A mechanistic framework for temperature effects on stage–structured populations

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- Every population is composed by individuals of varying ages
- Most of the time we don't care: our intuition (and our models) deal with total population sizes only
- When should we care?

- Different stages may affect growth and death rates very differently, e.g.
  - Newborn individuals don't reproduce
  - Competition for resources is important/involves only some stages
  - Maturation take a definite time, they are poorly modeled by constant rates

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Models for stage–structured populations

(Leslie) Matrix models

- Matrix models are nice and easy(-ier)
- They assume discrete time steps
- Sometimes that is not satisfactory

Continuous–time models

- Interacting species may have different peak times / scales
- Allow intra–generation (intra–annual) processes to be explicitly modeled and analyzed
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The Lotka equation

- Models the numbers of births only.
- The number of newborns at time t is given by the sum over the maternity rates of individuals of all ages.

$$B(t) = \int_{\alpha}^{\beta} B(t-\tau)S(\tau)m(\tau)d\tau$$

The McKendrick–von Foerster equation

- Follows cohorts in time
- It's nastier to solve, but it's more useful to derive delay differential equation models.

$$\frac{\partial n(t,a)}{\partial t} + \frac{\partial n(t,a)}{\partial a} = -\mu(t,a)$$

$$n(t,0) = \int n(t,a)m(a)da$$

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- We want to do those things. Really.
- Delay–differential equations (DDEs) are an easy(-ier) way to deal with that.
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A simple DDE

Let us model a population that consists of reproducing adults (A) and non–reproducing juveniles (J):

$$\frac{dJ(t)}{dt} = \frac{bA(t)}{1 + A(t)/K} - M_J(t) - d_J J(t)$$

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$$M_J(t) = \frac{bA(t - \tau(t))}{1 + A(t - \tau)/K} S(t)$$

$$S(t) = e^{-\mu_J \tau}$$

With $\tau \approx 0$, this looks a lot like the logistic equation. For larger τ, it's going to oscillate...

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Temperature effects on ectotherms

Temperature affects all life stages of ectotherms:

- fecundity
- survival
- development
- interactions (competition)

Thanks for your attention!