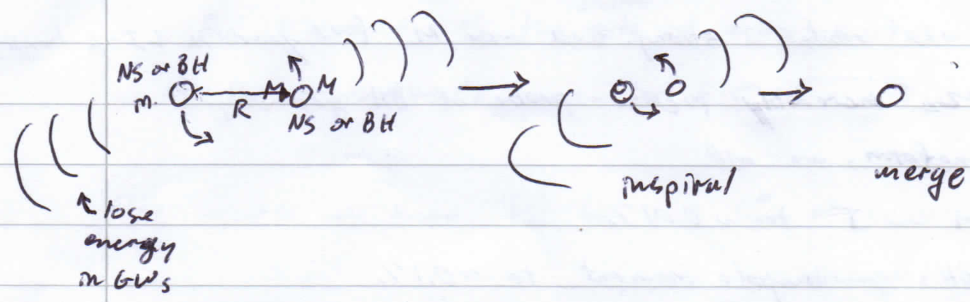
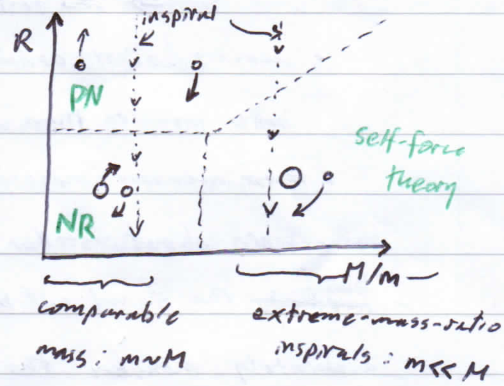
Introduction

Compact binaries: premiere sources of GWs



These binaries come in a few varieties:



Different types are observable by different detectors:
The frequency of a binary is $f \sim \sqrt{\frac{m+M}{R^3}}$

For simplicity, take $R \sim m+M \Rightarrow f \sim \frac{1}{M+m}$ or $f \sim \frac{c^3}{G(M+m)}$ (you can check this has dimension of 1/T

Ground-based detectors like LIGO & Virgo are sensitive to signals with $f \sim 100$ Hz

$\Rightarrow (M+m) \sim 10^{-2}s\,(c^3/G)$

$\sim 20\,M_\odot$

~~$\frac{1}{M} \sim 1kHz$~~
~~$\Rightarrow M \sim 10^{-3}s$~~
~~$\frac{GM}{c^3} \sim 10^{-3}s$~~
~~$\Rightarrow M$~~

Since m and M must be $\gtrsim M_\odot$ for a loud enough signal, LIGO can only detect comparable-mass binaries,

~~$\frac{c^3}{G} = \frac{[L]^3/[T]^3}{[L]^3/[M]/[T]^2} = [M]$~~

However, LISA, the space-based observatory, will be sensitive to signals with $f \sim 10^{-3}$ Hz

$\Rightarrow (M+m) \sim 10^5\,M_\odot$

$\Rightarrow$ LISA can detect comparable-mass binaries where $m \sim M \sim 10^5\,M_\odot$

__and__ EMRIs with $m \sim M_\odot$ and $M \sim 10^5\,M_\odot$

~~$\frac{10^{-2}\;3^3\times10^{24}\,m^3/s^3\,s}{6.7\times10^{-11}\,m^3/kg/s^2} \sim \frac{2.7\times10^{22}\,kg}{7\times10^{-11}} \sim 4\times10^{32}\,kg \sim 20\,M_\odot$~~

EMRIs :  M is a supermassive BH at centre of a galaxy ; m is a NS or BH

- orbit is very intricate : highly eccentric, inclined, triperiodic
- m spends $\sim \frac{M}{m} \sim 10^5$ orbits in strong field near M (~1 year in LISA band)

  $\Rightarrow$ precise probe of BH geometry
- from EMRI waveform, we will
  - measure M and J to $\sim 0.01\%$
  - measure BH's quadrupole moment to $\sim 0.1\%$

    $\Rightarrow$ test for deviations from Kerr geometry
  - measure other deviations from GR with order-of-magnitude better precision than other experiments
  - measure "mass function" $N(M)$
  - learn about stellar dynamics in galactic cores
- But! the signal will be buried under noise. To extract it and accurately measure the EMRI's parameters, we need our model of the waveform to agree with the buried signal's phase to $\ll 1$ radian over the EMRI's $\sim 10^5$ cycles

Modeling :

- Can't use PN because the system is highly relativistic
- can't use NR " we need so many orbits (also difficult to resolve different length scales)

We use BH perturbation theory / self-force theory :

- expand the metric as $g_{\alpha\beta} = g_{\alpha\beta} + \epsilon h^{(1)}_{\alpha\beta} + \epsilon^2 h^{(2)}_{\alpha\beta} + \mathcal{O}(\epsilon^3)$ we'll often think of $M \sim \epsilon^0$ and $m \sim \epsilon$

  where $g_{\alpha\beta}$ is the metric of the large BH, and $\epsilon = m/M$
- Say m traces out a worldline γ, with coords z^μ, in the Kerr background.

  At zeroth order, γ is a geodesic of $g_{\alpha\beta}$

  At subleading orders, it will have an EOM $\underbrace{\frac{D^2 z^\alpha}{d\tau^2}}_{\text{covariant acceleration wrt } g_{\alpha\beta}} = \epsilon f^\alpha_{(1)} + \epsilon^2 f^\alpha_{(2)} + \mathcal{O}(\epsilon^3)$

How high order must we go?

m loses kinetic energy at the rate that GWs carry energy out of the system (from conservation of energy)

⇒ $\dot{E} \sim$ GW energy flux $\sim \dot{h}\dot{h} \sim \epsilon^2$

But $E \sim m \sim \epsilon$ ⇒ the relative rate of energy loss is $\dot{E}/E \sim \epsilon$

⇒ the characteristic timescale of the inspiral is $\sim E/\dot{E} \sim 1/\epsilon$ (or M^2/m)

called "radiation-reaction" time t_{rr}

Now suppose we know $f^\mu_{(1)}$ but not $f^\mu_{(2)}$ ⇒ error $\delta \frac{D^2 z^\mu}{d\tau^2} \sim \epsilon^2$

⇒ error $\delta z^\mu \sim \epsilon^2 \tau^2$

⇒ after a time t_{rr}, $\delta z^\mu \sim \epsilon^2/\epsilon^2 \sim 1$

In particular, this error occurs in the orbital phase

⇒ it occurs in the GW phase

Since we need $\delta(\text{phase}) \ll 1$, this error is unacceptable

⇒ we need to ~~go to~~ include terms of order ϵ^2 in our calculations (but not ϵ^3)

I'll try to cover most of the core topics involved in EMRI modeling, from foundations to concrete calculations.

For more info, see the recent review article by Barack & Pound (arXiv:1805.1038

$$\partial^i \partial_i \bar{h}^{R}_{tt} = -16\pi\, \delta^3(x-z) - \partial^i\partial_i \bar{h}^{P}_{tt}$$

$$\Rightarrow \cancel{-\frac{\ell(\ell+1)}{r^2} \bar{h}^{R}_{tt}} = -\frac{16\pi}{r_0^2} Y^*_{\ell m}(\theta_0,\phi_0)\, \delta(r-r_0) - \cancel{\partial\partial}$$

$$\int \varphi^{\alpha\beta} E_{\alpha\beta}[h^{(1)}]\, dV = \int \varphi^{\alpha\beta} \partial^i\partial_j h^{(1)}_{\alpha\beta}\, dV = \lim$$

$$\bar{h}_{tt} = \frac{4m}{|x^i - x_p^i|} \Rightarrow \bar{h}^{\ell m}_{tt} =$$

$$\partial^i\partial_i \bar{h}_{tt} = 0 \quad \text{and} \quad \bar{h}_{tt} = \frac{\bar{h}^{(-1)}_{tt}}{r} + \bar{h}^{(0)}_{tt} + \dots$$

$$E_{\alpha\beta}[h](\varphi) = \lim_{\lambda \to 0} \int_r$$

$$\bar{h}_{tt} = \frac{4m}{r} + \bar{h}^{R}_{tt}$$

$$\bar{h}_{ti} = \bar{h}^{R}_{ti}$$

$$\bar{h}_{ij} = \bar{h}^{R}_{ij}$$

$$\bar{h}^{S,\ell m}_{tt} = \frac{16\pi m}{(2\ell+1)} Y^*_{\ell m}(\theta_0,\phi_0) \underbrace{\frac{r_<^{\ell}}{r_>^{\ell+1}}}_{}$$

$$r > r_0 : \frac{r_0^{\ell}}{r^{\ell+1}} \sim \frac{1}{r^{\ell+1}} \qquad \frac{r^\ell}{r_0^{\ell+1}}\theta(r_0 - r) + \frac{r_0^\ell}{r^{\ell+1}}\theta(r - r_0)$$

$$r < r_0 : \frac{r^\ell}{r_0^{\ell+1}}$$

$$\left(\partial_r^2 + \frac{2}{r}\partial_r - \frac{\ell(\ell+1)}{r^2}\right) \bar{h}^{S,\ell m}_{tt} \Big|_{\text{pointwise}} = 0 \quad \forall r \neq r_0 \Rightarrow S_{\text{eff}} = 0$$

$$\Rightarrow \left(\partial_r^2 + \frac{2}{r}\partial_r - \frac{\ell(\ell+1)}{r^2}\right) \bar{h}^{R,\ell m}_{\alpha\beta} = 0 \quad \forall r$$

$$\Rightarrow \bar{h}^{R,\ell m}_{\alpha\beta} = r^\ell A^{\ell m}_{\alpha\beta} + \frac{1}{r^{\ell+1}} B^{\ell m}_{\alpha\beta}$$

regularity at $r=0 \Rightarrow B=0$

decay at $r \to \infty \Rightarrow A = 0$

so $\bar{h}^{R}_{\alpha\beta} = 0$