2
0.3
0.4

(e)

Concluding remarks

- A self-bound BEC (dipolar, boson-boson, boson, boson-fermion) can be stabilized for a small repulsive three-body interaction and higher order correction.
- Experiments welcome for single-component BEC and for boson-fermion mixture

- Thank you for your attention