Quantum Gravity from the QFT perspective

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Lecture 5.

Advances topics in QG

- **Induced gravity concept.**
- **Effective QG: general idea.**
- **Effective QG as effective QFT.**
- **Where we are with QG?.**

Bibliography

S.L. Adler, Rev. Mod. Phys. **54** *(1982) 729.*

S. Weinberg, Effective Field Theory, Past and Future. arXive:0908.1964[hep-th];

J.F. Donoghue, The effective field theory treatment of quantum gravity. arXive:1209.3511[gr-qc];

I.Sh., Polemic notes on IR perturbative quantum gravity. arXiv:0812.3521 [hep-th].

I. Induced gravity.

The idea of induced gravity is simple, while its realization may be quite non-trivial, depending on the theory.
In any case, the induced gravity concept is something absolutely necessary if we consider an interaction of gravity with matter and quantum theory concepts.

I. Induced gravity from cut-off

Original simplest version.

Ya.B. Zeldovich, Sov. Phys. Dokl. 6 (1967) 883.
A.D. Sakharov, Sov. Phys. Dokl. 12 (1968) 1040.

Strong version of induced gravity is like that:
Suppose that the metric has no pre-determined equations of motion. These equations result from the interaction to matter.

Main advantage:
Since gravity is not fundamental, but induced interaction, there is no need to quantize metric.

And we already know that the semiclassical approach has no problems with renormalizability!

Suppose we have a theory of quantum matter fields $\Phi = (\varphi,\, \psi,\, A_\mu)$ interacting to the metric $g_{\mu\nu}$.

The action for matter fields depends also on gravity, $S_m(\Phi,\, g_{\mu\nu})$.

Originally, there is nor action for gravity, neither equations of motion. But after we intergrate out matter fields, we meet

$$e^{iS_{ind}(g_{\mu\nu})} = \int D\Phi e^{iS_m(\Phi,\, g_{\mu\nu})}\,.$$

After that we gain the dynamics of the gravitational field, which corresponds to the principle of the least action for

$$S_t = S_m(\Phi,\, g_{\mu\nu}) + S_{ind}(g_{\mu\nu})\,.$$

The scheme looks very nice, the question is how it can be put into practise.

Making derivatives expansion in $S_{ind}(g_{\mu\nu})$, we meet

$$S_{ind}(g_{\mu\nu}) = \int d^4x\sqrt{-g}\,\Big(-\rho_\Lambda^{ind} - \frac{1}{16\pi\,G_{ind}}\,R + \,...\Big)\,,$$

where ... indicate higher derivative and non-local terms, which are supposed to be irrelevant at low energies.

The natural questions are as follows:

- **How to evaluate the induced quantities like** $G,\ \rho_\Lambda,\ ...$ **?.**
- **What are the ambiguities in this evaluation?**
- **Is there certainty that the higher derivative and non-local terms in the induced action will not be important and will not contradict existing tests of GR?**
- **How to avoid massive ghosts in induced gravity?**

Nowadays we have several very different schemes to derive induced action, in all of them there are different ambiguities and the problems mentioned above are solved in different ways.

In the original paper by

Ya.B. Zeldovich, , Sov. Phys. Dokl. 6 (1967) 883

the derivation of ρ_Λ^{ind} **was performed in flat space, by means of integration over momentum, with the cut-off about** Λ_{QCD}**.**

As we shall see in what follows in the "purely induced" gravity it is impossible to go far with this choice of cut-off.

So, it is wise to keep the magnitude of the cut-off arbitrary and define it later on.

How one can implement a non-covariant regularization in curved space-time?

Example: cut-off regularization for the Energy-Momentum Tensor of vacuum.

B.S. DeWitt, Physics Reports - 1975.

E.K. Akhmedov, arXiv: hep-th/0204048.

$$\rho_{vac} = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \sqrt{\vec{k}^2 + m^2}\,,$$

$$p_{vac} = \frac{1}{6} \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^2}{\sqrt{\vec{k}^2 + m^2}}\,,$$

For each mode we have, in the massless limit, EOS of radiation. Naturally, after integration with cut-off we will get the EOS for the radiation in the quartic divergences.

But, Lorentz invariance requires the EOS to be $p_{vac} = -\rho_{vac}$.

Direct calculation of this sort gives

$$\rho = \frac{1}{16\pi^2}\left[\Omega^4 + m^2\Omega^2 + \frac{1}{8}m^4 - \frac{1}{2}m^4\log\frac{2\Omega}{m} + \mathcal{O}\left(\frac{m}{\Omega}\right)\right],$$

where Ω is a 3-momentum space cut-off. For the Planck-scale cut-off this gives the famouse "120 orders of magnitude discrepancy between theory and experiment."

However, the expression for the "pressure of vacuum" indicates that the situation is not that simple:

$$p = \frac{1}{48\pi^2}\left[\Omega^4 - m^2\Omega^2 - \frac{7}{8}m^4 + \frac{3}{2}m^4\log\frac{2\Omega}{m} + \mathcal{O}\left(\frac{m}{\Omega}\right)\right].$$

There is a radiation-like "equation of state" of the vacuum, instead of the one for the cosmological constant!

The reason is the use of the non-covariant regularization,

Asorey, Lavrov, Ribeiro & I.Sh. PRD (2012), arXive:1202.4235.

The problem with covariance can be solved if we use the covariant cut-off within EA & Schwinger-DeWitt approach. The one-loop contribution can be always presented as

$$\bar{\Gamma}^{(1)} = \frac{i}{2}\,\mathrm{Tr}\,\mathrm{Log}\,\big(\hat{H}\big)\,, \qquad \hat{H} = \hat{1}\Box + \hat{P}\,,$$

where the operator $\hat{P}$ depends on the kind of the field. Then

$$\bar{\Gamma}^{(1)}_L = \frac{1}{2}\int_{L^{-2}}^{\infty}\frac{ds}{s}\,\frac{1}{(4\pi s)^2}\,\mathrm{Tr}\,\Big\{\,\hat{1} + s\hat{a}_1 + s^2\,\hat{a}_2 + \dots\Big\}.$$

$$\langle T_{\mu\nu}(x)\rangle = -\frac{2}{\sqrt{-g(x)}}\,g_{\mu\alpha}(x)\,g_{\nu\beta}(x)\,\frac{\delta\Gamma}{\delta g_{\alpha\beta}(x)}\,.$$

Then, in the cosmological constant sector, we get

$$\langle T_{\mu\nu}(x)\rangle \sim g_{\mu\nu}\,L^4\,, \qquad P_\Lambda = -\,\rho_\Lambda\,,$$

in the perfect agreement with covariance.

Even after we arrive at the covariant result, the induced gravity approach in this original formulation is not free of problems.

Obviously, $\rho_\Lambda^{ind} \propto L^4$ **and** $\frac{1}{16\pi\, G_{ind}} \propto L^2\,.$

As far as all gravity is induced, we are forced to identify

$L \propto M_P\,,$ **then** $\rho_\Lambda^{ind} \propto L^4\,.$

Therefore the "calculated" value of the cosmological constant density is $M_P^4 \approx 10^{76}\, GeV^4$.

This is a way too much compared to the observed value

$$\rho_\Lambda^{obs} \approx 10^{-48}\, GeV^4\,.$$

For those interested in the cosmological constant problem, this is the unique way to get a famous "120 orders of magnitude discrepancy between theory and observations." And it is due to a very special choice of the theory: purely induced gravity.

Now, one may like to use renormalization theory instead of taking the cut-off value as a physical result. However, this requires the presence of vacuum terms with

$$\rho_\Lambda^{vac} \qquad \textbf{and} \qquad \frac{1}{16\pi\, G_{vac}},$$

and then we are out of the original induced gravity approach!

The huge discrepancy with the value of ρ_Λ^{ind} shows that the only way out is to introduce vacuum quantities, renormalize them and finally sum up with the induced ones:

$$\rho_\Lambda^{obs} = \rho_\Lambda^{vac} + \rho_\Lambda^{ind}\,, \qquad \frac{1}{16\pi\, G_{obs}} = \frac{1}{16\pi\, G_{vac}} + \frac{1}{16\pi\, G_{ind}}\,.$$

From the formal viewpoint this is fine, but the problem of QG was left aside and has not been solved, of course.

The consideration about induced gravity described above is valid independent on whether we introduce vacuum (classical) gravitational action or not.

At the quantum level both induced quantities

$$G_{ind} \qquad \text{and} \qquad \Lambda_{ind}$$

gain loop corrections.

A more ambitions version assumes that there are only quantum corrections, no classical induced terms.

One starts with initially massless theory, and all masses are the result of dimensional transmutation (Coleman-Weinber - type mechanism, or dynamical mass generation).

An important feature of these theories is an ambiguity in the induced quantities. Only one of them can be well-defined (see Adler's review).

RG equations for vacuum quantities, like cosmological constant density $\rho_\Lambda = \Lambda/(8\pi G)$ and Newton constant G:

$$(4\pi)^2 \, \mu \frac{d\, \rho_\Lambda^{vac}}{d\mu} = (4\pi)^2 \, \mu \, \frac{d}{d\mu} \left(\frac{\Lambda_{vac}}{8\pi G_{vac}} \right) = \frac{N_s m_s^4}{2} - 2 N_f m_f^4 \, .$$

$$(4\pi)^2 \, \mu \, \frac{d}{d\mu} \left(\frac{1}{16\pi G_{vac}} \right) = \frac{N_s m_s^2}{2} \left(\xi - \frac{1}{6} \right) + \frac{N_f m_f^2}{3} \, .$$

It is not clear how these equations can be used in cosmology, where the typical energies are very small.

However, even the UV running means the ρ_Λ^{vac} can not be much smaller then the fourth power of the typical mass of the theory.

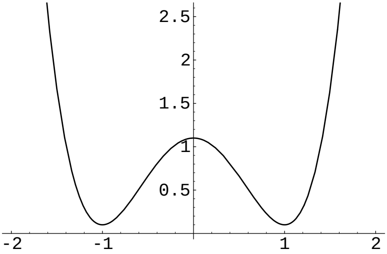
Consequence: the natural value from the MSM perspective is

$$\rho_\Lambda^{vac} \sim M_F^4 \sim 10^8 \, GeV^4 \, .$$

Cosmological Constant (CC) Problem in the Standard Model:

In the stable point of the Higgs potential $V = -m^2\phi^2 + f\phi^4$ **we meet** $\Lambda_{ind} = \langle V \rangle \approx 10^8\, GeV^4$ **– same order of magnitude as** Λ_{vac} **!**

This is induced CC, similar to the one found by Zeldovich (1968).



The observed CC is a sum $\rho_\Lambda^{obs} = \rho_\Lambda^{vac} + \rho_\Lambda^{ind}$. **Since** ρ_Λ^{vac} **is an independent parameter, the renormalization condition is**

$$\rho_\Lambda^{vac}(\mu_c) = \rho_\Lambda^{obs} - \rho_\Lambda^{ind}(\mu_c)\,.$$

Here μ_c **is the energy scale where** ρ_Λ^{obs} **is "measured".**

The main CC relation is

$$\rho_\Lambda^{obs} = \rho_\Lambda^{vac}(\mu_c) + \rho_\Lambda^{ind}(\mu_c)\,.$$

The ρ_Λ^{obs} which is likely observed in SN-Ia, LSS and CMB is

$$\rho_\Lambda^{obs}(\mu_c) \;\approx\; 0.7\,\rho_c^0 \;\propto\; 10^{-47}\,GeV^4.$$

The CC Problem is that the magnitudes of $\rho_\Lambda^{vac}(\mu_c)$ and $\rho_\Lambda^{ind}(\mu_c)$ are a huge 55 orders of magnitude greater than the sum!

Obviously, these two huge terms do cancel.

"Why they cancel so nicely" is the CC Problem (Weinberg, 1989).

The origin of the problem is the difference between the M_F scale of ρ_Λ^{ind} and ρ_Λ^{vac} and the μ_c scale of ρ_Λ^{obs}.

Obviously, CC Problem is nothing else but a sort of hierarchy problem, perhaps the most difficult one.

Intermediate conclusions: We have considered the "traditional" approaches to induced gravity.

This approach can be useful in many respects, including understanding the CC Problem.

But it is not completely successful for solving the problem of Quantum Gravity, especially because "pure" induced gravity is problematic.

Furthermore, at the next orders in derivative expansion we are going to meet higher derivatives and the same potential instabilities as we had in all versions of HDQG.

Does induced gravity solve the problem of ghosts?

In principle, the answer is negative.

The reason is that there is no way to restrict the emergence of higher derivative terms in the induced action.

And as far as these term are in the action of gravity, there is a problem of stability of low energy solutions in the presence of higher derivative terms.

At low energies one can simply ignore this problem, treating higher derivatives as small perturbations by definition.

But this approach fails in general, because there is no candidate for being the fundamental theory of QG.

String theory?

Ghost-free HD models of gravity

Consider an example of ghost-free HD model of QG.

- **In the (super)string theory, the object of quantization is a kind of non-linear sigma-model in two space-time dimensions.**

Both metric and matter fields are induced, implying unification of all fundamental forces.

The σ-model approach is close to QFT in curved space,

$$S_{str} = \int d^2\sigma\sqrt{g}\,\Big\{\frac{1}{2\alpha'}g^{\mu\nu}G_{ij}(X)\partial_\mu X^i\partial_\nu X^j$$

$$+\frac{1}{\alpha'}\,\frac{\varepsilon^{\mu\nu}}{\sqrt{g}}A_{ij}(X)\partial_\mu X^i\partial_\nu X^j + B(X)R + T(X)\Big\}\,, \quad i,j = 1,2,...,D\,.$$

The Polyakov approach: conditions of anomaly cancellation order by order in α'. Critical dimensions:

D=26 **for bosonic string,** D=10 **for superstrings.**

At the first order in α' the effective equations give GR !

E.S. Fradkin & A. Tseytlin (1985);
C. Callan, D. Friedan, E. Martinec, M. Perry, (1985).

- **Metric reparametrization remove ghosts at all orders in α' .**

In the torsionless case the effective action can be written as

$$S_M = \frac{2}{\kappa^2} \int d^D x \sqrt{G}\, e^{-2\phi} \Big\{ - R + 4\,(\partial\phi)^2$$

$$+\alpha' \left(a_1 R_{\lambda\mu\nu\rho} R^{\lambda\mu\nu\rho} + a_2 R_{\mu\nu} R^{\mu\nu} + a_3 R^2 \right) \Big\} + \ ...$$

In order to remove ghosts one performs reparametrization of the background metric $G_{\mu\nu}$

$$G_{\mu\nu} \longrightarrow G'_{\mu\nu} = G_{\mu\nu} + \alpha' \,(x_1\, R_{\mu\nu} + x_2\, R\, G_{\mu\nu}) + ...$$

where $x_{1,2,\dots}$ are specially tuned parameters.

B. Zweibach, S. Deser & A.N. Redlich, ... A. Tseytlin (1985-1987).

Ghost-killing reparametrization doesn't affect string S-matrix,

$$G_{\mu\nu} \longrightarrow G'_{\mu\nu} = G_{\mu\nu} + \alpha' \left(x_1 R_{\mu\nu} + x_2 R\, G_{\mu\nu} \right) + ...$$

At the same time, Zweibach reparametrization is ambiguous and this actually produce ambiguous physical solutions.

A. Maroto & I.Sh., PLB, hep-th/9706179.

- **Even more subtle point is that the effectively working ghost-killing transformation must be absolutely precise!**

Any infinitesimal change produce a ghost with a huge mass. Moreover, smaller violation of fine-tuning leads to a greater mass of the ghost, hence (according to a "standard wisdom") smaller violation of fine-tuning produce greater gravitational instability.

At low energies we know that the quantum effects are described by QFT, not string theory. Hence, string theory is ghost-free and unitary only if it completely controls QFT, even in the deep IR.

II. Effective low-energy gravity.

The effective approach is the cornerstone of the application of QFT to Particle Physics and to all Modern Physics.

This approach explains why we don't care about fundamental physical phenomena when dealing with low-energy ones.

For example, when we perform calculations of atomic spectra there is almost no need to care about what is going on in the atomic nuclei and absolutely no need to care about what is going at the level of quarks inside the nuclei.

The reason is that the energy scale of the two types of phenomena is very much different and the low energy scale is not sensible to the high energy interactions.

It looks natural to use it for QG, where we actually meet two very different energy scales: $M_P \approx 10^{19}\,GeV$ **and the energy scale of typical gravitational phenomena, e.g., in the present-day cosmology it is** $\mu_c \sim H_0 \approx 10^{-42}\,GeV$.

Standard approach to effective QG (up to 2012)

J.F. Donoghue, Phys. Rev. Lett. 72 (1994) 2996; Phys. Rev. D 50 (1994) 3874.

Correcting first set of mistakes:

H.W. Hamber, S. Liu, Phys. Lett. B357 (1995) 51;
I.J. Muzinich, S. Vokos, Phys. Rev. D52 (1995) 3472;
A.A. Akhundov, S. Bellucci, A. Shiekh, Phys. Lett. B395 (1997) 16.

Conflicting results:

N.E.J. Bjerrum-Bohr, J.F. Donoghue, B.R. Holstein, Phys. Rev. D67 (2003) 084033;
I.B. Khriplovich, G.G. Kirilin, J.Exp.Theor.Phys. 95 (2002) 981.

Earlier calculation:

Y. Iwasaki, Prog. Theor. Phys. 46 (1971) 1587.

Recent reviews:

C.P. Burgess, Living Rev. Rel. 7 (2004) 5;
J.F. Donoghue, arXive:1209.3511.

Our contributions:

J. A. Helayel-Neto, A. Penna-Firme and I. L. Shapiro, JHEP 0001, 009 (2000);

1-loop quantum corrections to the Newton's potential: A diagrammatic study of the gauge-dependence (Unpublished, 2002).

Correct analysis (my opinion!):

D.A.R. Dalvit, F.D. Mazzitelli, Phys. Rev. D56 (1997) 7779;

I. Sh., Polemic notes on IR perturbative quantum gravity. Int. J. Mod. Phys. A24 (2009) 1557; arXiv:0812.3521.

The main idea is that the low-energy effects are non-local and therefore completely separated from the high-energy, essentially local expressions related to the counterterms.

This approach is going to work very well in the situations when there is a well defined massive parameter. QG seems to be the "best possible case", just because the Planck mass is huge.

Technically the effective QG starts as usual QG.

$$S_t \;=\; S_{EH} + S_{gf} + S_{ghost} + S_{matter}\;,$$

where,

$$S_{EH} = -\frac{1}{\kappa^2}\int d^4x\sqrt{-g}\cdot R\,,$$

$$S_{gf} = \frac{1}{\alpha}\int d^4x\sqrt{-g}\cdot\chi_\lambda\,\chi^\lambda\,, \qquad \chi_\mu = \partial_\lambda\,h^\lambda_\mu - \beta\,\partial_\mu h^\lambda_\lambda\,;$$

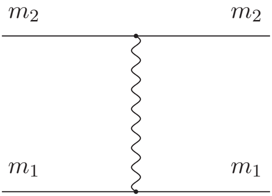
$$S_{ghost} = \int d^4x\sqrt{-g}\cdot\bar{C}_\mu\cdot\frac{\delta\chi^\mu}{\delta h_{\rho\sigma}}\cdot R^{\alpha}_{\cdot\rho\sigma}\cdot C_\alpha\,,$$

$$S_{matter} = \int d^4x\sqrt{-g}\cdot\left\{\frac{1}{2}\,g^{\mu\nu}\cdot\partial_\mu\phi\,\partial_\nu\phi\;-\;\frac{1}{2}\,m^2\phi^2\right\}\,.$$

The aim is to study the gravitational interaction between two scalars, of masses m_1 **and** m_2.

At the classical level the situation is simple.

The tree-level scattering amplitude has the form



$$T(q) \,=\, 4\pi\, \frac{G\, m_1\, m_2}{\vec{q}^{\,2}}\,.$$

In the static limit $q^0 = 0$ **and** $q^2 = -\vec{q}^{\,2}$. **After Fourier transform**

$$\int \frac{1}{\vec{q}^{\,2}}\, e^{i\vec{q}\cdot\vec{r}}\, d^3\vec{r} \,=\, \frac{1}{4\pi r}$$

we arrive at the Newton potential

$$V(r) = -G\,\frac{m_1\; m_2}{r},$$

which is the tree-level approximation to the potential for the interaction between two static sources.

One-graviton exchange between the two masses gives Newton law in the IR limit. What are the IR quantum corrections?

At the one-loop level there are two types of diagrams and two types of IR-relevant contributions.

I. P-type terms.

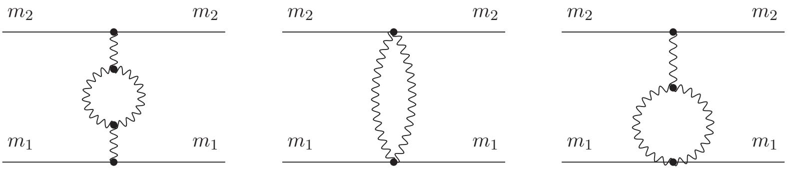
$$\int \frac{d^3q}{(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{r}}\, \frac{1}{\sqrt{\vec{q}^{\,2}}} \;=\; \frac{1}{2\pi^2\, r^2}\,.$$

II. L-type terms.

$$\int \frac{d^3q}{(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{r}}\; \ln \vec{q}^{\,2} \;=\; -\,\frac{1}{2\pi^2\, r^3}\,.$$

Definitely, from the phenomenological viewpoint the P-type terms look much more interesting. This type of terms are going to mix with the first post-Newtonian approximation and hence we have to expect that QG will simply reproduce here the classical GR result, e.g., the precession of the perihelion of Mercury.

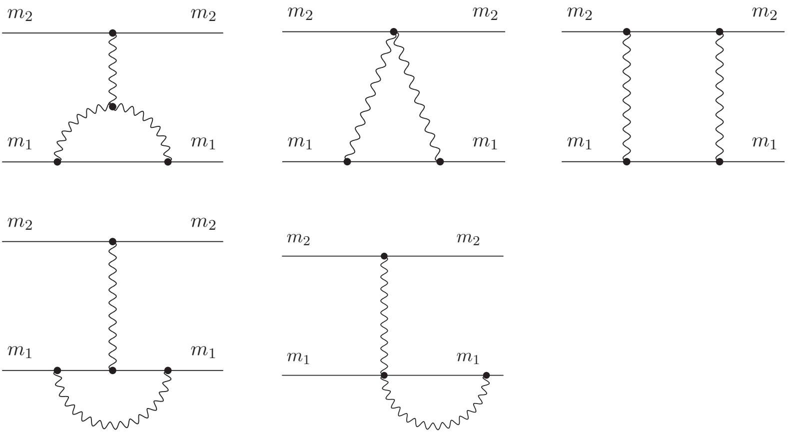
I. Graphs with only massless (gravitational) internal lines.



They contribute only to the L-type terms.

One can suppose that these diagrams correspond to the path integral over metric, when massive scalar field is an external classical source.

II. Graphs with both massless and massive (scalar field) internal lines.



They contribute to both L-type and P-type terms.

Classical or quantum?

The first 1994 paper: the contributions from the P-type diagrams reproduce the post-Newtonian limit of classical GR.

The consequent analysis of quantum corrections has shown that the original calculations had some mistakes, in particular one complicated diagram was omitted.

Without the full set of diagrams the quantum corrections are gauge fixing dependent and the calculation has no much sense.

For the full set of diagrams, after some changed sign and value, the P-terms are still the same (in fact, there are two conflicting results), fitting post-Newtonian approximation.

All this concerns interaction between two massive scalars. What about fermions and macroscopic bodies?

Polemic Note

I.Sh., IJMPA; arXiv:0812.3521 [hep-th].

The macroscopic bodies which take part in the relevant gravitational interactions are not made from a scalar field.

In reality, they do consist from a baryonic matter, that means interacting protons, neutrons and electrons.

These particles are not elementary (except electron) and none of them may be properly described by a scalar field.

Of course, nucleons consist from quarks and gluons, so one may think to replace the scalar field by the spinor one and try to obtain the quantum gravity corrections taking, e.g., mixed graviton-quark diagrams.

However, this would not be a right step, because quarks are not free particles.

One of the manifestation of this fact is that the total mass of the u, $\bar{u}$ **and** d **quarks is essentially smaller than the mass of the proton.**

If we calculate such (even tree-level) diagrams with quarks we have no chance to get a correct result.

Finally, we arrive at the conclusion that the “correct” set of diagrams includes only the L-type ones.

This is OK from the theoretical viewpoint, but then we are very far from any chance to have relevant observation of QG at low energies.

Conclusions

- **Singularities in GR may be a windows to the unknown fundamental physics, perhaps to some version of QG.**

- **We have very satisfactory models of QG, starting from QFT in curved space (semiclassical QG).**

- **There is no theoretically perfect model of QG.**

- **HDQG is the most realistic candidate, despite the ghost issue, and the last will be perhaps solved in one or another way.**

- **Finally, the main problem of QG is not theoretical, but experimental. More precisely, the real problem is that we have no experiments now and very small chances to have some in the visible future.**