

# Structure of the nucleon's low-lying excitations

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International Centre for Theoretical Physics South American Institute for Fundamental Research

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  - Explain the most important mass generating mechanism for visible matter in the Universe
  - Neither of these phenomena is apparent in QCD 's Lagrangian, HOWEVER, They play a dominant role in determining the characteristics of <u>real-world QCD</u>!

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### > Dyson-Schwinger equations

- ✓ A Nonperturbative symmetry-preserving tool for the study of Continuum-QCD
- ✓ Well suited to Relativistic Quantum Field Theory
- ✓ A method connects observables with long-range behaviour of the running coupling
- ✓ Experiment ↔ Theory comparison leads to an understanding of long-range behaviour of strong running-coupling

- Mesons: a 2-body bound state problem in QFT
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$$\Gamma_{q\bar{q}}(p;P) = -\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q+P) \Gamma_{q\bar{q}}(q;P) S(q) \frac{\lambda^a}{2} \gamma_\nu$$
  
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- (I=0, J^P=0^+): isoscalar-scalar diquark
- (I=1, J^P=1^+): isovector-pseudovector diquark
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#### PHYSICAL REVIEW C 96, 015208 (2017)

#### Parity partners in the baryon resonance spectrum

 Ya Lu,<sup>1,\*</sup> Chen Chen,<sup>2,†</sup> Craig D. Roberts,<sup>3,‡</sup> Jorge Segovia,<sup>4,§</sup> Shu-Sheng Xu,<sup>1,||</sup> and Hong-Shi Zong<sup>1,5,¶</sup>
<sup>1</sup>Department of Physics, Nanjing University, Nanjing, Jiangsu 210093, China
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<sup>3</sup>Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA
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<sup>5</sup>Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing, Jiangsu 210093, China (Received 10 May 2017; published 28 July 2017)

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Faddeev kernels: 22 × 22 matrices are reduced to 16 × 16 !

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  - ✓ gDB is the single free parameter in our study.

> Solution to the **50** year puzzle -- Roper resonance

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PRL 115, 171801 (2015)

PHYSICAL REVIEW LETTERS

week ending 23 OCTOBER 2015

#### Completing the Picture of the Roper Resonance

Jorge Segovia,<sup>1</sup> Bruno El-Bennich,<sup>2,3</sup> Eduardo Rojas,<sup>2,4</sup> Ian C. Cloët,<sup>5</sup> Craig D. Roberts,<sup>5</sup> Shu-Sheng Xu,<sup>6</sup> and Hong-Shi Zong<sup>6</sup> <sup>1</sup>Grupo de Física Nuclear and Instituto Universitario de Física Fundamental y Matemáticas (IUFFyM), Universidad de Salamanca, E-37008 Salamanca, Spain <sup>2</sup>Laboratório de Física Teórica e Computacional, Universidade Cruzeiro do Sul, 01506-000 São Paulo, SP, Brazil <sup>3</sup>Instituto de Física Teórica, Universidade Estadual Paulista, 01140-070 São Paulo, SP, Brazil <sup>4</sup>Instituto de Física, Universidad de Antioquia, Calle 70 No. 52-21, Medellín, Colombia <sup>5</sup>Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA <sup>6</sup>Department of Physics, Nanjing University, Nanjing 210093, China (Received 16 April 2015; revised manuscript received 29 July 2015; published 21 October 2015)

We employ a continuum approach to the three valence-quark bound-state problem in relativistic quantum field theory to predict a range of properties of the proton's radial excitation and thereby unify them with those of numerous other hadrons. Our analysis indicates that the nucleon's first radial excitation is the Roper resonance. It consists of a core of three dressed quarks, which expresses its valence-quark content and whose charge radius is 80% larger than the proton analogue. That core is complemented by a meson cloud, which reduces the observed Roper mass by roughly 20%. The meson cloud materially affects long-wavelength characteristics of the Roper electroproduction amplitudes but the quark core is revealed to probes with  $Q^2 \gtrsim 3m_N^2$ .

DOI: 10.1103/PhysRevLett.115.171801

PACS numbers: 13.40.Gp, 14.20.Dh, 14.20.Gk, 11.15.Tk

Introduction.—The strong-interaction sector of the standard model is thought to be described by quantum chromodynamics (QCD), a relativistic quantum field broken, express the intrinsic mass scale(s) and features associated with confinement and DCSB, and employ realistic kernels in baryon bound-state equations, which

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# **SOLUTIONS & THEIR PROPERTIES**

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#### PHYSICAL REVIEW D 97, 034016 (2018)

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(Received 9 November 2017; published 15 February 2018)

# **SOLUTIONS & THEIR PROPERTIES**

- The four lightest baryon (I=1/2, J^P=1/2^{+-}) isospin doublets: nucleon, roper, N(1535), N(1650)
- Masses
- Rest-frame orbital angular momentum
- Diquark content
- Pointwise structure

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Pseudoscalar and vector diquarks have no impact on the mass of the two positiveparity baryons, whereas scalar and pseudovector diquarks are important to the negative parity systems.

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# SOLUTIONS & THEIR PROPERTIES: Masses

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The relative difference is just 1.7%. We consider this to be a success of our calculation.





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(b) delivers the same qualitative picture as that presented in (a). Therefore, there is little mixing between partial waves in the computation of a baryon's mass.



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- These observations provide support in quantum field theory for the constituentquark model classifications of these systems.





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- From (a): The amplitudes associated with these negative-parity states contain roughly *equal fractions* of even and odd parity diquarks. Positive-parity states: negative-parity diquarks are almost ZERO.
- From (b): In each, there is a single dominant diquark component. There are significant interferences between different diquarks.



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- In their rest frames, these systems are predominantly P-wave in nature, but possess material S-wave components; and the first excited state in this negative parity channel—N(1650)1/2^-- has little of the appearance of a radial excitation, since most of the functions depicted in the right panels of the figure do not possess a zero.



# **Octet & Decuplet Baryons**

#### Spectrum and structure of octet and decuplet baryons and their positive-parity excitations

Chen Chen,<sup>1,\*</sup> Gastão Krein,<sup>1</sup> Craig D. Roberts,<sup>2,†</sup> Sebastian M. Schmidt,<sup>3</sup> and Jorge Segovia<sup>4</sup>

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<sup>3</sup>Institute for Advanced Simulation, Forschungszentrum Jülich and JARA, D-52425 Jülich, Germany

<sup>4</sup>Departamento de Sistemas Físicos, Químicos y Naturales,

Universidad Pablo de Olavide, E-41013 Sevilla, Spain

(Dated: 10 January 2019)

A continuum approach to the three valence-quark bound-state problem in quantum field theory is used to compute the spectrum and Poincaré-covariant wave functions for all flavour-SU(3) octet and decuplet baryons and their first positive-parity excitations. Such analyses predict the existence of nonpointlike, dynamical quark-quark (diquark) correlations within all baryons; and a uniformly sound description of the systems studied is obtained by retaining flavour-antitriplet-scalar and flavour-sextet-pseudovector diquarks. Thus constituted, the rest-frame wave function of every system studied is primarily S-wave in character; and the first positive-parity excitation of each octet or decuplet baryon exhibits the characteristics of a radial excitation. Importantly, every ground-state octet and decuplet baryon possesses a radial excitation. Hence, the analysis predicts the existence of positive-parity excitations of the  $\Xi$ ,  $\Xi^*$ ,  $\Omega$  baryons, with masses, respectively (in GeV): 1.75(12), 1.89(03), 2.05(02). These states have not yet been empirically identified. This body of analysis suggests that the expression of emergent mass generation is the same in all u, d, s baryons and, notably, that dynamical quark-quark correlations play an essential role in the structure of each one. It also provides the basis for developing an array of predictions that can be tested in new generation experiments.

# **Octet & Decuplet Baryons**



![](_page_94_Picture_1.jpeg)

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- The two lightest (I=1/2, J^P=1/2^+) doublets are dominated by scalar and pseudovector diquarks; the associated rest-frame Faddeev wave functions are primarily S-wave in nature; and the first excited state in this 1/2^+ channel has very much the appearance of a radial excitation of the ground state.

![](_page_96_Picture_3.jpeg)

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- In the two lightest (I=1/2, J^P=1/2^-) systems, TOO, scalar and pseudovector diquarks play a material role. In their rest frames, the Faddeev amplitudes describing the dressed-quark cores of these negative-parity states contain roughly equal fractions of even and odd parity diquarks; the associated wave functions of these negative-parity systems are predominantly P-wave in nature, but possess measurable S-wave components; and, the first excited state in this negative parity channel has little of the appearance of a radial excitation.

![](_page_97_Picture_4.jpeg)

# Thank you!

- > The dressed-quark propagator  $S(p) = -i\gamma \cdot p\sigma_V(p^2) + \sigma_S(p^2)$
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$$\bar{\sigma}_V(x) = \frac{1}{x + \bar{m}^2} [1 - \mathcal{F}(2(x + \bar{m}^2))], \qquad (A3b)$$

with 
$$x = p^2/\lambda^2$$
,  $\bar{m} = m/\lambda$ ,

$$\mathcal{F}(x) = \frac{1 - e^{-x}}{x},\tag{A4}$$

 $\bar{\sigma}_S(x) = \lambda \sigma_S(p^2)$  and  $\bar{\sigma}_V(x) = \lambda^2 \sigma_V(p^2)$ . The mass scale,  $\lambda = 0.566$  GeV, and parameter values,

$$\frac{\bar{m}}{0.00897} \quad \frac{b_0}{0.131} \quad \frac{b_1}{2.90} \quad \frac{b_2}{0.603} \quad \frac{b_3}{0.185}, \quad (A5)$$

associated with Eq. (A3) were fixed in a least-squares fit to light-meson observables [79,80]. [ $\epsilon = 10^{-4}$  in Eq. (A3a) acts only to decouple the large- and intermediate- $p^2$  domains.]

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FIG. 6. Solid curve (blue)—quark mass function generated by the parametrization of the dressed-quark propagator specified by Eqs. (A3) and (A4) (A5); and band (green)—exemplary range of numerical results obtained by solving the gap equation with the modern DCSB-improved kernels described and used in Refs. [16,81–83].

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- Compare with that computed using the DCSB-improved gap equation kernel (DB). The parametrization is a sound representation numerical results, although simple and introdu long beforehand.



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- Simple form. Just one parameter: diquark masses.
- Match expectations based on solutions of meson and diquark Bethe-Salpeter amplitudes.

> The diquark propagators

$$\Delta^{0^{\pm}}(K) = \frac{1}{m_{0^{\pm}}^2} \mathcal{F}(k^2/\omega_{0^{\pm}}^2),$$

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#### Diquarks are confined.

- free-particle-like at spacelike momenta
- > pole-free on the timelike axis
- This is NOT true of RL studies. It enables us to reach arbitrarily high values of momentum transfer.

$$\begin{split} \psi^{\pm}(p_{i},\alpha_{i},\sigma_{i}) &= [\Gamma^{0^{+}}(k;K)]_{\sigma_{1}\sigma_{2}}^{\alpha_{1}\alpha_{2}}\Delta^{0^{+}}(K)[\varphi_{0^{+}}^{\pm}(\ell;P)u(P)]_{\sigma_{3}}^{\alpha_{3}} \\ &+ [\Gamma_{\mu}^{1^{+}j}]\Delta_{\mu\nu}^{1^{+}}[\varphi_{1^{+}\nu}^{j\pm}(\ell;P)u(P)] \\ &+ [\Gamma^{0^{-}}]\Delta^{0^{-}}[\varphi_{0^{-}}^{\pm}(\ell;P)u(P)] \\ &+ [\Gamma_{\mu}^{1^{-}}]\Delta_{\mu\nu}^{1^{-}}[\varphi_{1^{-}\nu}^{\pm}(\ell;P)u(P)], \end{split}$$
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$$\varphi_{1^{-}\nu}^{\pm}(\ell;P) = \sum_{i=1}^{6} v_i^{\pm}(\ell^2, \ell \cdot P) \gamma_5 \mathcal{A}_{\nu}^i(\ell;P) \mathcal{G}^{\mp},$$

where  $\mathcal{G}^{+(-)}=\mathbf{I}_{D}(\gamma_{5})$  and

$$\begin{split} \mathcal{S}^{1} &= \mathbf{I}_{\mathrm{D}}, \qquad \mathcal{S}^{2} = i\gamma \cdot \hat{\ell} - \hat{\ell} \cdot \hat{P}\mathbf{I}_{\mathrm{D}} \\ \mathcal{A}_{\nu}^{1} &= \gamma \cdot \ell^{\perp} \hat{P}_{\nu}, \qquad \mathcal{A}_{\nu}^{2} = -i\hat{P}_{\nu}\mathbf{I}_{\mathrm{D}}, \qquad \mathcal{A}_{\nu}^{3} = \gamma \cdot \hat{\ell}^{\perp} \hat{\ell}_{\nu}^{\perp} \\ \mathcal{A}_{\nu}^{4} &= i\hat{\ell}_{\nu}^{\perp}\mathbf{I}_{\mathrm{D}}, \qquad \mathcal{A}_{\nu}^{5} = \gamma_{\nu}^{\perp} - \mathcal{A}_{\nu}^{3}, \qquad \mathcal{A}_{\nu}^{6} = i\gamma_{\nu}^{\perp}\gamma \cdot \hat{\ell}^{\perp} - \mathcal{A}_{\nu}^{4}, \end{split}$$

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<sup>2</sup>P:  $S^{2}$ ,  $\mathcal{A}_{\nu}^{1}$ ,  $(\mathcal{A}_{\nu}^{4} + \mathcal{A}_{\nu}^{6})$ ,  
<sup>4</sup>P:  $(2\mathcal{A}_{\nu}^{4} - \mathcal{A}_{\nu}^{6})/3$ ,  
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The scalar functions associated with these combinations of Dirac matrices in a Faddeev wave function possess the identified angular momentum correlation between the quark and diquark.



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- Modern diquarks are different from the old static, point-like diquarks which featured in early attempts to explain the so-called missing resonance problem.
- The number of states in the spectrum of baryons obtained is similar to that found in the three-constituent quark model, just as it is in today's LQCD calculations.