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Top-hat Spherical Collapose Model

Spherical density perturbation with constant over density



The equation of motion is very simple:

$$\frac{d^2r}{dt^2} = -GM(<\!r)/r^2$$

$$M(< r) = \frac{4}{3}\pi r^{3}(t)\rho(t)[1+\delta(t)]$$

Solutions:

$$r = a(1 - \cos \chi),$$

$$t = \left(\frac{a^3}{GM}\right)^{1/2} (\chi - \sin \chi),$$

Top-hat Spherical Collapose Model

$$r = a(1 - \cos \chi),$$
$$t = \left(\frac{a^3}{GM}\right)^{1/2} (\chi - \sin \chi),$$

(i) expansion $0 < \chi < \pi$

(ii) reaching maximum (trun-around, t_{ta}), r_{max} , at $\chi = \pi$

(iii) collapsing back between $\pi < \chi < 2\pi$

(iv) final stage at $\chi = 2\pi$ (t_{col} = 2 t_{ta})

In EdS universe:

$$\delta_{ta} \sim 5.55$$
 and $\delta_{col} = infinity$

Top-hat Spherical Collapose Model

More-realistic: system virializes

Virial theorem :

$$r_{vir} = r_{ta} / 2$$

In EdS universe system the above condition yields: $1+\delta_{vir} \approx 178$ (note: $\delta_{vir} = \Delta_{vir}$)

In non EdS universe with Lamda:

 $1 + \delta_{vir} \approx 360$

Another common adopted vallue: $1 + \delta_{vir} = \rho/\rho_{crit} = 200$



Power spectrum



Damping scale (
$$\lambda$$
) ~ m_p⁻¹

Hot dark matter : Relativistic at early times $m_p \sim 10 \text{ ev} \text{ (neutrinos)}$ $\lambda \sim 10^{15} \text{ M}_{sol} \text{ (galaxy clusters)}$

Cold dark matter: Non-relativistic at early times $m_p \sim 1 \text{ Gev}$ $\lambda \sim \text{ earth mass}$

warm dark matter: $m_p \sim 1 \text{ kev}$ $\lambda \sim \text{ massive galaxies}$

Hot dark matter (HDM) Clusters collapse first: top-down formation of structure

Cold dark matter (CDM) Dwarf galaxies collapse first: bottom-up (hierarchical) formatio

Numerical simulations and large scales

CfA redshift survey



Copyright SAO 1998





CfA redshift Survey sky projection

HDM sims.





CDM sims.

Millennium simulation



Virgo Consortium

Millennium simulation series





Millennium Sim. II



Assigning galaxies to dark matter halos based on semianalytic models

> Virgo Consortium Springel+2005 Boylan-Kolchin+2009

Millennium simulation fly-through





- cosmic web structure Voids Fillaments Sheets
- Bigger objects form at the intersection of fillaments
- Clumpiness
- Hierarchical growth ...
- Self-similarity ...

Aquarius project: simulations of Milky Way like halos



Springel et al. 2008 – Virgo Consortium









CDM self-similarity

One is a Milky-Way size halo $(mass \sim 10^{12} M_{sol});$ another a galaxy cluster halo $(mass \sim 10^{14} M_{sol})$



CDM self-similarity

Substructures mass function



density profile (mass profile)



Navarro, Frenk, White 1996, 1997

Wang+2012

Dark matter halo density profile in CDM

Density profiles of halos spanning 4 orders of magnitude in mass



Navarro, Frenk, White (NFW) 1996, 1997

Neto+2007, Maccio+2008, Ludlow+2014, Ludlow+2017

NFW profile

$$\frac{\rho(r)}{\rho_{\rm crit}} = \frac{\delta_c}{(r/r_s)(1 + r/r_s)^2}$$

Two-parmeters profile: r_s, ρ_s

M₂₀₀, C

Mass-concentration relation:



Halo mass function in LCDM



Increasing number of small structures

Dark matter



100 Mpc

Gas on large scales



color=temperature

Eagle simulation - Schaye et al. 2016

Gas condensation in the centre of potential of halos (White & Rees 1978)



Gaseous and stellar disk

Sizes are not proportional; e.g for a Milky-Way size galaxy: Stellar disk ~ 20 kpc; halo virial radius ~ 250kpc Dark matter is collisionless: Collapses into a spherical component

Gas is collisional:

momentum

loses energy due to friction, radiation, etc, Settles into a disk due to the conservation of angular

The EAGLE simulations

EVOLUTION AND ASSEMBLY OF GALAXIES AND THEIR ENVIRONMENTS A project of the Virgo consortium

Visible components: CDM

Credit: EAGLE collaboration – Virgo Consortium

Stellar halo and stellar streams



Bullock & Johnston 2005



NORTHERN SKY

MW stellar halo streams (Credit: A. Bonaca)



Andromeda stellar halo; PAndAS (McConnachie+2008)

10² M - II10 Millennium MXXL [Mpc³] 10 h³ Mpc⁻³ log₁₀M⁻¹* 10 Mb/nb 10-6 10 Σ 10⁻¹⁰ SDSS DR7 -6 log₁₀(¢ z=0triple Schechter 10^{-12} -7 single Schechter -81.5 0.2 N_{N-body}/N_{fit} (the put) 0.1 og₁₀($\Phi_{\rm spss}/$ 0.5 0.1 10¹⁰ 1014 10^{12} 10^{16} 10^{8} M M_o $\log_{10}(M_{\star} / h^{-2} M_{\odot})$

Halo mass function

Stellar mass function

& White 2009

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Note the difference in the slopes of HMF and SMF at both ends: Galaxy formation effeciency is different at different scales; different mechanisms play role in galaxy formation



Side note: Cosmic reionization important at smaller scales: $M_{halo} \sim 10^{10} M_{sol}$

"Abundance Matching" as a powerfull tool:

Assuming there is a one-to-one and monotonic relation between stellar mass and halo mass

Frenk+1983, Guo+2010, Behroozi+2013, Moster+2013





L^{*} galaxies (10¹² M_{sol} halos) are most efficient in retaining their baryons and turning them into stars!



Side note: Cosmic reionization important at smaller scales: $M_{halo} \sim 10^{10} M_{sol}$

EAGLE (Schaye+2015)











Ellipticals









Disk galaxies



Romeo



FIRE – (Garrison-Kimmel+2018)

Illustris (Vogelsberger+2014)