Structure of the nucleon’s low-lying excitations

CHEN CHEN

The Institute for Theoretical Physics, Sao Paulo State University (IFT- UNESP)
Non-Perturbative QCD:

➢ Hadrons, as bound states, are dominated by non-perturbative QCD dynamics – *Two emergent phenomena*
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➢ Explain the most important mass generating mechanism for visible matter in the Universe

➢ Neither of these phenomena is apparent in QCD 's Lagrangian, HOWEVER, They play a dominant role in determining the characteristics of real-world QCD!
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➢ **From a quantum field theoretical point of view**, these emergent phenomena could be associated with dramatic, dynamically driven changes in the analytic structure of QCD's Schwinger functions (propagators and vertices). The Schwinger functions are solutions of the quantum equations of motion (**Dyson-Schwinger equations**).
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\[
\begin{array}{c}
\text{-----} \quad -1 \quad \text{-----} \\
\text{-----} \\
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➢ Mass generated from the interaction of quarks with the gluon.
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➢ Responsible of the 98% of the mass of the proton and the large splitting between parity partners.
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Dyson-Schwinger equations (DSEs)
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Quark propagator:

\[ -1 = \text{propagator graph} -1 + \text{interaction graph} \]

Ghost propagator:

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Ghost-gluon vertex:

\[ \text{vertex graph} = \text{interaction graph} + \text{interaction graph} \]

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Dyson-Schwinger equations (DSEs)

- **Dyson-Schwinger equations**
  - ✓ A Nonperturbative symmetry-preserving tool for the study of Continuum-QCD
  - ✓ Well suited to Relativistic Quantum Field Theory
  - ✓ A method connects observables with long-range behaviour of the running coupling
  - ✓ Experiment ↔ Theory comparison leads to an understanding of long-range behaviour of strong running-coupling
Hadrons: Bound-states in QFT
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➢ **Mesons**: a 2-body bound state problem in QFT
  ➢ *Bethe-Salpeter Equation*
  ➢ K - fully amputated, two-particle irreducible, quark-antiquark scattering kernel
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![Diagram of Bethe-Salpeter Equation](image)

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   ➢ *Faddeev equation*: sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.
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**Diquark correlations:**
- In our approach: non-pointlike color-antitriplet and fully interacting.
- Diquark correlations are soft, they possess an electromagnetic size.
- Owing to properties of charge-conjugation, a diquark with spin-parity $J^P$ may be viewed as a partner to the analogous $J^{-P}$ meson.
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\Gamma_{q\bar{q}}(p; P) = - \int \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu\nu}(p - q) \frac{\lambda^a}{2} \gamma_\mu S(q + P) \Gamma_{q\bar{q}}(q; P) S(q) \frac{\lambda^a}{2} \gamma_\nu
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\[
\Gamma_{qq}(p; P) C^\dagger = - \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu\nu}(p - q) \frac{\lambda^a}{2} \gamma_\mu S(q + P) \Gamma_{qq}(q; P) C^\dagger S(q) \frac{\lambda^a}{2} \gamma_\nu
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2-body correlations: diquarks

- Quantum numbers:
  - \((l=0, J^P=0^+)\): isoscalar-scalar diquark
  - \((l=1, J^P=1^+)\): isovector-pseudovector diquark
  - \((l=0, J^P=0^-)\): isoscalar-pseudoscalar diquark
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Faddeev equation in rainbow-ladder truncation
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- **Three-body bound states**

Quark-Diquark two-body bound states

![Diagram of diquarks and quarks with momenta and labels]
QCD-kindred model
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- The dressed-quark propagator
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Parity partners in the baryon resonance spectrum

Ya Lu, 1, * Chen Chen, 2, † Craig D. Roberts, 3, ‡ Jorge Segovia, 4, § Shu-Sheng Xu, 1, ¶ and Hong-Shi Zong 1, 5, \|

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2 Instituto de Física Teórica, Universidade Estadual Paulista, Rua Dr. Bento Teobaldo Ferraz, 271, 01140-070 São Paulo, São Paulo, Brazil
3 Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA
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5 Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing, Jiangsu 210093, China
(Received 10 May 2017; published 28 July 2017)
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➢ Faddeev kernels: 22 \( \times \) 22 matrices are reduced to 16 \( \times \) 16!
There is an absence of spin-orbit repulsion owing to an oversimplification of the gluon-quark vertex when formulating the RL bound-state equations. We therefore employ a simple artifice in order to implement the missing interactions.
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We introduce a single parameter into the Faddeev equation for $J^P=1/2^{+-}$ baryons: $g_{DB}$, a linear multiplicative factor attached to each opposite-parity (-P) diquark amplitude in the baryon’s Faddeev equation kernel.
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$gDB$ is the single free parameter in our study.
QCD-kindred model
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➢ Solution to the 50 year puzzle -- Roper resonance
Completing the Picture of the Roper Resonance

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\(^2\)Laboratório de Física Teórica e Computacional, Universidade Cruzeiro do Sul, 01506-000 São Paulo, SP, Brazil

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\(^4\)Instituto de Física, Universidad de Antioquia, Calle 70 No. 52-21, Medellín, Colombia

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\(^6\)Department of Physics, Nanjing University, Nanjing 210093, China

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We employ a continuum approach to the three valence-quark bound-state problem in relativistic quantum field theory to predict a range of properties of the proton’s radial excitation and thereby unify them with those of numerous other hadrons. Our analysis indicates that the nucleon’s first radial excitation is the Roper resonance. It consists of a core of three dressed quarks, which expresses its valence-quark content and whose charge radius is 80\% larger than the proton analogue. That core is complemented by a meson cloud, which reduces the observed Roper mass by roughly 20\%. The meson cloud materially affects long-wavelength characteristics of the Roper electroproduction amplitudes but the quark core is revealed to probes with \(Q^2 \gtrsim 3m_N^2\).

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PACS numbers: 13.40.Gp, 14.20.Dh, 14.20.Gk, 11.15.Tk

Introduction.—The strong-interaction sector of the standard model is thought to be described by quantum chromodynamics (QCD), a relativistic quantum field theory. In nature, QCD is strongly coupled, and its solutions are only known at weak couplings. However, at some renormalization group fixed points, QCD is brought into a strongly coupled regime, and at these points, the strong coupling is broken, express the intrinsic mass scale(s) and features associated with confinement and DCSB, and employ realistic kernels in baryon bound-state equations, which
QCD-kindred model

➢ Solution to the 50 year puzzle -- Roper resonance
SOLUTIONS & THEIR PROPERTIES
Structure of the nucleon’s low-lying excitations

Chen Chen,¹,‡ Bruno El-Bennich,²,† Craig D. Roberts,³,‡ Sebastian M. Schmidt,⁴,§ Jorge Segovia,⁵,‖ and Shaolong Wan⁶,¶

¹Instituto de Física Teórica, Universidade Estadual Paulista, Rua Dr. Bento Teobaldo Ferraz, 271, 01140-070 São Paulo, São Paulo, Brazil
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⁶Institute for Theoretical Physics and Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, People’s Republic of China

(Received 9 November 2017; published 15 February 2018)
The four lightest baryon \((I=1/2, \ J^P=1/2^{\{\pm\}})\) isospin doublets: nucleon, roper, \(N(1535)\), \(N(1650)\)

- Masses
- Rest-frame orbital angular momentum
- Diquark content
- Pointwise structure
SOLUTIONS & THEIR PROPERTIES:
Masses
We choose $gDB = 0.43$ so as to produce a mass splitting of $0.1$ GeV (the empirical value) between the lowest-mass $P= -$ state ($N(1535)$) and the first excited $P=+$ state (Roper).
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Pseudoscalar and vector diquarks have no impact on the mass of the two positive-parity baryons, whereas scalar and pseudovector diquarks are important to the negative parity systems.
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The quark-diquark kernel omits all those resonant contributions which may be associated with meson-baryon final-state interactions that are resummed in dynamical coupled channels models in order to transform a bare baryon into the observed state.
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The Faddeev equations analyzed to produce the results should therefore be understood as producing the dressed-quark core of the bound state, not the completely dressed and hence observable object.
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where $M^0_B$ is the relevant bare mass inferred in the associated dynamical coupled-channels analysis.

The relative difference is just 1.7%. We consider this to be a success of our calculation.
SOLUTIONS & THEIR PROPERTIES:
Rest-frame orbital angular momentum
(a) Computed from the wave functions directly.
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These observations provide support in quantum field theory for the constituent-quark model classifications of these systems.
SOLUTIONS & THEIR PROPERTIES:

Diquark content

(a) Diquark fraction - $\Psi$

(b) Diquark fraction - mass
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SOLUTIONS & THEIR PROPERTIES:
Pointwise structure
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In their rest frames, these systems are predominantly P-wave in nature, but possess material S-wave components; and the first excited state in this negative parity channel—$N(1650)1/2^-$—has little of the appearance of a radial excitation, since most of the functions depicted in the right panels of the figure do not possess a zero.
Spectrum and structure of octet and decuplet baryons and their positive-parity excitations

Chen Chen,¹, *Gastão Krein,¹ Craig D. Roberts,², †Sebastian M. Schmidt,³ and Jorge Segovia⁴

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³Institute for Advanced Simulation, Forschungszentrum Jülich and JARA, D-52425 Jülich, Germany
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Universidad Pablo de Olavide, E-41013 Sevilla, Spain
(Dated: 10 January 2019)

A continuum approach to the three valence-quark bound-state problem in quantum field theory is used to compute the spectrum and Poincaré-covariant wave functions for all flavour-SU(3) octet and decuplet baryons and their first positive-parity excitations. Such analyses predict the existence of nonpointlike, dynamical quark-quark (diquark) correlations within all baryons; and a uniformly sound description of the systems studied is obtained by retaining flavour-antitriplet–scalar and flavour-sexet–pseudovector diquarks. Thus constituted, the rest-frame wave function of every system studied is primarily S-wave in character; and the first positive-parity excitation of each octet or decuplet baryon exhibits the characteristics of a radial excitation. Importantly, every ground-state octet and decuplet baryon possesses a radial excitation. Hence, the analysis predicts the existence of positive-parity excitations of the $\Xi$, $\Xi^*$, $\Omega$ baryons, with masses, respectively (in GeV): 1.75(12), 1.89(03), 2.05(02). These states have not yet been empirically identified. This body of analysis suggests that the expression of emergent mass generation is the same in all $u, d, s$ baryons and, notably, that dynamical quark-quark correlations play an essential role in the structure of each one. It also provides the basis for developing an array of predictions that can be tested in new generation experiments.
Octet & Decuplet Baryons
By including all kinds of diquarks, we performed a comparative study of the four lightest baryon \((I=1/2, J^P=1/2^{+/-})\) isospin doublets in order to both elucidate their structural similarities and differences.
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➢ The two lightest \((I=1/2, J^P=1/2^+)\) doublets are dominated by scalar and pseudovector diquarks; the associated rest-frame Faddeev wave functions are primarily \textit{S-wave} in nature; and the first excited state in this \(1/2^+\) channel has very much the appearance of a radial excitation of the ground state.
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In the two lightest \((I=1/2, J^P=1/2^-)\) systems, \textit{TOO}, scalar and pseudovector diquarks play a material role. In their rest frames, the Faddeev amplitudes describing the dressed-quark cores of these negative-parity states contain roughly equal fractions of even and odd parity diquarks; the associated wave functions of these negative-parity systems are predominantly \textit{P-wave} in nature, but possess measurable \textit{S-wave} components; and, the first excited state in this negative parity channel has little of the appearance of a radial excitation.
Thank you!
The dressed-quark propagator

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algebraic form:
QCD-kindred model

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\bar{\sigma}_S(x) = 2\bar{m} \mathcal{F}(2(x + \bar{m}^2)) \\
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\]

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with \( x = p^2/\lambda^2, \bar{m} = m/\lambda, \)

\[
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\( \bar{\sigma}_S(x) = \lambda \sigma_S(p^2) \) and \( \bar{\sigma}_V(x) = \lambda^2 \sigma_V(p^2). \) The mass scale, \( \lambda = 0.566 \text{ GeV}, \) and parameter values,

\[
\begin{array}{cccccc}
\bar{m} & b_0 & b_1 & b_2 & b_3 \\
0.00897 & 0.131 & 2.90 & 0.603 & 0.185
\end{array}
\]

associated with Eq. (A3) were fixed in a least-squares fit to light-meson observables \([79,80]. \) \([\epsilon = 10^{-4} \text{ in Eq. (A3a) acts only to decouple the large- and intermediate-} p^2 \text{ domains.} \]
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- Parameters in quark propagators were fitted to a diverse array of meson observables. **ZERO** parameters changed in study of baryons.
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FIG. 6. Solid curve (blue)—quark mass function generated by the parametrization of the dressed-quark propagator specified by Eqs. (A3) and (A4) (A5); and band (green)—exemplary range of numerical results obtained by solving the gap equation with the modern DCSB-improved kernels described and used in Refs. [16,81–83].
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Compare with that computed using the DCSB-improved gap equation kernel (DB). The parametrization is a sound representation numerical results, although simple and introduced long beforehand.

![Graph](image1)

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Diquark amplitudes: five types of correlation are possible in a $J=1/2$ bound state: isoscalar scalar ($I=0, J^P=0^+$), isovector pseudovector, isoscalar pseudoscalar, isoscalar vector, and isovector vector.
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\begin{align*}
\Gamma^{0+}(k;K) &= g_0^+ \gamma_5 C \tau^2 \tilde{H} \mathcal{F}(k^2/\omega_{0+}^2), \\
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\Gamma^{1-}_{\mu}(k; K) &= g_{1-}\gamma_\mu\gamma_5 C\tau^2 \tilde{H}F(k^2/\omega_{1-}^2), \\
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Simple form. Just one parameter: diquark masses.

Match expectations based on solutions of meson and diquark Bethe-Salpeter amplitudes.
The diquark propagators

\[ \Delta^0_{\pm}(K) = \frac{1}{m^2_{0\pm}} \mathcal{F}(k^2/\omega^2_{0\pm}) , \]

\[ \Delta^1_{\mu\nu}(K) = \left[ \delta_{\mu\nu} + \frac{K_{\mu}K_{\nu}}{m^2_{1\pm}} \right] \frac{1}{m^2_{1\pm}} \mathcal{F}(k^2/\omega^2_{1\pm}) . \]
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The **\( \mathcal{F} \)-functions**: Simplest possible form that is consistent with infrared and ultraviolet constraints of confinement (IR) and \( 1/q^2 \) evolution (UV) of meson propagators.
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Diquarks are confined.
QCD-kindred model

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  ➢ free-particle-like at spacelike momenta
QCD-kindred model

➢ The diquark propagators

\[ \Delta^{0\pm}(K) = \frac{1}{m_{0\pm}^2} \mathcal{F}(k^2 / \omega_{0\pm}^2), \]

\[ \Delta^{1\pm}_{\mu\nu}(K) = \left[ \delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1\pm}^2} \right] \frac{1}{m_{1\pm}^2} \mathcal{F}(k^2 / \omega_{1\pm}^2). \]

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  - This is \textbf{NOT} true of RL studies. It enables us to reach arbitrarily high values of momentum transfer.
$$\psi^\pm(p_i, \alpha_i, \sigma_i) = [\Gamma^0_+ (k; K)]_{\alpha_1 \alpha_2} \Delta^{1+}(K)[\varphi^\pm_{0^+}(\ell; P)u(P)]_{\sigma_3}$$

$$+ [\Gamma^1_+ \Delta^1_+ \varphi^{1+}_{1+\nu}(\ell; P)u(P)]$$

$$+ [\Gamma^0_0 \Delta^0_0 \varphi^{0^+}_{0^+}(\ell; P)u(P)]$$

$$+ [\Gamma^1_- \Delta^1_- \varphi^{1-}_{1-\nu}(\ell; P)u(P)],$$ (9)
\[ \psi^{\pm}(p_i, \alpha_i, \sigma_i) = [\Gamma^{0^+}(k; K)]_{\alpha_1 \alpha_2}^{0^+} \Delta^{0^+}(K) [\phi^{\pm}_0(\ell; P) u(P)]_{\sigma_3} \]
\[ + [\Gamma^{1^+}_{\mu}] \Delta^{1^+}_{\mu\nu} [\phi^{\pm}_1(\ell; P) u(P)] \]
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The Faddeev amplitudes:

\[
\psi^{\pm}(p_i, \alpha_i, \sigma_i) = \left[ \Gamma^{0+}(k; K) \right]^{\alpha_i \alpha_2 \sigma_2}_{\sigma_1 \sigma_3} \Delta^{0+}(K) \left[ \varphi^{\pm}_{0^+}(\ell; P) u(P) \right]^{\sigma_3}_{\sigma_1} \\
+ \left[ \Gamma^{1^+}_{\mu} \right]^{1^+}_{\mu} \Delta^{1^+}_{\mu\nu} \left[ \varphi^{\pm}_{1^+\nu}(\ell; P) u(P) \right] \\
+ \left[ \Gamma^{0^-} \right]^{0^-} \Delta^{0^-} \left[ \varphi^{\pm}_{0^-}(\ell; P) u(P) \right] \\
+ \left[ \Gamma^{-}_{\mu} \right]^{1^-}_{\mu} \Delta^{1^-}_{\mu\nu} \left[ \varphi^{\pm}_{1^-\nu}(\ell; P) u(P) \right], \quad (9)
\]

Quark-diquark vertices:
The Faddeev amplitudes:

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\psi_{\pm}(p_i, \alpha_i, \sigma_i) = \left[ \Gamma^{0^+}(k; K) \right]_{\sigma_1 \sigma_2} \Delta^{0^+}(K) \left[ \varphi_{0^+}^\pm(\ell; P) u(P) \right]_{\sigma_3} \\
+ \left[ \Gamma^{1^+}_\mu \right] \Delta^{1^+}_{\mu \nu} \left[ \varphi_{1^+}^{\pm\nu}(\ell; P) u(P) \right] \\
+ \left[ \Gamma^0 \right] \Delta^0 \left[ \varphi_{0^+}^\pm(\ell; P) u(P) \right] \\
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\]

Quark-diquark vertices:

\[
\varphi_{0^+}^\pm(\ell; P) = \sum_{i=1}^{2} \beta_i^\pm(\ell^2, \ell \cdot P) S^i(\ell; P) G^\pm,
\]

\[
\varphi_{1^+}^{\pm\nu}(\ell; P) = \sum_{i=1}^{6} \alpha_i^\pm(\ell^2, \ell \cdot P) \gamma_5 A_{\nu}^i(\ell; P) G^\pm,
\]

\[
\varphi_{0^-}(\ell; P) = \sum_{i=1}^{2} \rho_i^\pm(\ell^2, \ell \cdot P) S^i(\ell; P) G^\mp,
\]

\[
\varphi_{1^-}^{\pm\nu}(\ell; P) = \sum_{i=1}^{6} \nu_i^\pm(\ell^2, \ell \cdot P) \gamma_5 A_{\nu}^i(\ell; P) G^\mp,
\]
QCD-kindred model

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\psi^{\pm}(p_i, \alpha_i, \sigma_i) = \left[ \Gamma^{0+}(k; K) \right]^\alpha_1^\alpha_2 \Delta^{0+}(K) \left[ \varphi^{\pm}_0(\ell; P) u(P) \right]^\alpha_3 \\
+ \left[ \Gamma^{1+}_{\mu, j} \right] \Delta^{1+}_{\mu, \nu} \left[ \varphi^{j+}_{1, \nu}(\ell; P) u(P) \right] \\
+ \left[ \Gamma^0 \right] \Delta^0 \left[ \varphi^{\pm}_0(\ell; P) u(P) \right] \\
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\]

\[
\varphi^{\pm}_0(\ell; P) = \sum_{i=1}^{2} \rho^{\pm}_i(\ell^2, \ell \cdot P) S^i(\ell; P) G^{\mp},
\]

where \( G^{\pm(-)} = I_D(\gamma_5) \) and

\[
S^1 = I_D, \quad S^2 = i \gamma \cdot \ell - \ell \cdot \hat{P} I_D,
\]

\[
A^1_{\nu} = \gamma \cdot \ell \cdot \hat{P} I_D, \quad A^2_{\nu} = -i \hat{P} I_D, \quad A^3_{\nu} = \gamma \cdot \ell \cdot \hat{\ell} \cdot \hat{\nu},
\]

\[
A^4_{\nu} = i \hat{\ell} \cdot \hat{P} I_D, \quad A^5_{\nu} = \gamma_{\nu} - A^3_{\nu}, \quad A^6_{\nu} = i \gamma_{\nu} \gamma \cdot \ell \cdot \hat{\nu} = A^4_{\nu},
\]
QCD-kindred model
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➢ Both the Faddeev amplitude and wave function are Poincare covariant, i.e. they are qualitatively identical in all reference frames.
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\begin{align*}
2S &: S^1, A^2_v, (A^3_v + A^5_v), \\
2P &: S^2, A^1_v, (A^4_v + A^6_v), \\
4P &: (2A^4_v - A^6_v)/3, \\
4D &: (2A^3_v - A^5_v)/3,
\end{align*}
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$$^2S: S^1, A^2_u, (A^3_u + A^5_u),$$

$$^2P: S^2, A^1_u, (A^4_u + A^6_u),$$

$$^4P : (2A^4_u - A^6_u)/3,$$

$$^4D : (2A^3_u - A^5_u)/3,$$

➢ The scalar functions associated with these combinations of Dirac matrices in a Faddeev wave function possess the identified angular momentum correlation between the quark and diquark.
Quark-diquark picture
➢ A baryon can be viewed as a Borromean
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Modern diquarks are different from the old static, point-like diquarks which featured in early attempts to explain the so-called missing resonance problem.

The number of states in the spectrum of baryons obtained is similar to that found in the three-constituent quark model, just as it is in today's LQCD calculations.