ΛCDM - Default conjecture

$$\rho_\Lambda = \Omega_\Lambda \frac{3H_0^2}{8\pi G}$$

use $\hbar c = 197$ MeV-fm
$H_0^{-1} = 3000\, h^{-1}$ Mpc
Mpc $= 3 \times 10^{22}$ m

$\sim (0.002\,\mathrm{eV})^4$

why so small? why worry?

Refs: S. Weinberg, RMP 61 1 (1989)
Nobbenhuis, gr-qc/0609011

EXERCISE: ANALYZE RECENT SN DATA

- DOWNLOAD UNION 2.1 (supernova.lbl.gov/union)
- COMPARE m(data) vs. m(thy)

$$m_{THY}(z) = 5\log_{10}(H_0 d_L(z)) + \mathcal{M}$$

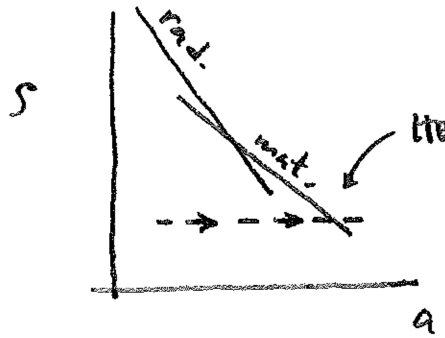
$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + (1-\Omega_m)} \quad \text{in ΛCDM}$$

- FIND Ω_m THAT MINIMIZES $\chi^2 = \sum_i \left(\frac{m^i_{DAT} - m_{THY}(z^i)}{\delta m^i}\right)^2$

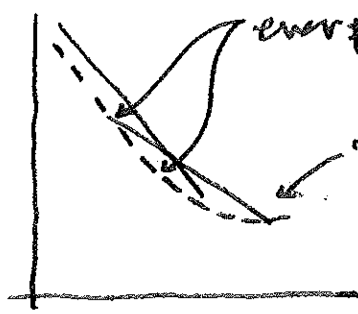
where $\mathcal{M}$ is a nuisance parameter
[for each Ω_m, minimize χ^2 w/r/t $\mathcal{M}$]

COINCIDENCE / WHY NOW?



HOW DOES THIS COINCIDENCE ARISE?

Is there a more satisfactory explanation?



ever present

until
"inevitable" transition to domination,
late in matter era.

without fine tuning.

Wetterich 1988 NPB 302 668 (1988)

THY w/ DIMENSIONLESS PARAMETERS (eg SM w/ $\lambda \to 0$)

IS DESCRIBED BY DILATATION-INVARIANT (CLASSICAL) ACTION

"FUNDAMENTAL" CONSTANTS w/ DIMENSION (m_e, m_p, etc)

ARE INDUCED BY VEV'S OF SCALAR FIELDS, QM EFFECTS

Propose action $S = \int d^4x \sqrt{-\tilde{g}}\, \tilde{\mathcal{L}}$

$$\tilde{\mathcal{L}} = \frac{1}{\ell^2}\tilde{R} - \frac{4\omega}{\ell^4}\left(\tilde{g}^{\mu\nu}\partial_\mu \ell\, \partial_\nu \ell\right) + \mathcal{L}_{SM}(\lambda \to 0)$$

include scalar field w/ some constant vacuum potential at minimum

$$S = \int d^4x \sqrt{\tilde{g}}\left[\tilde{\mathcal{L}} - \frac{1}{2}\tilde{g}^{\mu\nu}(\partial_\mu\phi\, \partial_\nu\phi) - V(\phi)\right]$$

BTW I USE SAME CONVENTIONS AS SEAN CARROLL.

Perform conformal transformation

tools $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ $\qquad g_{\mu\nu} = \Omega^{-2}\tilde{g}_{\mu\nu}$

$$\tilde{g}^{\mu\nu} = \Omega^{-2} g^{\mu\nu}$$

$$\sqrt{-\det\tilde{g}} = \Omega^4\sqrt{-\det g}$$

$$\tilde{R} = \Omega^{-2}R - 6\Omega^{-3}\Box\Omega$$

$$S = \int d^4x \sqrt{-g}\; \Omega^4 \Big[\Omega^{-2} \ell^{-2} R - 6\Omega^{-3} \ell^{-2} \Box\Omega - 4\omega \ell^{-4} \Omega^{-2} (\partial \ell) - \tfrac{1}{2} \Omega^{-2} (\partial\phi)^2 - V + \mathcal{L}_{sm}(\lambda \to 0) \Big]$$

Next, define $\Omega = \ell / \sqrt{16\pi G}$ $\quad \{ \text{BTW } M_p = 1/\sqrt{G} \}$

so $\quad \Omega^2/\ell^2 = 1/16\pi G$

$$6\Omega \ell^{-2} \Box\Omega = \frac{1}{16\pi G}\left(\frac{6}{\Omega} \Box\Omega \right) \xrightarrow{\text{ibp}} -\frac{1}{16\pi G}\left(6 \frac{(\partial\Omega)^2}{\Omega^2} \right)$$

$$4\omega \ell^{-4} \Omega^2 (\partial\ell)^2 = \frac{1}{16\pi G}\, 4\omega \,\frac{(\partial\Omega)^2}{\Omega^2}$$

$$= \int d^4x \sqrt{g} \left[\frac{R}{16\pi G} - \frac{1}{16\pi G}\left(\frac{6+4\omega}{\Omega^2} (\partial\Omega)^2 \right) - \frac{1}{2}\Omega^2 (\partial\phi)^2 - \Omega^4 V + \Omega^4 \mathcal{L}_{sm}(\lambda \to 0) \right]$$

provided $\omega \neq -3/2$ then define $\Omega = A \exp\left(\sqrt{\frac{8\pi G}{6+4\omega}}\, \chi \right)$

Eliminate A by shifting $\chi \to \Omega = \exp\left(\frac{\alpha}{4M_p} \chi \right)$

$$= \int d^4x \sqrt{g} \left[\frac{R}{16\pi G} - \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}(\partial\chi)^2 e^{\frac{1}{2}\frac{\alpha}{M_p}\chi} + (\mathcal{L}_{sm} - V)\, e^{\frac{\alpha}{M_p}\chi} \right]$$

very nice! Let us suppose that ϕ is stabilized s.t. $(\partial\phi)^2 = 0$, but $V_{min} = V_0 \neq 0$ [ABSORB ANY ANOMALOUS $\mathcal{L}_{sm}(\lambda \to 0)$]

$$S = \int d^4x \sqrt{g} \left[\frac{R}{16\pi G} - \frac{1}{2}(\partial\chi)^2 - V_0\, e^{\frac{\alpha}{M_p}\chi} \right]$$

Wetterich's sol'n to cosmological constant problem, ca. 1988, is to dynamically adjust it.

$$\rho_\Lambda = V_0 \quad \rightarrow \quad -\frac{1}{2}(\partial\chi)^2 + V_0 e^{\alpha\chi/M_p}$$

In detail: $\quad \Box\chi = V'$

In RW: $ds^2 = -dt^2 + a^2(t)\,d\vec{x}^2$

$$\frac{d^2\chi}{dt^2} + 3H\frac{d\chi}{dt} + \frac{\alpha}{M_p}V_0 e^{\alpha\chi/M_p} = 0$$

for power-law expansion $a(t) \propto t^n$, $n = \frac{2}{3(1+w_B)}$

ansatz $\chi = A \ln Bt$

can show $A = -2\frac{M_p}{\alpha}$, $\quad B^2 = \frac{\alpha^2 V_0}{2M_p^2(3n-1)}$

so $\rho_\chi = \frac{1}{2}\dot{\chi}^2 + V = \left(\frac{M_p}{\alpha}\right)^2 \frac{6n}{t^2}$

$p_\chi = \quad \left(\frac{M_p}{\alpha}\right)^2 \frac{4-6n}{t^2}$

$$w_\chi = \frac{p_\chi}{\rho_\chi} = \frac{4-6n}{6n} = w_B \quad \checkmark$$

$$\Omega_\chi = \frac{\rho_\chi}{\frac{3}{8\pi G}H^2} = \frac{16\pi}{\alpha^2 n} = \frac{24\pi}{\alpha^2}(1+w_B) \quad \checkmark$$

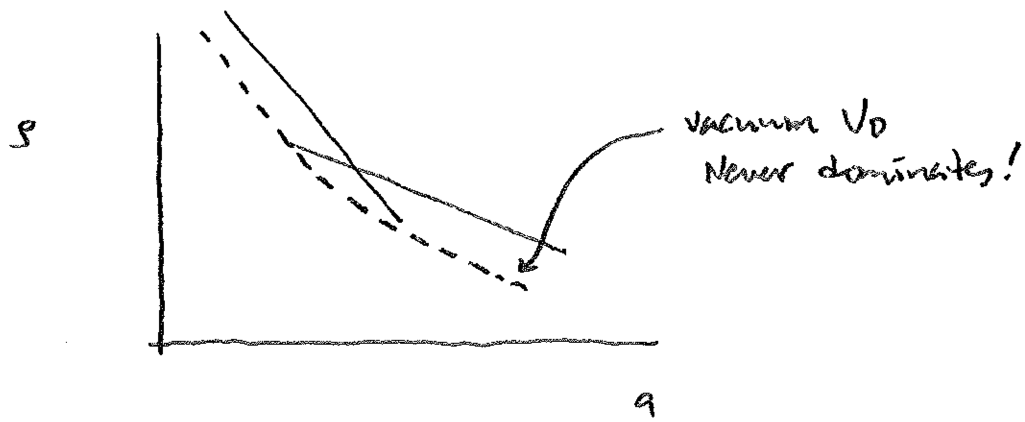
Provided $\alpha^2 > 24\pi(1+w_B)$ then $\Omega < 1$

Wetterich discovered <u>SCALING SOLUTION</u>

since $w_{rad} (= \frac{1}{3}) > w_{matter} (= 0)$

then a solution that scales in radn era

also scales in matter era.



Fine soln ... until discovery of cosmic acceleration

But if $\alpha^2 < 24\pi(1+w_B)$ then scaling sol'n is invalid.

Instead, $w \to -1$ so that $\Omega \to 1$

Adapt Wetterich's scaling solution

→ somehow lower α at late times

how? couple χ to a "trigger"?

design a potential?

Also Historic: Peebles + Ratra PRD37, 3406 (1988)

Dark Energy as a cosmic scalar field

ϕ: a pioneer for new physics!

Zeldovich: most important field for cosmology

"QUINTESSENCE" $\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 - V(\phi)$ [RC [illegible]
PRL 80 1582 (1998)]

quick list of examples

$V = \frac{1}{2}m^2\phi^2$, $M^4(1+\cos\phi/f)$, $M^{4+n}\phi^{-n}$, ...

examples nearly as numerous as models of inflation

[MANY REFS.]

Properties / Problems

Since we require $w \approx -1$, then $\dot\phi^2 \ll V$

non-clustering, then $m = \sqrt{V''} \lesssim H$

dominant $V \simeq M_p^2 H^2$

so $\frac{V''}{V} M_p^2 \simeq 1$

which for $V = \frac{1}{2}m^2\phi^2$ means $\phi \simeq M_p$

EXTREME!

It is a challenge to build a particle physics model of such a light field, which has Planckian ϕ, and remain DARK

[Carroll, PRL 81, 3067 (1998); Peccei hep-ph/0009030; Kolda + Lyth PLB 458, 197 (1999)]

EQUATION OF STATE

$$w = \frac{P}{\rho}$$ (NOT A TRUE E.O.S. SINCE ρ, P ARE HOMOG.)

reverse engineer: given $w(a)$

$$\rho(a) = \rho(a_0) \exp\left[3 \int_a^{a_0} \frac{da'}{a'} (1 + w(a'))\right]$$

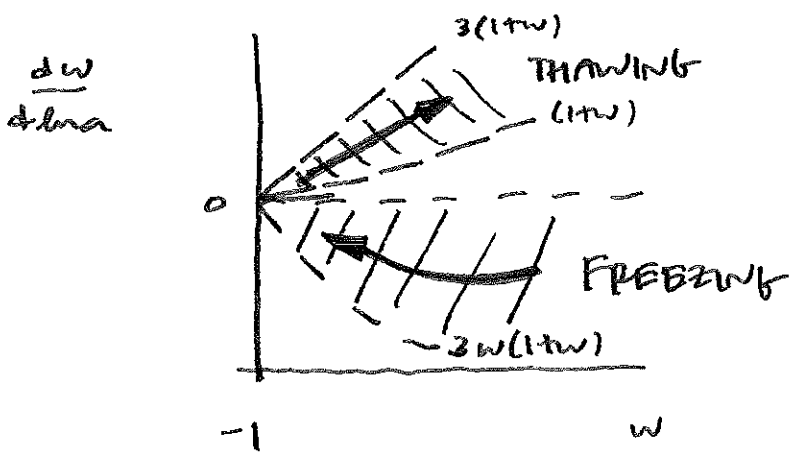
<u>SIMPLISTIC</u>: $w = \text{constant}$, $\rho = \rho_0 (a_0/a)^{3(1+w)}$

<u>POPULAR MODEL</u>: $w(a) = w_0 + w_a(1 - a/a_0)$

use to diagnose for $w' \neq 0$

$$\text{so} \quad \rho = \rho_0 (a_0/a)^{3(1+w_0+w_a)} \exp\left[3 w_a \left(\frac{a_0}{a} - 1\right)\right]$$

<u>BROAD CLASSIFICATION</u>: THAWING VS. FREEZING



BOUNDARIES ARE PHENO. BASED ON VARIOUS MODELS.

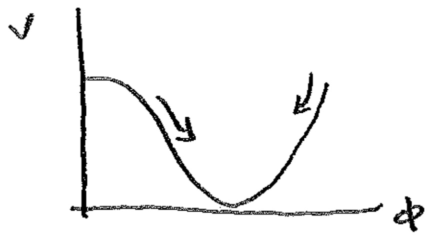
[Re+Linder PRL 95, 141301 (2005)]

THAWING: SUGGESTS $\frac{dw}{d\ln a} \simeq C(1+w)$ $\quad 1 \lesssim C \lesssim 3$

$$\text{or} \quad w = -1 + (1 + w_0)\left(\frac{a}{a_0}\right)^C$$

POTENTIALS

$V = \frac{1}{2}m^2\phi^2, \; \lambda(\phi^2-\sigma^2), \; M^4(\cos\frac{\phi}{f}+1), \; \dots$



rolling slowly today

how did they start?

must choose $\phi, \dot\phi$ + parameters $\to$ DELICATE

However, H-friction damps ϕ motion at early times

$|\dot\phi/\phi| \ll H$

so it seems fair to set $\dot\phi \to 0$ at early times

call such models "Frozen" $\to$ $\boxed{\text{THAWING}}$

$V = M^{4+n}\phi^{-n}$ "TRACKER"

This model has $w < w_B$ while $\Omega_\phi \ll 1$

so it eventually catches up to dominate.

Into the future $w \to -1$ $\boxed{\text{FREEZING}}$

let's see: $\ddot\phi + 3H\dot\phi + V' = 0$

use $H = 2/3(1+w_B)t$ & $\phi = At^B$

show $\dot\phi^2, \phi^{-n} \propto t^{-2n/(2+n)}$

$w = -1 - \frac{1}{3}\frac{1}{H}\frac{d\ln\rho}{dt} = -1 + \frac{n}{2+n}(1+w_B)$

or more generally $w = \dfrac{w_B - 2(\Gamma-1)}{1+2(\Gamma-1)}$, $\Gamma \equiv \dfrac{V''V}{(V')^2}$

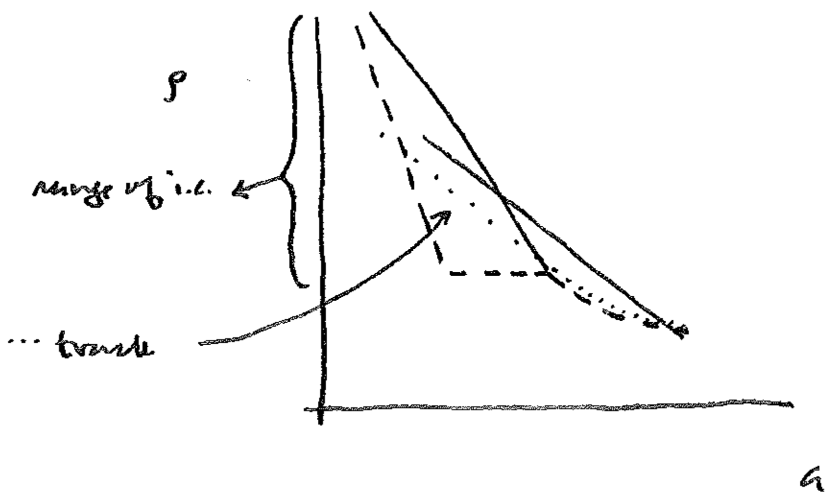
tracking conditions: $\Gamma \sim$ constant)

$\Gamma > 1$ for $w < w_B$

features: broad insensitivity to initial conditions

tracker is an attractor.

challenge: Need $0 < n \ll 1$ or $\Gamma \gg 1$ to get a sufficiently negative w by today.



see Steinhardt et al, PRD59, 123504 (1999)

Further examples

$$V = \sum_{i=1}^{N} V_i\, e^{-\alpha_i \phi / M_p}$$ one field, many exponential potentials

suppose $N=2$ with $\alpha_1 > \sqrt{24\pi}$ & $\alpha_2 < \sqrt{24\pi}$

so at early times ϕ scales

" late " dominates

good example of <u>early</u> dark energy

$$V = \sum_i V_i\, e^{-\alpha_i \phi_i / M_p}$$ many fields

For all the fields that are in play, ϕ_i's behave as a single fluid with

$$\frac{1}{\alpha_{eff}^2} = \sum_i \frac{1}{\alpha_i^2}$$

[NOT IN PLAY? $V_i \ll V_{others}$]

Asymptotic EOS: $w \to -1 + \frac{\alpha_{eff}^2}{24\pi}$

"assisted inflation" Liddle et al, PRD <u>58</u> 061301 (1998)

UNIMODULAR GRAVITY

GR with constraint $\sqrt{-g} = 1$

At classical level, identical to GR except Λ is an integration constant, not a fundamental parameter.

see here $S = \int d^4x \sqrt{g} \left(\frac{R}{16\pi G} + \mathcal{L}_m \right)$

$$\delta S = \int d^4x \sqrt{g} \; \delta \left(\frac{R}{16\pi G} + \mathcal{L}_m \right) = 0$$

leads to $R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = 8\pi G \left(T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} T \right)$

[see Einstein 1919; Anderson + Finkelstein AmJP 39 901 (1971); and van der Bij, van Dam, Ng Physica 116A, 307 (1982).

Observe Bianchi identities and $\nabla^\mu T_{\mu\nu} = 0$ still true. So taking divergence of both sides

$$\nabla^\mu \left(R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R \right) = \nabla^\mu \left(G_{\mu\nu} + \frac{1}{4} g_{\mu\nu} R \right)$$
$$= \frac{1}{4} \nabla_\nu R = -\frac{1}{4} 8\pi G \nabla_\nu T$$

$$\nabla_\nu (R + 8\pi G T) = 0$$
$$R + 8\pi G T = 4\Lambda \text{, an integration constant}$$

whereby $G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$ ✓

UNIMODULAR ...

The classical eq's of motion resulting from an action

$$S = \int d^4x \sqrt{-\hat{g}} \left(\frac{\hat{R}}{16\pi G} + \hat{\mathcal{L}}_m \right), \quad \text{NO COSMO. CONST.}$$

where "^" means $\sqrt{-\hat{g}} = 1$ constraint is satisfied, are equivalent to those equations resulting from

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{R}{16\pi G} + \mathcal{L}_m \right) + \Lambda_i \right]$$

where Λ_i is an integration constant

What's so great about this? If Λ is due to QM, then we are compelled to explain why it is not $(M_p)^4$. If Λ is due to a mere integration constant, then there is less burden to explain its size. Perhaps.

Higgs-Dilaton Unimodular Gravity.

$$S = \int d^4x \sqrt{-\hat{g}} \left[\tfrac{1}{2}(\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger \varphi)\hat{R} + \hat{\mathcal{L}}_{SM}(\lambda \to 0) - \tfrac{1}{2}\hat{g}^{\mu\nu}\partial_\mu \chi \partial_\nu \chi - V(\chi, \varphi) \right]$$

NO COSMO. CONST.

$\hat{\mathcal{L}}_{SM}(\lambda \to 0)$ includes Higgs kinetic term

NO MASSES — TRACELESS

so any vacuum potential or anomalous trace does not make Λ!

Instead there is only a Λ_i as an integration constant.