'back' to physics

Approach: GR+Mag-Hydrodynamics

- Einstein equations
  - Generalized Harmonic formulation: $\nabla^a \nabla_a x^u = H^u$
  - Constraints : $C_a = \Gamma_a + H_a$
  - Einstein eqns: $R_{ab} = \nabla_{(a} C_{b)} + {}^{TR}T_{ab} + \kappa\{2n_{(a} C_{b)} - g_{ab} n^c C_c\}$
  - Expressed in terms of g_{ab}; H_a *from stationary solution.*

- GRHydro: Eqns determined by:

$$\nabla_a T^{ab} = 0 \quad ; \quad \nabla_a (\rho u^a) = 0$$

$$T_{ab} = (\rho_0(1+\varepsilon) + P) u_a u_b + P g_{ab} + F_a^c F_{bc} - \tfrac{1}{4} F^{cd} F_{cd}$$

  - Expressed in terms of *conservative variables, (use of HRSC)*

  e.g of simple eqn of state: $P = (\Gamma - 1)\rho_o \varepsilon \qquad (though,\ P = k\rho_0^{\Gamma}\ for\ ID)$

  - Tabulated, piece-wise polytropes, etc.

In explicit form

- 'Extended' ideal MHD equations

- Terms added for :
  - Divergence cleaning (c_r , c_h)
  - Ensure strong hyperbolicity even if no div cleaning used.
  - if no added field, 'eight' wave formulation. (but has sources with derivatives)

$$\partial_t \tilde{D} + \partial_i \left[\alpha \tilde{D}\left(v^i - \frac{\beta^i}{\alpha}\right)\right] = 0,$$

$$\partial_t \tilde{S}_j + \partial_i \left[\alpha\left(\tilde{S}_j\left(v^i - \frac{\beta^i}{\alpha}\right) + \sqrt{h}\, P\, h^i{}_j\right)\right]$$
$$= \alpha\, {}^3\Gamma^i{}_{jk}\left(\tilde{S}_i v^k + \sqrt{h}\, P h_i{}^k\right) + \tilde{S}_a \partial_j \beta^a$$
$$-\partial_j \alpha\,(\tilde{\tau} + \tilde{D})$$
$$-\zeta\alpha(\tilde{B}_i W^{-2} + v_i v_j \tilde{B}^j)\partial_k \tilde{B}^k,$$

$$\partial_t \tilde{\tau} + \partial_i \left[\alpha\left(\tilde{S}^i - \frac{\beta^i}{\alpha}\tilde{\tau} - v^i \tilde{D}\right)\right]$$
$$= \alpha\left[K_{ij}\tilde{S}^i v^j + \sqrt{h} K P - \frac{1}{\alpha}\tilde{S}^a \partial_a \alpha\right],$$
$$-\zeta\alpha v_j \tilde{B}^j \partial_k \tilde{B}^k$$

$$\partial_t \tilde{B}^b + \partial_i \left[\tilde{B}^b\left(v^i - \frac{\beta^i}{\alpha}\right) - \tilde{B}^i\left(v^b - \frac{\beta^b}{\alpha}\right)\right]$$
$$= -h^{bj}\partial_j \Psi - \alpha\sqrt{h} h^{ij}\partial_j \Psi - \zeta\alpha v^i \partial_j \tilde{B}^j$$

$$\partial_t \Psi = -c_r \alpha \Psi - c_h \frac{\alpha}{\sqrt{h}}\partial_i \tilde{B}^i + (\beta^i - \alpha v^i)\partial_i \Psi$$

Example 1: Electrovac (GR+EM) (e.g. plasmas)

- Einstein equations
  - Generalized Harmonic formulation: $\nabla^a \nabla_a x^u = H^u$
  - Constraints : $C_a = \Gamma_a + H_a$
  - Einstein eqns: $R_{ab} = \nabla_{(a} C_{b)} + {}^{TR}T_{ab} + \kappa \{ 2 n_{(a} C_{b)} - g_{ab} n^c C_c \}$
  - Expressed in terms of g_{ab}; H_a *(some 'good' prescription needed!)*

- Maxwell equations: (no currents)
  - Extended div. cleaning formulation:
  - Expressed in terms of E^i, B^i, ϕ, ψ

$$\nabla_a (F^{ab} + g^{ab}\psi) = -\chi n^b \psi$$
$$\nabla_a ({}^*F^{ab} + g^{ab}\phi) = -\chi n^b \phi$$

  - *Currents? Charges? … e.g. force-free*

- Even simpler, GR + scalar field (real or complex: e.g. axions)

$$\mathcal{L} = -\frac{1}{16\pi}R + \frac{1}{2}\left[g^{ab}\partial_a\bar{\phi}\partial_b\phi + V\left(|\phi|^2\right)\right] .$$

$$R_{ab} - \frac{1}{2}g_{ab}R = 8\pi T_{ab} \quad , \tag{2}$$

$$g^{ab}\nabla_a\nabla_b\Phi = \frac{dV}{d|\Phi|^2}\Phi \quad , \tag{3}$$

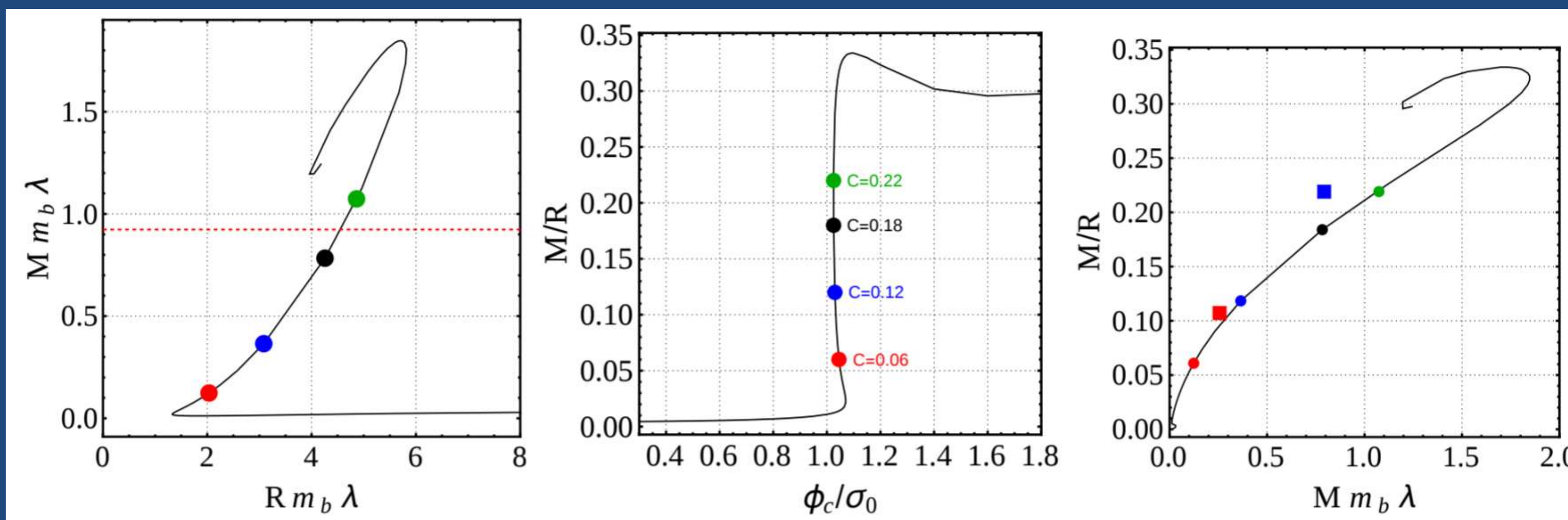
where T_{ab} is the scalar stress-energy tensor

$$T_{ab} = \nabla_a\Phi\nabla_b\Phi^* + \nabla_a\Phi^*\nabla_b\Phi - g_{ab}\left[\nabla^c\Phi\nabla_c\Phi^* + V\left(|\Phi|^2\right)\right] .$$

Alternate compact object: Boson Star

- $\lambda = \sigma_0\sqrt{8\pi}$

$$V\left(|\Phi|^2\right) = m_b^2|\Phi|^2\left[1 - \frac{2|\Phi|^2}{\sigma_0^2}\right]^2$$



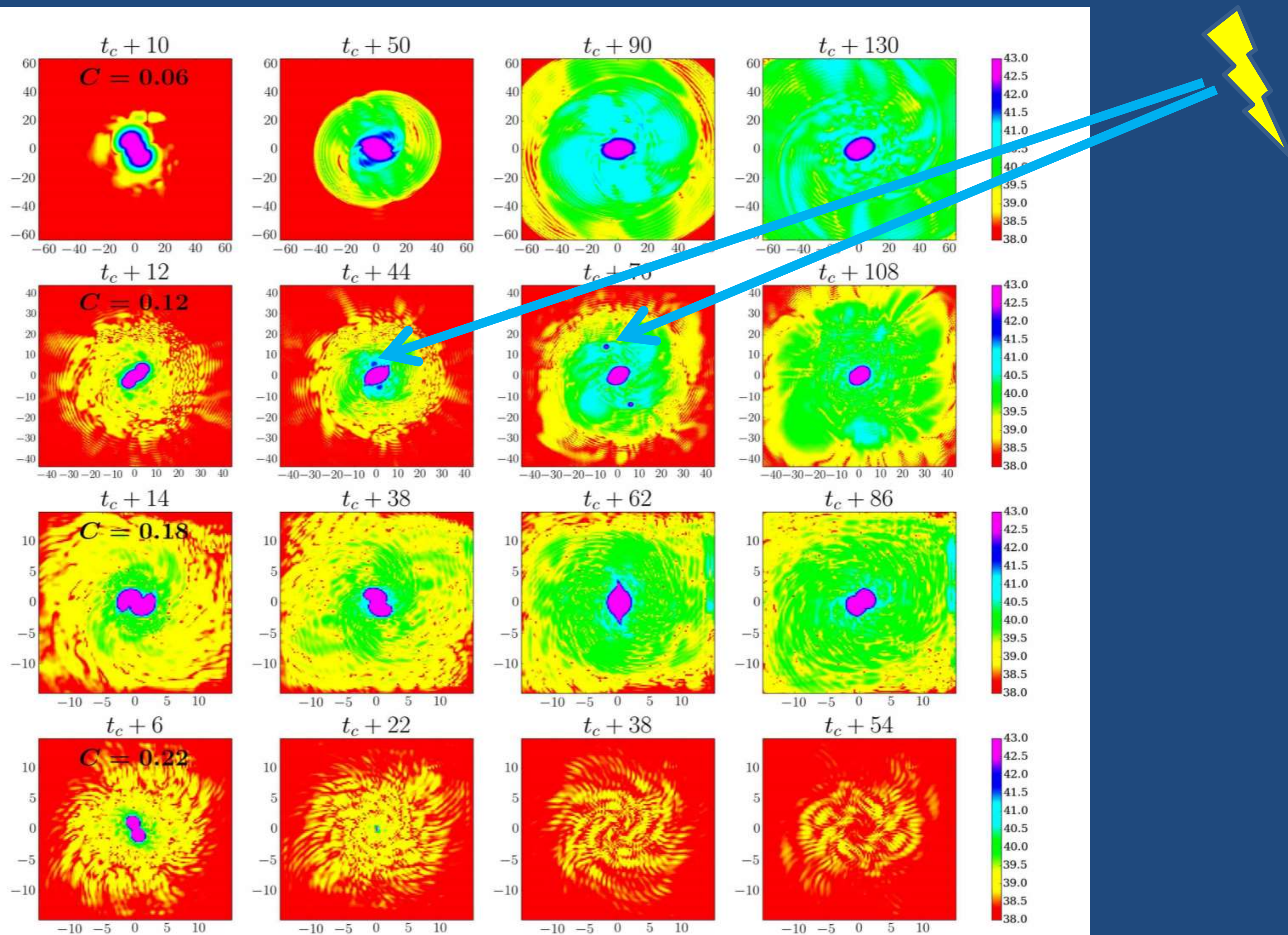


FIG. 3. *Coalescence of binary BSs*. Time snapshots of $||\phi||$ in the plane $z = 0$ in log-scale. Each row corresponds to the different coalescence of BSs cases studied here. In each case the scalar field emitted during the evolution increases after the merger.

Beyond GR ?

Beyond GR I?

- Restricting to theories known to allow for well-posed problems. I.e. those where one can show $|u(T)| \leq ae^{bt}|u(0)|$
- Few options known to be amenable to well defined initial (boundary) value problems. Examples: Scalar-Vector-Tensor theories.

Scalar-Tensor (ST) *{many incarnations}*

$$S = \int d^4x \frac{\sqrt{-g}}{2\kappa} \left[\phi R - \frac{\omega(\phi)}{\phi} \partial_\mu \phi \partial^\mu \phi \right] + S_M[g_{\mu\nu}, \psi]$$

$$S^{\mathrm{E}} = \int d^4x \sqrt{-g^{\mathrm{E}}} \left(\frac{M_{\mathrm{Pl}}^2}{2} R^{\mathrm{E}} - \frac{1}{2} \partial_\mu \varphi \, \partial^\mu \varphi - V(\varphi) \right) + S^{\mathrm{E,\,M}}[g^{\mathrm{E}}_{\mu\nu} \, \phi(\varphi)^{-1}, \psi]$$

- Scalar tensor

$$G^E_{\mu\nu} = \kappa\left(T^{\varphi}_{\mu\nu} + T^E_{\mu\nu}\right),$$

$$\Box^E\varphi = \frac{1}{2}\frac{\mathrm{d}\log\phi}{\mathrm{d}\varphi}T_E,$$

$$\nabla^E_\mu T^{\mu\nu}_E = -\frac{1}{2}T_E\frac{\mathrm{d}\log\phi}{\mathrm{d}\varphi}g^{\mu\nu}_E\partial_\mu\varphi,$$

where

$$T^{\mu\nu}_E = \frac{2}{\sqrt{-g^E}}\frac{\delta S_M}{\delta g^E_{\mu\nu}} \quad \text{and}$$

$$T^{\varphi}_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - \frac{g^E_{\mu\nu}}{2}g^{\alpha\beta}_E\partial_\alpha\varphi\partial_\beta\varphi$$

What's new? : *'phase' transitions*

- A non-trivial solution $\varphi \neq 0$ can develop for sufficiently dense objects and the asymptotic boundary condition on φ [Damour-Esposito-Farese '96]

- In 'gravitational terms' the scalar field endows stars with a 'scalar charge' that introduces several effects:
  - – Newtonian constant is renormalized $G_e \sim G\,(1+a_1\, a_2)$
  - – Dipolar radiation

$$\dot{E}_{\rm dipole} = \frac{G_N}{3c^3}\left(\frac{G_{\rm eff}m_1m_2}{r^2}\right)^2(\alpha_1-\alpha_2)^2+\mathcal{O}\left(\frac{1}{\omega_0}\right)$$

NSNS- but now in ST

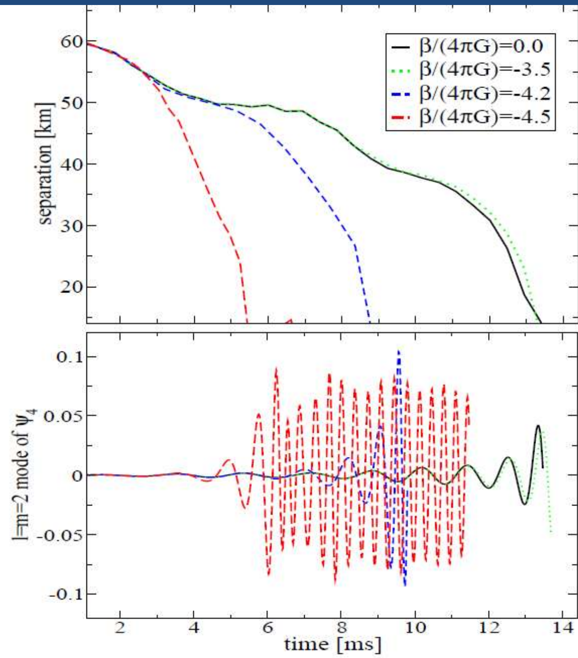


FIG. 1: The separation and the dominant mode of the ψ_4 scalar (encoding the effect of GWs) for a binary with gravitational masses $\{1.58, 1.67\}M_{\odot}$, and for different values of β.

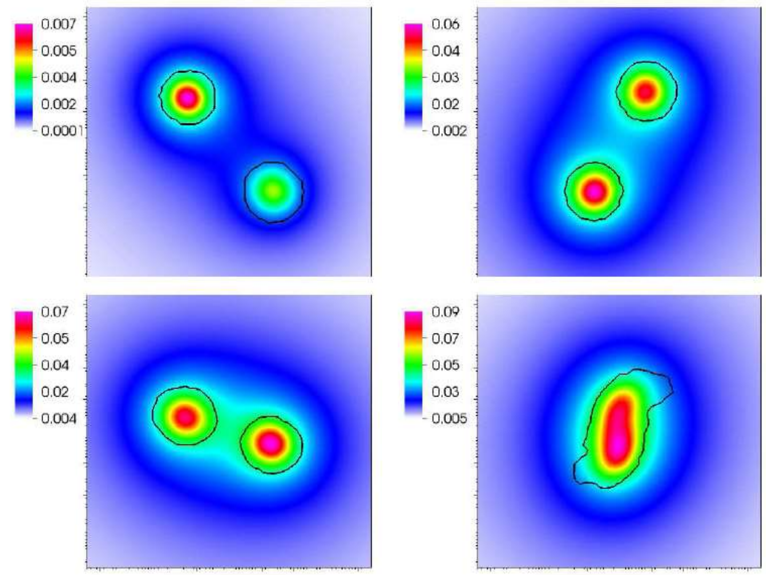


FIG. 2: The scalar field $\varphi G^{1/2}$ (color code) and the NS surfaces (solid black line) at $t = \{1.8, 3.1, 4.0, 5.3\}$ms for $\beta/(4\pi G) = -4.5$, and the binary of Fig. 1

- Dynamical/induced scalarization → non-monotonicity of scalar charges! *–pressure on parameterized deviations--*

What else? *(just some examples to discuss, not exhaustive at all!)*

- Beyond ScalarTensor, and a EMD theory, no full IMR for compact binaries done [extensions to GR]
- Beyond boson stars (and neutron stars) no full IMR compact binaries done [beyond BHs]

Is this really all?

$$S_{æ} = -\frac{1}{16\pi G_{æ}} \int \left(R + \frac{1}{3} c_\theta \theta^2 + c_\sigma \sigma_{\mu\nu}\sigma^{\mu\nu} + c_\omega \omega_{\mu\nu}\omega^{\mu\nu} + c_a a_\mu a^\mu \right) \sqrt{-g}\, d^4x$$

- Slowly rotating BHs in EinsteinAether theory (Barausse-Sotiriou+)

→ Lorentz violating theory! A further 'specialized' field to 'restore' gauge invariance

- *BHs in some quadratic gravity…*

$$S \equiv \int d^4x\sqrt{-g}\{\kappa R + \alpha_1 f_1(\vartheta)R^2 + \alpha_2 f_2(\vartheta)R_{ab}R^{ab} + \alpha_3 f_3(\vartheta)R_{abcd}R^{abcd} + \alpha_4 f_4(\vartheta)R_{abcd}{}^{*}R^{abcd} - \frac{\beta}{2}\left[\nabla_a\vartheta\nabla^a\vartheta + 2V(\vartheta)\right] + \mathcal{L}_{\rm mat}\}.$$

EoMs in : Einstein-Dilaton-Gauss-Bonet [Let's count derivatives!]

$$G_{ab} + 16\pi\alpha_{\rm EdGB}\mathcal{D}^{(\vartheta)}_{ab} - 8\pi T^{(\vartheta)}_{ab} = 8\pi T^{\rm mat}_{ab},$$

where

$$T^{(\vartheta)}_{ab} = \nabla_a\vartheta\nabla_b\vartheta - \frac{1}{2}g_{ab}\nabla_c\vartheta\nabla^c\vartheta,$$

is the stress-energy tensor of the scalar field and

$$\mathcal{D}^{(\vartheta)}_{ab} \equiv -2R\nabla_a\nabla_b\vartheta + 2(g_{ab}R - 2R_{ab})\nabla^c\nabla_c\vartheta + 8R_{c(a}\nabla^c\nabla_{b)}\vartheta - 4g_{ab}R^{cd}\nabla_c\nabla_d\vartheta + 4R_{acbd}\nabla^c\nabla^d\vartheta.$$

The scalar field equation is given by

$$\Box\vartheta = -\alpha_{\rm EdGB}\left(R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}\right) = -\alpha R_{\rm GB}.$$

- Dynamical Chern-Simons

$$G_{ab} + 2\,\alpha\kappa\, C_{ab} = \kappa^2 T_{ab}\,, \tag{6}$$

where G_{ab} is the Einstein tensor, and the traceless 'C-tensor' is defined as

$$C^{ab} = (\nabla_c \vartheta)\ \epsilon^{cde(a}\nabla_e R^{b)}{}_d + (\nabla_c \nabla_d \vartheta)\ {}^*R^{d(ab)c}\,. \tag{7}$$

The stress-energy tensor decomposes linearly into a term that depends only on the matter degrees of freedom and a term that depends only on the scalar field, i.e. $T_{ab} = T_{ab}^{\rm Mat} + T_{ab}^{\vartheta}$, where the latter is

$$T_{ab}^{\vartheta} := (\nabla_a \vartheta)\,(\nabla_b \vartheta) - \frac{1}{2} g_{ab}\,(\nabla_c \vartheta)\,(\nabla^c \vartheta)\ . \tag{8}$$

Variation of the action with respect to the scalar field yields its evolution equation

$$\Box \vartheta = \frac{\alpha}{4\kappa}\,{}^*RR\,, \tag{9}$$

(Yunes, Yagi, Stein, Pretorius +)