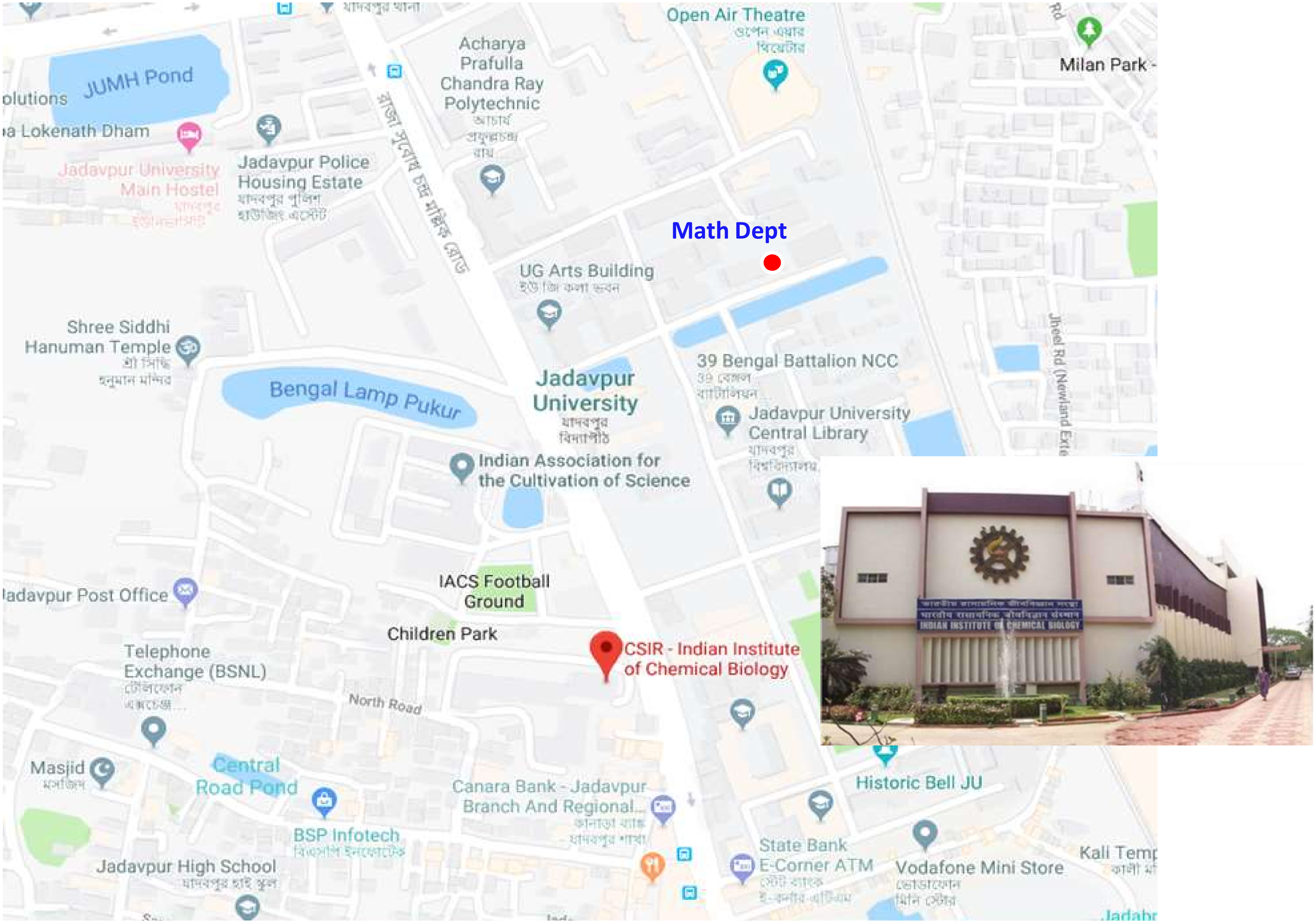
ICTP-SAIFR Colloquium
Sao Paulo, November 15, 2018

যাদবপুর বিশ্ববিদ্যালয়
JADAVPUR UNIVERSITY

Sudden Transition to Large Amplitude Oscillation
Routes to Extreme Events

Syamal Kumar Dana
Department of Mathematics, Jadavpur University
Kolkata 700032, India


Math Dept
Open Air Theatre
Milan Park
JUMH Pond
Lokenath Dham
Jadavpur University Main Hostel
Jadavpur Police Housing Estate
Acharya Prafulla Chandra Ray Polytechnic
UG Arts Building
Shree Siddhi Hanuman Temple
Bengal Lamp Pukur
Jadavpur University
39 Bengal Battalion NCC
Jadavpur University Central Library
Indian Association for the Cultivation of Science
IACS Football Ground
Children Park
Jadavpur Post Office
Telephone Exchange (BSNL)
North Road
CSIR - Indian Institute of Chemical Biology
Masjid
Central Road Pond
Canara Bank - Jadavpur Branch And Regional...
BSP Infotech
Historic Bell JU
Jadavpur High School
State Bank E-Corner ATM
Vodafone Mini Store
Jheel Rd (Newland Exte
INDIAN INSTITUTE OF CHEMICAL BIOLOGY

Jadavpur University, Kolkata

Hokkalorab (make some noise)

Sudden Transition to Large Amplitude Oscillation

Routes to Extreme Event

PHYSICAL REVIEW E **96**, 052204 (2017)

Extreme events in the forced Lién

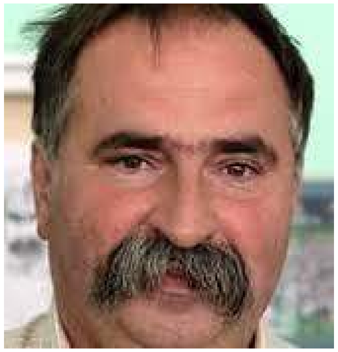
PHYSICAL REVIEW E **97**, 062311 (

Dragon-king-like extreme events in coupled b

Collaborators

Faculties

Ulrike Feudel
University of Oldenburg, Germany



Tomasz Kapitaniak
Politechnika Lódzka, Poland

Pinaki Pal
National Institute of Technology-Durgapur, India

K. Thamilmaran,
Bharathidasan University, Trichy, India

Collaborators

Graduate Students

Arindam Mishra
Jadavpur University,
Kolkata, India

Suman Saha
Jadavpur University,
Kolkata

Leo Kingston
Bharathidasan University
Trichy, India

M. Vigneshwaran
Central Instrumentation,
CSIR-Indian Institute of
Chemical Biology, Kolkata

Motivation

Nonlinear map

$$X_{n+1} = f(X_n, r)$$

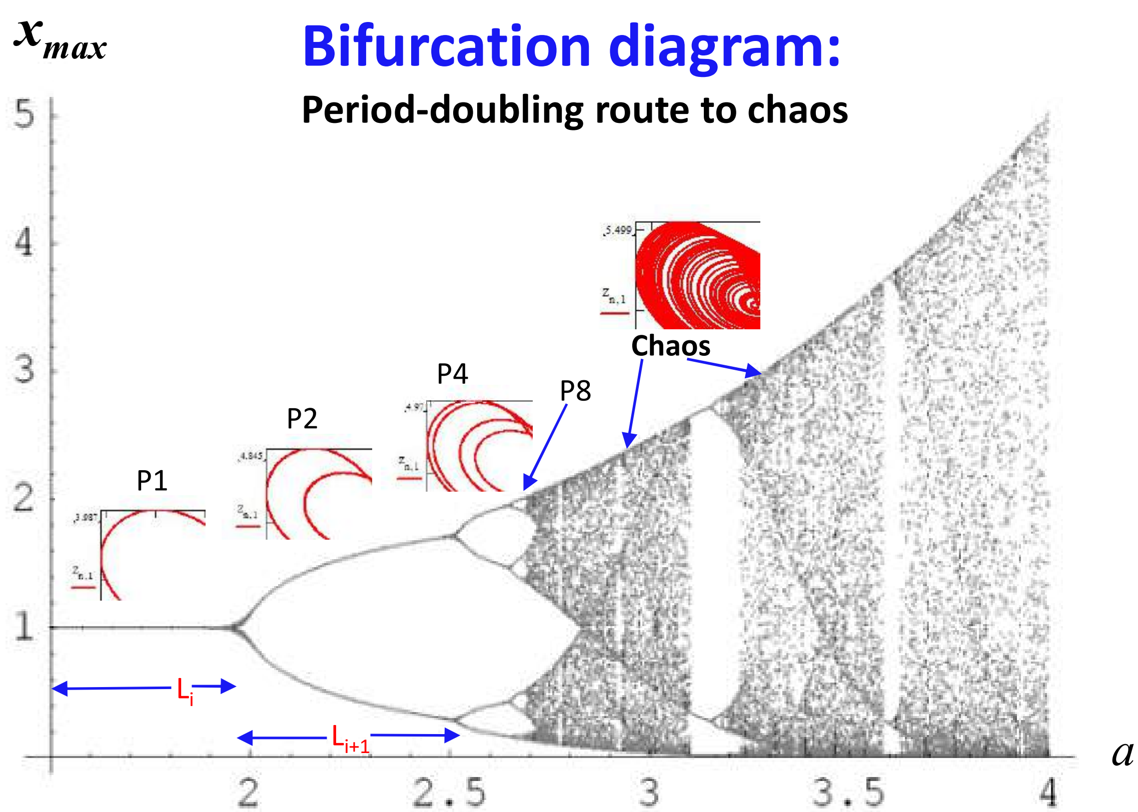
$e.g., f(X)=2X(1-X)$

Nonlinear Ordinary Differential Equations: Rössler oscillator

$$\dot{x} = -y - z$$
$$\dot{y} = x + a\,y$$
$$\dot{z} = b + (x - c)z$$

Bifurcation diagram:

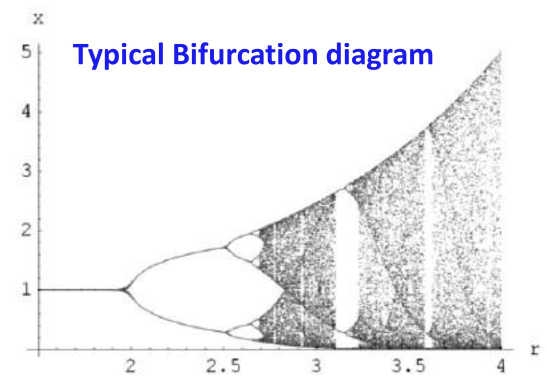
Period-doubling route to chaos



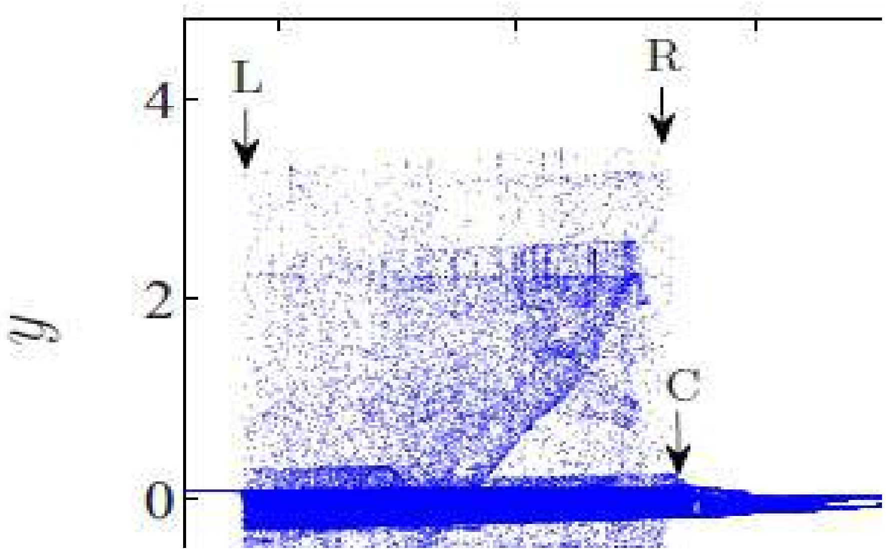
Feigenbuam number=L_i/L_{i+1}=4.669201…..

Motivation:

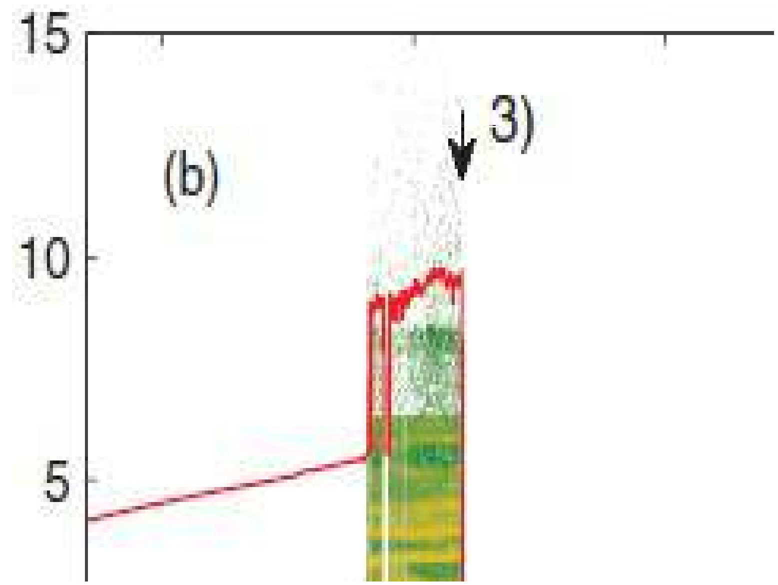
Sudden large change in amplitude of oscillation in dynamical systems



Forced Liénard system



Laser Model



Kingston, Thamilmaran, Pal, Feudel, S.K.Dana
PRE 96, 052204 (2017)

Reinoso, Zamora-Munt, Masoller,
PRE 87, 062913 (2013)

Questions

(1) Dynamical processes of Sudden Transition to large amplitude oscillation in response to parameter change
(2) Classification of the processes (Dynamics and Statistics)
(3) If they indicate Extreme Events

Extreme Events

- Floods **(Kerala flood, India, 2018)**
- Rogue Waves
- Tsunami
- Algal Bloom
- Drought
- Earthquakes
- Epilepsy
- Power black-out
- Share market crashes

Rogue Waves

Extreme Events

Tsunami, 2004

Kerala Flood, India 2018

Algal Bloom

Drought

Extreme Events: Dynamical Systems

- **Extraordinary large events**
- **Rare**

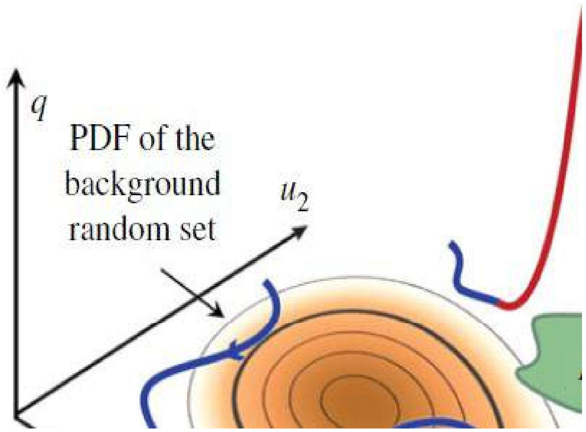
Prediction? **Timing and Intensity**

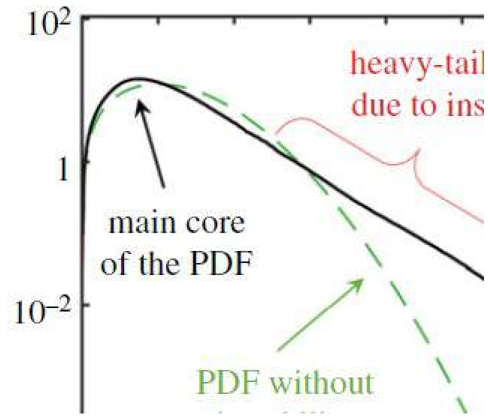
Dynamic Origin!!

- **Simple systems: Laser, electronic circuits**
- **Forced or unforced systems**
- **Coupled systems**
- **Networks of systems**

Extreme Events in Dynamical System

Probability Distribution





$$\dot{u} = f(u, t). \qquad f : U \times I \to \mathbb{R}^n$$

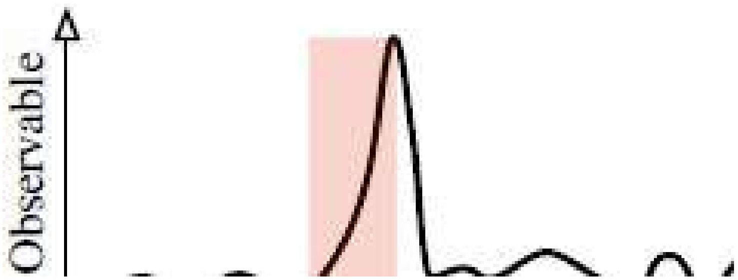
Observable $q(\mathrm{t})=T(u(\mathrm{t}))$

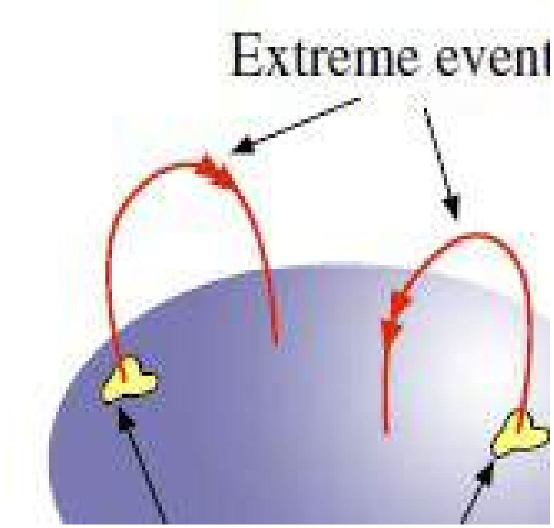
T P Sapsis, *Phil. Trans. R. Soc.* A 376: 20170133 (2018);
M.Farazmand, T. Sapsis, *Sci Adv.* 3, e1701533 (2017)

Visualization of Extreme events

Instability regions

Observable $q(t)=T(u(t))$





Examples : (1) regular

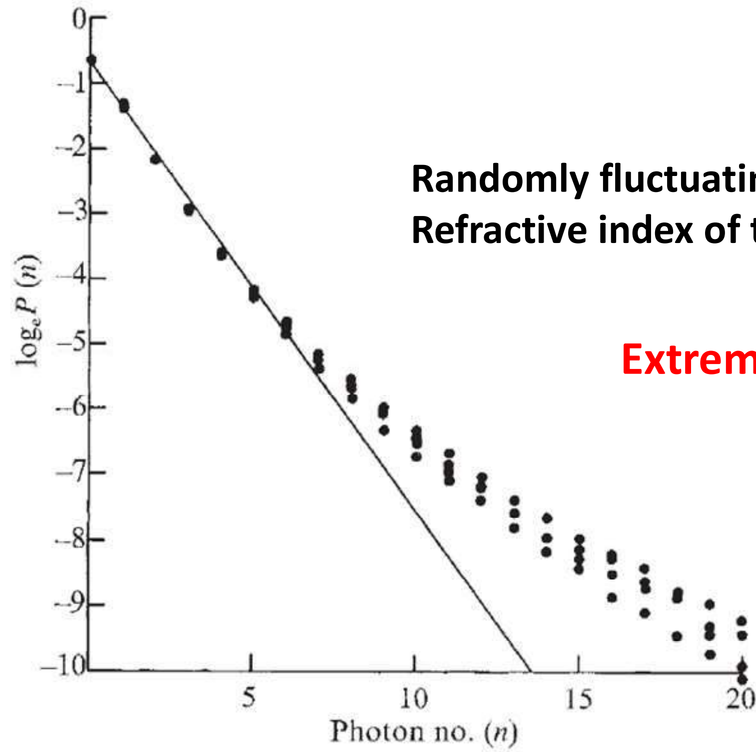
(2) complex dynamics

with occasional instabilities leading to EE

Photon correlation study of stellar scintillation

E. JAKEMAN
E. R. PIKE
P. N. PUSEY

Nature 263, September 16, 1976



Randomly fluctuating
Refractive index of the atmosphere

Extreme events??

M V Berry **Focusing and twinkling: critical exponents from catastrophes in non-Gaussian random short waves**

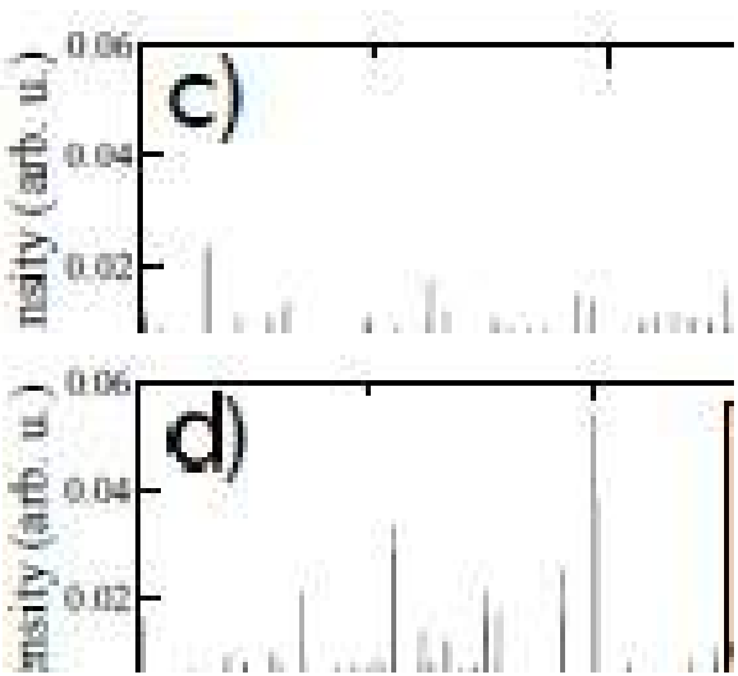
J. Phys. A: Math. Gen., Vol. 10, No. 12, 1977

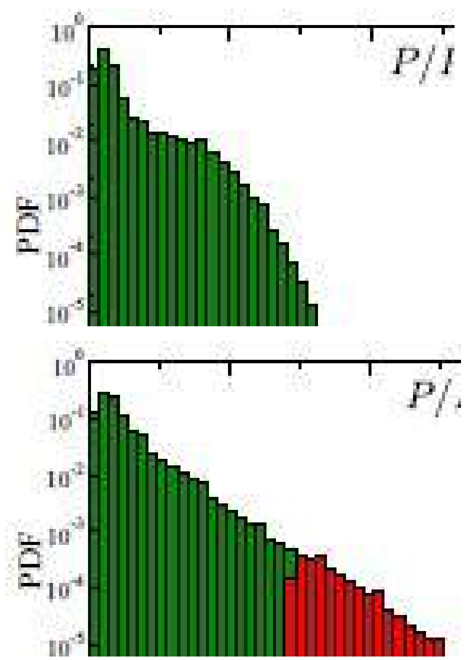
Extreme Events in Laser system

PRL 116, 013901 (2016) PHYSICAL REVIEW LETTEI

Spatiotemporal Chaos Induces Extreme Events in an F

F. Selmi,[1] S. Coulibaly,[2] Z. Loghmari,[1] I. Sagnes,[1] G. Beaudoin,[1] M. G. Clerc,[3] and S. Barbay[1,*]

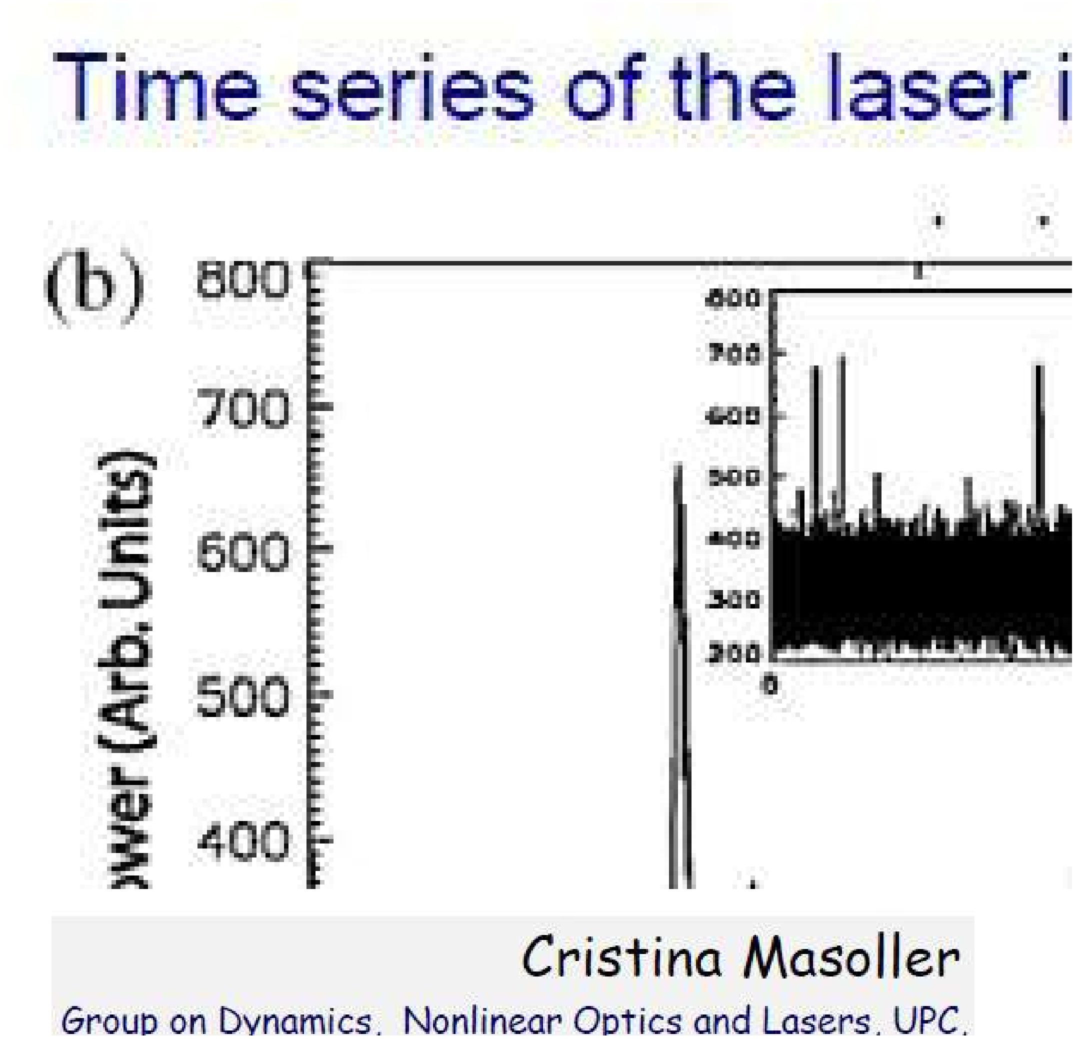




Gaussian

Non-Gaussian: long-tail

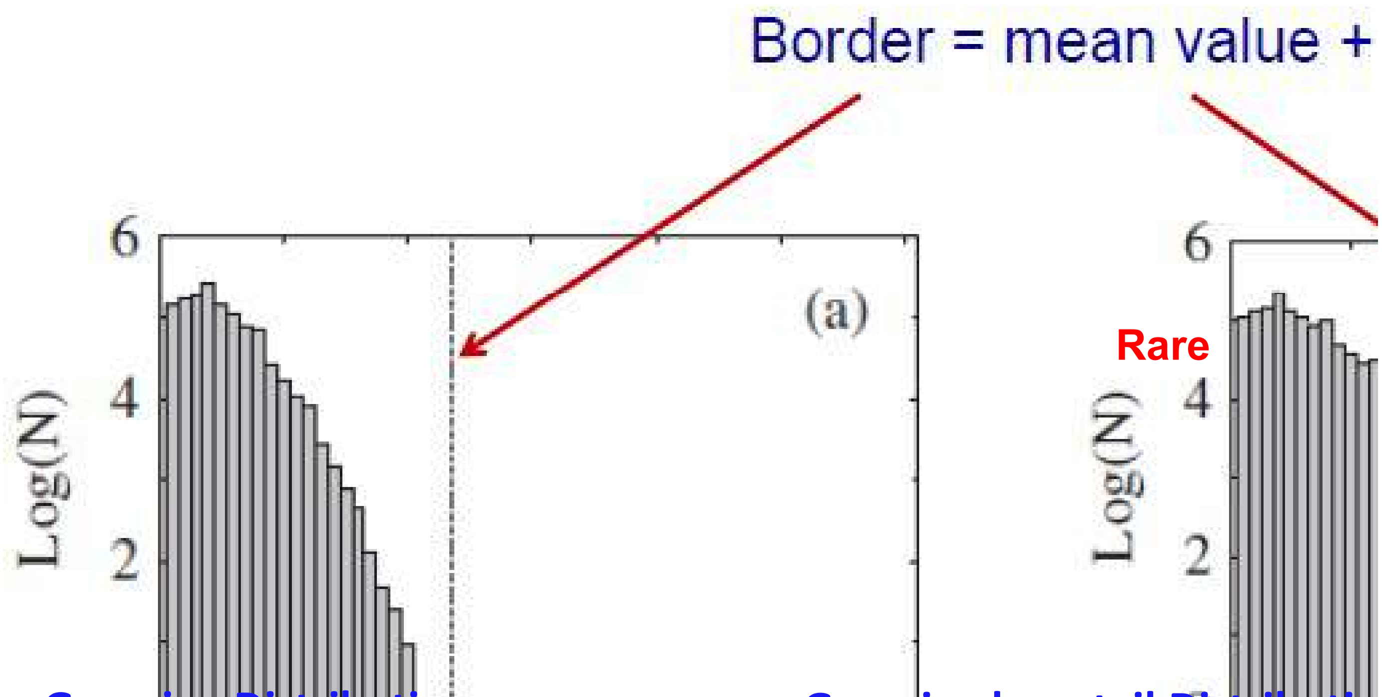
Extreme events in Laser system



Complex Dynamics

Distribution of Extreme Events

Significant Height= H_S=Mean + 8σ

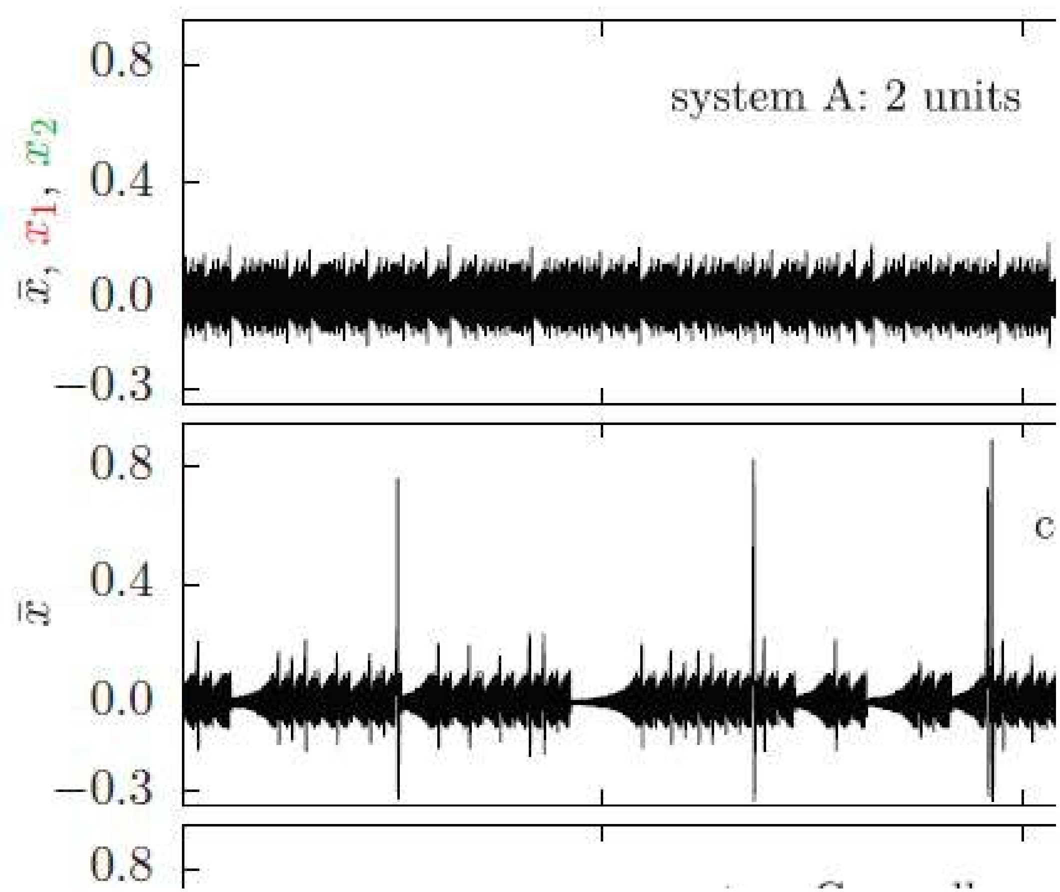


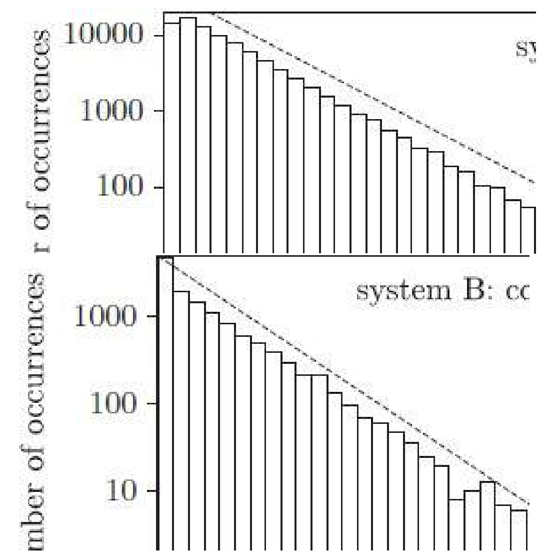
Gaussian Distribution non-Gaussian long-tail Distribution

Extreme events in excitable systems

PHYSICAL REVIEW E 88, 052911

Extreme events in excitable systems and mecha





Outline

Dynamical Models: Exreme Events

- Forced Liénard System, PRE 96, 052204 (2017)
- Coupled Hindmarsh-Rose Neuron Model, PRE 97, 062311 (2018)

Sudden transition to large amplitude oscillation

1. Period-doubling route to Chaos followed by Crisis
2. Pomeau-Manneville Intermittency followed by Crisis
3. Quasiperiodicity route to Chaos Crisis??

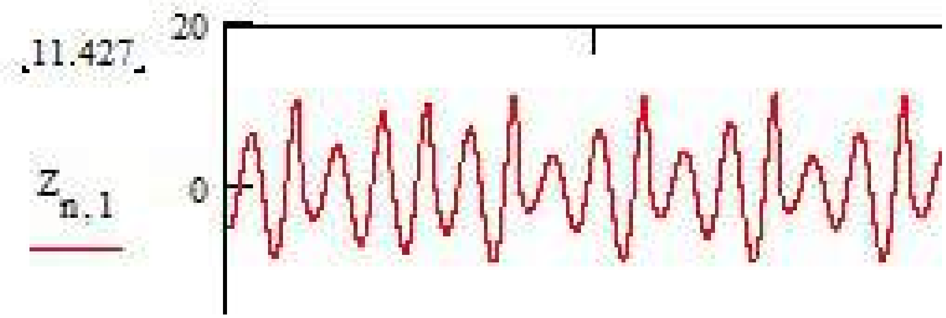
G. Nicolis, V.Balakrishnan, Nicolis, PRL 97, 210602 (2006)

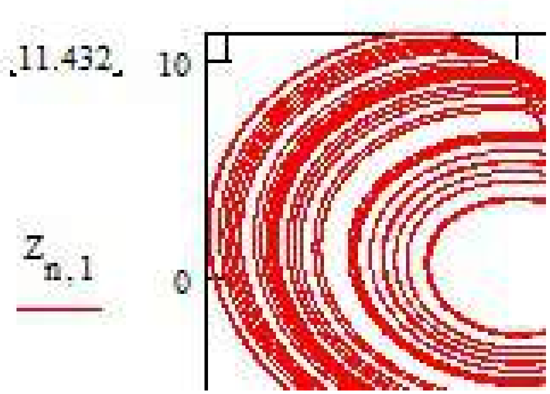
Chaotic Dynamics: Brief Introduction

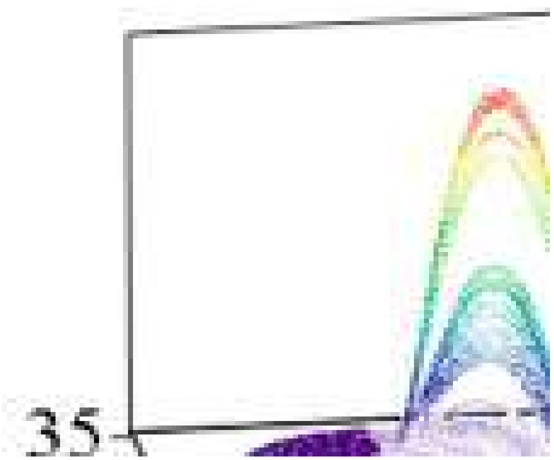
Rössler System

$$\dot{x} = -y - z$$
$$\dot{y} = x + a\,y$$
$$\dot{z} = b + (x - c)z$$

a=0.2, b=0.2, c=5.7







Period-Doubling route to Chaos

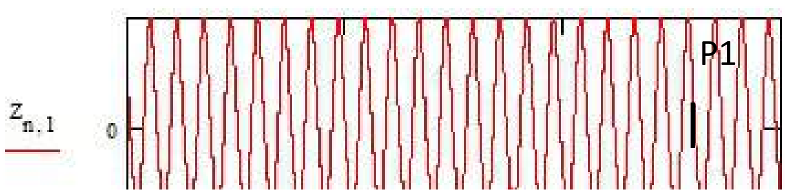
Rössler System

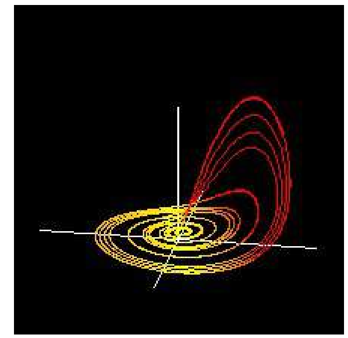
b=2, c=4

Period-doubling sequence

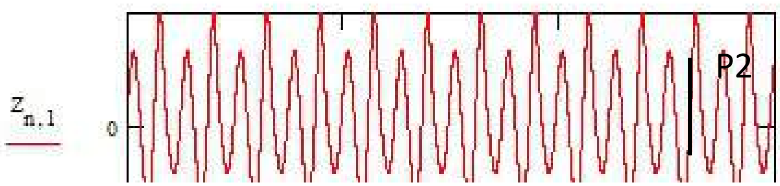
P1---P2---P4---P8---P16---P32---P64.....Chaos

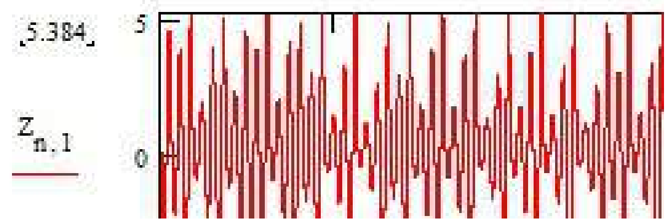
a=0.3



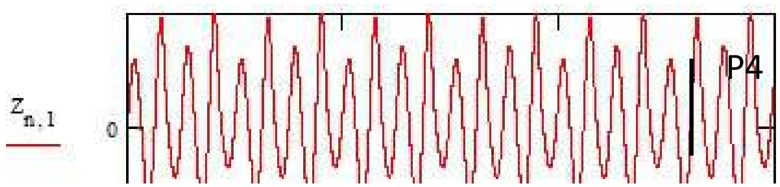


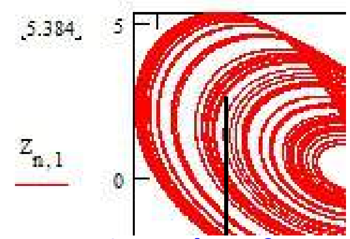
a=0.375





a=0.38

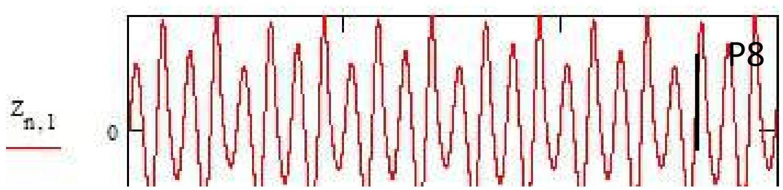




a=0.42

Poincaré surface of section

a=0.385

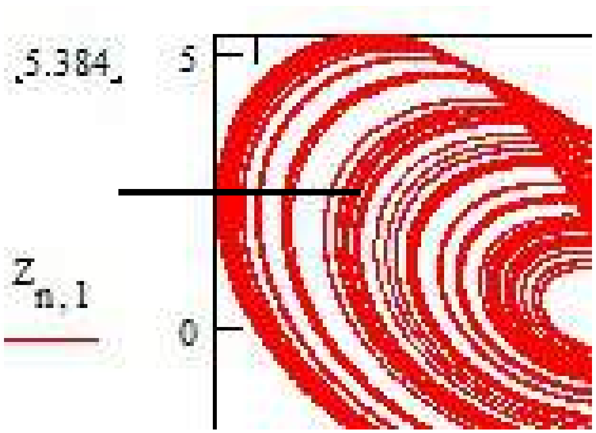


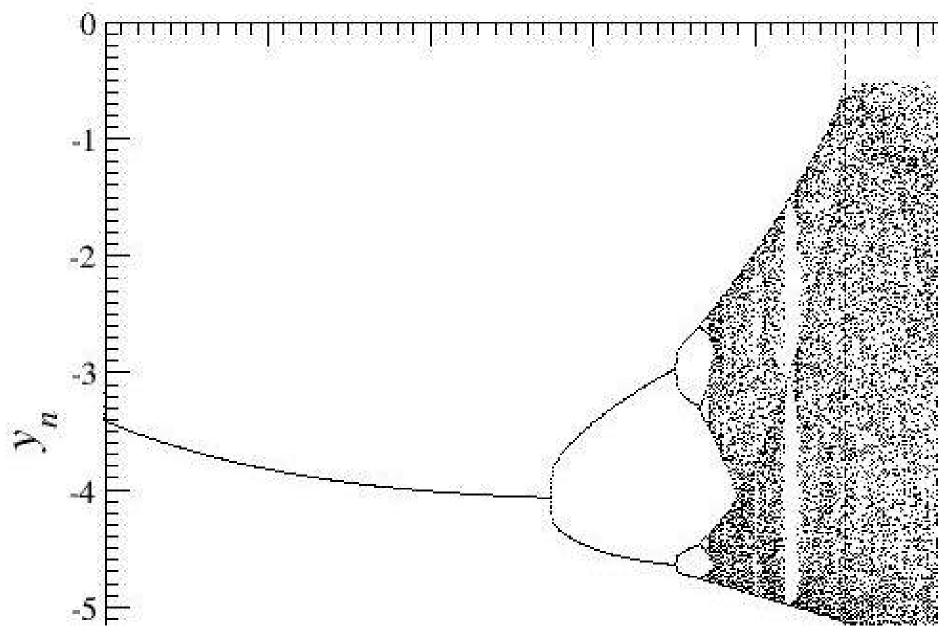
Bifurcation Diagram: Rössler Oscillator

Period-doubling route to Chaos

Rössler Oscillator

$$\dot{x} = -y - z$$
$$\dot{y} = x + a\,y$$
$$\dot{z} = b + (x - c)z$$





Route to Chaos:
Pomeau-Manneville (PM) Intermittency

$$(\text{chaos}) \rightarrow \begin{bmatrix} \text{approximately} \\ \text{periodic} \end{bmatrix} \rightarrow (\text{chaos})$$

Y. Pomeau and P. Manneville, Commun. Math. Phys. 74, 189 (1980); Physica 1D, 219 (1980).

PM Intermittency

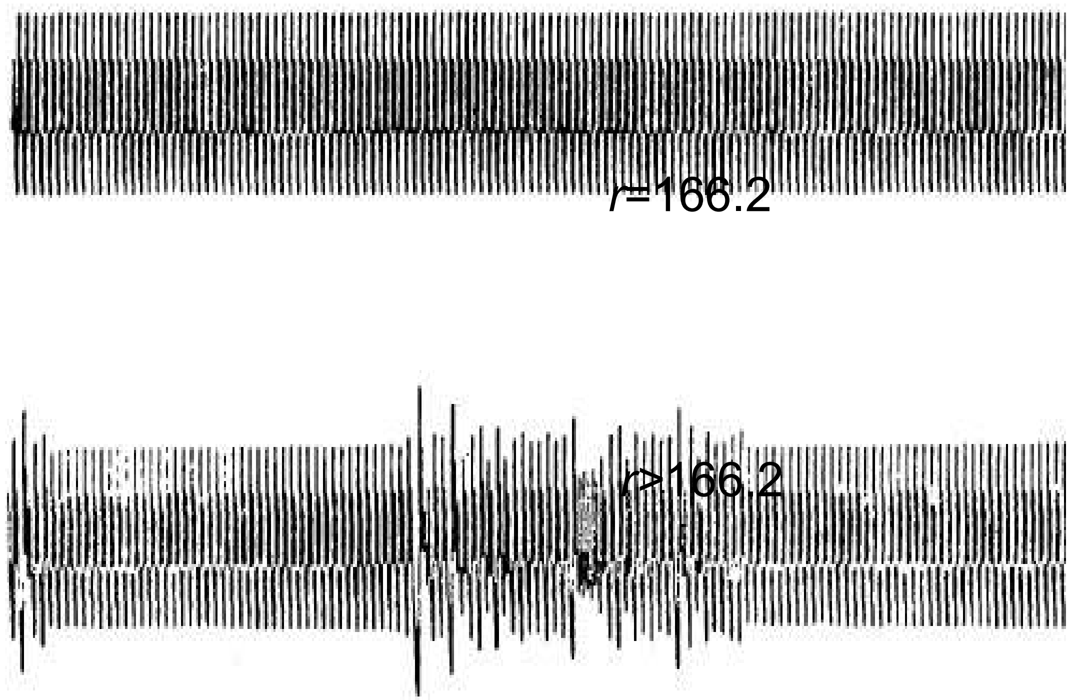
Lorenz System

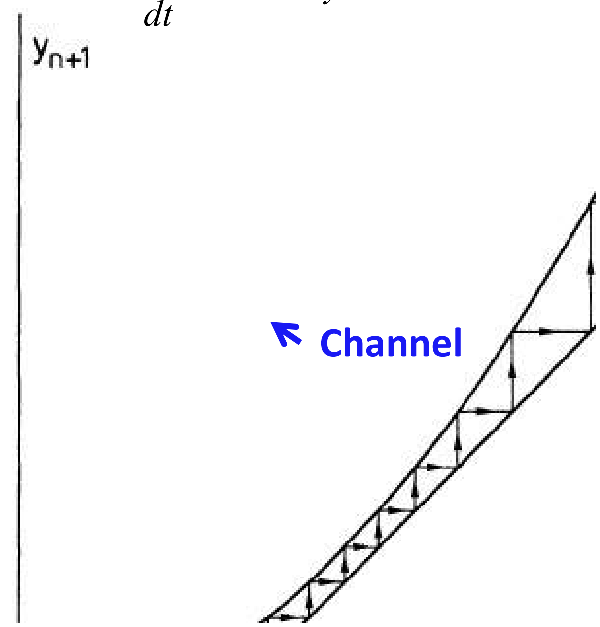
$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = rx - y - xz$$

$$\frac{dy}{dt} = -bz + xy$$

r=166

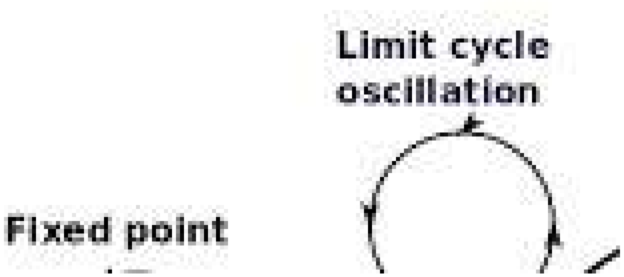




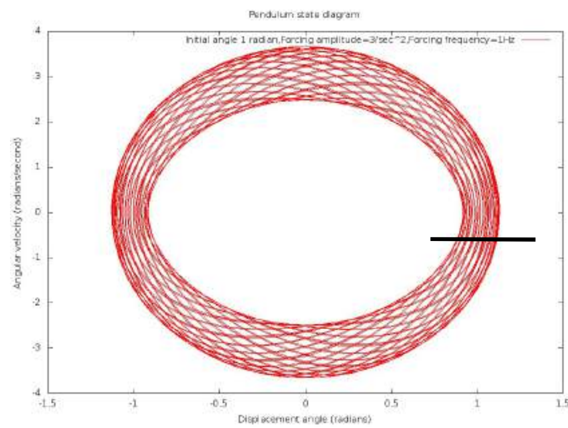
Chaos, r>>166.2

Pomeau, Manneville, Commun. Math. Phys. 74, 189 (1980)

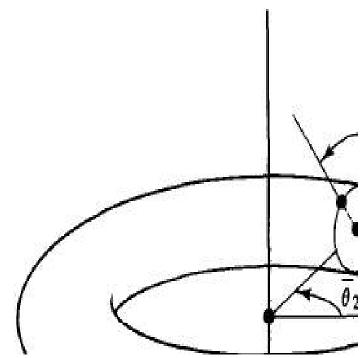
Quasiperiodic route to chaos

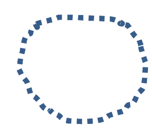


Torus- ω_1/ω_2=n
Commensurate

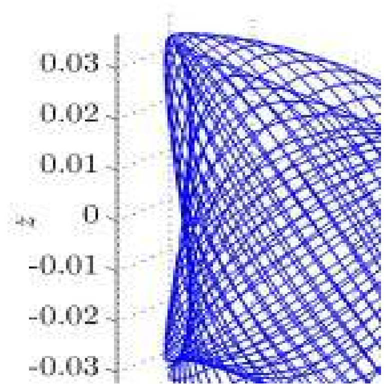


Quasipeirodicity- $\omega_1/\omega_2 \neq$n
Incommensurate



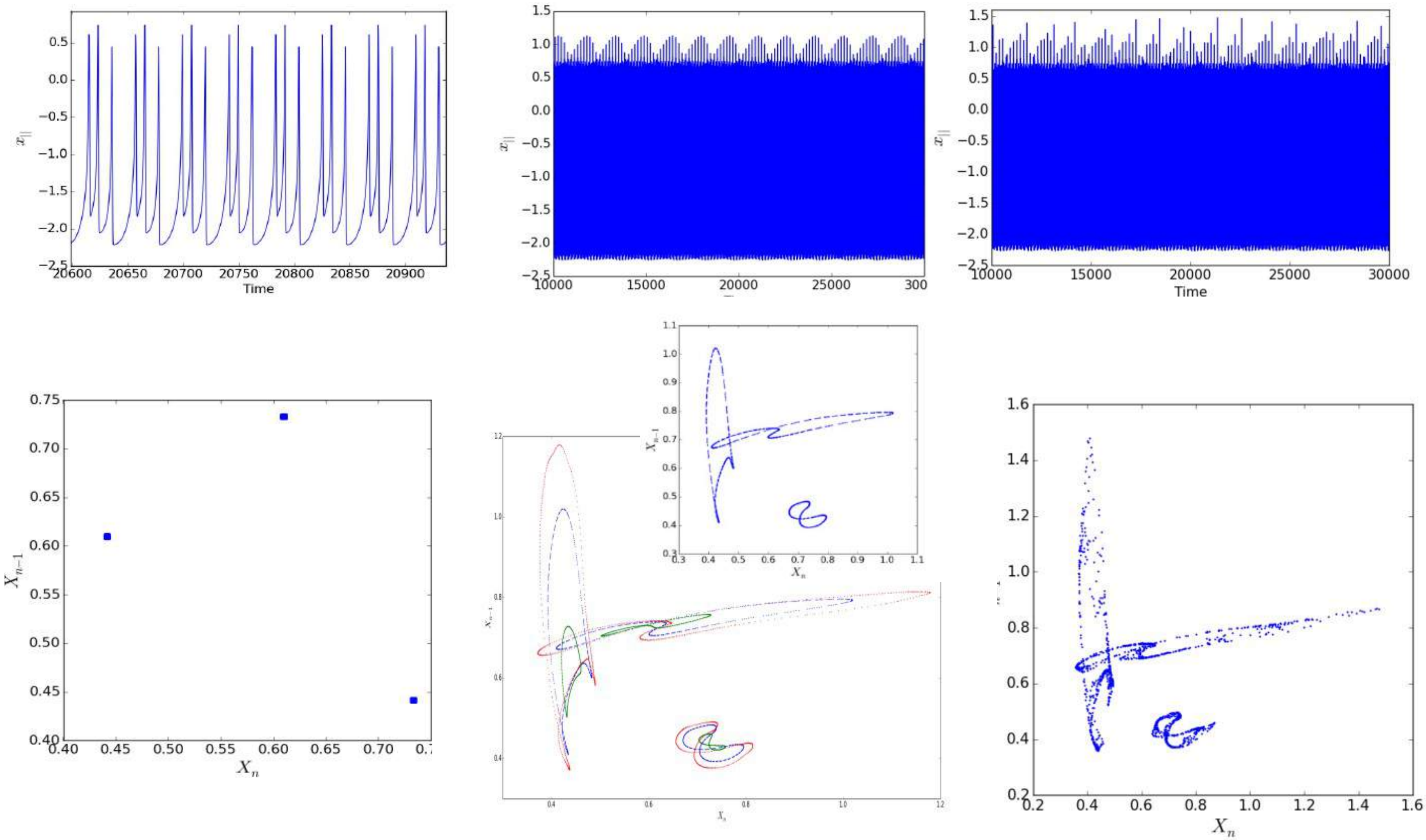


Poincare surface of section



Breakdown to QP to Chaos…

Quasiperiodic Breakdown to Chaos



Crisis-Induced-Intermittency

Crisis-induced Intermittency

Saddle

$$(\text{chaos})_1 \to (\text{chaos})_2 \to (\text{chaos})_1 \to (\text{chaos})_2 \to \cdots .$$

Grebogi, Ott, Yorke, PRL 48, 1507 (1982); Physica 7D, 181 (1983)
Grebogi, Ott, Romeiras, Yorke, PRA 36, 5365 (1987)

Pomeau-Manneville (PM) Intermittency

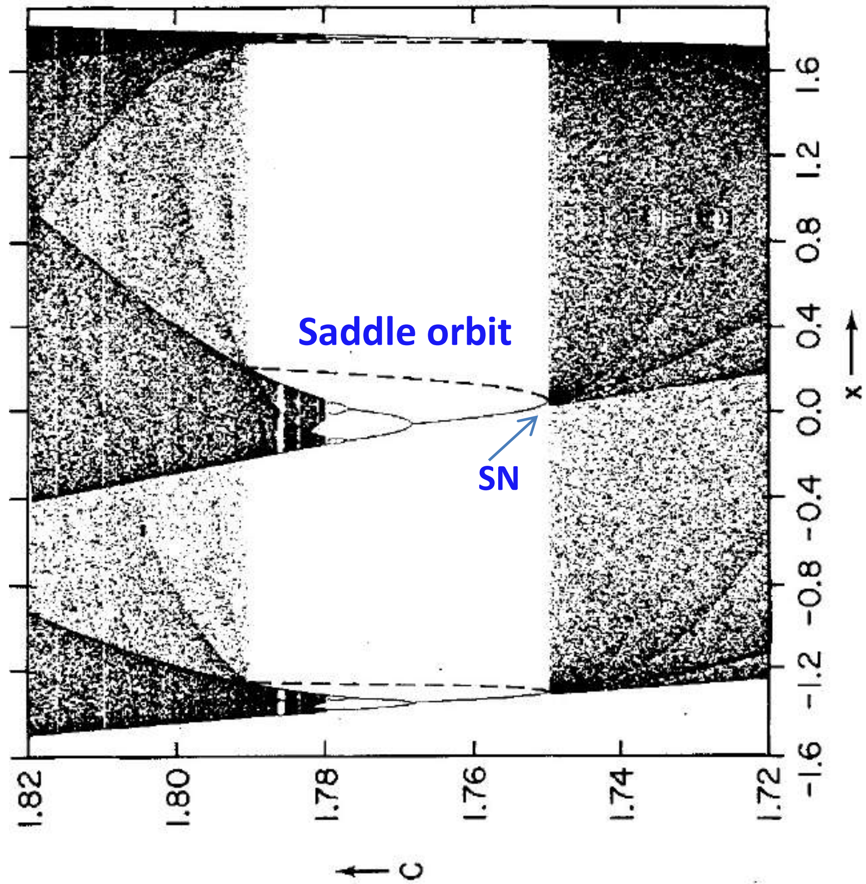
$$(\text{chaos}) \to \begin{bmatrix} \text{approximately} \\ \text{periodic} \end{bmatrix} \to (\text{chaos})$$

Channel

Y. Pomeau and P. Manneville, Commun. Math. Phys. 74, 189 (1980); Physica 1D, 219 (1980).

Crisis induced Intermittency

Interior Crisis



Grebogi, Ott, Yorke, Physica 7D, 181 (1983)

Extreme Events: Crisis Induced Intermittency

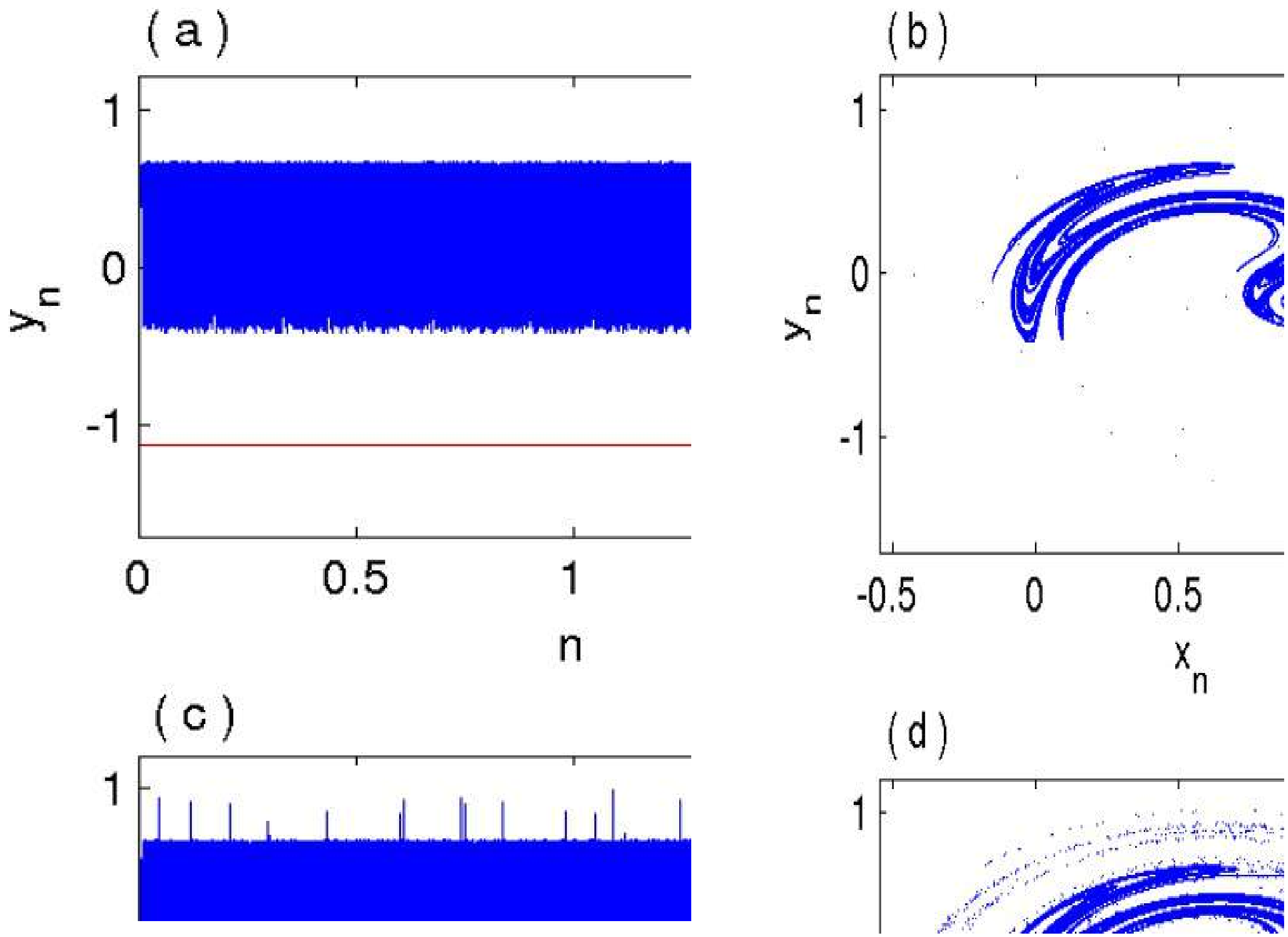
Ikeda Map $z_{n+1} = A + Bz_n \exp[i\kappa - ip/(1+|z_n|^2)]$

Grebogi, Ott, Romeiras, Yorke, PRE 36, 5365 (1987)

H_s=significant height=mean -5σ

p=7.26<p$_c$

p=7.2788>p$_c$



Expansion of Attractors

Our First Example

Forced Liénard System

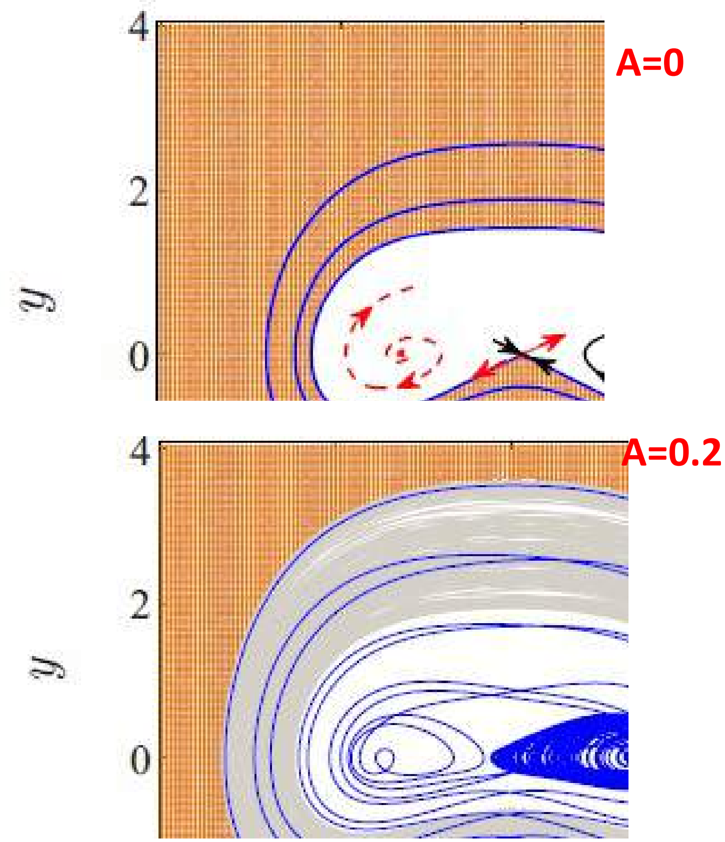
PHYSICAL REVIEW E 96, (

Extreme events in the forced Lién

Liénard System

$$\dot{x} = y,$$

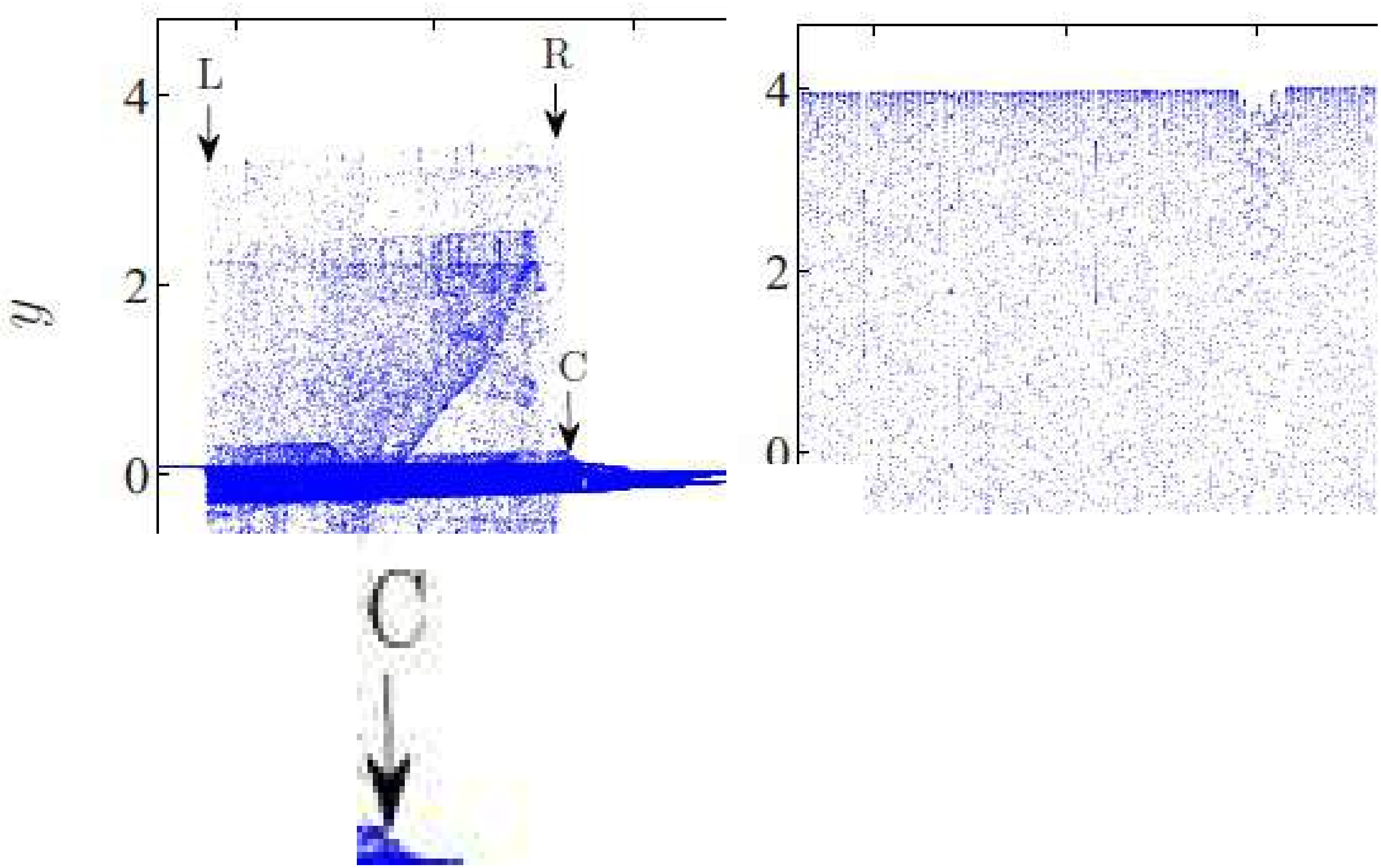
$\alpha = 0.45$, $\beta = 0.50$, and $\gamma = -0.50$



(1) Stable Focus becomes unstable and LC is born
(2) Saddle becomes saddle orbit
(3) Saddle focus remains unchanged
(4) Neutrally stable orbits become quasi-periodic

Forced Liénard System: Bistable

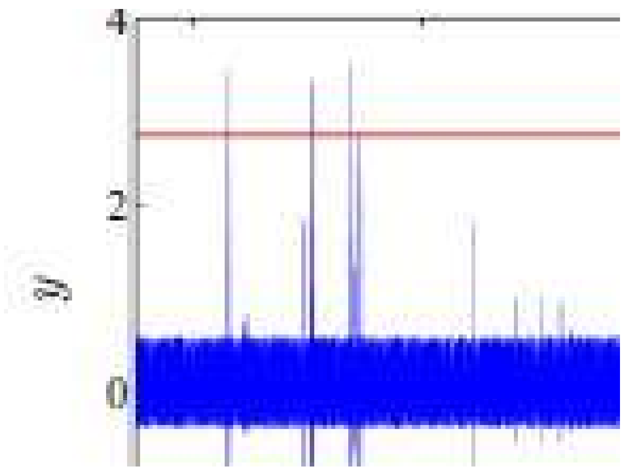
Bifurcation diagrams

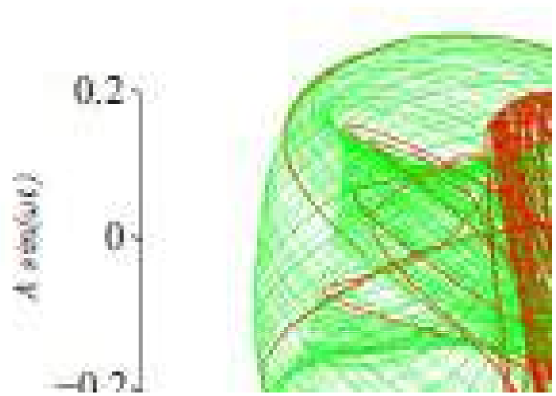


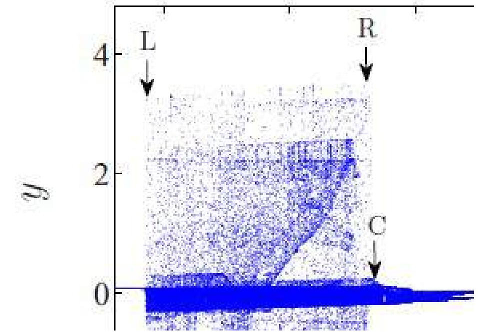
Forced Liénard System

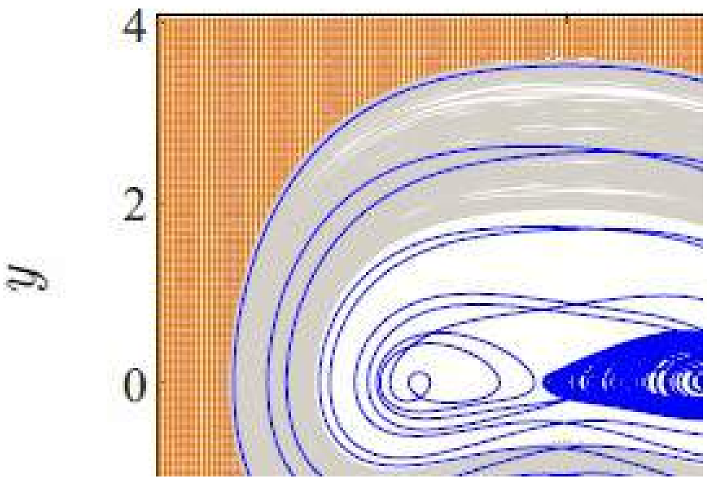
Region R

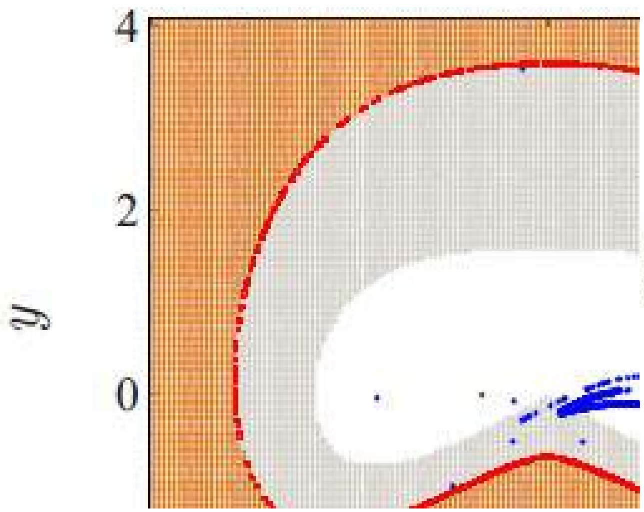
H_s=Significant height=mean+6σ



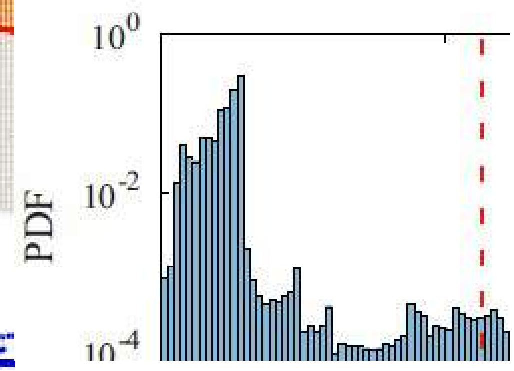






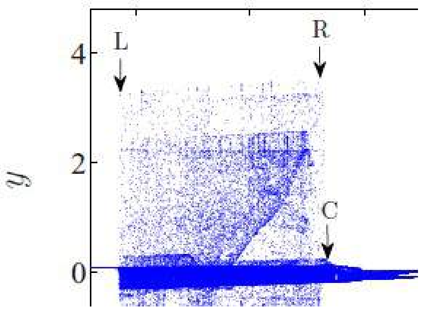


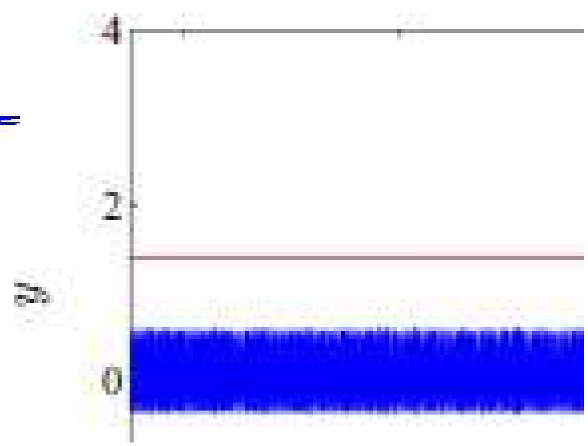
Long-tail distribution

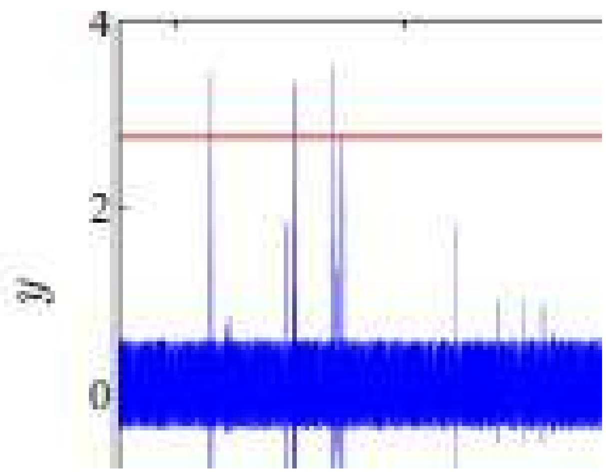


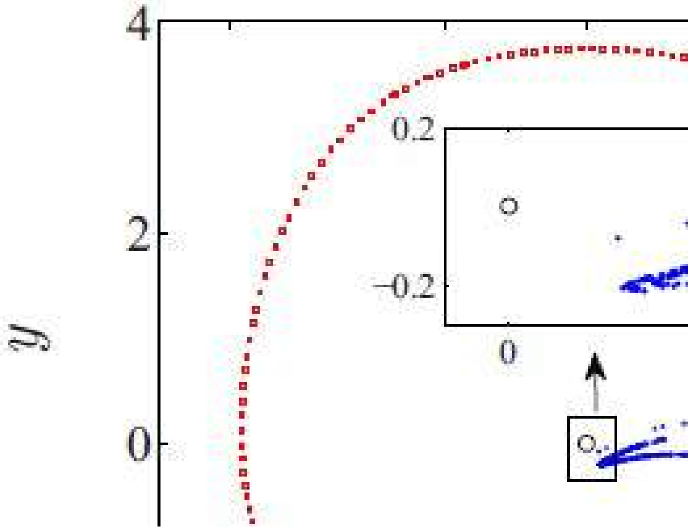
Extreme Events

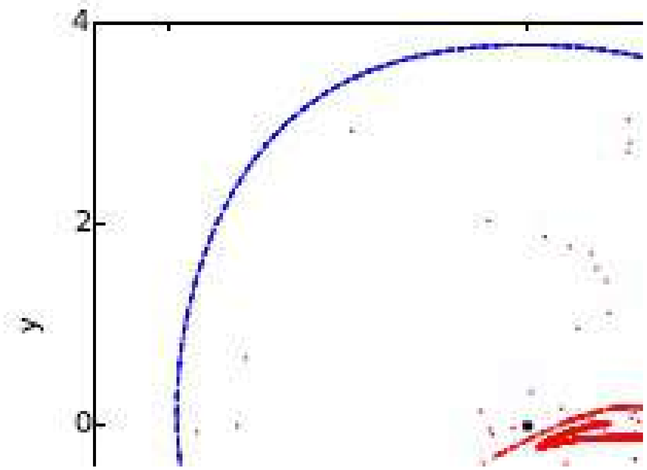
Crisis Induced Intermittency







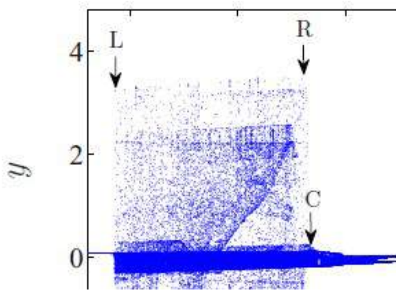


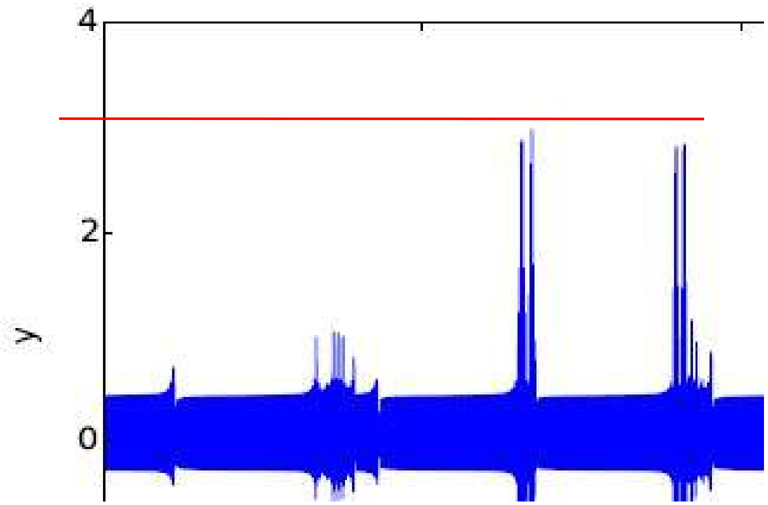


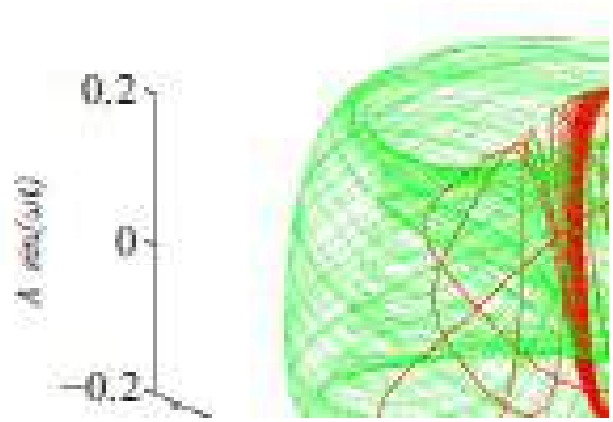
Extreme Events

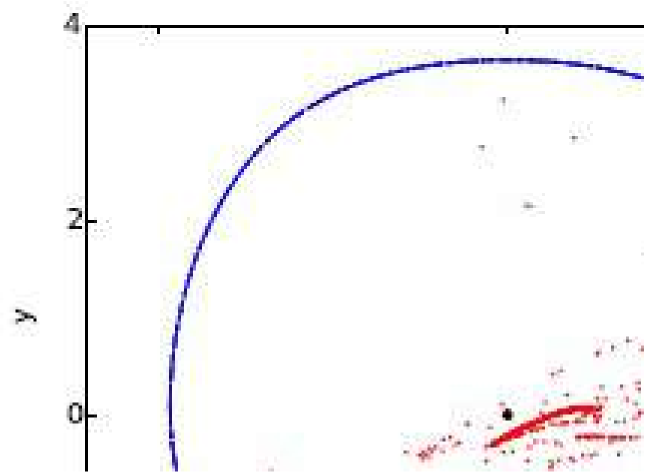
PM Intermittency followed by Interior Crisis

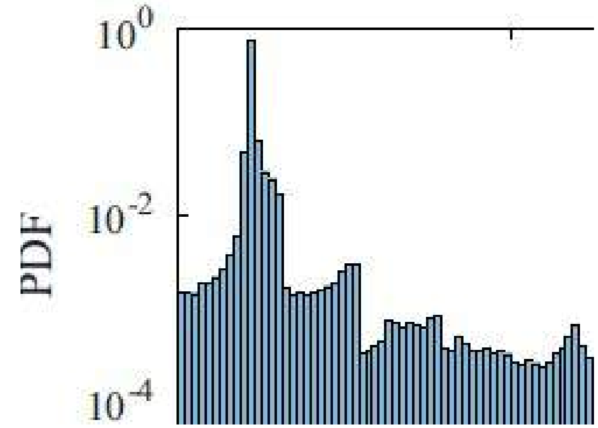
Region L





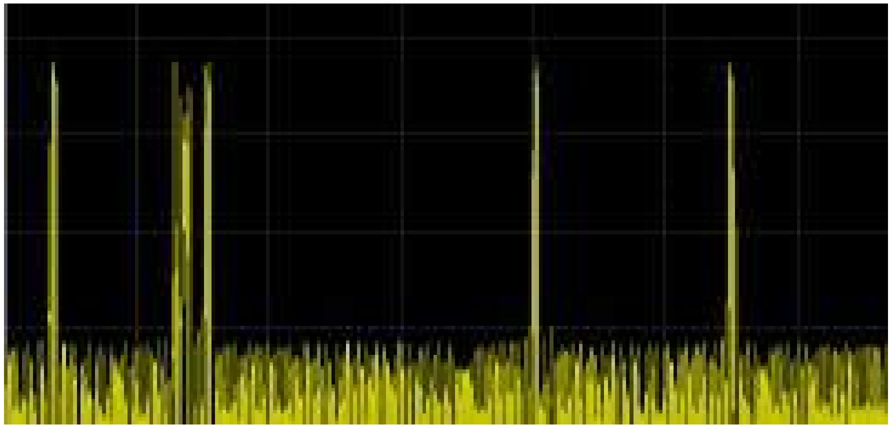


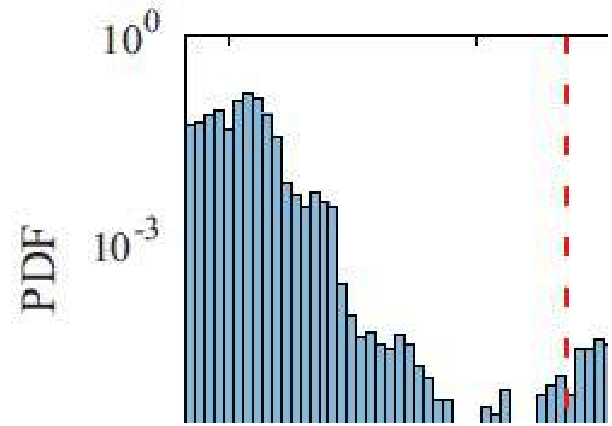




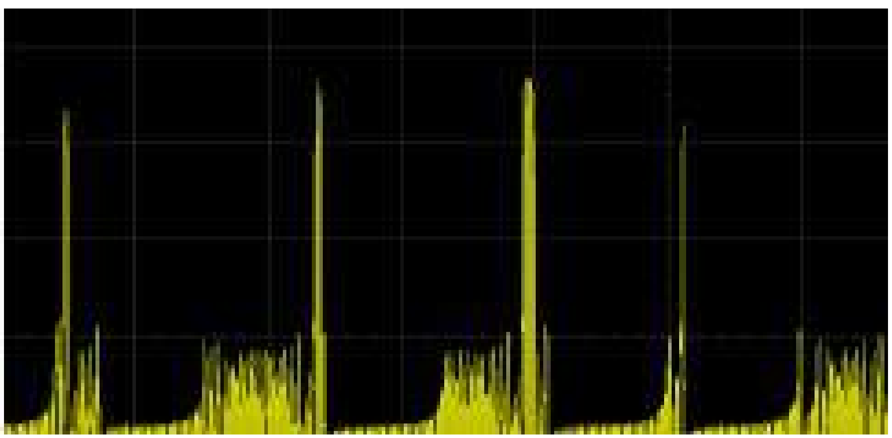
Experimental Verification

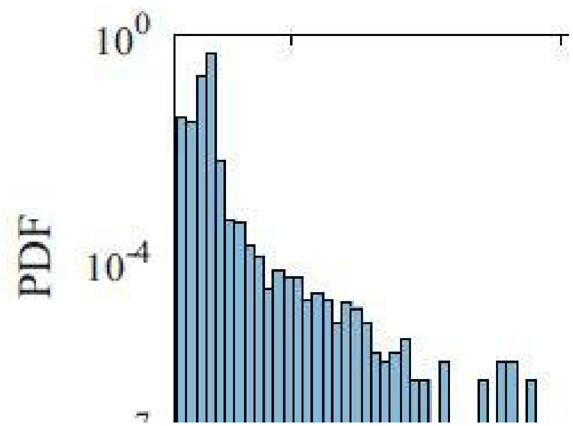
Period-doubling to Chaos followed by Crisis





PM Intermittency followed by Crisis





PHYSICAL REVIEW E 96 (

Extreme events in the forced Lién

Summary

1. Period-doubling to Chaos followed by Crisis

2. PM Intermittency followed by Crisis

Large Expansion of Attractors

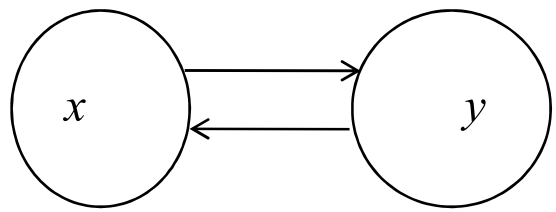
Coupled oscillators

Collective Dynamics of Coupled Oscillators

- **Synchronization**
- **Clustering**
- **Oscillation quenching**
- **Chimera states**

Collective Dynamics of Coupled Oscillators

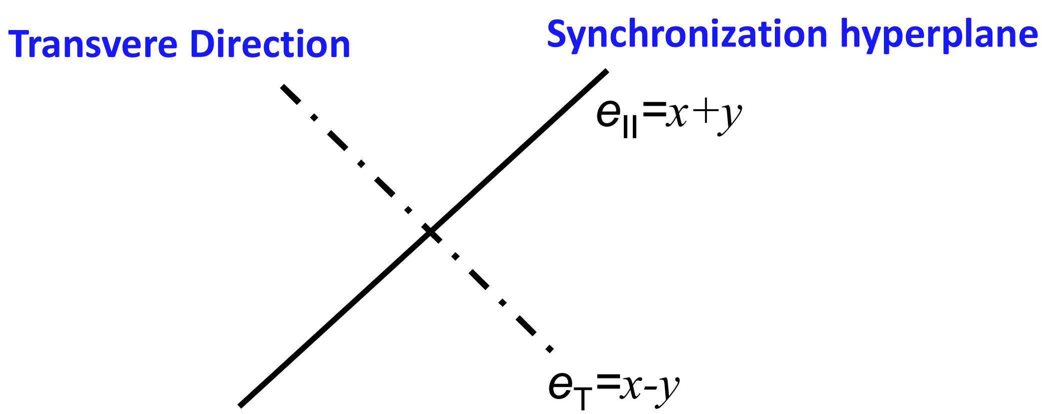
Brief Story: Synchronization



Periodic or Chaotic systems

$$\dot{x} = f(x) + \varepsilon(y - x)$$
$$\dot{y} = f(y) + \varepsilon(x - y)$$

n-dimensional system



Complete Synchronization

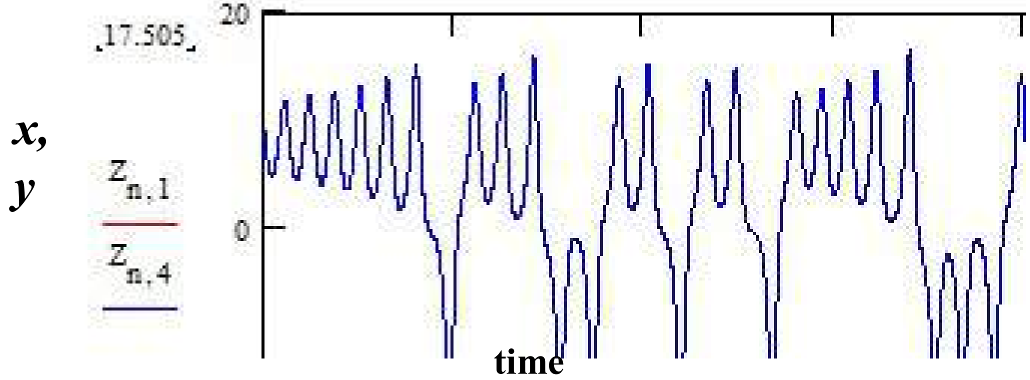
$$e_T = x - y = 0$$

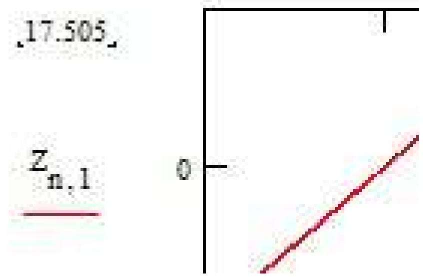
Anti-Synchronization

$$e_{||} = x + y = 0$$

Variety of Synchronization

Complete Synchronization: Lorenz System

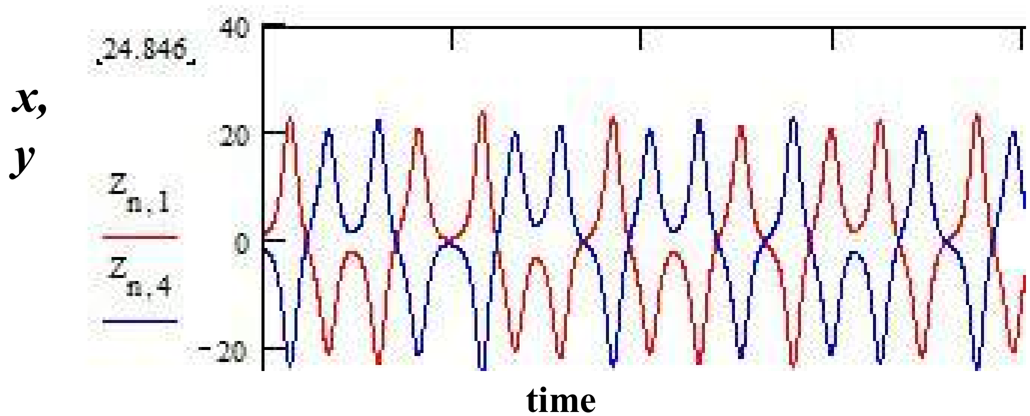


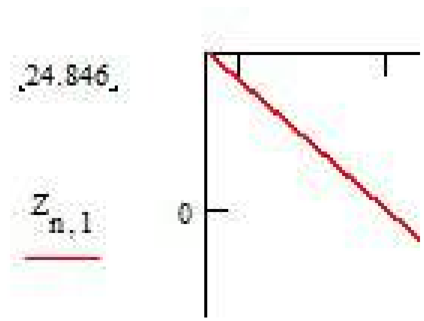


$e_T = x - y = 0$

Identical amplitude and phase

Anti-synchronization: Lorenz System





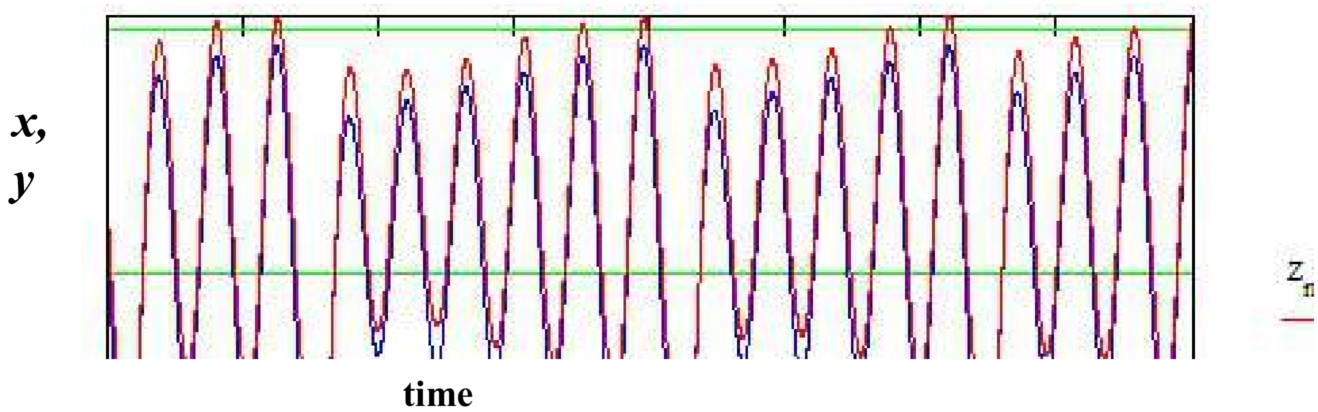
$e_{II} = x + y = 0$

Identical amplitude, but Opposite phase

Phase Synchronization

Amplitude almost identical
but phase difference almost zero

In-phase Synchronization: Chua Oscillator

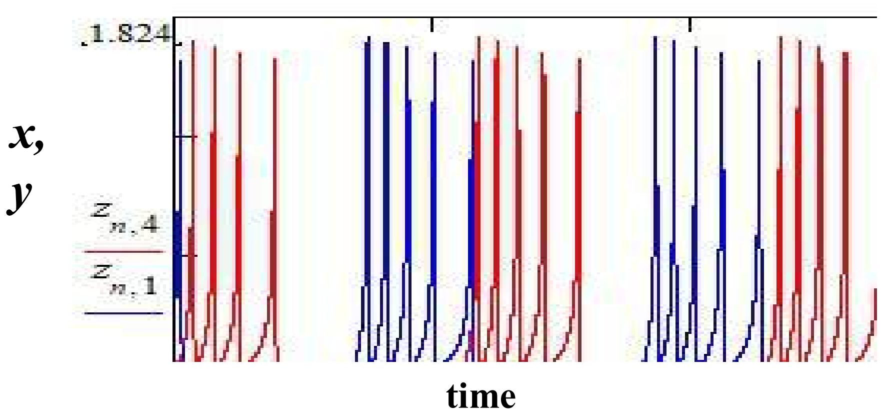


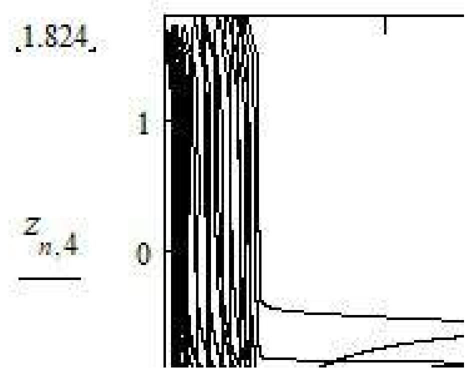
$e_T = x - y \neq 0$

$\Phi_1 - \Phi_2 = 0$

Antiphase Synchronization: HR Model

Amplitude almost identical
but out-of-phase





$e_{II} = x + y \neq 0$

$\Phi_1 - \Phi_2 \approx \pi$

Selected for a Viewpoint in *Physics*

Hugo L. D. de S. Cavalcante* and Marcos C

Grupo de Física Atômica e Lasers-DF, Universidade Federal da Paraíba, Caixa Postal 50

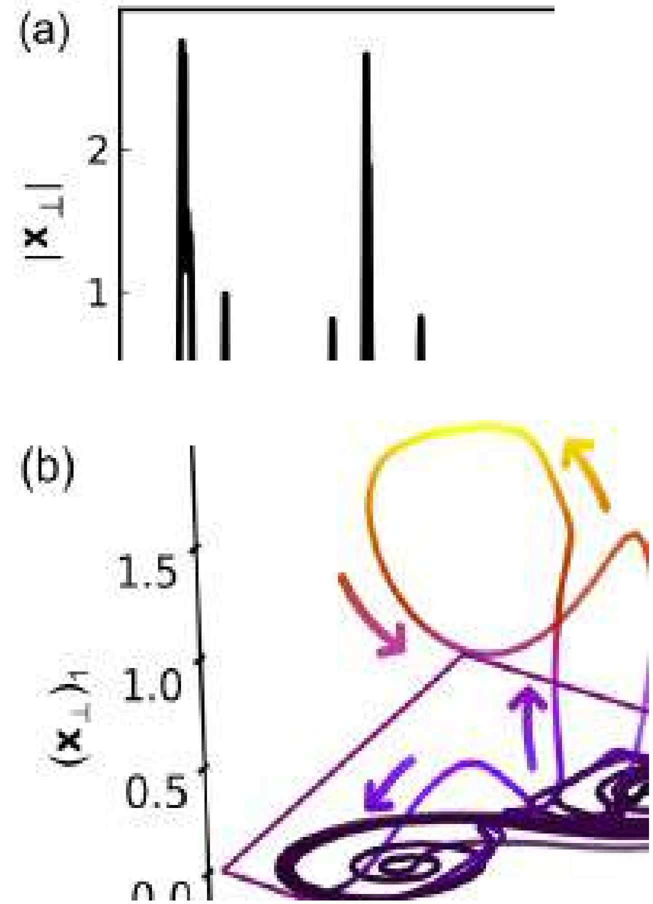
Didier Sornette

ETH Zurich, Department of Management, Technology and Economics, Scheuchz

Edward Ott

On-off Intermittency

Bubbling transition



Model: M-master, S-slave

$$\boxed{\begin{aligned} \dot{\mathbf{x}}_M &= \mathbf{F}[\mathbf{x}_M], \\ \dot{\mathbf{x}}_S &= \mathbf{F}[\mathbf{x}_S] + c\mathbf{K}(\mathbf{x}_M - \mathbf{x}_s), \end{aligned}}$$

Instability in Complete Synch

$$\mathbf{x}_\perp = (\mathbf{x}_M - \mathbf{x}_S)/2$$

$$\dot{V}_{1i} = \frac{V_{1j}}{R} - g[V_1$$

$$\dot{V}_{2i} = g[V_{1i} - V_{2i}] - I_i + \delta_2$$

$$\dot{I}_i = V_{2i} -$$

$$g[V] = \frac{V}{R} + I_r[\exp(\alpha_f V) -$$

P. Ashwin, J. Buescu and I.N. Stewart, Phys. Lett. A 193 (1994) 126.

E. Ott and J.C. Sommerer, Phys. Lett. A 188 (1994) 39.

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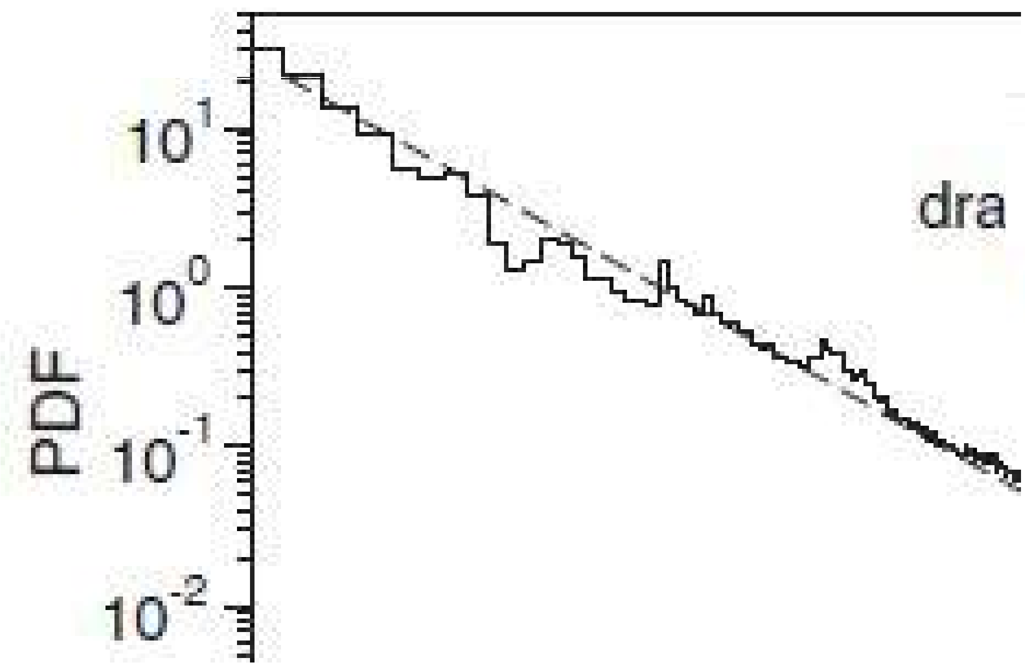
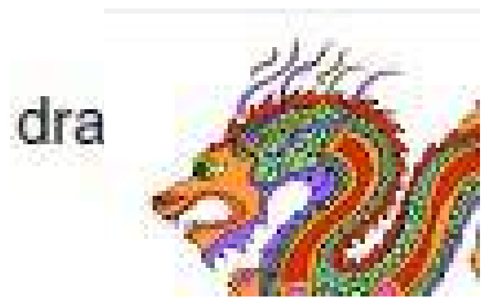
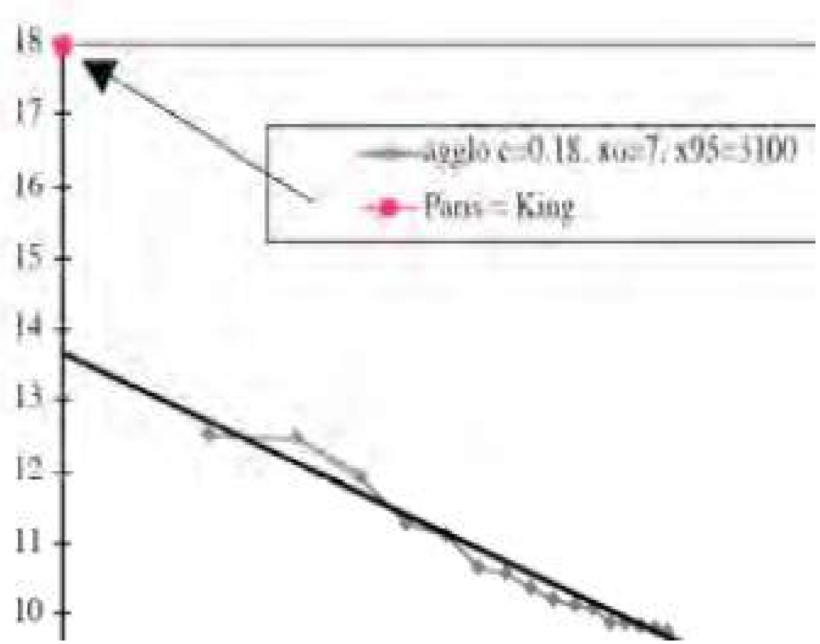




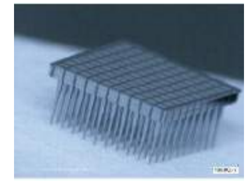
Possibility of prediction is higher??

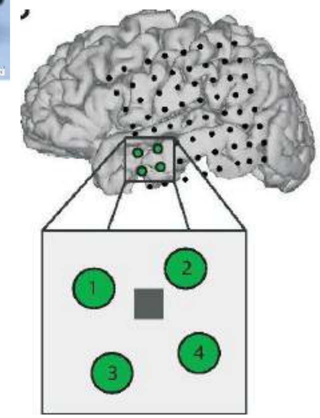
Dragon-King

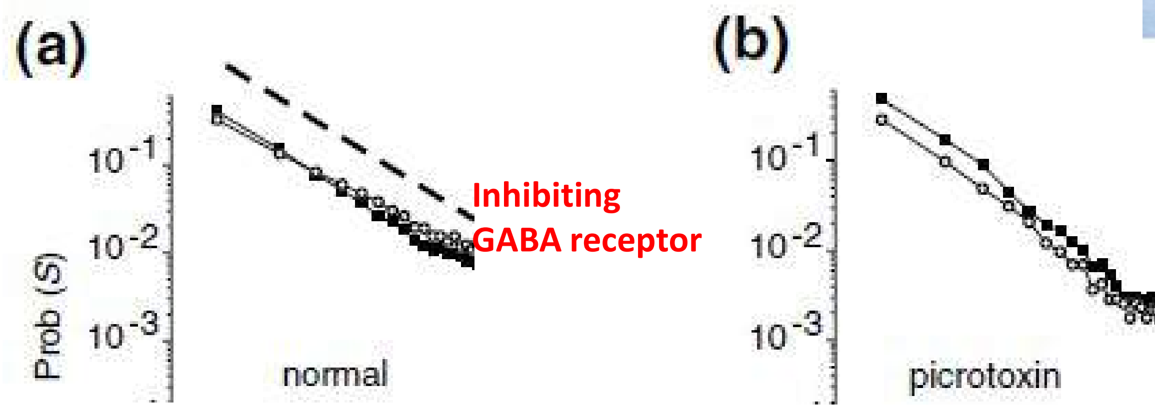
“when a log–log graph of the populations of French cities versus the population rankings of those cities is plotted, all the points fall on a straight line with the exception of Paris. The French capital has a much greater population than is predicted by the log–log "Zipf plot" and it is therefore a Dragon-king”----------Didier Sornette



Neuronal Avalanche







frontiers

REVIEW AR

published: 07 J

FIG. 1. Power law distribution of burst
(a) Normal cultures correspond to $\sigma = 1$ i
line gives slope $= -3/2$. (b) Cultures m

[7] J. M. Beggs and D. Plenz, J. Neurosci. **23**, 11167 (2003).
PRL **94**, 058101 (2005)

L.De Arcangelis, Journal of Physics: Conference Series 297, 012001 (2011)

"Dragon-kings …. appear in the cultures that realize a **high level of burst synchronization**…., **bicuculline increases burst synchronization** leading to a large population of dragon-king avalanches. **Dragon-king events may therefore affect the ability of the brain to give good performances in different tasks**"

PHYSICAL REVIEW E 97, 062311 (

Dragon-king-like extreme events in coupl

Arindam Mishra,[1,*] Suman Saha,[2] M. Vigneshwaran,[3] Pinaki Pal,[4] Tomasz Kapitaniak,[5] and Syamal K. Dana[2]

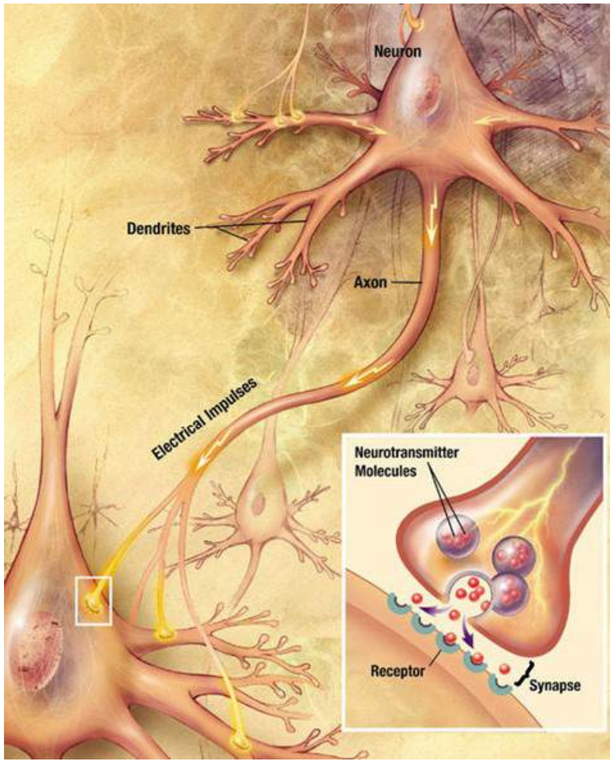
Our Second Example

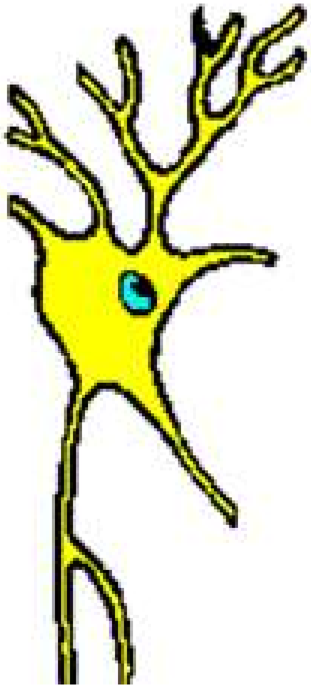
Coupled Neurons

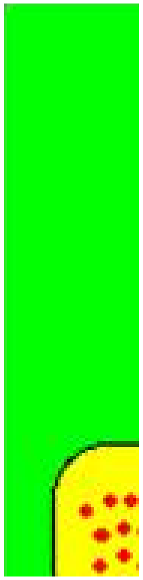
Inhibiting GABA receptor

Inhibitory coupling in addition to excitatory coupling

Coupling: Chemical Synapses







Hindmarsh-Rose Model

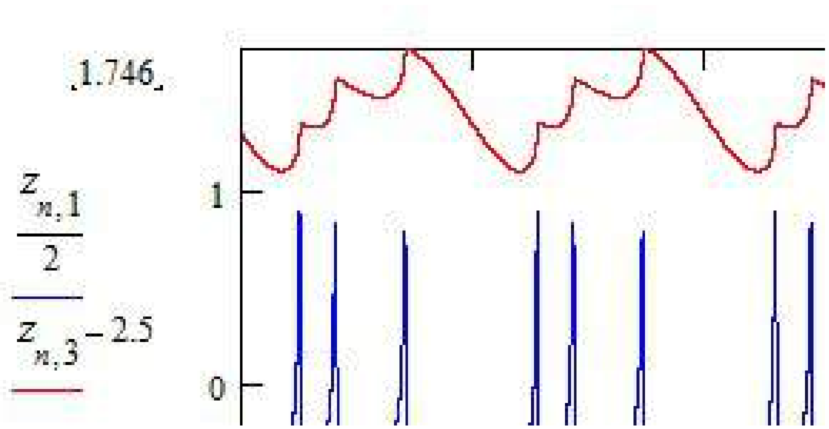
$$\dot{x}_i = y_i - ax_i^3 + bx_i^2 - z_i + I$$

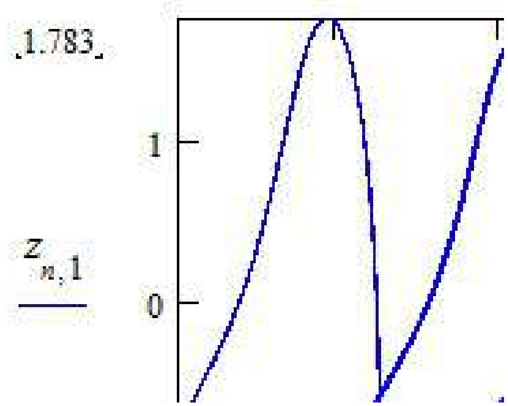
$$\dot{y}_i = c - dx$$

$$\dot{z}_i = r[s(x_i - x_R) - z_i]$$

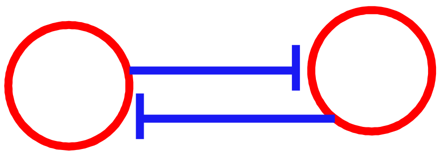
$a = 1,\ b = 3,\ c = 1,\ d = 5$

$r = 0.01,\ \ S = 5,\ \ I = 4, x_R = -1.6$

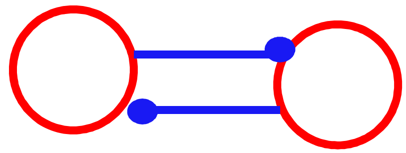




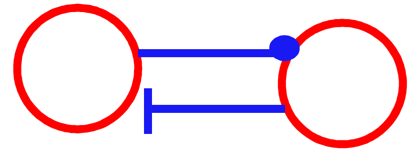
Synaptic Coupling



Inhibitory synapses



Excitatory synapses



Inhibitory and Excitatory synapses

$$\dot{x}_i = y_i + bx_i^2 - ax_i^3 - z_i + I - k_{1,2}(x_i - V_s)g(x_j)$$

$$\dot{\mathrm{y}}_{\mathrm{i}} = c - dx_i^2 - y_i$$

$$z_i = r[s(x_i + 1.6) - z_i]$$

$$a = 1, b = 3, \mathrm{c} = 1, \mathrm{d} = 5$$

$$\mathrm{r} = 0.01,\ \mathrm{S} = 5,\ \mathrm{I} = 4,$$

$$\lambda = 10, \theta = -0.25, V_s = 2$$

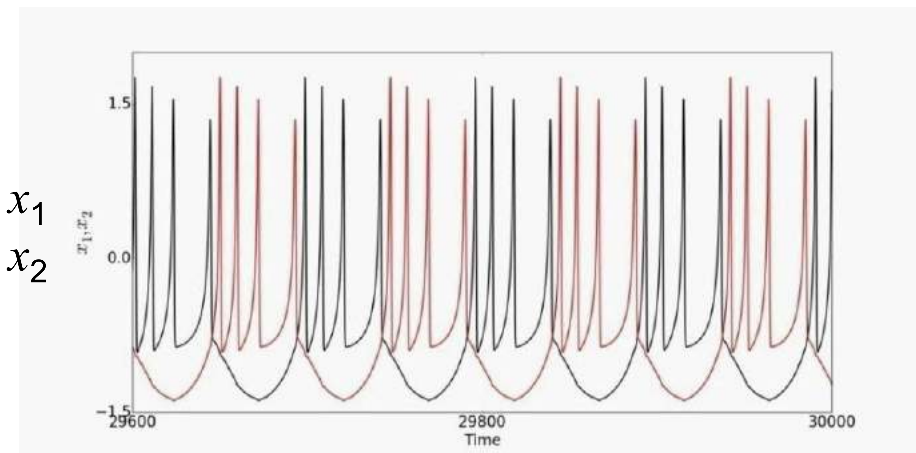
$$i = 1, 2$$

$$g(x) = \frac{1}{1 + e^{-\lambda(x-\theta)}}$$

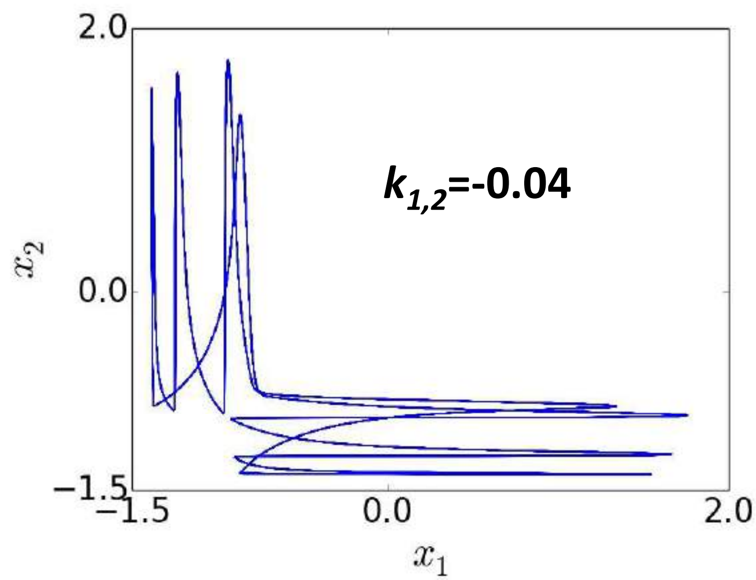
$$x_1 + x_2 = x_{||}$$

$$x_1 - x_2 = x_T$$

Two coupled Hindmarsh-Rose Neurons



Time series



Inhibitory coupling

Antiphase Burst Synchronization

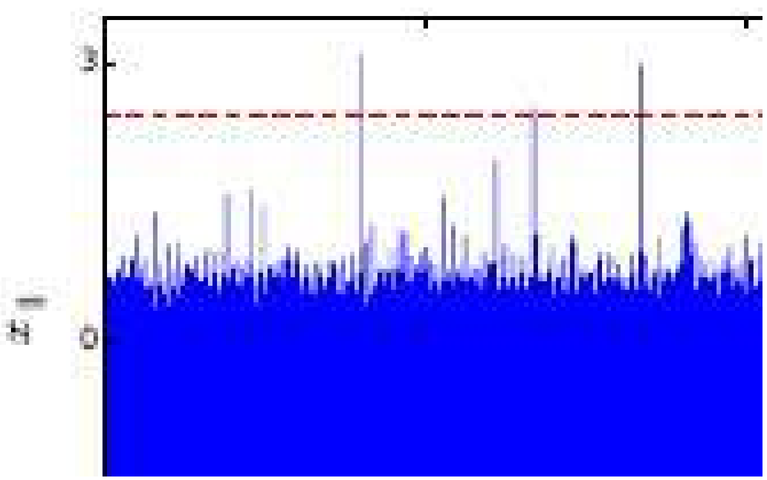
180^0 out-of-phase

Two coupled HR Systems

Inhibitory coupling

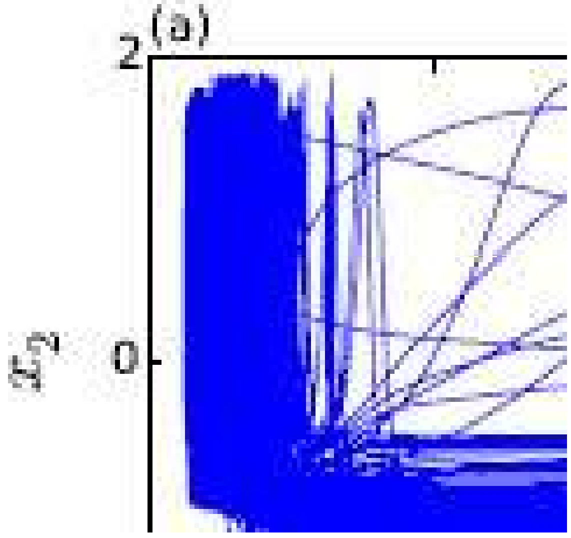
EE>Mean height +6σ

$x_{||}$



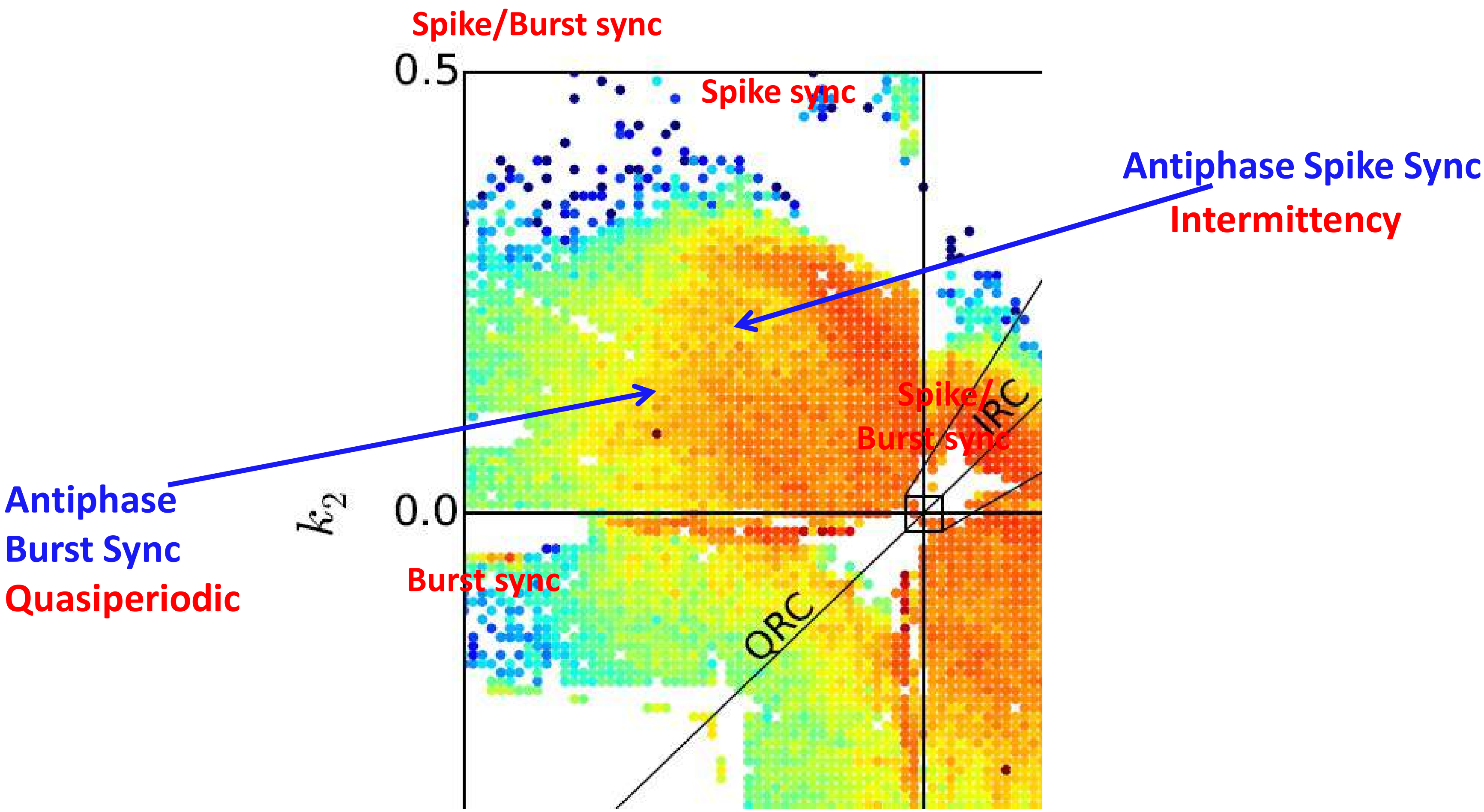
$$x_{||} = x_1 + x_2$$

$\boldsymbol{k_{1,2}}$**=-0.07**



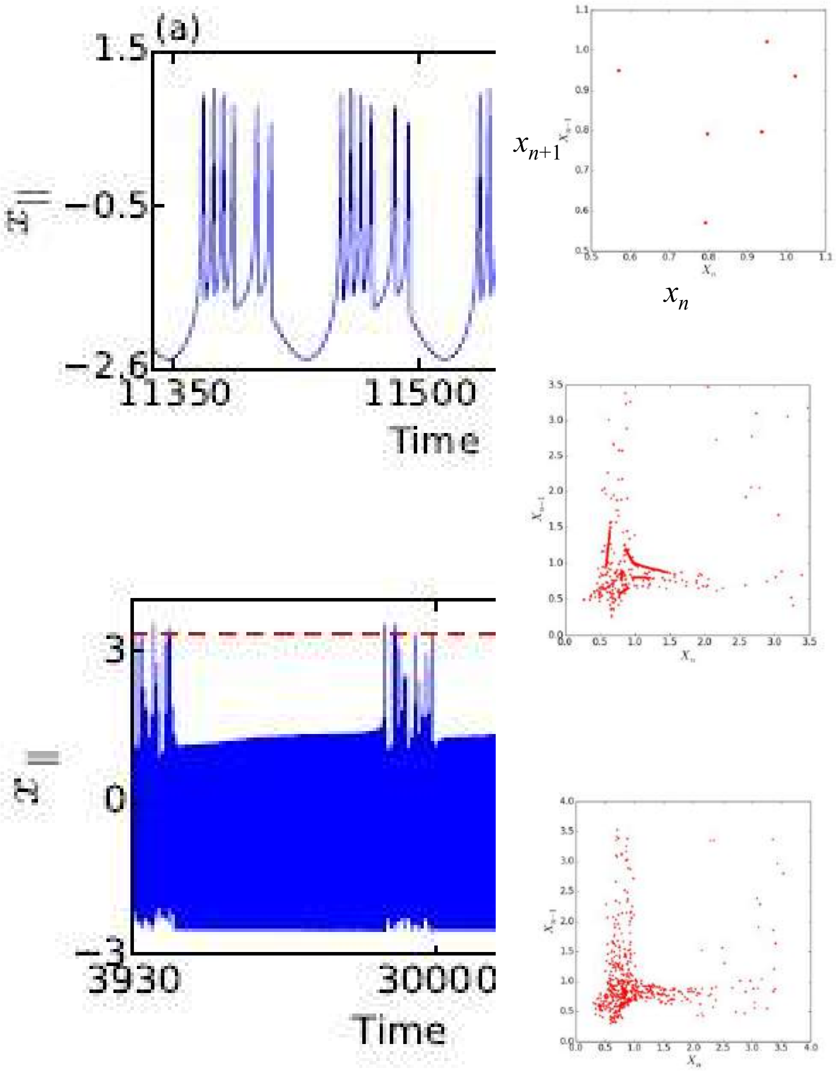
Instability in Antiphase Sync

Phase Diagram



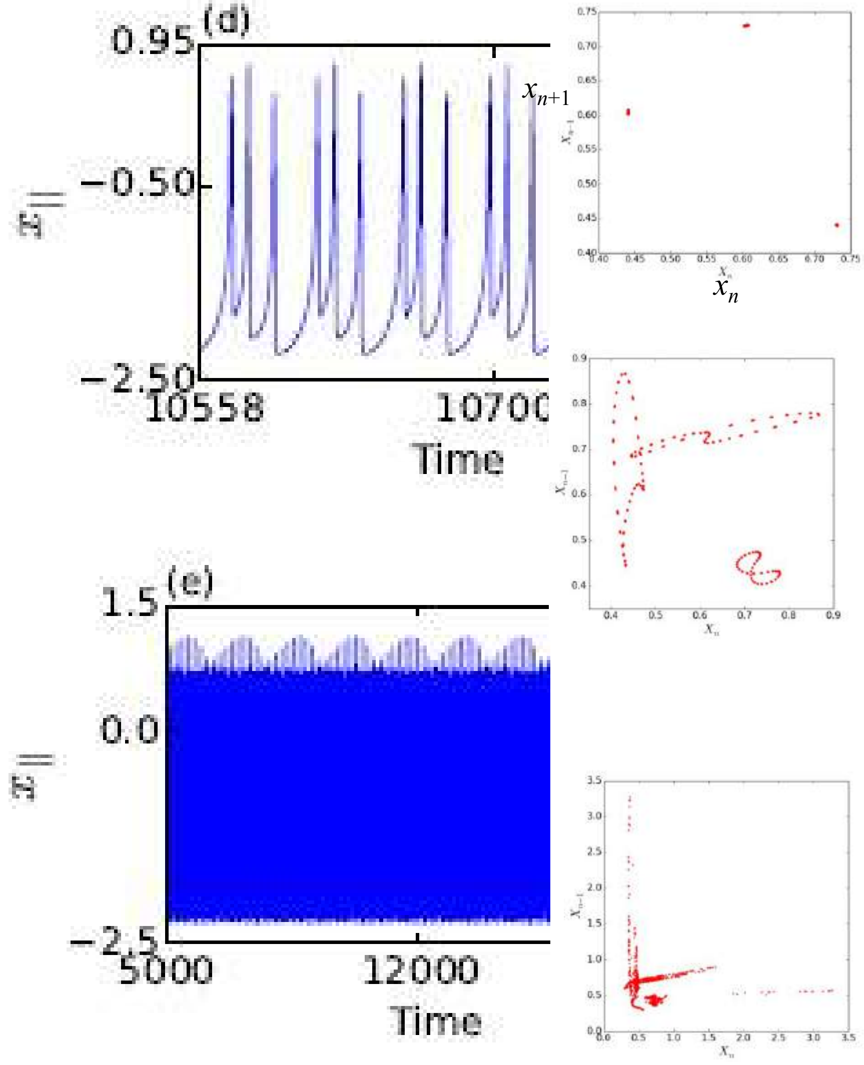
Extreme Events: Routes to Complexity

PM Intermittency



$k_{1,2} = 0.04, 0.0532, 0.07$

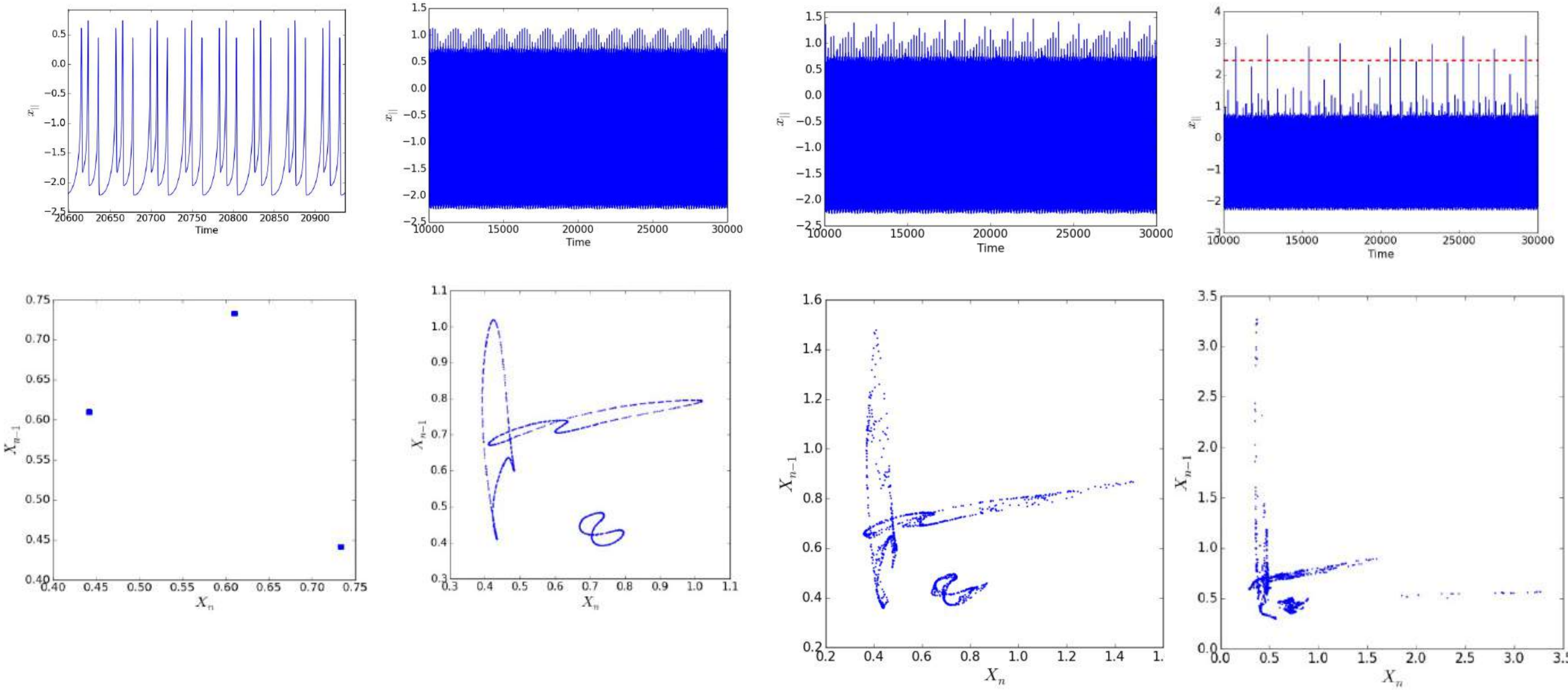
Quasiperiodicity



$k_{1,2} = -0.04, -0.06, -0.07$

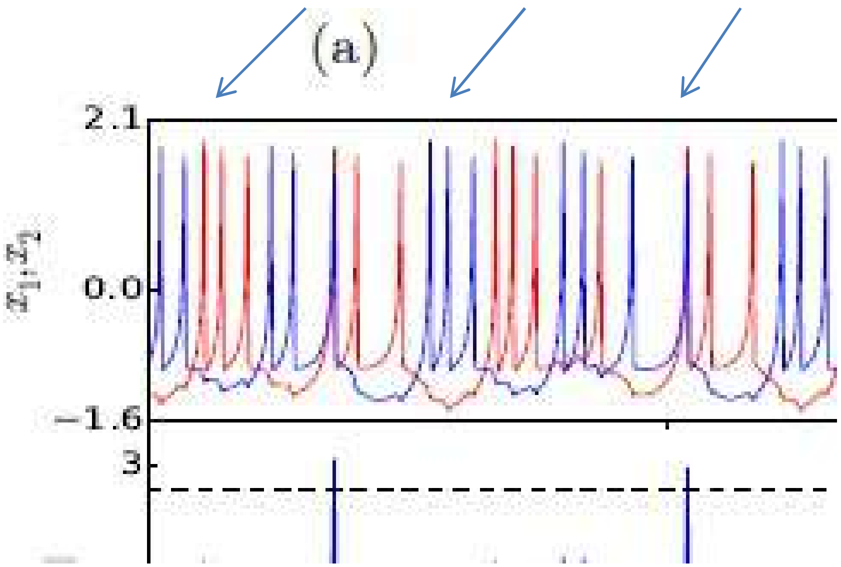
Extreme Events: Quasiperiodic Routes

Third Route



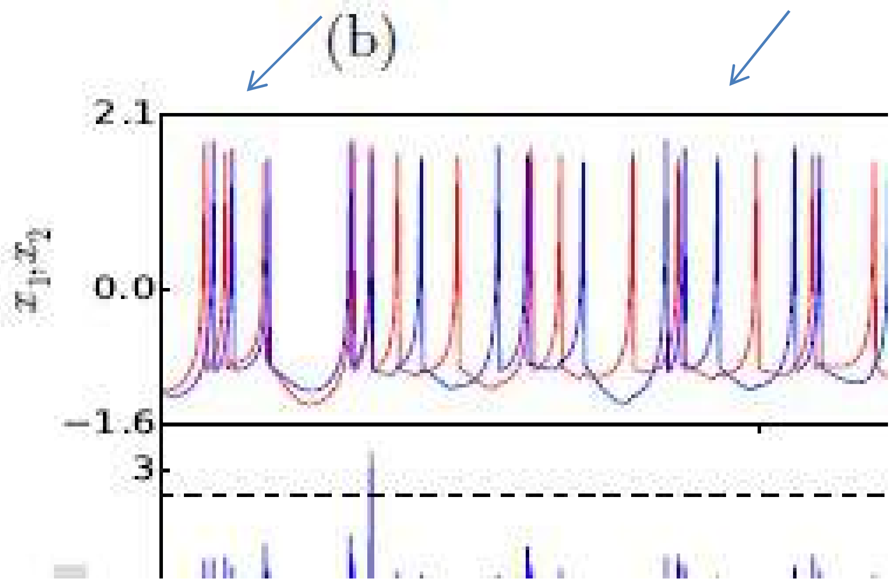
Antiphase Synchronization
Instability

Antiphase burst Synchronization



Quasiperiodicity route
Inhibitory coupling

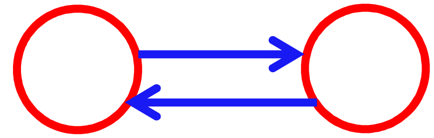
Antiphase spike Synchronization

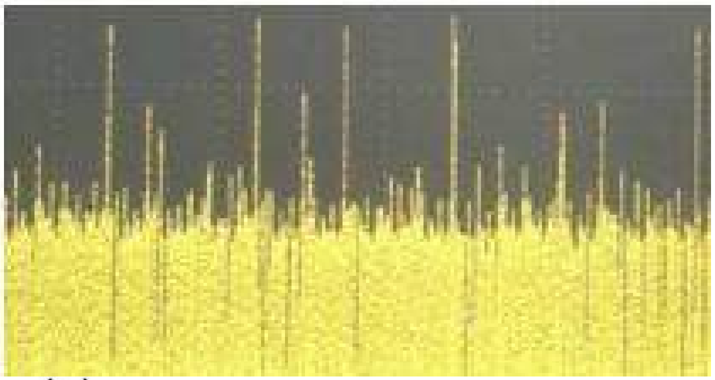


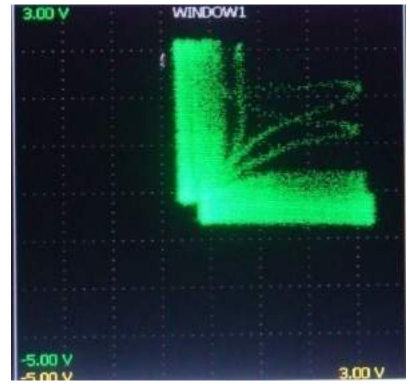
Intermittency route
Excitatory coupling

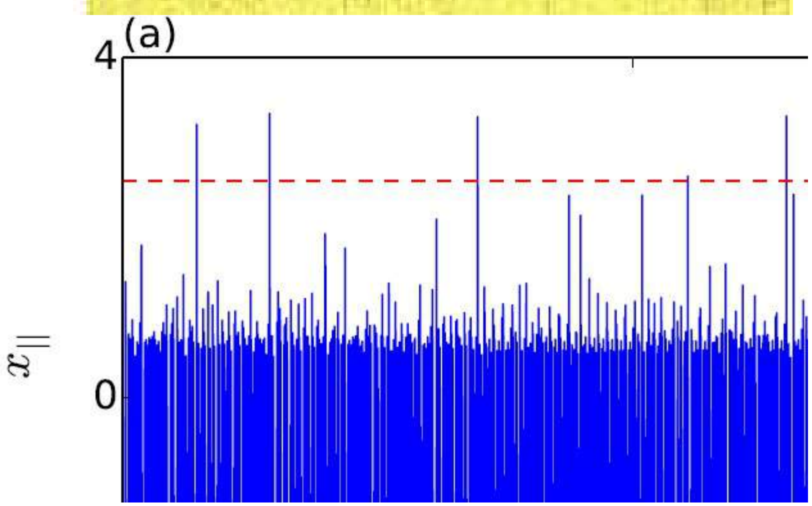
Experimental Results

Diffusive and Repulsive coupling

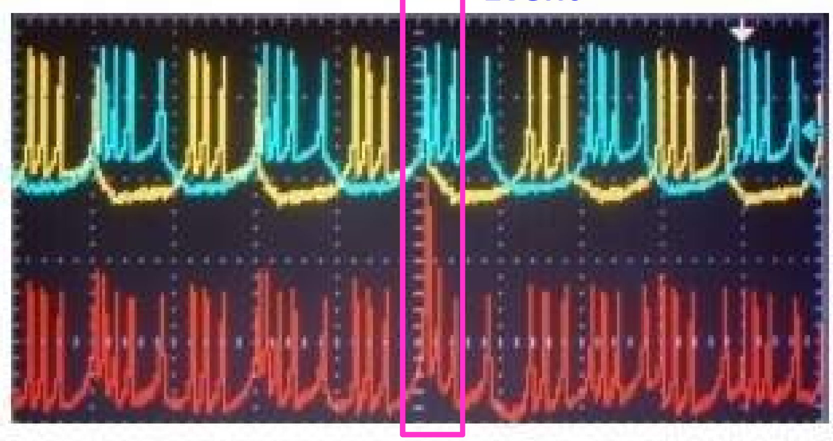








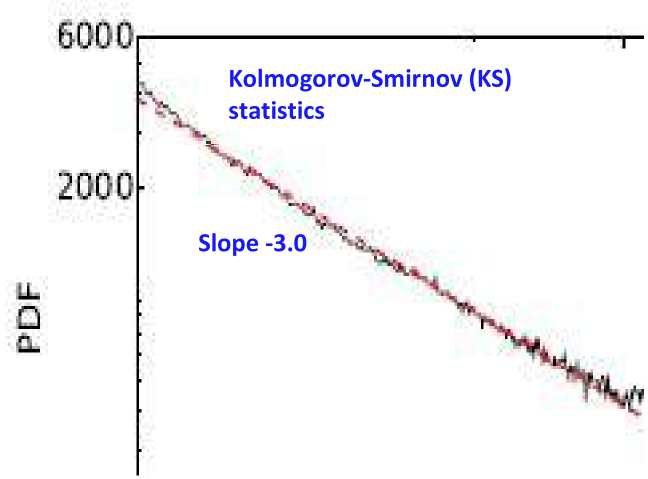
Event



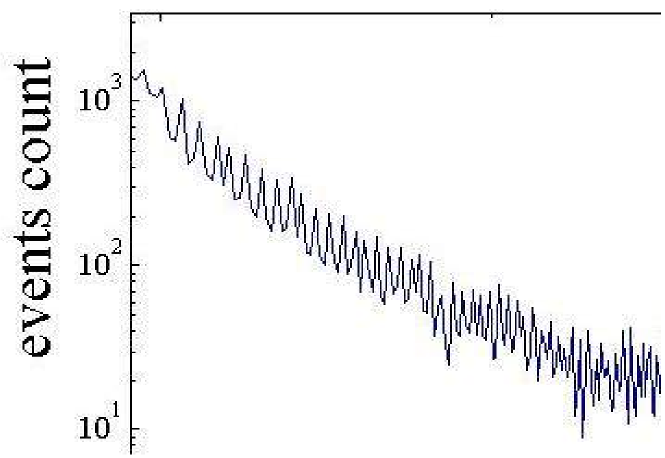
Quasiperiodicity route to chaos

Dragon-king Behavior

Numerical Result



Experimental Result



Summary

Extreme events

1. **Period-doubling to chaos followed by Interior Crisis**
   (Liénard System, Laser systems)
2. **PM Intermittency followed by Interior Crisis**
   (Liénard System, coupled Neurons)
3. **Quasiperiodic breakdown to chaos Crisis!!**
   (Coupled Neurons)

Future direction

How statistical distribution varies with different routes?