1. Problems for Maldacena's lectures

1.1. Problem 1

a) Consider the SYK model at large temperatures, on a small thermal circle with length β, such that $\beta J \ll 1$. Using perturbation theory compute the first correction to the free energy. What is the first non-zero power of (βJ) that appears? This can be computed from the path integral point of view, by inserting a couple of interaction terms.

b) Compare the answer to what you would get by using the following other method. Compute

$$J \partial_J \log Z = \frac{J^2}{q} \beta \int_0^\beta d\tau G(\tau)^q \tag{1.1}$$

and then use the free expression for $G(\tau) = \frac{1}{2}\text{sign}(\tau)$, to obtain the first order correction.

1.2. Problem 2

Given a solution $G(\tau)$, or $G(t_1, t_2)$, we would like to be able to read off the energy.

Prove that the energy is proportional to

$$E \propto \lim_{t_1 \to t_2^+} [\partial_{t_1} G(t_1, t_2)] \tag{1.2}$$

and find the proportionality constant. Hint: Express the derivative $\dot{\psi}^i \propto [H, \psi^i]$, compute the commutator, insert it into $\dot{\psi}^i \psi^i$ and see what you get.

Is this an approximate equation or is it exact ? What correlator should you use in order for this equation to be exact (one of the fermions, sum over all fermions)?

1.3. Problem 3

Consider the Schwarzian action

$$S \propto -\int dt \{f(t), t\} \ , \qquad \{f(t), t\} = \frac{f'''}{f'} - \frac{3}{2} \frac{f''^2}{f'^2} \tag{1.3}$$

a) It is invariant under the infinitesimal transformation, $f \to f + c$ with a very small c. Find the associated conserved charge. You can use Noether's procedure. If you have time, you can also find the charges for the infinitesimal transformations $f \to f + \gamma f$ and $f \to f + \sigma f^2$. The finite form of these symmetries is $f \to \frac{af+b}{cf+d}$, with $ad - bc = 1$.

b) It is also invariant under time translations, $t \to t + \alpha$. Compute the associated conserved energy.

You can find the most general solution of the equations of motion for (1.3)?. You can try by brute force, or you can use the above symmetries to first start form a simple solution and act with the above symmetries.

1.4. *Problem 4*

Define $f(t) = \tan \frac{\varphi(\tau)}{2}$ and find the action for φ, by computing $\{f(t), t\} = \{\tan \frac{\varphi(t)}{2}, t\}$. You might find it useful to derive a "chain" rule for the schwarzian $\{f(g(t)), t\} = \{f, g\}g'^2 + \{g, t\}$.

1.5. *Problem 5*

Consider the reparametrized expression for the correlator

$$G_f(t_1, t_2) = \left[\frac{f'(t_1)f'(t_2)}{(f(t_1) - f(t_2))^2} \right]^{\Delta} \tag{1.4}$$

Expand t_1 around t_2 and look at the first term that depends on f. You will find that this term is related to the energy in the Schwarzian approximation. Is this related to problem 2?

Can you think of this limit as an OPE ?

What is the two point function of two energy insertions. Could these possibly depend on their time separation ? Do they behave as a stress tensor with dimension $\Delta = 2$?

1.6. *Problem 6*

Consider AdS_2 (or H_2) in terms of embedding coordinates Y^M living in $R^{2,1}$ with the constraint $-Y_{-1}^2 - Y_0^2 + Y_1^2 = -1$.

Understand the changes of coordinates from Poincare, to Rindler and global coordinates where the metric is

$$ds^2 = \frac{-dt^2 + dz^2}{z^2} \;, \qquad ds^2 = -d\tau^2 \sinh^2 \rho + d\rho^2 \;, \qquad ds^2 = -d\tilde{\tau}^2 \cosh^2 \tilde{\rho} + d\tilde{\rho}^2 \tag{1.5}$$

respectively. One way to do it is to identify the isometry with the right properties. For example, shifts in $\tilde{\tau}$ correspond to rotations in the (-1)0 plane of $R^{2,1}$, and then write the Y^M in terms of each of the sets of coordinates. Plot the Penrose diagram and regions covered by each of the coordinates.

Understand the change of coordinates near the boundary of the space.

1.7. Problem 7

a) Consider a geodesic in AdS_2. Show that the trajectory of the particle is given by $Y.a = 0$ where a is constant vector. You can argue this by consider a particular geodesic and then performing general $SL(2)$ transformations. Define by Q^M the $SL(2)$ charges of this particle. Show that $a \propto Q$. This should be simplest for a geodesic near the origin.

b) Show that the general solution of a particle in an electric field moving in AdS_2 is given by $Y.Q$ =constant. You could argue by first finding particular solutions and then $SL(2)$ transforming them by the symmetries to general solutions. Here Q are the $SL(2)$ charges of the particle. There is a point in AdS_2 which lies at the tip of the causally accessible wedge from this particle trajectory. Find its location in terms of Q^M.

c) Now consider a system with two boundary particles (which that behave as particles in an electric field) and then some matter in the middle. Use the equation $Q_L^M + q_{matt}^M + Q_R^M = 0$ to find a configuration where we have matter at rest at the center of AdS_2, with some energy, and then the two boundary particles. Check that the causal wedges of the two boundary trajectories do not overlap. What property of matter ensures that they do not overlap ?

1.8. Problem 8

Find the expression for the entropy as a function of the temperature for a charged black hole (you can do it in 4d or any dimension) and expand it around extremality (you can find the solution on the web). Did you get a term proportional to the temperature ?. Find the coefficient of the Schwarzian action in terms of the parameters of the black hole (the charge, the Netwon's constant, etc.)