II Southern-Summer School on Mathematical Biology

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Lecture II

São Paulo, January 2013

Outline

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Nota bene

There is also the **amensalism** (negative for one species, neutral for the other) and the **comensalism** (positive for one species and neutral for the other). Not to speak of **neutralism**.

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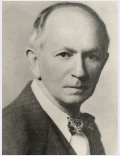
Lotka and Volterra

Muoiono gl'imperi, ma i teoremi d'Euclide conservano eterna giovinezza (Volterra)

Vito Volterra (1860-1940), an Italian mathematician, proposed the equation now known as the Lotka-Volterra one to undestand a problem proposed by his futer son-in-law , Umberto d'Ancona, who tried to explain oscillations in the quantity of predator fishes captured at the certain ports of the Adriatic sea.



Alfred Lotka (1880-1949),was an USA mathematician and chemist,born in Ukraine, who tried to transpose the principles of physical-chemistry to biology. He published his results in a book called "Elements of Physical Biology", dedicated to the memory of Poynting. His results are independent from the work of Volterra.

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The Lotka-Volterra equations

Let

- $N(t)$ be the number of predators,
- $V(t)$ the number of preys.

In what follows, a, b, c e d are positive constants

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The Lotka-Volterra equations

O number of prey will increase when there are no predators:

$$\frac{dV}{dt} = aV$$

The Lotka-Volterra equations

But the presence of predators should lower the growth rate of prey:

$$\frac{dV}{dt} = V(a - bP)$$

The Lotka-Volterra equations

On the other hand the population of predators should decrease in the absence of prey :

$$\frac{dV}{dt} = V(a - bP)$$

$$\frac{dP}{dt} = -dP$$

The Lotka-Volterra equations

and presence of prey will increase the number of predators:

$$\frac{dV}{dt} = V(a - bP)$$

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The Lotka-Volterra equations

These two coupled equations ate known as

The Lotka-Volterra equations

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Let's study them!

Lotka-Volterra: analysis

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- So that:

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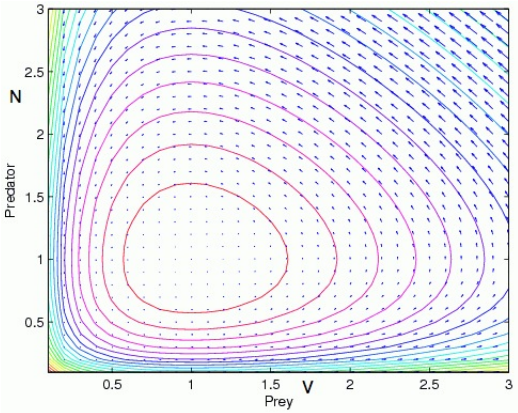
Phase trajectories



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The phase trajectories of the Lotka-Volterra equations, with $a = b = c = d = 1$. Each curve corresponds to a given value of H. The curves obey: $c\mathbf{V}(\mathbf{t}) - b\mathbf{P}(\mathbf{t}) + a \ln \mathbf{P}(\mathbf{t}) + d \ln \mathbf{V}(\mathbf{t}) = H$

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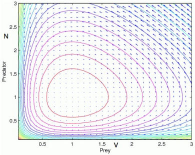

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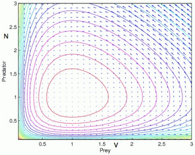

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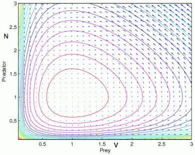

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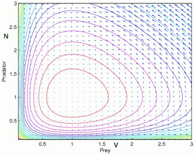

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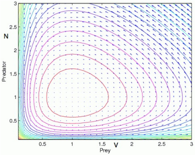

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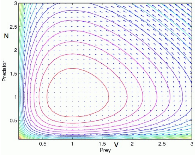

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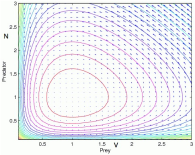

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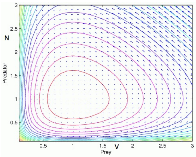

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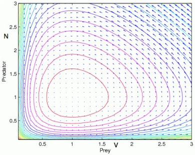

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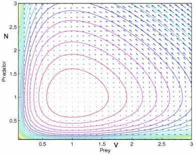

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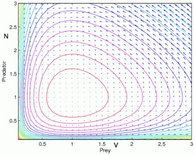

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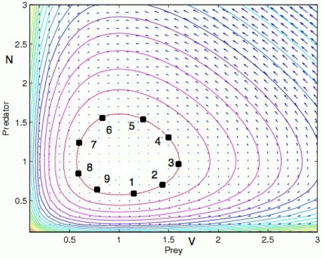

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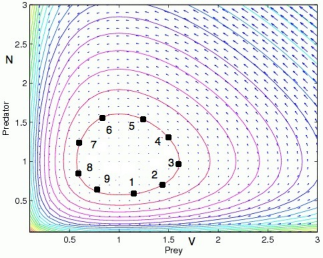

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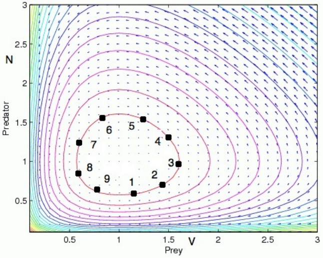

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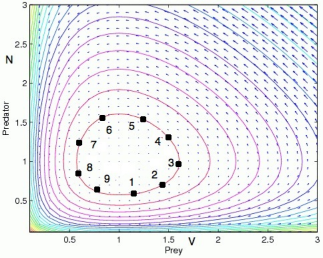

- Let us see how the variable V evolves (prey).
- from 1 to 3 it increases.
- from 3 to 8 it decreases.

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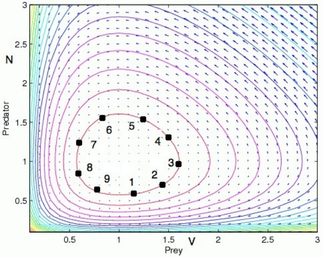

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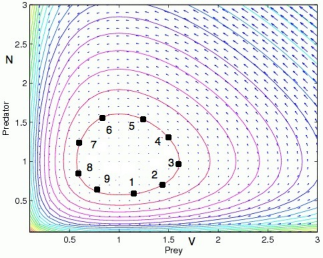

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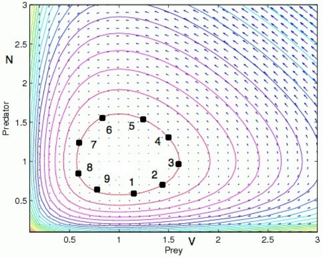

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The number of prey oscillates periodically in time.

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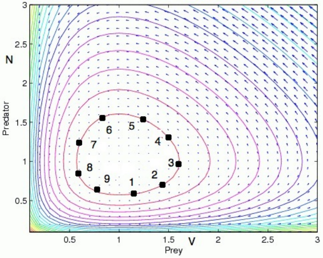


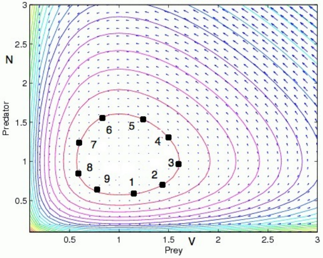
- Let us see how the variable V evolves (prey).
- from 1 to 3 it increases.
- from 3 to 8 it decreases.
- and from 8 to 3 it increases again
- and so on.

The number of prey oscillates periodically in time.

and the predators so the same.

Oscillations II

- Ok, the system is **periodic.**
- Let's take a closer look.
- Take a point in the $P \times V$ plane and follow it in time:



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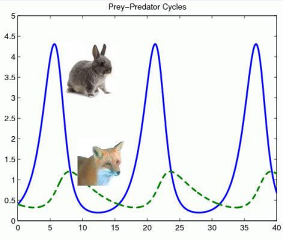

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In words...

- Lotka-Volterra equations tell us that:
  - Given a small number of predators and a certain number (not small) of prey ;
  - The availability of prey makes the population of predators grow;
  - And therefore the prey population will grow slower. After a certain amount of time, it will begin to decrease ;
  - And predators attain a maximal population, and – because the lack of enough prey – it's population begins to decrease;
  - Meanwhile, prey get to a minimum and begin to recover, as the number of predators has decreased;
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- Makes sense!
- **But, is it true?**

The real world

- Does the Lotka-Volterra equations describe real situations?
- Partially.
- There are some elements that are clearly not realistic:
    - The growth of prey in the absence of predator is exponential; it does not saturate.
        - No big deal. Just put a logistic term there. We can still have oscillating solutions. Great!
    - On the other hand... the growth rate of the predator is given by $(cV - d)$.
    - The larger V, the higher the rate. This predator is voracious!
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  - It would be rather natural to suppose that the conversion rate also satures. An effect of the predators becoming satieted or because there is handling time to consume prey.
  - We can modify the above equations to take this into account.
- Cycling can still be present.
- So, the lesson of the Lotka-Volterra equation is: although being an oversimplified equation for predator-prey system it captures an important feature: this kind of system exhibits oscillations – which are intrinsic to the dynamics.

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Further beyond the Lotka-Volterra equations

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- Simple question
  - Whereupon does the prey feed?
  - This is not taken into account in the Lotka-Volterra equations.
  - If resource availability for prey is approximatively constant than a (generalized) Lotka-Volterra dynamics is maybe a good model.
  - But, on the other hand, the possibility exists that the prey and its resource are dynamically coupled... In this case we need to consider at least three species.
  - But beware!!! Do not try to put all species in a model.
- In summary, the Lotka-Volterra equations are rather a staring point than a final point for predator-prey models. .

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A last comment

Host-parasitoid relations

- In close relation to the predator-prey dynamics there is the relation a parasitoid and its host ,
- The parasitoid plays a role analogous to the one of the predator and the host, that of the prey.
- Although these may be seen as different biological interactions, the dynamics is similarly described.

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- The parasitoid plays a role analogous to the one of the predator and the host, that of the prey.
- Although these may be seen as different biological interactions, the dynamics is similarly described.
- Note, however, that many insect species have non-overlaping generations.
- which takes us to the realm of discrete-time equations, or coupled mappings.

What I should remember

- Two-species interactions are the building blocks of larger networks of interactions:
- In a rough way, we can divide them as:
  - **predator-prey;**
  - **competition;**
  - **mutualism.**
- Predator-Prey tend to produce oscillations.
- Just don't forget that not every oscillation comes from a predator-prey dynamics.

Thank you for your attention.

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