A generalized non-Gaussian consistency relation for single field inflation

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Outline

- Motivation.
- Slow roll single field inflation & the consistency relation.
- Ultra-slow roll Inflation & breakdown of the consistency relation.
- Generalization of the consistency relation.
- An application: the vanishing of the bispectrum.
- Conclusions.

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Motivation

The scale-dependent halo bias

$$\Delta b(k) = 2(b-1) f_{NL}^{loc} \delta_c \frac{3\Omega_m}{2ag(a) r_H^2 k^2}$$

Seljak (2008)
Dalal, Doré, Huterer & Shirokov (2008)
Chan, Hamaus & Biagetti (2018)

Sensitive probe of primordial non-Gaussianity from LSS surveys

f_{NL}^{loc} ⟷ Initial conditions

Motivation

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Sensitive probe of primordial non-Gaussianity from LSS surveys

f_{NL}^{loc} ⟷ Inflation 🙄

What kind of inflationary models can produce a detectable amount of local non-Gaussianity?

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Slow roll inflation: background

Inflation → $$\frac{d}{dt}(aH)^{-1} < 0$$ ↔ $$\epsilon \equiv -\frac{\dot{H}}{H^2} < 1$$ Expansion

$$\eta \equiv \frac{\dot{\epsilon}}{H\epsilon} < 1$$ Duration of the expansion

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$$

Slow roll → $$3H\dot{\phi}_0 + \partial_{\phi_0}V(\phi) = 0$$

Attractor solution

Slow roll inflation: perturbations

ADM formalism for the metric **Arnowitt, Deser & Misner (1959)**

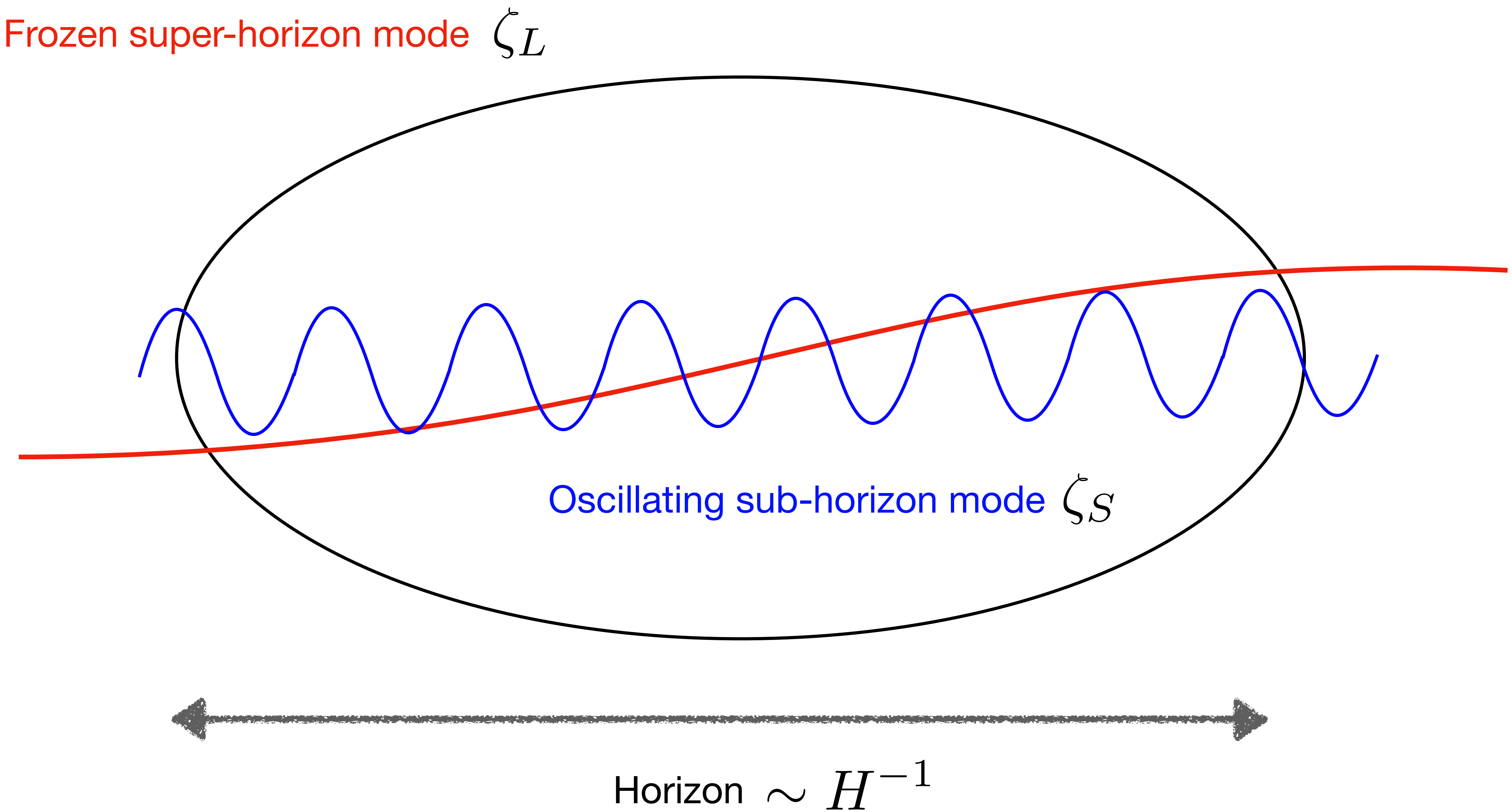
$$ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$\gamma_{ij} \equiv a^2(t)(1 + 2\zeta(t, \mathbf{x}))\delta_{ij}$$

$$\delta N = \frac{1}{\mathcal{H}}\partial_0\zeta \qquad N_i = -\frac{1}{\mathcal{H}}\partial_i\zeta + \epsilon\frac{\partial_i}{\partial^2}\partial_0\zeta$$

$$S_2 = \int d\tau d^3x a^2\epsilon((\zeta')^2 - (\partial\zeta)^2)$$

Sub- and super-horizon modes of ζ



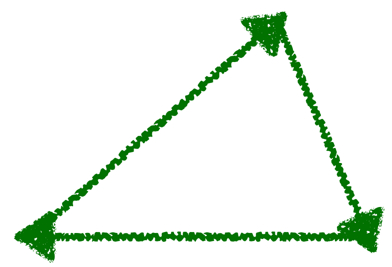
Observables: correlation functions

The two point function

$$\langle 0|\zeta(\vec{k}_1)\zeta(\vec{k}_2)|0\rangle \equiv (2\pi)^3\delta(\vec{k}_1+\vec{k}_2)P_\zeta(k_1)$$

$$\frac{k^3}{2\pi^2}P_\zeta(k) = A_s\left(\frac{k}{k_0}\right)^{n_s-1}$$

The three point function



$$\langle 0|\zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)|0\rangle \equiv (2\pi)^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)B_\zeta(k_1,k_2,k_3)$$

The squeezed limit $k_3 \to 0$ in the local configuration

$$f_{NL}^{loc} = -\frac{5}{12}\lim_{\mathbf{k}_3\to 0}\frac{B_\zeta(k_1,k_2,k_3)}{P_\zeta(k_1)P_\zeta(k_3)} \qquad P_\zeta(\mathbf{k}) = \frac{H^2}{8\pi^2\epsilon M_{Pl}^2}$$

The Maldacena's consistency relation

$$\lim_{\mathbf{k}_3 \to 0} \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = -(2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)(n_s - 1) P_\zeta(k_L) P_\zeta(k_S)$$

Maldacena (2003)

Obtained from the cubic order action for curvature perturbations

$$S_3 = \int dt d^3x \Big\{ a^3 \epsilon^2 \zeta \dot{\zeta}^2 + a\epsilon^2 \zeta (\partial\zeta)^2 - 2a\epsilon \dot{\zeta} \partial\zeta \partial\chi + \frac{a^3\epsilon}{2} \dot{\eta} \zeta^2 \dot{\zeta} + \frac{\epsilon}{2a} \partial\zeta \partial\chi \partial^2\chi + \frac{\epsilon}{4a} \partial^2\zeta (\partial\chi)^2 + 2f(\zeta) \left.\frac{\delta \mathcal{L}}{\delta \zeta}\right|_1 + \mathcal{L}_b \Big\} , \qquad \partial^2\chi \equiv a^2 \epsilon \dot{\zeta}$$

Indicates how two short modes are modulated by the long mode

A crucial test for all single field attractor models of inflation

$$\boxed{f_{NL}^{loc} = \frac{5}{12}(n_s - 1)} \sim 0.014$$

$$f_{NL}^{loc} = 0.8 \pm 5.0 \quad (68\% CL)$$

Planck collaboration (2015)

Symmetry based derivation of the consistency relation

$$ds^2 = a^2(\tau)[-e^{\delta N} d\tau^2 + 2N_i dx^i d\tau + e^{2\zeta} dx^2]$$

The metric is invariant under

$$x_i \to e^{-\zeta_L} x_i'$$

$$\tau \to \tau'$$

$$\zeta(\tau, \mathbf{x}) \to \zeta'(\tau', \mathbf{x}') + \zeta_L$$

Same solution as ζ

If we split $\zeta(\tau, \mathbf{x}) = \zeta_S(\tau, \mathbf{x}) + \zeta_L$ $\longrightarrow$ $\zeta_S(\tau, \mathbf{x}) = \zeta'(\tau, e^{\zeta_L}\mathbf{x})$

let's correlate two short modes

$$\langle \zeta_S(\tau, \mathbf{x}) \zeta_S(\tau, \mathbf{y}) \rangle = \langle \zeta' \zeta' \rangle (\tau, e^{\zeta_L} |\mathbf{x} - \mathbf{y}|)$$

expanding in powers of the long mode

$$\langle \zeta_S(\tau, \mathbf{x}) \zeta_S(\tau, \mathbf{y}) \rangle = \langle \zeta' \zeta' \rangle (\tau, |\mathbf{x} - \mathbf{y}|) + \zeta_L \frac{d}{d \ln |\mathbf{x} - \mathbf{y}|} \langle \zeta' \zeta' \rangle (\tau, |\mathbf{x} - \mathbf{y}|) + \ldots$$

in Fourier space

$$\langle \zeta_S \zeta_S \rangle (\mathbf{k}_1, \mathbf{k}_2) = \langle \zeta' \zeta' \rangle (\mathbf{k}_1, \mathbf{k}_2) - \zeta_L(\mathbf{k}_L)(n_s - 1) P_\zeta(k_S)$$

Symmetry based derivation of the consistency relation

Correlating with a long mode $\zeta_L(\mathbf{k}_3)$

$$\langle \zeta_L(\mathbf{k}_3) \langle \zeta_S \zeta_S \rangle (\mathbf{k}_1, \mathbf{k}_2) \rangle = -\langle \zeta_L(\mathbf{k}_3) \zeta_L(\mathbf{k}_L) \rangle (n_s - 1) P_\zeta(k_S)$$

Creminelli & Zaldarriaga (2004)

The squeezed limit appear as

$$\lim_{\mathbf{k}_3 \to 0} (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\zeta(k_1, k_2, k_3) = \langle \zeta_L(\mathbf{k}_3) \langle \zeta_S \zeta_S \rangle (\mathbf{k}_1, \mathbf{k}_2) \rangle$$

then

$$B_\zeta(k_1, k_2, k_3) = -(n_s - 1) P_\zeta(k_L) P_\zeta(k_S)$$

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Ultra-slow roll inflation: $V(\phi) = V_0$

Mechanism to generate **P**rimordial **B**lack **H**oles

Germani & Prokopec (2017)
Biagetti, Franciolini, Kehagias & Riotto (2018)
Atal & Germani (2018)

The curvature perturbation satisfy the adiabaticy condition and evolves on super-horizon scales

$$\zeta \propto a^3$$

Mooij & Palma (2015)
Romano, Mooij & Sasaki (2016)

Non-attractor solution: the background depends on the initial conditions

Almost scale invariant $n_s \simeq 1$

Realistic? Transition between ultra-slow roll and slow roll? **Cai, Chen, Namjoo, Sasaki, Wang & Wang (2018)**

Why the model is interesting?

Breakdown of the consistency relation

For USR

Computed a la Maldacena

$$B_\zeta(k_1, k_2, k_3) = 6P_\zeta(k_L)P_\zeta(k_S)$$

Chen, Firouzjahi, Namjoo & Sasaki (2013)
Namjoo, Firouzjahi & Sasaki (2013)

$$B_\zeta(k_1, k_2, k_3) \neq -(n_s - 1)P_\zeta(k_L)P_\zeta(k_S)$$

$$f_{NL}^{loc} = \frac{5}{2}$$

Or

$$B_\zeta(k_1, k_2, k_3) = -(n_s - 1)P_\zeta(k_L)P_\zeta(k_S) + (\quad ? \quad)$$

Attractor — Non-attractor

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Again: symmetry based derivation

$$ds^2 = a^2(\tau)[-e^{\delta N} d\tau^2 + 2N_i dx^i d\tau + e^{2\zeta} dx^2]$$

The metric and the cubic order action are invariant under

$$x_i \to e^{-\zeta_L(\tau_*)} x_i'$$

$$\tau \to e^{(\zeta_L(\tau') - \zeta_L(\tau_*))} \tau'$$

$$\zeta(\tau, \mathbf{x}) \to \zeta'(\tau', \mathbf{x}') + \zeta_L(\tau')$$

splitting $\zeta(\tau, \mathbf{x}) = \zeta_S(\tau, \mathbf{x}) + \zeta_L(\tau)$ → $\boxed{\zeta_S(\tau, \mathbf{x}) = \zeta'(e^{-[\zeta_L(\tau) - \zeta_L(\tau_*)]}\tau, e^{\zeta_L(\tau_*)}\mathbf{x})}$

if $\zeta_L(\tau)$ does not evolve, then $\zeta_L(\tau) = \zeta_L(\tau_*)$ and we recover the attractor result!

correlating, expanding in time and space, go to Fourier

$$\langle \zeta_S \zeta_S \rangle(\mathbf{k}_1, \mathbf{k}_2) = \langle \zeta' \zeta' \rangle(\mathbf{k}_1, \mathbf{k}_2) - [\zeta_L(\mathbf{k}_L) - \zeta_L^*(\mathbf{k}_L)] \frac{d}{d\ln\tau} P_\zeta(\tau, k_S)$$

$$-\zeta_L^*(\mathbf{k}_L)(n_s - 1) P_\zeta(\tau, k_S)$$

correlating with the long mode, one has

The generalized consistency relation

$$\langle \zeta_L(\mathbf{k}_3)\langle \zeta_S \zeta_S \rangle(\mathbf{k}_1, \mathbf{k}_2)\rangle = \langle \zeta_L(\mathbf{k}_3)[\zeta_L(\mathbf{k}_L) - \zeta_L^*(\mathbf{k}_L)]\rangle \frac{d}{d\ln\tau} P_\zeta(\tau, k_S)$$
$$- \langle \zeta_L(\mathbf{k}_3)\zeta_L^*(\mathbf{k}_L)\rangle (n_s - 1) P_\zeta(\tau, k_S)$$

When super horizon modes freeze, we end up with the Maldacena's standard attractor result.

If $\zeta_L(\tau)$ grows on super horizon scales fast enough for $\zeta_L(\tau_*)$ to become subdominant

$$B(k_1, k_2, k_3) = -P_\zeta(k_L) \frac{d}{d\ln\tau} P_\zeta(k_S)$$

RB, Mooij, Palma & Pradenas (2017)
Finelli, Goon, Pajer & Santoni (2017)

Under a substantial super-horizon growth, the squeezed limit is dominated by a time derivative of the power spectrum.

$$\zeta \propto \tau^{-3} \longrightarrow P_\zeta \propto \tau^{-6}$$

$$B_\zeta(k_1, k_2, k_3) = 6 P_\zeta(k_1) P_\zeta(k_2)$$

USR as an exact symmetry $\epsilon \neq 0$

In this case, one has to consider the full metric

$$ds^2 = a^2(\tau)[-e^{\delta N} d\tau^2 + 2N_i dx^i d\tau + e^{2\zeta} dx^2]$$

the change of coordinates

$$x_i \to e^g x_i'$$

$$a(\tau) \to e^f a(\tau')$$

$$\zeta \to \zeta' + \Delta\zeta$$

leads

$$ds^2 = a^2(\tau')[-e^{\delta N'} d\tau'^2 + 2(N_i' + \Delta N_i) dx'^i d\tau' + e^{2\zeta'} dx'^2]$$

$$\Delta N_i = -\epsilon \mathcal{H} f x_i' + \frac{1}{3} x_i' \epsilon (\epsilon \mathcal{H} f + \partial_0 f)$$

USR as an exact symmetry $\epsilon \neq 0$

The metric (and the action) will not be invariant unless $\Delta N_i = 0$

the difference with the original metric is of order ϵ

this condition could be achieved independent of the size of ϵ

$$\Delta N_i = 0 \iff \partial_0(a^{-2}\mathcal{H}^{-1} f) = 0$$

$f = C\mathcal{H}a^2$ ⟶ must be compatible with $\Delta\zeta = \zeta_L$

the condition is possible only if $\dot{\zeta}_L = 3CH^2a^3 \propto \dfrac{1}{\epsilon a^3}$

which is in agreement with the linear equation for ζ_L on super-horizon scales!

$$\frac{d}{dt}\left(\epsilon a^3 \dot{\zeta}_L\right) = 0$$

The transformation of coordinates is an exact symmetry of the metric for the USR curvature perturbation

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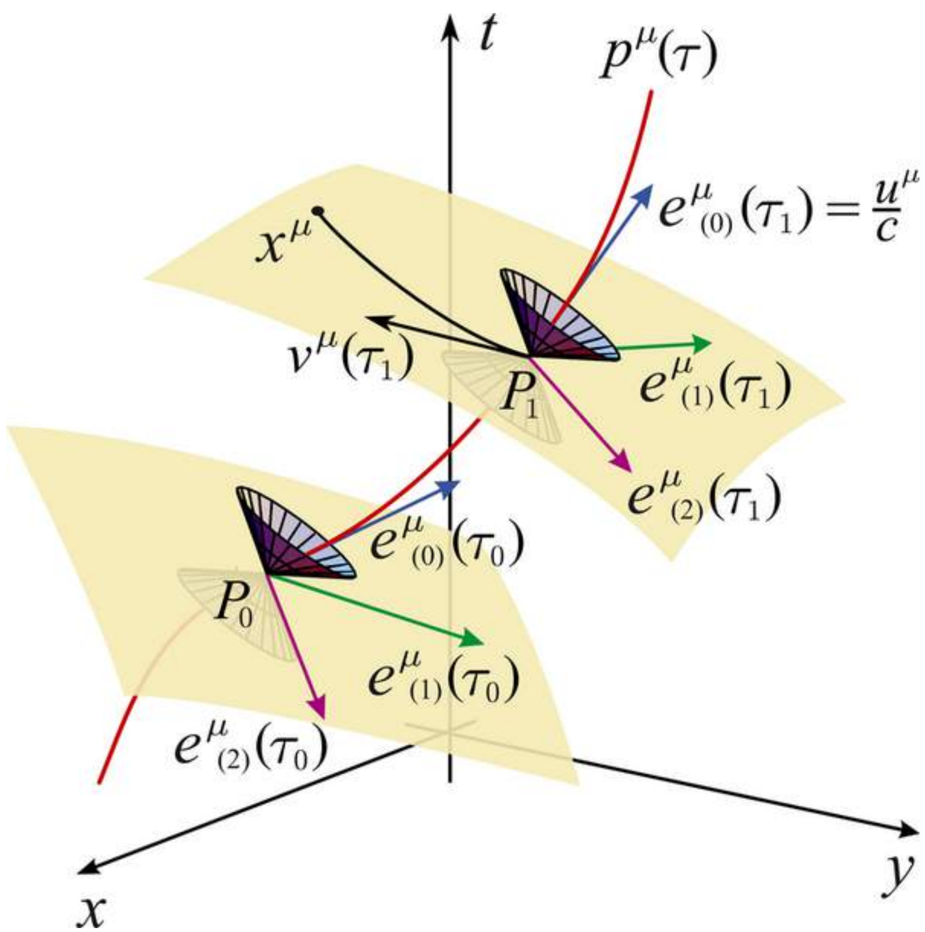
Conformal Fermi Coordinates

It is possible to construct a coordinate system in which the observers can measure genuine observable quantities ?



- This coordinates parametrize the local environment of observers on a FLRW spacetime

$$g^F_{00} = a^2_F(\tau_F)\left[-1 - R_{0k0l}\; x^k_F x^l_F + \mathcal{O}(x^3_F)\right]$$

$$g^F_{0j} = a^2_F(\tau_F)\left[-\frac{2}{3}R_{0kjl}\; x^k_F x^l_F + \mathcal{O}(x^3_F)\right]$$

$$g^F_{ij} = a^2_F(\tau_F)\left[\delta_{ij} - \frac{1}{3}R_{ikjl}\; x^k_F x^l_F + \mathcal{O}(x^3_F)\right]$$

Slow roll $\quad x^i = (1-\zeta_L)\bar{x}^i \quad \tau = \bar{\tau}$

Ultra-slow roll $\quad x^i = \bar{x}^i \quad \tau = (1+\zeta_L)\bar{\tau}$

$$+\mathcal{O}(\bar{x}^2)$$

Vanishing of the bispectrum

The three point correlation function in Conformal Fermi Coordinates

$$\langle \bar{\zeta}_L(\mathbf{k}_3)\langle \bar{\zeta}_S \bar{\zeta}_S \rangle(\mathbf{k}_1, \mathbf{k}_2)\rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$
$$+ (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(\tau, k_3) \frac{\partial}{\partial \ln \tau} P_\zeta(\tau, k_S)$$
$$- \langle \zeta_L(\mathbf{k}_3) \zeta_*(\mathbf{k}_L)\rangle \left[\frac{\partial}{\partial \ln \tau} - [n_s(k_S, \tau) - 1] \right] P_\zeta(\tau, k_S)$$

$$B_{CFC} = B_{CM} + \Delta B$$

$$B_{CM} = -\Delta B$$

Slow roll → Tanaka & Urakawa (2011)
Pajer, Schmidt & Zaldarriaga (2013)

↑

Valid for all canonical single field models (attractor or not)

RB, Mooij, Palma & Pradenas (2017)

$$f_{NL}^{obs} = 0 + \mathcal{O}\left(\frac{k_L}{k_S}\right)^2$$

Symmetries!

The symmetries used to derive the consistency relations

$$x^i \to x'^i = x + \zeta_L x$$

$$\tau \to \tau' = \tau - \zeta_L \tau$$

The map between CFC and comoving coordinates

$$x^i \to \bar{x}^i = x - \zeta_L x$$

$$\tau \to \bar{\tau} = \tau + \zeta_L \tau$$

The modulation effect of the long mode is canceled by the re-scaling of the coordinates and it is independent of the behavior of the background!

Higher order terms

$$x^i = (1 - \zeta_L)\bar{x}^i - \bar{x}^j \partial_j \zeta_L \bar{x}^i + \frac{1}{2}\partial^i \zeta_L \bar{x}^2 + \mathcal{O}(\bar{x}^3) + \ldots$$

Dilation + Special Conformal Transformation

?

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Conclusions

- We have generalized the well known consistency relation for models where the curvature perturbation evolves on super horizon scales. For both models, attractor and non-attractor, the modulation of small modes by long ones, in comoving gauge, can be understood as the result of a symmetry.

- We have showed that the observable squeezed bispectrum vanishes at leading order, for all canonical single field models of inflation.

- If tomorrow we measure a sizable amount of local non-Gaussianity, from where does it come from? Multi-field? Non BV vacuum models? Non-canonical kinetic terms?